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The Quality of Blume and Vasicek Betas for forecasting systematic risk: Evidence from a German stock portfolio

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Abstract

In both practical applications and empirical capital market research the beta factor is often estimated for the future period using an OLS regression analysis based on historical data. However, these betas are often not reliable for the future. For this reason, methods have been developed to correct or adjust the (raw) beta. The aim of this study is to find out to what extent the forecasting quality of a stock beta can be improved by Blume Beta, Vasicek Beta and a simple variant of Blume Beta ("Adjusted Beta"). For this purpose, an empirical analysis is carried out on the basis of 10 stocks included in the German stock index DAX. For the entire period under review, it can be observed that all three adjustment methods tend to improve the forecast quality compared to the raw beta, with the simple "Adjusted beta" leading to the best values. The examination of two subperiods, one covering the financial and economic crisis of 2008/2009 and the other the Corona pandemic, shows that the forecasting quality tends to be lower when looking at individual stocks compared with the overall period. However, according to the values (i.e. averages of all stocks) of the forecasting quality measures used in this paper, the adjustment procedures mostly produce better values than raw beta in both subperiods.

JEL classification numbers: G11.
Keywords: Adjusted beta, Blume beta, Vasicek beta, Mean Squared Error.

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1. Introduction

In the practice of equity management, the beta factor, which is based on the theoretical Capital Asset Pricing Model (CAPM), is usually used to quantify systematic (market) risks. In both practical applications and empirical capital market research, beta is often estimated using an OLS regression analysis based on historical data for the future period without any adjustments being made. In this case, the term "raw beta" can be used (Scheld, 2013, Ziemer, 2018).

The CAPM assumes a constant beta factor over time during the period under consideration. Nevertheless, empirical studies have shown that the linear relationship assumed in the OLS regression is often not stable over time (Zimmermann, 1997, Rudolph and Zimmermann, 1998, Meitner and Streitferdt, 2015). This means that the beta values often are not independent of the estimation period and not constant for successive periods. Thus, it appears that a reliable raw beta for the future cannot be estimated from past data.

For this reason, methods have been developed to correct or adjust the raw beta of which the Blume and Vasicek methods are among the best known (Scheld, 2013, Bruns and Meyer-Bullerdiek, 2020).

In this context, the question arises to what extent the adjusted stock betas are suitable as forecast values for the future (actual) betas compared to the raw beta. The aim of this paper is therefore to determine whether for individual stocks and a stock portfolio the beta adjustments according to Blume and Vasicek lead to better forecast values than the raw betas. Special attention is paid to the extreme crisis situations of the financial crisis in 2008/2009 and the Corona pandemic.

The paper is organized as follows: Chapter 2 discusses the two adjustment methods. In chapter 3, the procedure and the results of the empirical investigation are presented. In the last chapter, some concluding remarks and ideas for future research are given.

2. Beta Adjustment according to Blume and Vasicek

2.1 Blume Beta

It was found by Blume that the betas of U.S. stocks and stock portfolios exhibit an autoregressive tendency in successive periods that do not overlap. Blume calls this “a tendency to regress towards the grand mean of all betas, namely one” (Blume, 1975). The regression tendency states that the beta of a stock in the subsequent period tends to be closer to the mean of the betas of all stocks than in the previous period. Betas above the mean should be lower on average in the following period, and betas below the mean should be higher on average. An estimate of the regression trend in betas can be made using a cross-sectional regression of stock betas computed for two consecutive periods. Here, the betas of stocks in the cross sectional sample in period t ($ß_{i,t}$) are regressed on the betas of the previous period t-1 ($ß_{i,t-1}$) (Zimmermann, 1997, Blume, 1971, Klemkowsky and Martin, 1975):

$$ß_{i,t} = a_t + b_t \cdot ß_{i,t-1} + \varepsilon_{i,t} \qquad \text{for } i = 1,2, \dots, n \tag{1}$$

where a_t and b_t are the regression coefficients in period t, n is the number of stocks in the cross sectional sample and $\varepsilon_{i,t}$ is a random disturbance variable.

In order to take into account the above-mentioned autoregressivity when forecasting beta factors, appropriate adjustment methods can be used, whereby autoregressive methods take into account the dependence of the current beta values on the beta factors of the previous period. This is intended to improve beta forecasting. In Blume's basic procedure, the parameters in the above formula are estimated using the betas of periods t and t-1. Thus, the beta factor ($ß_{i,t+1}$) of a stock forecast for the next period t+1 is calculated as follows (Zimmermann, 1997, Blume, 1971, Klemkosky and Martin, 1975):

$$ß_{i,t+1}^{\text{Blume}} = a_t + b_t \cdot ß_{i,t} \tag{2}$$

Here, relatively stable values for a_t and b_t are assumed over time. Blume estimates values between 0.265 and 0.526 for a and between 0.489 and 0.750 for b (depending on the subperiod considered), which are then used for forecasting. He concludes from his analysis that the beta factors show a tendency to move toward the mean. Thus, the corrected forecast would better capture future risk. Corresponding studies for other periods or other return intervals yielded similar results (Blume, 1971, Ziemer, 2018).

The relationship determined by Blume has been slightly modified in practice and has become established under the name "adjusted beta". Accordingly, the adjusted beta is calculated as follows (Kern and Mölls, 2010, 2010, Ziemer, 2018, Bodie, Kane and Marcus, 2023):

$$ß_{i,t+1}^{\text{adjusted}} = \frac{1}{3} \cdot ß_{m,t} + \frac{2}{3} \cdot ß_{i,t} = \frac{1}{3} + \frac{2}{3} \cdot ß_{i,t} \tag{3}$$

According to this equation, the beta estimator is weighted 1/3 with the beta factor of the market ($ß_m$, which assumes the value 1) and 2/3 with the raw beta of the stock. This adjusted beta is reported by some data providers as part of beta estimates. It is also used in company valuations. Nevertheless, there is also criticism of this method. For example, Meitner, 2021 points out that the Blume beta is conceptually inconsistent neither with the general understanding nor with the basic principles of business valuation. However, Kern and Mölls, 2010 point out that due to the prevalence in the market, this approach definitely reflects market expectations. Anyway, a continuous critical review of the weighting parameters in the light of new findings is generally recommended. On the other hand, the adjustment is also seen positively because it takes into account the autoregression (Spremann and Ernst, 2011).

2.2 Vasicek Beta

In addition to the basic autoregressive adjustment procedure proposed by Blume, a procedure for adjusting the raw beta that is also used in practice goes back to Vasicek. In contrast to Blume, this method does not perform the same adjustment for each stock. Rather, the statistical quality of the regression is taken into account and thus a security-specific modification is made. Included is the consideration that the beta adjustment is stronger for companies with a high beta standard error than for companies with a lower beta standard error. Formally, the Vasicek adjustment can be represented as follows (Vasicek, 1973, Klemkosky and Martin, 1975, Zimmermann, 1997, Lally, 1998, Scheld, 2013, Gray, Hall, Diamond and Brooks, 2013, Ziemer, 2018):

$$ß_{i,t+1}^{Vasicek} = \overline{ß}_t \cdot \frac{s_{ß_i}^2}{\sigma_{\overline{ß}_t}^2 + s_{ß_i}^2} + ß_{i,t} \cdot \frac{\sigma_{\overline{ß}_t}^2}{\sigma_{\overline{ß}_t}^2 + s_{ß_i}^2} \quad (4)$$

where $ß_{i,t+1}^{Vasicek}$ is the Vasicek estimator of stock i's beta, $\overline{ß}_t$ is the average beta across the sample of stocks in period t, $ß_{i,t}$ is the OLS regression coefficient of the time series regression, or OLS estimate of stock i's beta measured in period t (raw beta), $\sigma_{\overline{ß}_t}^2$ is the cross-sectional variance of the distribution of the beta estimates over the sample of stocks, $s_{ß_i}^2$ is the variance in the estimate of $ß_{i,t}$ which is the squared standard error of the regression coefficient $ß_{i,t}$.

As equation (4) shows, a weighted average of the raw beta and the beta of the stock sample is calculated for beta adjustment. If the standard error of $ß_i$ is small compared to the cross-sectional variance of all betas ($\sigma_{\overline{ß}_t}^2$), the beta value of stock i can be estimated relatively reliably. This leads to a higher weighting of the stock beta compared to the cross-sectional mean of all beta values. In the opposite case, i.e. when the estimate of the beta value of stock i is unreliable (with a high standard error), a higher weighting of the cross-sectional mean of all beta values takes place, so that the beta value of stock i is changed more towards the mean value. Accordingly, the cross-sectional variance ($\sigma_{\overline{ß}_t}^2$) is reduced by this adjustment procedure, which incidentally also applies to the Blume procedure (Zimmermann, 1997).

Based on equation (4), a similarity between Vasicek and Blume adjustment can be shown:

$$ß_{i,t+1}^{Vasicek} = \left(1-x\right)\cdot \overline{ß}_t + x\cdot ß_{i,t} \tag{5}$$

$$x = \frac{\sigma_{\overline{ß}_t}^2}{\sigma_{\overline{ß}_t}^2 + s_{ß_i}^2} \quad , \quad 1-x = 1-\frac{\sigma_{\overline{ß}_t}^2}{\sigma_{\overline{ß}_t}^2 + s_{ß_i}^2} = \frac{\sigma_{\overline{ß}_t}^2 + s_{ß_i}^2}{\sigma_{\overline{ß}_t}^2 + s_{ß_i}^2} - \frac{\sigma_{\overline{ß}_t}^2}{\sigma_{\overline{ß}_t}^2 + s_{ß_i}^2} = \frac{s_{ß_i}^2}{\sigma_{\overline{ß}_t}^2 + s_{ß_i}^2} \tag{6}$$

One advantage of the Vasicek method over the Blume method is that the beta can be adjusted not only against the market as a whole, but also against a specific stock sample, or the sector average. If, for example, the industry to which the stock under consideration belongs has a beta far below the market average of one, an adjustment in the direction of the industry average is more appropriate (Scheld, 2013).

3. Empirical Analysis

3.1 Research Design

The forecasting quality of the Blume beta and the Vasicek beta compared to the raw beta ("naive forecast") is to be tested in the following for a portfolio consisting of 10 DAX stocks. In order to cover as long a period as possible, taking into account the available data, the analysis refers to stocks from the German DAX index which have been listed in the index without interruption since December 30, 1999 (Boerse.de, 2024): Allianz, BASF, Bayer, BMW, Deutsche Bank, E.ON, Henkel Vz, RWE, Siemens, Volkswagen Vz.

E.ON is a DAX starter stock that has remained in the DAX as a result of mergers. The Mercedes-Benz Group (formerly Daimler) and Linde are also DAX starter stocks. However, adjusted stock prices were not available for the entire period (December 30, 1999 - December 31, 2021) for either stock, so they are not included in the analysis. All stock prices used are taken from the ariva.de website and are adjusted for dividends, stock splits and subscription rights proceeds, so that the stock returns calculated from them are comparable with the DAX total return index. The basis for the beta estimate is the adjusted closing prices at the end of a month. The monthly returns are then determined from these prices. By using monthly returns, price adjustment lags can be smoothed out, so that the beta factors should be more meaningful (Berner, Rojahn, Kiel and Dreimann, 2005).

The risk-free returns of the previous month (broken down to a monthly basis) are then deducted from the monthly stock returns. For this purpose, the 3-month Euribor rates available on the boerse.de website are divided by 12. Thus, in line with the single index model, only excess returns are used for the regression analysis.

In addition to the raw betas of the above-mentioned stocks, the beta factors adjusted according to equations (2), (3), and (4) are determined (Blume beta, the “adjusted beta”, and Vasicek beta). The procedure is as follows: For each month-end from

Dec. 30, 2002, the raw betas and their standard errors are calculated on the basis of the past 36 monthly excess returns, so that raw beta values are available for 229 rolling 3-year periods up to and including Dec. 31, 2021. For Blume beta, the above parameters a_t and b_t are determined by regressing the respective betas (of the stocks in the portfolio) of period "t+36 months" on the betas (of the stocks in the portfolio) of period t. Accordingly, these parameters are only available from December 30, 2005 on. Thus, the Blume beta can be determined for the first time as of December 30, 2005. The beta value determined in this way is regarded as the forecast value for the following 36-month period, so that the associated actual beta value for this period, which is used for comparison, cannot be determined until December 30, 2008. The same procedure is used for the following rolling periods (i.e. determination of the Blume beta as of January 31, 2006 and comparison with the actual beta value on January 30, 2009, etc.).

To determine the Vasicek betas, the raw beta, the cross-sectional mean of the betas of the 10 DAX stocks (equally weighted) and the variance of these beta values are determined at each month-end. In addition, the squared standard error of the regression coefficient (raw beta of the stock) determined from the 36 previous monthly excess returns of the stock and DAX is included. In order to establish comparability with the Blume beta, the Vasicek betas are also determined for the first time as of December 30, 2005.

In addition, the above-mentioned "adjusted betas" (equation 3), which are frequently used in valuation practice, are included in the analysis.

The study will be conducted for the entire period as well as for two sub-periods, one covering the 2008/2009 financial crisis and the other the Covid pandemic. In particular, the beta forecasts of the individual stocks, but also of a portfolio equally weighted from these stocks, are considered.

To assess the extent to which the adjusted stock betas can provide better forecast values for the future (actual) betas compared with the raw beta, the mean squared error (MSE) between the forecast values and the actual values (which are calculated 36 months later) is first determined. The deviations between the beta values are additionally analyzed in more detail on the basis of a time series regression (regression of the actual betas on the predicted betas in each month) by splitting the MSE into three components which represent bias (the mean component), inefficiency (the slope component), and the random disturbance (the residual component) (Mincer and Zarnowitz, 1969, Zimmermann, 1997, Klemkosky and Martin, 1975, Auer and Rottmann, 2020):

$$\text{MSE} = \frac{1}{n} \cdot \sum_{t=1}^{n} \left(ß_{t,r} - ß_{t,p} \right)^2 \tag{7}$$

$$\text{MSE} = \underbrace{\left(\overline{ß}_r - \overline{ß}_p \right)^2}_{\text{bias}} + \underbrace{\left(1 - b\right)^2 \cdot \sigma_{ß_p}^2}_{\text{inefficiency}} + \underbrace{\left(1 - R^2\right) \cdot \sigma_{ß_r}^2}_{\text{random disturbance}} \tag{8}$$

where t is the number of periods, $ß_{t,r}$ is the actual (realized) beta in period t (calculated from the excess returns of the previous 36 months), $ß_{t,p}$ is the predicted beta (computed beta of period t-1 that is used as the predictor of beta for period t), $\overline{ß}_r, \overline{ß}_p$ are the means of the realizations and predictions, b is the slope of the regression of the actual betas ($ß_r$) on the predicted betas ($ß_p$), $\sigma^2_{ß_p}, \sigma^2_{ß_r}$ are the sample variances of $ß_p$ and $ß_r$, R^2 is the coefficient of determination of the regression. For the calculation of $\sigma^2_{ß_p}$ and $\sigma^2_{ß_r}$, the formulas for population variance are used. Accordingly, the sum of the squared variances is divided by n (and not by n-1), unlike the sampling variance. If there are non-zero values for the bias and inefficiency components, systematic forecast errors are present. Since the factor (1-R^2) expresses the proportion of the unexplained variance of the (actually occurred) beta values $ß_{t,r}$, the random component is the unexplained, random part of the deviations between the forecast values and the actually occurred values for the beta factor (Zimmermann, 1997).

In addition to MSE, the root MSE (RMSE) and the mean absolute error (MAE) are used as further forecasting quality measures in this paper to assess the beta forecasts. This gives an additional impression of the magnitude of the deviations, which the MSE cannot do because it weights larger deviations more heavily than smaller ones and involves squaring. The corresponding equations are (Zimmermann, 1997, Auer and Rottmann, 2020):

$$RMSE = \sqrt{\frac{1}{n} \cdot \sum_{t=1}^{n} \left(ß_{t,r} - ß_{t,p}\right)^2} \tag{9}$$

$$MAE = \frac{1}{n} \cdot \sum_{t=1}^{n} \left|ß_{t,r} - ß_{t,p}\right| \tag{10}$$

For a direct comparison of the adjusted betas with the raw betas as naive forecasts, the Theil's Inequality Coefficient (TIC) is used (Auer and Rottmann, 2020):

$$TIC = \frac{RMSE}{RMSE_{naive}} \tag{11}$$

With a value for TIC < 1, the prediction quality of the considered method (e.g. Vasicek method) is better than the naive estimation in the form of the raw beta.

3.2 Results of the Empirical Analysis

In the following, the results of the empirical analysis are first presented for the total period. For the mean squared error (MSE) between the predicted and the actual beta values as well as for the MSE components, the results are shown in Table 1.

Table 1: MSE and MSE components, total period

	Raw beta	Blume beta	"Adjusted beta"	Vasicek beta
MSE				
MSE average of all stocks	0.09352	0.08098	0.07618	0.08365
MSE Standard deviation	0.04269	0.04133	0.03763	0.04240
Equally weighted portfolio	0.00681	0.00775	0.00555	0.00681
Bias				
Average of all stocks	0.00364	0.02009	0.01382	0.01337
Standard deviation	0.00274	0.01953	0.01292	0.01327
Equally weighted portfolio	0.00003	0.00009	0.00097	0.00003
Inefficiency				
Average of all stocks	0.05693	0.02745	0.02941	0.03759
Standard deviation	0.03304	0.01763	0.02076	0.02227
Equally weighted portfolio	0.00527	0.00574	0.00307	0.00527
Random disturbance				
Average of all stocks	0.03295	0.03344	0.03295	0.03268
Standard deviation	0.02159	0.02142	0.02159	0.01999
Equally weighted portfolio	0.00151	0.00193	0.00151	0.00151

With regard to MSE average of all stocks, it can be seen that the naive beta or raw beta performs worst in terms of MSE average. The Vasicek beta follows in second last place. In contrast, the "Adjusted Beta" leads to the lowest MSE on average, although for some stocks the Blume Beta leads to lower MSE values. For comparison, the data for the equally weighted portfolio are also included, whose MSE values are much lower than the values for the individual stocks. The forecasting quality of this portfolio's beta is therefore relatively good, which is due to the fact that the forecasted and actual beta values fluctuate relatively closely around one in each case; this is because the price movement of the portfolio is very strongly related to the DAX movement. But also for the equally weighted portfolio it can be seen that the "Adjusted beta" shows the lowest MSE. In this portfolio, the Vasicek beta leads to the same values as the raw beta, because the cross-sectional mean of the selected stock sample (stock portfolio) corresponds in this case to the raw beta of the portfolio and the two weightings in equation (4) add to one (Cloete, Jonah and de Wet, 2002). This leads to equal beta factors in each period:

$$\frac{s_{ß_i}^2}{\sigma_{ß_t}^2 + s_{ß_i}^2} + \frac{\sigma_{ß_t}^2}{\sigma_{ß_t}^2 + s_{ß_i}^2} = \frac{s_{ß_i}^2 + \sigma_{ß_t}^2}{\sigma_{ß_t}^2 + s_{ß_i}^2} = 1 \tag{13}$$

It should be noted that the mean values for bias, inefficiency and random disturbance shown in Table 1 add up to the mean MSE values indicated. The non-zero values for the bias and inefficiency components indicate systematic forecast errors. However, there are also random, unexplained deviations between the predicted beta values and the beta values that actually occurred. Strikingly, the values for the random disturbance are exactly the same for the raw beta and the "Adjusted beta". This is due to the fact that the coefficient of determination resulting from the regression of the actual beta on the predicted beta is identical in both cases. With regard to the equally weighted portfolio, the values for the Vasicek beta are also identical to the raw beta for the individual components, because the individual beta values are the same in the respective periods for the reasons mentioned above. Table 2 presents the values for RMSE, MAE, and TIC.

Table 2: RMSE, MAE, and TIC, total period

	Raw beta	**Blume beta**	**"Adjusted beta"**	**Vasicek beta**
RMSE				
Average of all stocks	0.29824	0.27521	0.26792	0.28068
Standard deviation	0.07132	0.07630	0.06992	0.07356
Equally weighted portfolio	0.08255	0.08805	0.07451	0.08255
MAE				
Average of all stocks	0.24569	0.22299	0.21598	0.22918
Standard deviation	0.05963	0.06454	0.05663	0.06153
Equally weighted portfolio	0.06681	0.07287	0.05408	0.06681
TIC				
Average of all stocks	1.00000	0.91883	0.89646	0.93780
Standard deviation	0.00000	0.10911	0.07193	0.04749
Equally weighted portfolio	1.00000	1.06663	0.90260	1.00000

The results presented in Table 2 also show that the "Adjusted beta" obviously leads to the best values, at least on average. This is also confirmed by Theil's inequality coefficient, which assumes the lowest values here. Thus, the forecast quality of the "Adjusted beta" is best compared to the naive estimation (raw beta).

In terms of the total period, it can be stated that all three adjustment methods presented in this paper tend to improve the forecast quality compared to the raw beta, with the "Adjusted beta", which is also the easiest to determine, leading to the best values. Whether this also applies to periods characterized by particular crises is analyzed below.

First, the period Dec. 30, 2009 – Dec. 28, 2012 is considered ("subperiod 1"), which can be seen as a 3-year period after the 2008/2009 financial crisis. The corresponding (3-year) beta forecast values were calculated in the period Dec. 29, 2006 - Dec. 30, 2009 and thus include the financial crisis. Tables 3 and 4 show the results.

Table 3: MSE and MSE components, subperiod 1

	Raw beta	Blume beta	"Adjusted beta"	Vasicek beta
MSE				
MSE average of all stocks	0.10926	0.08912	0.09030	0.08814
MSE Standard deviation	0.08223	0.07935	0.07966	0.06438
Equally weighted portfolio	0.01246	0.01096	0.01160	0.01246
Bias				
Average of all stocks	0.04552	0.05828	0.05021	0.04449
Standard deviation	0.03838	0.06311	0.05185	0.04440
Equally weighted portfolio	0.00297	0.00039	0.00582	0.00297
Inefficiency				
Average of all stocks	0.05582	0.02434	0.03217	0.03573
Standard deviation	0.06076	0.02707	0.03749	0.03503
Equally weighted portfolio	0.00939	0.01045	0.00569	0.00939
Random disturbance				
Average of all stocks	0.00792	0.00651	0.00792	0.00791
Standard deviation	0.01035	0.00616	0.01035	0.00997
Equally weighted portfolio	0.00010	0.00011	0.00010	0.00010

Compared to the total period, the averages of the individual MSE values for subperiod 1 are higher. This applies both to the raw beta and to the various adjusted betas. Thus, the actual beta values could not be forecast as well as when looking at the overall period, i.e. the beta values obtained during the period encompassing the financial and economic crisis were not such good forecast values for the coming 3 years. It is striking that – unlike when considering the total period – the "Adjusted beta" now shows a slightly worse MSE value compared with the other two adjustment methods. Compared to the overall period, the mean bias values are significantly higher, while the inefficiency component shows only relatively slight deviations. In contrast, the values for the random component are significantly lower compared to the overall period.

Table 4: RMSE, MAE, and TIC, subperiod 1

	Raw beta	Blume beta	"Adjusted beta"	Vasicek beta
RMSE				
Average of all stocks	0.31072	0.27234	0.27685	0.27884
Standard deviation	0.11887	0.12888	0.12317	0.10742
Equally weighted portfolio	0.11162	0.10469	0.10772	0.11162
MAE				
Average of all stocks	0.26912	0.23810	0.23778	0.23883
Standard deviation	0.09702	0.11799	0.10537	0.09535
Equally weighted portfolio	0.10210	0.09703	0.08760	0.10210
TIC				
Average of all stocks	1.00000	0.87735	0.88324	0.89748
Standard deviation	0.00000	0.27234	0.13229	0.10938
Equally weighted portfolio	1.00000	0.93795	0.96507	1.00000

The values (averages of all stocks) for RMSE and MAE shown in Table 4 are similar to those in the total period, although they are mostly higher. In this subperiod, the adjustment procedures again lead to lower values (i.e. lower averages of all stocks) than the raw beta. The better forecasting quality of the adjustment methods is again expressed in Theil's coefficient of inequality. It is not clear which adjustment method performed best in this subperiod.

Subperiod 2 is considered to be the period from Dec. 28, 2019 - Dec. 30, 2021. It thus includes the Corona pandemic. The corresponding betas from the period Dec. 30, 2016 - Dec. 28, 2019 were used for the respective forecast. The results in detail are shown in Tables 5 and 6.

Table 5: MSE and MSE components, subperiod 2

	Raw beta	Blume beta	"Adjusted beta"	Vasicek beta
MSE				
MSE average of all stocks	0.10070	0.10739	0.09563	0.10741
MSE Standard deviation	0.05581	0.08223	0.07373	0.09269
Equally weighted portfolio	0.00358	0.00678	0.00183	0.00358
Bias				
Average of all stocks	0.05839	0.06545	0.06536	0.07532
Standard deviation	0.04855	0.07205	0.06777	0.08284
Equally weighted portfolio	0.00112	0.00100	0.00008	0.00112
Inefficiency				
Average of all stocks	0.03247	0.02945	0.02042	0.02137
Standard deviation	0.03191	0.02383	0.02085	0.02021
Equally weighted portfolio	0.00168	0.00490	0.00097	0.00168
Random disturbance				
Average of all stocks	0.00985	0.01249	0.00985	0.01072
Standard deviation	0.00672	0.01007	0.00672	0.00799
Equally weighted portfolio	0.00078	0.00088	0.00078	0.00078

Table 6: RMSE, MAE, and TIC, subperiod 2

	Raw beta	Blume beta	"Adjusted beta"	Vasicek beta
RMSE				
Average of all stocks	0.30625	0.30582	0.28469	0.30022
Standard deviation	0.08765	0.12412	0.12729	0.13859
Equally weighted portfolio	0.05985	0.08235	0.04279	0.05985
MAE				
Average of all stocks	0.26909	0.27148	0.24805	0.26639
Standard deviation	0.08597	0.12254	0.12117	0.13195
Equally weighted portfolio	0.04640	0.06365	0.03382	0.04640
TIC				
Average of all stocks	1.00000	0.97035	0.89879	0.94592
Standard deviation	0.00000	0.14534	0.22467	0.21086
Equally weighted portfolio	1.00000	1.37591	0.71492	1.00000

In subperiod 2, too, the mean values for MSE are higher than in the overall period, and even higher for the adjusted betas than in subperiod 1. It is striking that the mean MSE values for the Blume beta and the Vasicek beta are larger than for the raw beta. The "Adjusted beta" again performs best in this respect. Again, regarding the average of all stocks, the much higher mean bias values compared to the total period are striking, which are also even higher than in subperiod 1. As in subperiod 1, there are significantly lower values for the random component compared to the total period.

The values (averages of all stocks) for RMSE and MAE in subperiod 2 are also similar to those in the total period, although they are somewhat higher. The "Adjusted beta" performs best here as well. Although the adjustment procedures also lead to TIC values (average of all stocks) below one in this subperiod, they are higher compared to subperiod 1 and also to the total period. Thus, although the forecasting quality of the adjustment methods seems to be better than that of the raw beta, it is not as distinct.

Overall, it can be stated that the simple adjustment method ("Adjusted beta") seems to have the highest forecasting quality of the beta estimators considered in this paper. However, it should be noted that the study only covers a relatively small number of stocks and only considers the German market. Moreover, only rolling 3-year periods (36 monthly returns) are consistently used. Thus, more extensive analyses over other time periods and markets are required in order to be able to generalize the results obtained in this paper.

4. Conclusion

Methods have been developed to improve the forecasting quality of the stock beta ("raw beta"), of which the Blume beta and the Vasicek beta have received the most attention in literature and practice. Both methods, together with a simplified Blume beta ("Adjusted beta") frequently used in practice, are tested in this study for selected DAX stocks over a long period with rolling periods. The aim is to find out to what extent the forecasting quality of a stock beta (raw beta) can be improved by these adjustment methods.

The Mean Squared Error (MSE), the Root MSE (RMSE), the Mean Absolute Error (MAE) and Theil's uncertainty coefficient are used to assess the forecasting quality. The MSE is further decomposed into the components bias, inefficiency, and random disturbance.

For the total period, it can be seen that all three adjustment methods tend to improve the forecast quality compared to the raw beta, with the "Adjusted beta" leading to the best values. Furthermore, it can be observed that the forecasting quality of the equally weighted portfolio's beta is considerably better than for the individual stocks due to the beta values fluctuating relatively close around one.

The examination of two subperiods, one covering the financial and economic crisis of 2008/2009 and the other the Corona pandemic, shows that the forecasting quality tends to be lower when looking at individual stocks compared with the total period. According to the values (i.e. averages of all stocks) of the forecasting quality measures used in this paper, the adjustment procedures again mostly produce better values than raw beta in both subperiods. It is striking that, compared to the overall period, the mean bias values are significantly higher in both subperiods (and thus indicate larger systematic forecast errors), while the values for the random component are significantly lower.

In summary, according to the present analysis the simple "Adjusted beta" seems to be the best overall procedure for beta adjustment. However, the analysis only covers a relatively small number of stocks and only considers the German market. In addition, only rolling 3-year periods are used over the entire period. Further analyses of other time periods and markets are required in order to be able to generalize the results obtained in this paper.

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