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Numerical computation and series solution for mathematical model of HIV/AIDS

Deborah O. Odulaja[1], L.M. Erinle-Ibrahim[1] and Adedapo C. Loyinmi[2]

Abstract

In this paper, a mathematical *model* of HIV/AIDS model was examined and of particular interest is the stability of equilibrium solutions. The characteristic equation which gives the Eigen values was examined. By series solution method the behaviour of the viruses and $CD4^+$Tcells was looked into. It was shown that if the recovery rate is high enough, the healthy $CD4^+$Tcell may never die out completely. Hence the patient that test HIV positive, may never develop into full-blown AIDS.

Keyword: $CD4^+$Tcell

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[1] Department of Mathematics, Tai Solarin University of Education, Ogun State. Nigeria.
[2] Department of Mathematical Sciences, University of Liverpool, England.
E-mail: a.c.loyinmi@liv.ac.uk

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1 Introduction

AIDS is caused by Human Immunodeficiency Virus (HIV) infection and is characterized by a severe reduction in $CD4^+$ Tcells, which means an infected person develops a very weak immune system and becomes vulnerable to contracting life-threatening infection (such as pneumocysticcarinii pneumonia). AIDS (Acquired immunodeficiency syndrome) occurs late in HIV disease. The first cares of AIDS were reported in the United States in the spring of 1981. By 1983 HIV had been isolated. Several mathematicians have proposed models to describe the dynamics of the HIV/AIDS infection of $CD4^+$Tcells. In particular Ayeni etal [1,2] proposed the following model:

$$\frac{dX}{dt} = -(d_1 + k_1 V_*)X + \mu Y - k_1 T_* Z \tag{1.1}$$

$$\frac{dY}{dt} = k_1 V_* X - (d_2 + \mu)Y + k_1 T_* Z \tag{1.2}$$

$$\frac{dZ}{dt} = k_2 Y - CZ \tag{1.3}$$

where:

T = Population of $CD4^+$T cells; T_i =Population of infected $CD4^+$ T cells;

V = Virus; π = Production rate of $CD4^+$T cells; d_1 = National death rate of healthy $CD4^+$T cells; d_2 = Death rate of infected $CD4^+$T cells; k_1 = Viral infection rate of $CD4^+$T cells; k_2 = Viral production rate for $CD4^+$T cells; C = Viral clearance rate.

Clearance

$Td_1 \rightarrow$ Death of normal $CD4^+$T cells; $T_i d_2 \rightarrow$ Death of infected $CD4^+$T cells

$VC \rightarrow$ Viral clearance rate.

Ayeni (2010) replaced equation (1.1) by

$$\frac{dT}{dt} = \pi - d_1 T - \frac{k_1 TV}{1 + \alpha V} + \mu T_i \tag{1.4}$$

and (1.2) by

$$\frac{dT_i}{dt} = \frac{k_1 TV}{1+\alpha V} - d_2 T_i - \mu T_i \tag{1.5}$$

where $\alpha =$ Disease related to death rate .

The mathematical model of (1.1) – (1.3) and (1.4) and (1.5) have not been fully established in literature. So the research goes on and on this basis we propose the following model:

2 Mathematical Formulation

A model of HIV infection similar to (1.1) and (1.2) but using

$\frac{k_1 TV}{1+\alpha V}$ for infection CD4$^+$T cells is proposed .

Thus the model is

$$\begin{aligned}
\frac{dT}{dt} &= \pi - d_1 T - \frac{k_1 TV}{1+\alpha V} + \mu T_i, & T(0) &= T_0 \\
\frac{dT_i}{dt} &= \frac{k_1 TV}{1+\alpha V} - d_2 T_i - \mu T_i, & T_i(0) &= T_{i(0)} \\
\frac{dV}{dt} &= k_2 T_i - CV, & V(0) &= V_0
\end{aligned} \tag{2.1}$$

3 Models

3.1 Method of Solution

To obtain the critical point, we set in infected free equilibrium then,

$$\frac{dT}{dt} = \frac{dT_i}{dt} = \frac{dV}{dt} = 0.$$

Equation (2.1) becomes

$$\begin{aligned} &\pi - d_1 T - \frac{k_1 TV}{1+\alpha V} + \mu T_i = 0 \\ &\frac{k_1 TV}{1+\alpha V} - d_2 T_i - \mu T_i = 0 \\ &k_2 T_i - CV = 0 \end{aligned} \tag{3.1}$$

when there is no CD4[+]T cells infection then $V = T_i = 0.$

And equation (3.1) becomes $\pi - d_1 T = 0$, with $T = \frac{\pi}{d_1}.$

So the un- infected equilibrium is $(T,0,0) = (\frac{\pi}{d_1},0,0)$.

The infected equilibrium when there is CD4[+]Tcells infections is V ≠0 ,T_I≠ 0.

$$\frac{dT}{dt} = \pi - d_1 T - \frac{k_1 TV}{1+\alpha V} + \mu T_i \qquad T(0) = T_0 \tag{3.1a}$$

$$\frac{dT_i}{dt} = \frac{k_1 TV}{1+\alpha V} - d_2 T_i - \mu T_i \qquad T_i(0) = T_{i(0)} \tag{3.1b}$$

$$\frac{dV}{dt} = k_2 T_i - CV \qquad V(0) = V_0 \tag{3.1c}$$

Then equation (3.1c) becomes

$$\begin{aligned} &CV = k_2 T_i \\ &V = \frac{k_2 T_i}{C} \end{aligned} \tag{3.2}$$

Substituting (3.2) in (3.1b)

$$\frac{k_1 T\left(\frac{k_2 T_i}{C}\right)}{1+\alpha\left(\frac{k_2 T_i}{C}\right)} - d_2 T_i - \mu T_i = 0.$$

Then

$$T = \frac{(d_2 + \mu)(C + \alpha k_2 T_i)}{k_2 k_1} \tag{3.3}$$

Substituting (3.2) and (3.3) in equation (3.1a)

$$\pi - d_1\left(\frac{(d_2+\mu)(C+\alpha k_2 T_i)}{k_2 k_1}\right) - \frac{k_1\left(\frac{(d_2+\mu)(C+\alpha k_2 T_i)}{k_2 k_1}\right)\left(\frac{k_2 T_i}{C}\right)}{1+\alpha\left(\frac{k_2 T_i}{C}\right)} + \mu T_i = 0.$$

Then the equation becomes

$$T_i = \frac{\pi k_2 k_1 - C d_1 (d_2+\mu)}{\alpha d_1 k_2 (d_2+\mu) + k_2 k_1 d_2}.$$

Then

$$V = \frac{\pi k_2 k_1 - C d_1 (d_2+\mu)}{\alpha C d_1 (d_2+\mu) + C k_1 d_2}.$$

Now T becomes

$$T = \frac{(d_2+\mu)C}{k_2 k_1} + \frac{\alpha k_2 (d_2+\mu)}{k_2 k_1}\left(\frac{\pi k_2 k_1 - C d_1 (d_2+\mu)}{\alpha d_1 k_2 (d_2+\mu) + k_2 k_1 d_2}\right).$$

Then the infected equilibrium is

$$\left(\frac{(d_2+\mu)C}{k_2 k_1} + \frac{\alpha k_2 (d_2+\mu)}{k_2 k_1}\left(\frac{\pi k_2 k_1 - C d_1 (d_2+\mu)}{\alpha d_1 k_2 (d_2+\mu) + k_2 k_1 d_2}\right), \frac{\pi k_2 k_1 - C d_1 (d_2+\mu)}{\alpha d_1 k_2 (d_2+\mu) + k_2 k_1 d_2}, \frac{\pi k_2 k_1 - C d_1 (d_2+\mu)}{\alpha C d_1 (d_2+\mu) + C k_1 d_2}\right)$$

3.2 Reduction to origin

Let

$$x = T - T_* \qquad T = x + T_*$$

$$y = T_i - T_{i*} \qquad T_{i*} = y + T_{i*}$$

$$z = V - V_* \qquad V = z + V_*$$

where (T_*, T_{i*}, V_*) is the infected equilibrium such that

$$\pi - d_1 T_* - \frac{k_1 T_* V_*}{1+\alpha V_*} + \mu T_{i*} = 0$$

$$\frac{k_1 T_* V_*}{1+\alpha V_*} - d_2 T_{i*} - \mu T_{i*} = 0 \tag{3.2.1}$$

$$k_2 T_{i*} - C V_* = 0$$

Then

$$\frac{dx}{dt} = \frac{dT}{dt}, \frac{dy}{dt} = \frac{dT_i}{dt}, \frac{dz}{dt} = \frac{dV}{dt} \tag{3.2.2}$$

Substituting (3.2. 1) and (3.2.2) in (3.1)

$$\frac{dx}{dt} = \pi - d_1 \left(x + T_*\right) - \frac{k_1 \left(x + T_*\right)\left(z + V_*\right)}{1 + \alpha\left(z + V_*\right)} + \mu(y + T_{i*})$$

$$\frac{dy}{dt} = \frac{k_1 \left(x + T_*\right)\left(z + V_*\right)}{1 + \alpha\left(z + V_*\right)} - d_2 \left(y + T_{i*}\right) - \mu\left(y + T_{i*}\right) \tag{3.2.3}$$

$$\frac{dV}{dt} = k_2 \left(y + T_{i*}\right) - C\left(z + V_*\right)$$

Then equation (3.2.3) becomes,

$$\frac{dx}{dt} = \left(-d_1 + \frac{k_1 V_*}{1+\alpha V_*}\right)x + \mu y - \frac{k_1 T_* z}{1+\alpha V_*} + nonlinearterms$$

$$\frac{dy}{dt} = \frac{k_1 V_*}{1+\alpha V_*} x - \left(d_2 + \mu\right)y + \frac{k_1 T_* z}{1+\alpha V_*} + nonlinearterms$$

$$\frac{dz}{dt} = k_2 y - Cz + nonlinearterms$$

So

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{pmatrix} = \begin{pmatrix} \left(-d_1 + \frac{k_1 V_*}{1+\alpha V_*}\right) & \mu & \frac{k_1 T_*}{1+\alpha V_*} \\ \frac{k_1 V_*}{1+\alpha V_*} & -\left(d_2 + \mu\right) & \frac{k_1 T_*}{1+\alpha V_*} \\ 0 & k_2 & -C \end{pmatrix} + \left(nonlinearterms\right)$$

Then

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{pmatrix} = A \begin{pmatrix} x \\ y \\ z \end{pmatrix} + (nonlinearterms)$$

$$|A - \lambda I| = 0$$

where

$$A = \begin{pmatrix} \left(-d_1 + \frac{k_1 V_*}{1+\alpha V_*}\right) & \mu & \frac{k_1 T_*}{1+\alpha V_*} \\ \frac{k_1 V_*}{1+\alpha V_*} & -(d_2 + \mu) & \frac{k_1 T_*}{1+\alpha V_*} \\ 0 & k_2 & -C \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} \left(-d_1 + \frac{k_1 V_*}{1+\alpha V_*}\right) - \lambda & \mu & \frac{k_1 T_*}{1+\alpha V_*} \\ \frac{k_1 V_*}{1+\alpha V_*} & -(d_2 + \mu) - \lambda & \frac{k_1 T_*}{1+\alpha V_*} \\ 0 & k_2 & -C - \lambda \end{vmatrix}$$

Let $\pi = 50$, $d_1 = d_2 = 0.01$, $k_1 = k_2 = 0.03$, $C = 0.01$, $\mu = 0.01$ and $\alpha = 2$.

Substituting the parameters in (3.4).

Then

$(T_*, T_{i*}, V_*) = (2857.24,\ 2142.76,\ 6428.28)$.

So

$$|A - \lambda I| = \begin{vmatrix} -0.0249 - \lambda & 0.01 & -0.0067 \\ 0.0149 & -0.02 - \lambda & 0.0067 \\ 0 & 0.03 & -0.01 - \lambda \end{vmatrix} = 0$$

$\lambda^3 + 0.0549\,\lambda^2 + 0.000597\lambda + 0.000001255 = 0$

The eigenvalues of the system are check by MATLAB function and it gives

λ_1 = -0.002774, λ_2 = -0.041125, λ_3 = - 0.0110004.

Therefore the Eigenvalues of this system are λ = - 0.0028, - 0.0411 and – 0.011, hence this system is asymptotically stable.

The conclusion of this system is similar to

Theorem (Derrick and Grossman 1976)

Let $V(x_1, x_2, x_3)$ be a lyapunov function the system

$x_1^1 = f_1(x_1, x_2, x_3)$

$x_2^1 = f_2(x_1, x_2, x_3)$

$x_3^1 = f_3(x_1, x_2, x_3)$

Then,

$V^1 = (x_1 x_2 x_3)$ is negative semi definite the origin is stable

$V^1 = (x_1 x_2 x_3)$ is negative definite, the origin is asymptotically stable

$V^1 = (x_1 x_2 x_3)$ is positive definite the origin is unstable

4 Numerical solution

Substituting (3.2.1) and (3.2.2) in (1.1)-(1.3), it becomes

$$\frac{dx}{dt} = \left(-d_1 + \frac{k_1 V_*}{1+\alpha V_*}\right)x + \mu y - \frac{k_1 T_* z}{1+\alpha V_*} + nonlinearterms$$

$$\frac{dy}{dt} = \frac{k_1 V_*}{1+\alpha V_*}x - (d_2 + \mu)y + \frac{k_1 T_* z}{1+\alpha V_*} + nonlinearterms$$

$$\frac{dz}{dt} = k_2 y - Cz + nonlinearterms$$

$$\frac{dx}{dt}=\frac{k_1\left(x+T_*\right)\left(z+V_*\right)}{1+\alpha\left(z_0+V_*\right)}$$
$$\frac{dy}{dt}=\frac{k_1V_*x\left(x+T_*\right)\left(z+V_*\right)-}{1+\alpha\left(z_0+V_*\right)} \qquad (4.1)$$
$$\frac{dz}{dt}=k_2y-cz$$

In order to find the approximate solution to the model, power series solution is used.

Let the solution to the system (1) be

$$x(t)=x_0+a_1t+a_2t^2+\cdots+a_nt^n$$

$$y(t)=y_0+b_1t+b_2t^2+\cdots+b_nt^n$$

$$z(t)=z_0+m_1t+m_2t^2+\cdots+m_nt^n$$

Let

$$x_1(t)=x_0+a_1t$$
$$y_1(t)=y_0+b_1t \qquad (4.2)$$

$$z_1(t)=z_0+m_1t$$
$$x_1^2(t)=a_1,\ \ y_1^2(t)=b_1,\ \ z_1^2(t)=m_1 \qquad (4.2.1)$$

Substituting (4.2) and (4.2.1) in (4.1), the system becomes

$$a_1=\frac{-\left[d_1\left(1+\alpha z_0+\alpha V_*\right)+k_1V_*\right]x_0+\mu y_0\left(1+\alpha z_0+\alpha V_*\right)-k_1T_*z_0}{1+\alpha\left(z_0+V_*\right)}$$
$$b_1=\frac{k_1V_*x_0-\left[\left(d_2+\mu\right)\left(1+\alpha z_0+\alpha V_*\right)\right]y_0+k_1T_*z_0}{1+\alpha\left(z_0+V_*\right)} \qquad (4.2.2)$$
$$m_1=k_2y_0-Cz_0$$

Then equation (4.2) can be written as

$$x_1\left(t\right)=x_0+\left(\frac{-\left[d_1\left(1+\alpha z_0+\alpha V_*\right)+k_1V_*\right]x_0+\mu y_0\left(1+\alpha z_0+\alpha V_*\right)-k_1T_*z_0}{1+\alpha\left(z_0+V_*\right)}\right)t$$

$$y_1(t)=y_0+\left(\frac{k_1V_*x_0-\left[(d_2+\mu)(1+\alpha z_0+\alpha V_*)\right]y_0+k_1T_*z_0}{1+\alpha(z_0+V_*)}\right)t$$

$$z_1(t)=z_0+(k_2y_0-Cz_0)t$$

Let

$$x_2(t)=x_0+\left(\frac{-\left[d_1(1+\alpha z_0+\alpha V_*)+k_1V_*\right]x_0+\mu y_0(1+\alpha z_0+\alpha V_*)-k_1T_*z_0}{1+\alpha(z_0+V_*)}\right)t+a_2t^2$$

$$y_2(t)=y_0+\left(\frac{k_1V_*x_0-\left[(d_2+\mu)(1+\alpha z_0+\alpha V_*)\right]y_0+k_1T_*z_0}{1+\alpha(z_0+V_*)}\right)t+b_2t^2 \qquad (4.2.3)$$

$$z_2(t)=z_0+(k_2y_0-Cz_0)t+m_2t^2$$

Perturb (4.2.3) and substitute into (1), we have

$$a_2=\frac{1}{2}\{\frac{-\left[d_1(1+\alpha z_0+\alpha V_*)+k_1V_*\right]}{1+\alpha(z_0+V_*)}$$
$$\times(\frac{-\left[d_1(1+\alpha z_0+\alpha V_*)+k_1V_*\right]x_0+\mu y_0(1+\alpha z_0+\alpha V_*)-k_1T_*z_0}{1+\alpha(z_0+V_*)})$$
$$+\mu(\frac{k_1V_*x_0+(d_1+\mu)(1+\alpha z_0+\alpha V_*)y_0+k_1T_*z_0}{1+\alpha(z_0+V_*)})-\frac{k_1T_*(k_2y_0-Cz_0)}{(1+\alpha z_0+\alpha V_*)}\}$$

$$b_2=\frac{1}{2}\{\frac{k_1T_*}{(1+\alpha z_0+\alpha V_*)} \qquad (4.2.4)$$
$$\times(\frac{-\left[d_1(1+\alpha z_0+\alpha V_*)+k_1V_*\right]x_0+\mu y_0(1+\alpha z_0+\alpha V_*)-k_1T_*z_0}{1+\alpha(z_0+V_*)})$$
$$-(d_1+\mu)(\frac{k_1V_*x_0-(d_1+\mu)(1+\alpha z_0+\alpha V_*)y_0+k_1T_*z_0}{1+\alpha(z_0+V_*)})+\frac{k_1T_*(k_2y_0-Cz_0)}{(1+\alpha z_0+\alpha V_*)}\}$$

$$m_2=\frac{1}{2}\{k_2(\frac{k_1V_*x_0-(d_1+\mu)(1+\alpha z_0+\alpha V_*)y_0+k_1T_*z_0}{1+\alpha(z_0+V_*)})-C(k_2y_0-Cz_0)\}$$

Substituting (4.2.2) into (4.2.4)

$$a_2=\frac{1}{2}(\frac{-\left[d_1(1+\alpha z_0+\alpha V_*)+k_1V_*\right]}{1+\alpha(z_0+V_*)}a_1+\mu b_1-\frac{k_1T_*}{(1+\alpha z_0+\alpha V_*)}m_1)$$

$$b_2=\frac{1}{2}(\frac{k_1T_*}{(1+\alpha z_0+\alpha V_*)}a_1-(d_1+\mu)b_1+\frac{k_1T_*}{(1+\alpha z_0+\alpha V_*)}m_1)$$

$$m_2 = \frac{1}{2}(k_2 b_1 - Cm_1)$$

Let

$$x_0 \neq 0, \quad y_0 = 0, \; z_0 = 0, \; T_* = T_* \neq 0, \; T_{i*} = 0, \quad V_* = 0 .$$

Substituting case 1 in (4.2.2)

$$a_1 = -d_1 x_0 , \;\; b_1 = 0 , \;\; m_1 = 0$$

$$a_2 = \frac{d_1}{2}(-d_1 x_0) = -\frac{d_1^2}{2} x_0 , \; b_2 = 0, \; m_2 = 0 .$$

Then the series become

$$x(t) = x_0 + x_0 d_1 t + x_0 \frac{d_1^2 t^2}{2} + \cdots + x_0 \frac{d_1^n t^n}{n}$$

when $x_0 = 20$, $d_2 = 0.01$

$$x(t) = 20(1 - 0.01t + \frac{0.01^2}{2} t^2 + \cdots)$$

when $x_0 = 10$, $d_2 = 1$

$$x(t) = 10(1 - t + \frac{t^2}{2} + \cdots)$$

Case 2

Let $x_0 \neq 0, \quad y_0 = 0, \; z_0 = 0, \; T_* = T_* \neq 0, \; T_{i*} = 0, \quad V_* = 0$

Substituting case 2 in (4.2.2)

$$a_2 = \frac{1}{2}(d_1 a_1 + \mu b_1 - \frac{k_1 T_0}{\left(1 + \alpha z_0\right)} m_1)$$

$$b_2 = \frac{1}{2}(-\left(d_1 + \mu\right) b_1 + \frac{k_1 T_0}{\left(1 + \alpha z_0\right)} m_1)$$

$$m_2 = \frac{1}{2}(k_2 b_1 - Cm_1)$$

If $x_0 = 10$, $y_0 = 2$, $z_0 = 2$, $T_0 = 5$ and considering other parameter $k_1 = k_2 = 3$, $d_1 = d_2 = 1$, and $c = 1$.

We have the following solutions:

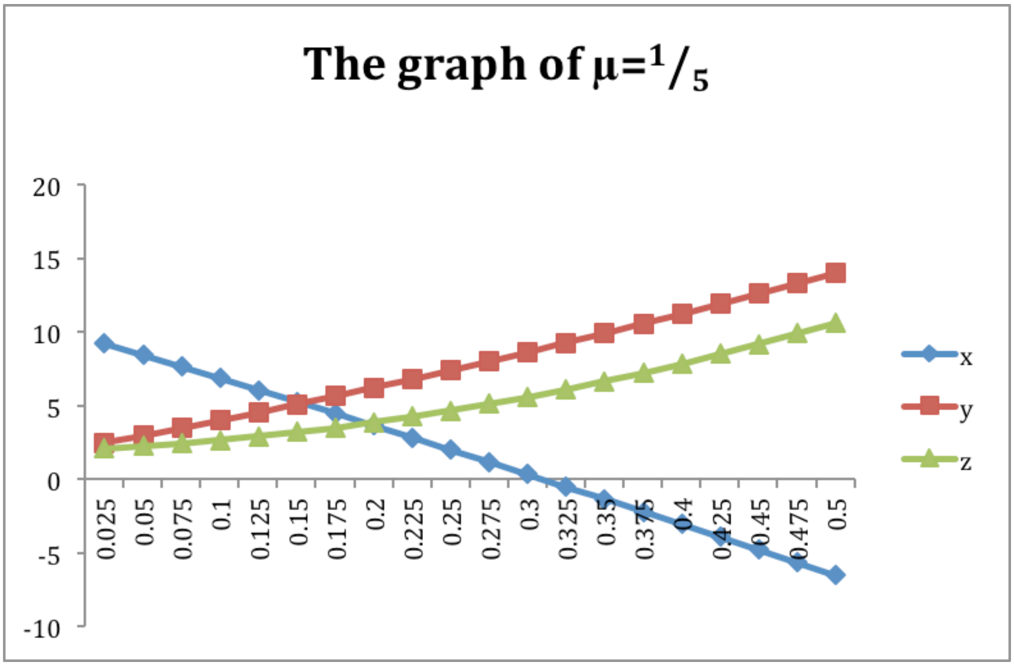

Figure 1: Graph of x (CD4+T cells), y (infected cells), z (virus) against time at $\mu = {}^{1}/_{5}$ and $\alpha = {}^{1}/_{5}$

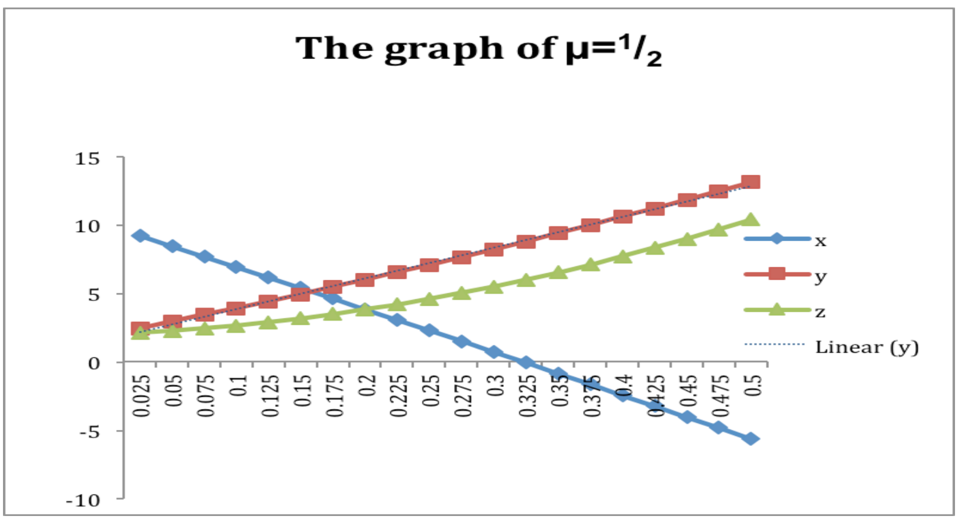

Figure 2: Graph of x (CD4+T cells), y (infected cells), z (virus) against time at $\mu = ½$ and $\alpha = 1/5$

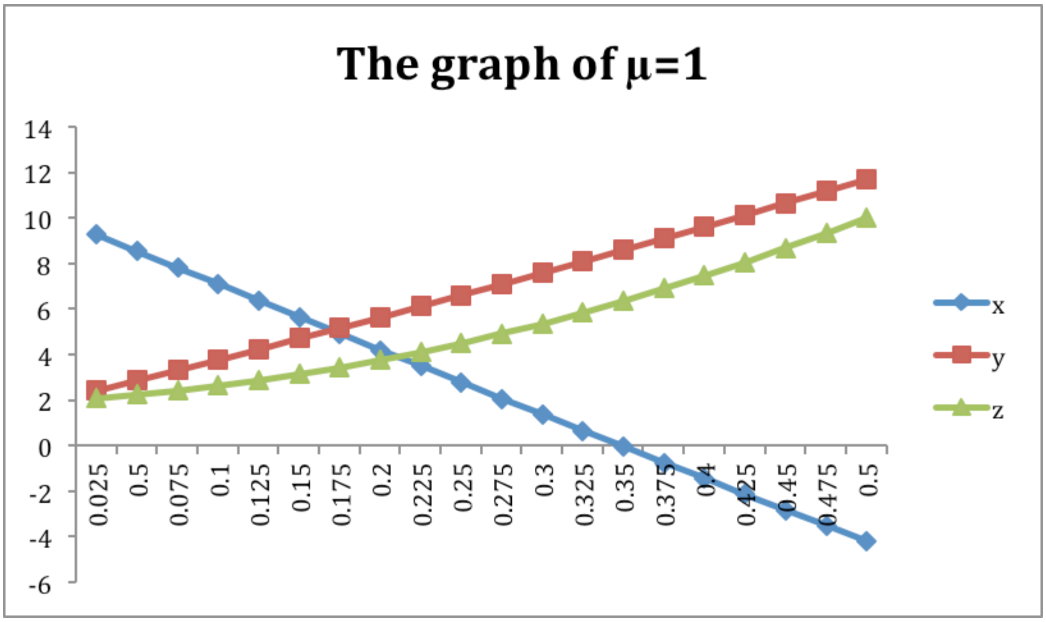

Figure 3: Graph of x (CD4+T cells), y (infected cells), z (virus) against time at

$\mu = 1$ and $\alpha = 1/5$

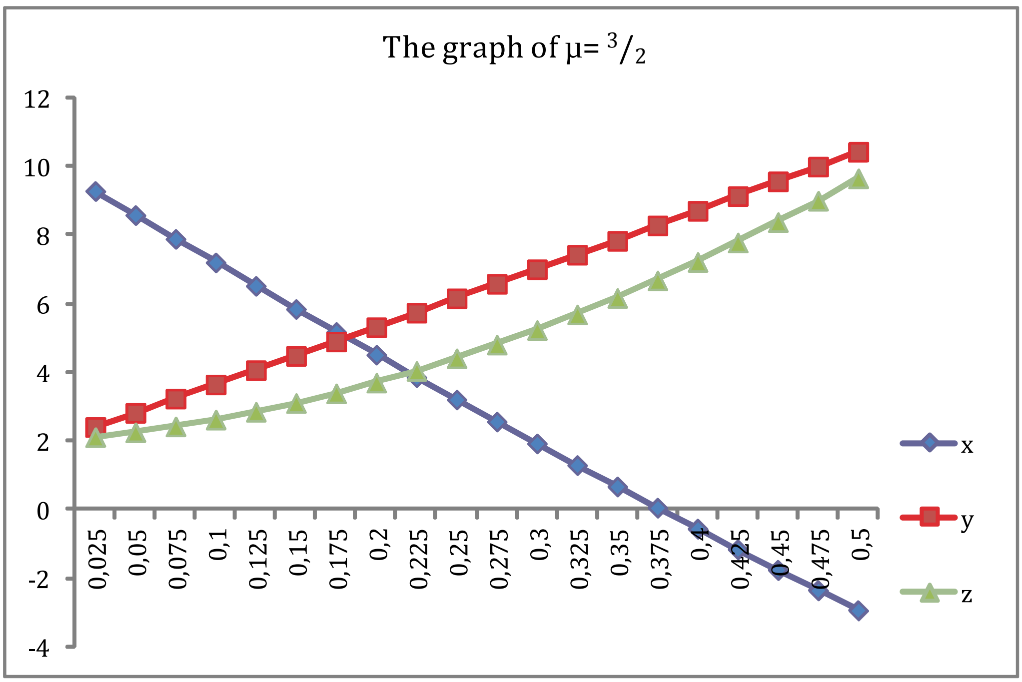

Figure 4: Graph of x (CD4+T cells), y (infected cells), z (virus) against time at

$\mu = {}^{3}/_{2}$ and $\alpha = {}^{1}/_{5}$

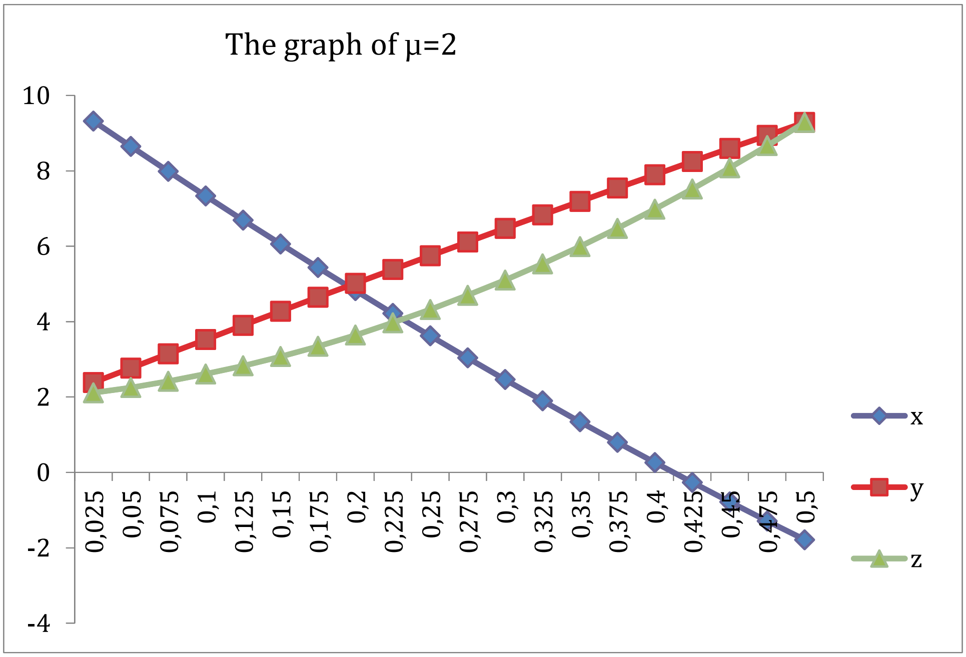

Figure 5: Graph of x (CD4+T cells), y (infected cells), z (virus) against time at $\mu = 2$ and $\alpha = {}^{1}/_{5}$

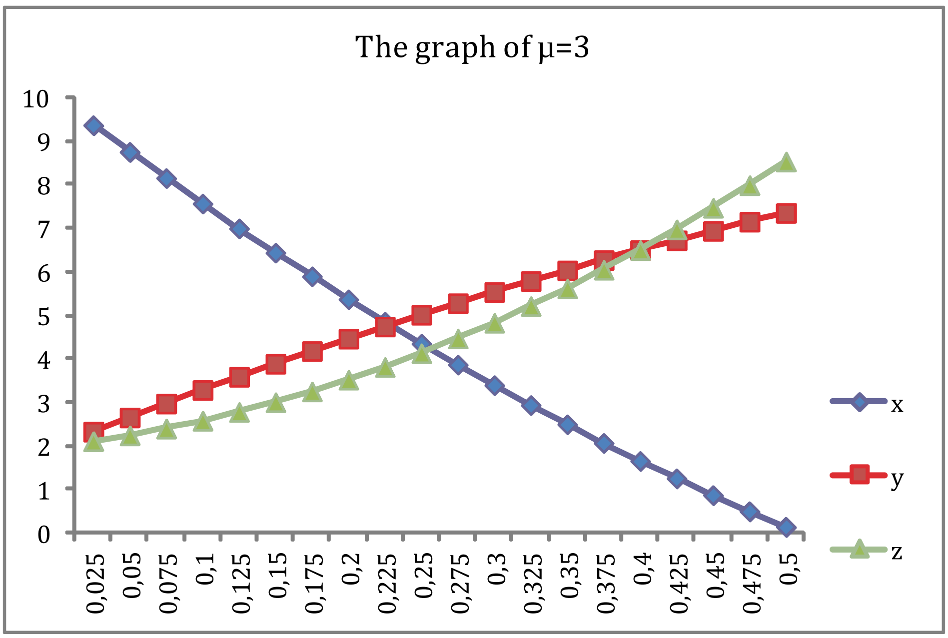

Figure 6: Graph of x (CD4+T cells), y (infected cells), z (virus) against time at $\mu = 3$ and $\alpha = {}^{1}/_{5}$

5 Conclusion

Figure 1 shows that the healthy CD4+T cells are all infected around t = 0.325, when μ=0.2

Figure 2 shows that the healthy CD4+T cells are all infected around t = 0.3125, when μ=0.5

Figure 3 shows that the healthy CD4+T cells are all infected around t = 0.3625, when μ=1

Figure 4 shows that the healthy CD4+T cells are all infected around t = 0.3875, when μ=1.5

Figure 5 shows that the healthy CD4+T cells are all infected around t = 0.4, when μ=2

Figure 6 shows that the healthy CD4+T cells are all infected around t = 0.4875, when μ=3

The above results show that as μ increases, the duration of time for all the $CD4^{+}T$ cells to get infected also increases μ is the recovery rate of the $CD4^{+}T$ cells. This implies that when the recovery rate μ is high, $CD4^{+}T$ cells take longer time for all to get infected.

In this paper, we modified an existing HIV/AIDS model. We investigated the characteristic equation and discussed the stability of equilibrium points by finding the eigenvalues of the model that were previously considered.

We solved existing characteristic equations numerically using realistic values for the parameters and we interpreted the graphs that resulted from the numerical solution.

The stability criteria showed that HIV may not lead to full blown AIDS since the healthy $CD4^{+}T$ cells may never die out completely.

References

[1] R.O. Ayeni, T.O. Oluyo and O.O. Ayandokun, Mathematical Analysis of the global dynamics of a model for HIV infection of CD4+ T cells, *JNAMP*, **11**, (2007), 103-110.

[2] R.O. Ayeni, A.O. Popoola and J.K. Ogunmola, Some new results on affinity hemodialysis and T cell recovery, *Journal of Bacteriology Research*, **2**, (2010), 74-79.

[3] W.R. Derrick and S. Grossman, *Elementary Differential Equations*, Addison Wesley, Reading, 1976.

[4] L.B. Steven and A.S. Peter, Substance Abuse Treatment For Persons with HIV/AIDS, U.S. Department of health and services, (2008), 27-29.

[5] D.O. Odulaja, *Numerical Computation and Series solution for mathematical model of HIV/AIDS*, M.ed dissertation, Tai Solarin University of Education, Ogun State, Nigeria, 2013.