Journal of Statistical and Econometric Methods, vol.8, no.3, 2019, 21-34
ISSN: 2241-0384 (print), 2241-0376 (online)

Time Series Forecasting: A Comparative Study of VAR ANN and SVM Models

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Abstract

Modeling and forecasting time series data has primary significance to numerous practical areas. Numerous significant models have been proposed in texts for enlightening the correctness and effectiveness of modeling and predicting time series data. The goal of this work is to discovery an appropriate model to forecast time series data. Firstly, we applied the most popular multivariate time series model is Vector Autoregressive model, with its frequently used four criteria of VAR order selection such as AIC, HQ, SC and FPE, and the asymptotic Portmanteau test of VAR order selection. We also checked the accuracy of forecast performance based on impulse response function. Secondly, we applied the most popular time series modeling and forecasting machine learning techniques, such as artificial neural network and support vector machine. In this study, we applied the three multivariate time series models, viz. VAR, ANN and SVM, compared together with their inherent forecasting strengths based on the five forecast performance measures: mean squared error, mean absolute deviation, root mean squared error, mean absolute percentage error and Theil's U-statistics. Finally, we found that the artificial neural network, time series modeling and forecasting machine learning technique, is the best technique for time series modeling and forecasting.

Keywords: Forecasting, VAR, ANN, SVM, Forecast Performance Measures.
Mathematics Subject Classification: 60G10, 60G15, 82C05, 53A17

1. Introduction

The modeling of time series is a versatile study field which has engrossed devotions of scholars' civic over last few years. The time series modeling key objective is to wisely gather and thoroughly investigate the past time series data to improve a suitable model which defines the essential arrangement of the sequences. This model is then would like to produce upcoming observations for the sequences, such as to create projections. The predicting of time series can be labelled as the performance of forecasting the upcoming by comprehending the previous (Brockwell and Davis (2002). Because of the inevitable significance of time sequence projecting in various real-world areas viz. commercial, finances, economics, science and industrial, etc. (Enders (2008), Wei (2006)), appropriate maintenance should be taken to adequate an acceptable model to the core time

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Article Info: *Received*: November 2, 2018. *Revised* : February 5, 2019
Published online : May 10, 2019

sequences. A fruitful time sequence projecting rely on a suitable model fitting (Zhang (2003)). Lots of efforts have been completed by investigators over several ages for the expansion of well-organized models to progress the predicting correctness. Consequently, numerous significant time series predicting models have been developed in texts (Enders (2008), Box and Jenkins (1970), Cao, Francis and Tay (2003), Hyndman and Athanasopoulos, (2014), Wei (2006)). One of the most common and regularly used predicting time series models is the Vector Autoregressive (VAR) model (Enders (2008), Calomiris, et al. (2013), Robertson and Tallman (1999), Sims (1980), Wang (2018)). According to Sims, if there is real simultaneity midst a set of variables, they would all be preserved on an identical stability; there should not be any a priori dissimilarity between endogeneous and exogeneous variables. It is in this sprit that Sims established his VAR model. Since the different factors in the projected VAR models are frequently challenging to understand, the researchers of this method frequently evaluate the supposed impulse response function (IRF) (Enders (2008)). The IRF vestige out the reply of the endogeneous variable in the VAR models to shocks in the disturbance expressions. Although the function of such IRF exploration has been examined by investigators, it is the focus of the VAR analysis (Wei (2006)). Occasionally, econometrician used to identify the order of VAR by using Portmanteau (asymptotic) Test. For an assessment of the evaluation of VAR with other forecasting methods, several of the investigators select to apply prediction evaluation procedures such as MAD, MAPE, MSE, RMSE, and Theil's U-statistic (Brockwell and Davis (2002), Wei (2006)).

Recently, artificial neural networks (ANNs) have played attention enhancing devotions in the field of time series predicting (Hamzacebi, (2008), Zhang, Patuwo and Hu (1998), Kihoro, et al. (2006)). Although primarily biologically motivated, but advanced on ANNs have been fruitfully functioned in versatile domain, specifically for predicting and grouping intentions (Zhang (2007), Kihoro, et al. (2006)). The outstanding architecture of ANNs, when functioned to time series predicting difficulty is their integral proficiency of non-linear modeling, without any assumption about the statistical distribution followed by the values. The proper model is adaptively made depend on the given data. Because of this purpose, ANNs are data-driven and self-adaptive by nature (Zhang, Patuwo and Hu (1998), Zhang (2003)). During the past few years a significant volume of investigation have been conducted towards the utilizations of neural networks for time series modeling and predicting.

A key development in the domain of time series predicting happened with the invention of Vapnik's support vector machine (SVM) conception (Cao, Francis and Tay (2003), Gestel, et al. (2001)). Vapnik and his co-workers architected SVM at the AT & T Bell laboratories in 1995 (Raicharoen, et al. (2003)). The primary goal of SVM was to explain pattern recognition difficulties but subsequently they have been broadly applied in versatile areas such as signal processing, function estimation, regression and time series forecasting problems (Cao, Francis and Tay (2003)). The noteworthy feature of SVM is that it is not only designed for good classification but also planned for a better simplification of the training data. For this purpose the SVM architecture has developed one of the renowned methods, particularly for time series predicting difficulties in current ages.

The goal of this research is to assess a ample investigation around the three commonly used methods for time series predicting, such as, VAR, ANN and SVM methods. This study covers four segments, which are prepared as follows: section 1 gives an introduction to the elementary ideas of time series modeling, together with some related designs such as VAR, ANN and SVM. Section 2 is arranged to discuss about the numerous time series models. These include the VAR models, Impulse Response Function (IRF) of VAR, the selection criteria of order VAR, the Portmanteau test(Asymptotic) of the selection of order VAR, Artificial Neural Networks (ANN) time series model, Support Vector Machine (SVM) as well as Five forecast performance criterions, often used in texts. In section 3 presents our investigational forecasting outcomes in terms of afore-mentioned models & performance criterions, achieved on real time series dataset, together with the associated forecast plots. After completion of these sections, we have given a brief conclusion of our work.

2. Methods & Materials

2.1. Vector Auto-regressive (VAR) Model

There are a various procedures presented for predicting economic variables. One frequent type of forecasting technique is Vector auto-regression modeling for multivariate Time Series methodology. This type of forecast is chiefly in financial and economic analysis. A VAR model is a valuable and flexible methodology to define the lively performance of financial movement and economic time series dataset; that is, a vector of time series. In this structure, we suppose one equation for one variable as dependent variable with constant and lags. Each variable is supposed to effect with each other in the arrangement, which marks straight explanation of the projected quantities very problematic (Enders (2008)).

We write a multi-dimensional VAR (p) as:

$$Y_t = \alpha + \varphi_1 \begin{bmatrix} LMVb_{t-1} \\ CCI_{t-1} \\ CPI_{t-1} \\ PP_{t-1} \\ UR_{t-1} \\ PMI_{t-1} \end{bmatrix} + \varphi_2 \begin{bmatrix} LMVb_{t-2} \\ CCI_{t-2} \\ CPI_{t-2} \\ PP_{t-2} \\ UR_{t-2} \\ PMI_{t-2} \end{bmatrix} + \cdots + \varphi_p \begin{bmatrix} LMVb_{t-p} \\ CCI_{t-p} \\ CPI_{t-p} \\ PP_{t-p} \\ UR_{t-p} \\ PMI_{t-p} \end{bmatrix} + e_t$$

Where, e_t are white noise processes, $E(e_t) = 0$ and

$$E(e_t e_t') = \begin{cases} 0 & when\, t = \tau \\ \Omega & when\, t \neq \tau \end{cases} :$$

$$Y_t = \begin{bmatrix} LMVb_1 \\ CCI_t \\ CPI_t \\ PP_t \\ UR_t \\ PMI_t \end{bmatrix}, \; \varphi_1 = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \cdots & \varphi_{1p} \\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ \varphi_{p1} & \varphi_{p2} & \cdots & \varphi_{pp} \end{bmatrix}$$

A "VAR in levels" is known as the series modeled are stationary; we predict them straightly by estimating a VAR to the data. In this case, the models and coefficients are estimated equation by equation using the principle of least squares.

2.1.1. Impulse Response Function of VAR

Based on the vector autoregressive models, we can write the impulse response function (Enders (2008)) as:

$$x_t = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i}$$

Where, $x_t = Y_t$, $\mu =$ column mean of Y_t, $\phi_i =$ coefficient matrix and $\varepsilon_{t-i} =$ shocks or impulse response.

One important matter about the impulse response functions is that they are created using the projected coefficient. Since each coefficient is projected inaccurately, the impulse responses also hold disturbance. The matter is to build confidence intervals about the impulse responses that permit for the parameter ambiguity characteristic in the estimation procedure.

2.1.2. Information Criteria for VAR Order Selection

Information criteria (IC) are statistics that assess the distance between observations and model classes (Brockwell and Davis (2002)). If the IC value is small, the distance is small and the model class holds a good descriptor of the Land Market values. The information criteria for a VAR (p) process are defined as:

Table 1: Order selection Criteria of VAR model

Selection Method	Statistic
AIC(p)	$In \mid \tilde{\sum}_u (m) \mid + {}^{2p}\!/_{T}\, k^2$
HQ(p)	$In \mid \tilde{\sum}_u (m) \mid + {}^{2p}\!/_{T}\, In(InT)$
SC(p)	$In \mid \tilde{\sum}_u (m) \mid + {}^{p}\!/_{T}\, InT$
FPE(p)	$In \mid \tilde{\sum}_u (m) \mid + {}^{T+p}\!/_{T-P}$

Where, m = the number of free parameters, T = sample size, $\tilde{\sum}_u$ denotes the ML estimate of the error variance matrix based on using the given model class with m free parameters, VAR (p) models have $m = k^2 p + k + k(k+1)/2$ free parameters, p value is central, the dimension K is kept constant.

2.1.3. Portmanteau (Asymptotic) test for VAR Order Selection

For testing the lack of serial correlation in the disturbances of a VAR (p), a Portmanteau test is applied in the function (Hyndman and Athanasopoulos, (2014)). The Portmanteau statistic is well-defined as:

$$Q_h = T \sum tr(\hat{C}_j^T \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1})$$

Where $\hat{C}_j = {}^{1}\!/_{T} \sum_{t=i+1}^{T} \hat{e}_t \hat{e}'_{t-j}$. The test statistic has an approximate $\chi^2(k^2 h - p)$ distribution, and p is the number of coefficients excluding deterministic terms of a VAR (p).

2.2. Artificial Neural Networks (ANNs)

The ANN is a mathematical structure decorated after the matching series of the human brain (Enders (2008), Cottrell, et al. (1995)). A feed forward network (FFN), one of the frequently used features of the ANN, is commonly digest of three layers of input, hidden and output (Figure: 1). Each layer of the ANN has a certain number of nodes and each node in a layer is connected to other nodes in the next layer with a specific weight and bias (Zhang (2007), Cottrell, et al. (1995)). Below we shall remark the hidden layer of ANNs, which create them fairly preferred for time series analysis and forecasting.

The most extensively practiced ANNs in predicting difficulties are multi-layer perceptron's (MLPs), which practice a single hidden layer feed forward network (FNN) (Hamzacebi, (2008), Zhang (2003)). The model is described by a network of three layers, such as input, hidden and output layer, associated by acyclic links. There may be more than one hidden layer. The nodes in numerous layers are also recognized as treating features. The three-layer feed forward features of ANN models can be architected as below:

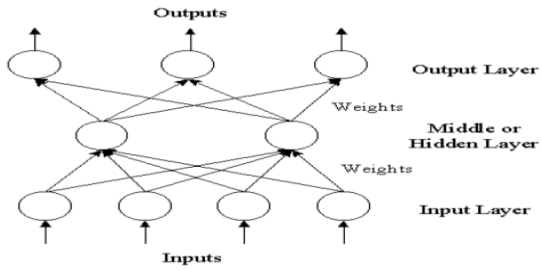


Figure 1: The three-layer feed forward ANN architecture

The yield of the model is calculated using the following mathematical model (Cottrell, et al. (1995)):

$$y_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j g\left(\beta_{0j} + \sum_{i=1}^{p} \beta_{ij} y_{t-j} \right) + e_t , \quad \forall\, t \qquad (1)$$

Here $y_{t-i} (i = 1,2,..., p)$ are the p inputs and y_t is the output. The integers p, q are the number of input and hidden nodes respectively. $\alpha_j \left(j = 0,1,2,..., q \right)$ and $\beta_{ij} \left(i = 0,1,2,...., p \,;\, j = 0,1,2,..., q \right)$ are the connection weights and e_t is the random shock; α_0 and β_{0j} are the bias terms. Usually, the logistic sigmoid function $g(x) = 1 / (1 + e^{-x})$ is applied as the nonlinear activation function. Other activation functions, such as linear, hyperbolic tangent, Gaussian, etc. can also be used (Kihoro, et al. (2006)).

The feed forward ANN model (1) in fact executes a non-linear functional mapping from the previous values of the time series to the upcoming observation, i.e $y_t = f(y, y_{t-2},..., y_{t-p}, w) + e_t$ where **w** is a vector of all parameters and *f* is a function determined by the network construction and joining weights.

To estimate the joining weights, non-linear least square processes are used, which are based on the minimization of the error function (Gestel, et al. (2001)):

$$F(\psi) = \sum_t e_t^2 = \sum_t (y_t - \hat{y}_t)^2 \qquad (2)$$

Here Ψ is the space of all connection weights. The optimization methods used for minimizing the error function (2) are referred as *Learning Rules*.

2.3. Support Vector Machine (SVM)

Currently, a new statistical learning theory, such as the Support Vector Machine (SVM) has been unloading enhancing consideration for grouping and predicting (Cao, Francis and Tay (2003), Raicharoen, et al. (2003), Gestel, et al. (2001)). But soon we found widespread uses in other fileds, such as function approximation, regression estimation and time series prediction problems.

Consider a training data set of N points $\{Y_{t-i}, y_i\}_{i=1}^{N}$ with input data $Y_{t-i} \in \Re^n$ and the responses $y_i \in \Re$. Then we have the optimization problem:

$$\text{Minimize } \underset{w,b,e}{J(w,e)} = \tfrac{1}{2} w'w + \tfrac{1}{2}\gamma \sum_{i=1}^{N} e_i^2 \tag{3}$$

$$\text{Subject to } y_i = w'\phi(x_i) + b + e_i; \forall i = 1,2,......, N$$

Here ϕ is the non-linear mapping to a higher dimensional space and γ is the regularization parameter. The primal space model of the optimization problem (3) is given by:

$$y = w'\phi(x) + b$$

For computational simplicity and avoiding the case of infinite dimensionality of the weight vector **w** the optimization operations are performed in the dual space (Raicharoen, et al. (2003), Gestel, et al. (2001)).

The Lagrangian for the problem (3) is given by:

$$L(w,b,e;\alpha) = J(w,e) - \sum_{i=1}^{N} \alpha_i \{w'\phi(x_i) + b + e_i - y_i\}$$

$\alpha = [\alpha_1, \alpha_2, ..., \alpha_N]'$, where $\alpha_i \geq 0 \ \forall i = 1,2,..., N$ are the Lagrange multipliers.

Applying the conditions of optimality, one can calculate the partial derivatives of L with respect to w, b, e_k, α_k equate them to zero.

2.4. Forecast Performance Measures

Due to the vital significance of time series forecasting in numerous applied circumstances, suitable maintenance should be occupied while choosing a certain model. For this reason, numerous assessment procedures are projected in texts (Chatfield (1996), Enders (2008), Brockwell and Davis (2002), Hyndman and Athanasopoulos (2014), Wei (2006)) to estimate forecast precision and to evaluate different models. These are also recognized as assessment metrics. Each of these criterions is a function of the actual and forecasted values of the time series. In this section we shall mention few significant performance criterions which are commonly used by investigators. In each of the forthcoming definitions, y_t is the actual value, f_t is the forecasted value, $e_t = y_t - f_t$ is the forecast error and n is the size of the test set.

Table 2: Forecast Performance Measures

Method	Statistic	Criteria
Mean Absolute Deviation (MAD)	$MAD = \frac{1}{n}\sum_{t=1}^{n} \lvert e_t \rvert$	Small errors
The Mean Absolute Percentage Error (MAPE)	$MAPE = \frac{1}{n}\sum_{t=1}^{n} \lvert \frac{e_t}{y_t} \rvert \times 100$	To have a minimum bias
The Mean Squared Error (MSE)	$MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^2$	Close to zero
The Root Mean Squared Error (RMSE)	$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2}$	As small as possible
The Theil's U-statistics	$U = \dfrac{\frac{1}{n}\sum_{t=1}^{n} e_t^2}{\frac{1}{n}\sum_{t=1}^{n} f_t^2 \;\frac{1}{n}\sum_{t=1}^{n} y_t^2}$	0 ≤ U ≤ 1;U = 0 means a perfect fit

3. Experimental Results & Discussions

After gaining a reasonable knowledge about time series modeling and forecasting from the previous sections, we are now going to assess them on real life dataset. In this current section, till now we have considered land market dataset which contains six variables, taken from research work (Wang (2018)). All the associated programs are written in R version 3.2.4 (2016-03-10). To judge forecast performances of different methods, the measures MAD, MAPE, MSE, RMSE, and Theil's U-statistic are considered. We have presented our obtained results in tabular form. Also in this section we have used the term forecast plot to mean the graph showing the actual and forecasted data points. In each forecast plot, the solid and dotted line respectively represents the actual and forecasted observations.

Firstly we have checked the performance of forecast accuracy based on different types of VAR models. We also applied the selection criterion of VAR models to identify the actual order of VAR and again to check the validity of the order of VAR based on impulse response function and based on asymptotic chi- square portmanteau test. The outcomes of the VAR models in the land market values in USA are show in the following below:

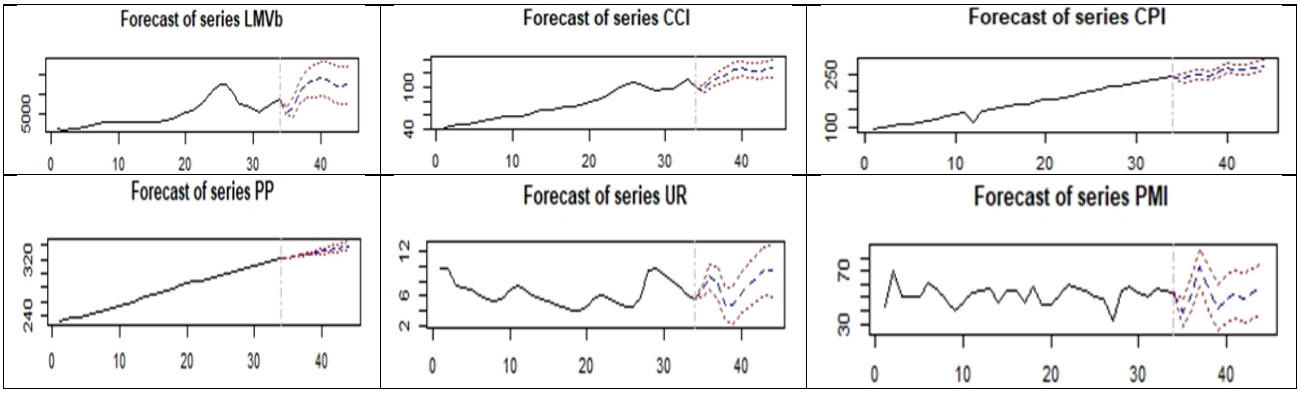


Figure: 2. Forecast Plot of VAR (2) model

From the depicted Figure: 2, we can see that VAR (2) generated the forecast but not quite good for this dataset. Therefore we can say that this is the only forecast.
We have also fitted the VAR (3) model to this dataset. Our obtained forecast performance calculated on the dataset by using the above mentioned model in presented is Figure: 3.

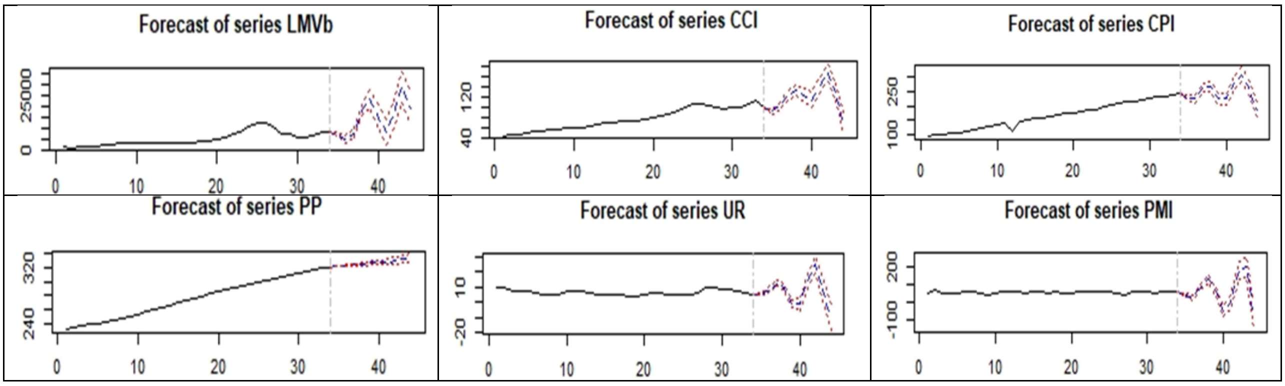


Figure: 3. Forecast Plot of VAR (3) model

The forecast plot in Figure: 3 show the success of our applied techniques to produce forecasts for the land market dataset. Finally the said mentioned figure depicted the excellent forecasting performance of VAR (3) for this dataset.

The forecast for the land market data set is shown in the Figure: 4 below.

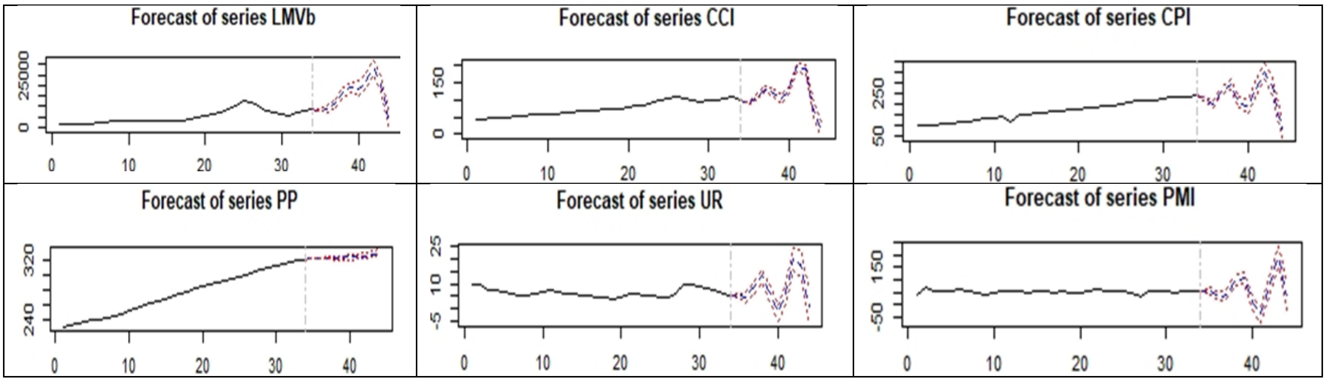


Figure: 4. Forecast Plot of VAR (4) model

Form Figure: 4 it can be seen that the performance of forecast plot of VAR (4) is same as VAR(3) as well as we could not visualize the accuracy of forecast performance based on impulse response function because the calculated data matrix is not positive definite.

The VAR order selection performance measures, we obtained for the land market dataset by using the above mentioned selection criteria are presented in table 3.

Table 3: VAR Model Selection Criteria

Selection Criteria	No of Lags									
	1	2	**3**	4	5	6	7	8	9	10
AIC(n)	1.7797e+01	1.5719e+01	**-2.7139e+01**	-Inf	-Inf	-Inf	-Inf	-Inf	-Inf	-Inf
HQ(n)	1.8344e+01	1.6734e+01	**-2.5655e+01**	-Inf	-Inf	-Inf	-Inf	-Inf	-Inf	-Inf
SC(n)	1.9859e+01	1.9547e+01	**-2.1543e+01**	-Inf	-Inf	-Inf	-Inf	-Inf	-Inf	-Inf
FPE(n)	5.9543e+07	1.4612e+07	**4.9498e-11**	0	0	0	0	0	0	0

From the above Table 3 we can see that the order selection of VAR (3) model is the best in our experiment for the land market values in USA dataset.

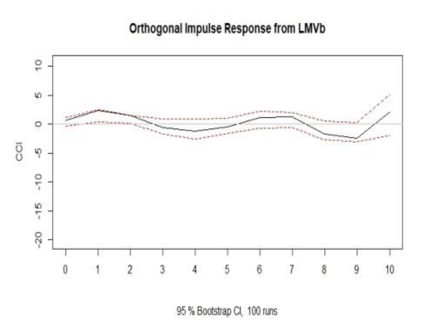
The results of asymptotic Portmanteau test of the selection of order VAR are shown in the following Table 4.

Table 4: Portmanteau Test of VAR Test

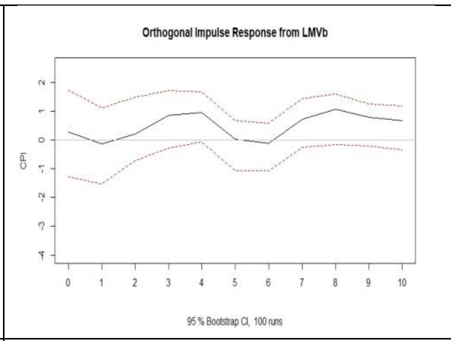
Portmanteau Test (Asymptotic)				
VAR (p)	Degrees of freedom	Chi-square Calculated Value	Tabulated Value	P-Value
VAR(2)	288	296.74	328.5804	0.349
VAR(3)	**252**	**329.23**	**290.0285**	**0.0007633**
VAR (4)	219	361.17	254.5232	0.000104

From Table 4 it can be seen that the best forecast performance for this dataset is obtained by fitting VAR (3) model. The forecast performance of VAR (4) model is over fitting, while VAR (2) model is not up to the expectation.

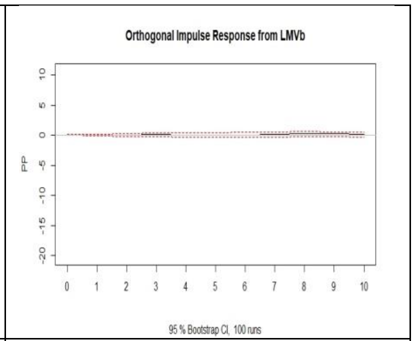
The impulse response function for VAR (3) model is shown in the following Figure: 5 below:



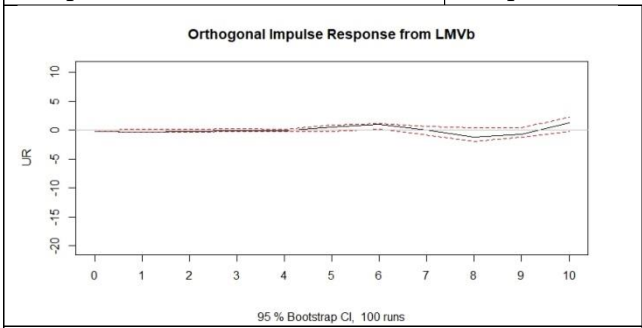
Response of LMVb to CCI



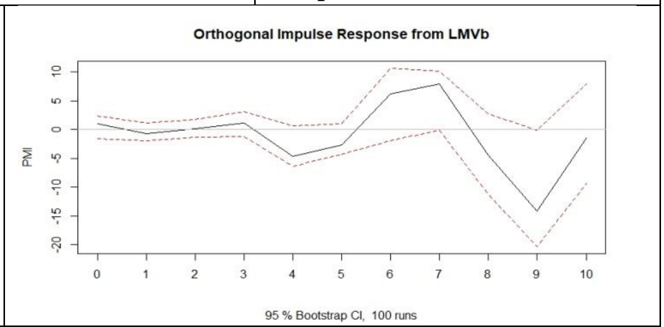
Response of LMVb to CPI



Response of LMVb to PP



Response of LMVb to UR



Response of LMVb to PMI

Figure: 5. Impulse Responses for the VAR (3) Model

From Figure: 5 show the impulse response function of the land market value in USA. For clarity, it is an interesting matter to note that the intervals contain the actual values.

The forecast values based on the selected VAR (3) model is presented in Table 5.

Table 5: Forecast Values based on selected VAR(3) model

Year	LMVb	CCI	CPI	PP	UR	PMI
1916	7437.972	90.92760	227.5619	322.3968	5.540241	36.586740
1917	4951.890	102.6244	224.7998	323.0180	7.952287	22.377582
1918	7923.805	122.8603	263.5032	323.9361	12.49748	78.832155
1919	18205.296	132.4255	267.0007	325.5470	9.501358	125.19297
2020	23114.658	122.2564	223.2512	327.0240	-1.00481	54.197345
2021	15338.115	114.0929	223.4565	327.8661	-1.46357	-53.89102
2022	8207.217	140.2537	275.6233	328.5739	14.56797	-6.17667
2023	16375.677	167.1195	306.2461	330.2816	24.58697	177.81063
2024	28929.927	129.8234	266.3373	332.7865	9.133926	198.53282
2025	19179.651	68.62179	185.1658	333.8277	-10.5453	-77.7058

We have also fitted the ANN model (varying the number of hidden nodes to 1,4,5,6 and 10) to this dataset. This can also be clarified from its forecast plots and hidden layer diagrams as given in the Figure: 6 below:

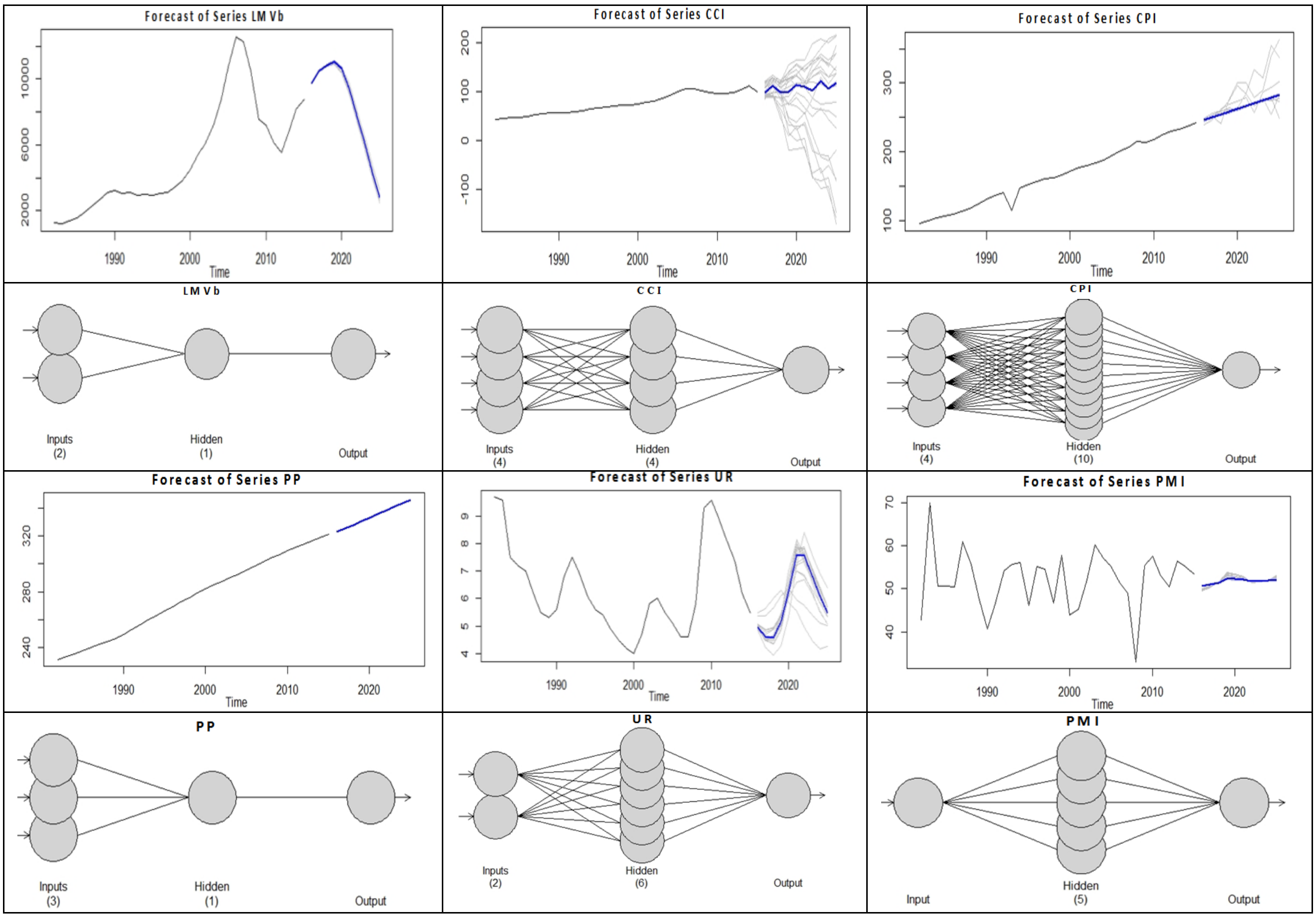


Figure: 6. Forecast Plot of ANN model

From the above Figure: 6, forecast plots and hidden layer diagrams, we can get an excellent visual idea about the forecast accuracy of the mentioned ANN model for this dataset. We think that the forecast performance of ANN model is excellent.

We have fitted to this time series are ANN model, the forecast values of this model is shown in the following table 6 below:

Table 6: Forecast Values based on ANN Model

Year	LMVb	CCI	CPI	PP	UR	PMI
1916	9738.1090	98.7689	246.0828	323.1920	4.9648	50.6210
1917	10489.252	111.4003	250.3459	325.4462	4.5980	51.0022
1918	10834.509	97.9887	254.2492	327.8728	4.5899	51.4289
1919	11037.579	101.0349	258.5430	330.3790	5.1681	52.3330
2020	10630.803	113.8616	262.8166	332.9828	6.2563	52.2306
2021	9409.2899	110.9941	267.2469	335.5947	7.5837	52.1209
2022	7774.0667	102.8703	271.4787	338.2178	7.5619	51.7668
2023	6078.4447	122.5727	275.5904	340.7841	6.7998	51.7908
2024	4405.2006	106.4942	279.3225	343.3394	6.0937	51.92323
2025	2823.3309	117.2057	283.4597	345.9065	5.5000	52.0439

Here we applied the most popular forecasting time series SVM model in this dataset. The outcomes of the SVM time series forecasting model is shown in the following Figure: 7:

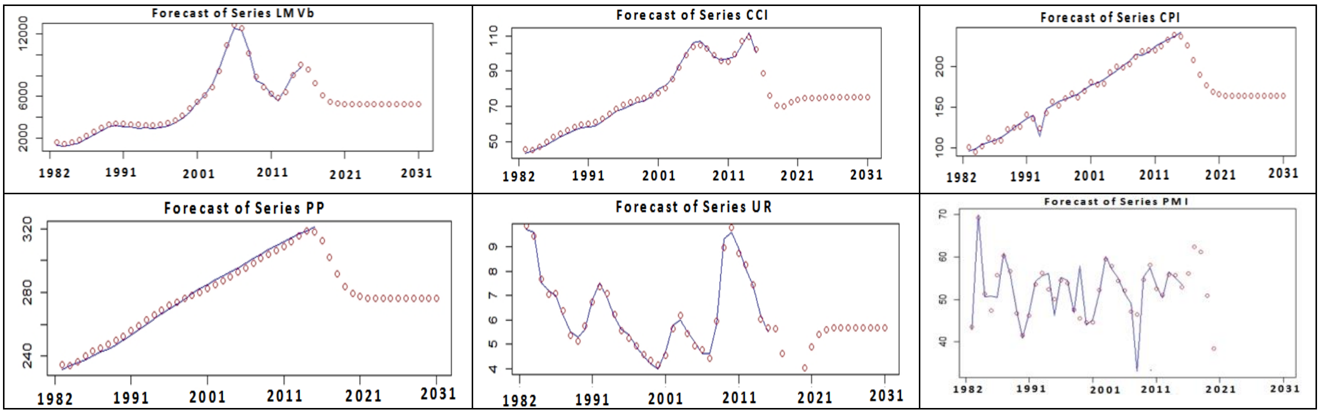


Figure: 7. Forecast Plot of SVM model

From the depicted figure: 7, we can visualize that the forecast performance of SVM model is moderate for this dataset.

The forecasted values of SVM model are shown in the following Table 7 below:

Table 7: Forecast Values based on SVM Model

Year	LMVb	CCI	CPI	PP	UR	PMI
1916	8623.070	88.66925	226.28332	312.2104	5.640260	56.15273
1917	7294.876	76.25164	208.46007	302.2066	4.597390	62.38355
1918	6112.583	70.42718	190.39681	291.7565	3.358578	61.18558
1919	5496.364	70.24544	176.95521	283.8565	3.223589	50.89713
2020	5297.872	72.15501	169.26160	279.2966	4.024170	38.41264
2021	5267.782	73.78282	165.79943	277.2352	4.893260	29.58361
2022	5273.669	74.59831	164.55945	276.4952	5.401749	25.25992
2023	5278.944	74.88867	164.20329	276.2823	5.603432	23.69580
2024	5280.768	74.96676	164.12085	276.2330	5.662027	23.26394
2025	5281.190	74.98307	164.10541	276.2238	5.674978	23.17116
2026	5281.262	74.98576	164.10307	276.2224	5.677204	23.15545
2027	5281.272	74.98612	164.10278	276.2222	5.677506	23.15334
2028	5281.273	74.98615	164.10275	276.2222	5.677539	23.15311
2029	5281.273	74.98616	164.10275	276.2222	5.677541	23.15309
2030	5281.273	74.98616	164.10275	276.2222	5.677542	23.15309
2031	5281.273	74.98616	164.10275	276.2222	5.677542	23.15309

Out of the total 34 observations in the series, we have used the observations of first twenty four years for training and the last ten years for testing. Thus the first 24 observations are considered for training and the remaining 10 for testing. We have applied the five different forecast performance measures that we mentioned in Table 2. These measures are calculated based on the three different models such as VAR (3), ANN and SVM. The outcomes of the performance measures are shown in the following table 8 below:

Table 8: Performance of Forecast Result for Land Market dataset

Method	MAD	MAPE	MSE	RMSE	Theil's U Statistic
			LMVb		
VAR(3)	156418.37100	2082.67876	51086007692.71650	226022.13983	0.00003
ANN	**3280.91560**	**38.47706**	**12090454.22499**	**3477.13305**	**0.00002**
SVM	15216.31200	210.51746	305145840.66241	17468.42410	0.00003
			CCI		
VAR(3)	381.95346	371.26891	305739.50766	552.93716	0.00272
ANN	**31.41539**	**30.69903**	**1046.92246**	**32.35618**	**0.00140**
SVM	63.57991	62.55502	5369.13389	73.27437	0.00133
			CPI		
VAR(3)	56.68590	24.72650	5746.81074	75.80772	0.00062
ANN	**7.91289**	**3.54830**	**67.96632**	**8.24417**	**0.00005**
SVM	72.15233	31.99040	5730.37367	75.69923	0.00071
			PP		
VAR(3)	15.12761	4.80577	326.31716	18.06425	0.00006
ANN	**1.88914**	**0.59715**	**6.41973**	**2.53372**	**0.00001**
SVM	43.54612	13.94573	2093.00238	45.74934	0.00017
			UR		
VAR(3)	130.30970	1981.15083	33193.28982	182.19026	0.04492
ANN	**1.52921**	**19.81740**	**3.69407**	**1.92200**	**0.01395**
SVM	1.96857	24.39393	6.19996	2.48997	0.02086
			PMI		
VAR(3)	661.60616	1242.00136	891119.85808	943.99145	0.00585
ANN	**4.08555**	**9.59990**	**39.71736**	**6.30217**	**0.00074**
SVM	16.75111	36.91315	498.81752	22.33422	0.00227

It can be seen form the above Table 8 that the best forecast performance is obtained by using ANN model. We also see that the minimum forecast performance measure is obtained by the fitted VAR (3) and SVM models.

4. Conclusion

Our considerable receiving almost the careful predicting models and their successful used can be identified from the five performance criterions and the forecast plots; we obtained for each of land market value in USA dataset. From the three forecast plots of VAR (3), ANN and SVM, depicted in Figure 3, 5, 6; we have a graphical comparison between the actual and forecast observations for the land market values in USA dataset. It can be seen that in Figure 5 of ANN time series forecasting model forecasted the series closely resembles with the original one. We also found that ANN is the best technique of modeling and forecasting multivariate time series dataset based on the five popular forecast performance measures.

References

[1] Box, G.E.P., and Jenkins, G., Time Series Analysis, Forecasting and Control, Holden-Day, San Francisco, CA, (1970).

[2] Brockwell, P.J. and Davis, R.A., Introduction to Time Series and Forecasting. 2nd Edn., Springe International Publishing, New York, (2002).

[3] Calomiris, C.W., Longhofer S.D., and Miles, W.R., The foreclosure-house price nexus: A panel VAR model for U.S. States, 1981-2009. Real Estate Econom., 41, (2013), 709-746.

[4] Crawford, G.W. and Fratantoni, M.C., Assessing the forecasting performance of regime-switching, arima and garch models of house prices. Real Estate Econom., 31 (2003), 223 - 243.

[5] Chatfield, C., Model uncertainty and forecast accuracy, J. Forecasting, 15, (1996), 495–508.

[6] Cao, L.J. Francis, and Tay, E.H., Support Vector Machine with Adaptive Parameters in Financial Time Series Forecasting, IEEE Transaction on Neural Networks, 14(6), (2003), 1506-1518.

[7] Cottrell, M., Girard, B., Girard, Y., Mangeas, M., Muller, C., Neural modeling for time series: a statistical stepwise method for weight elimination, IEEE Trans. Neural Networks 6, (1995), 1355–1364.

[8] Enders, W., Applied Econometric Time Series, 2nd Edn, John Wiley & Sons, (2008), 264-318.

[9] Faraway, J., Chatfield, C., Time series forecasting with neural networks: a comparative study using the airline data, Applied Statistics 47, (1998), 231–250.

[10] Gestel, T.V., Suykens, J.A.K. Baestaens, D. A. Lambrechts, G. Lanckriet, B. Vandaele, B. Moor, D., and Vandewalle, J. “Financial time series prediction using least squares support vector machines within the evidence framework”, IEEE Trans. Neural Networks, 12(4), (2001), 809 - 821.

[11] Hamzacebi, C., Improving artificial neural networks’ performance in seasonal time series forecasting, Information Sciences, 178, (2008), 4550-4559.

[12] Hyndman, R.J. and Athanasopoulos, G., Forecasting: Principles and Practice. 1st Edn., OTexts, (2014), 291.

[13] Kihoro, J.M., Otieno, R.O. Wafula, C., Seasonal Time Series Forecasting: A Comparative Study of ARIMA and ANN Models, African Journal of Science and Technology (AJST), 5(2), (2006), 41-49.

[14] Kuminoff, N.V. and Pope, J.C., x The value of residential land and structures during the great housing boom and bust. Land Econom., 89, (2013), 1-29.

[15] Miles, W., Boom-bust cycles and the forecasting performance of linear and non-linear models of house prices. J. Real Estate Finance Econom., 36, (2008), 249-264.

[16] Robertson, J.C. and Tallman, E.W., Vector autoregressions: Forecasting and reality. Federal Resene Bank Atlanta Econom. Rev., 84, (1999), 4-18.

[17] Raicharoen, T., Lursinsap, C., Sanguanbhoki, P., Application of critical support vector machine to time series prediction, Circuits and Systems, Proceedings of the 2003 International Symposium, 5, (2003), 741-744.

[18] Sims, C.A., Macroeconomics and reality. Econometrica, 48, (1980), 1-48.

[19] Wei, W., Time Series Analysis: Univariate and Multivariate Methods. 2nd Edn. Pearson Addison Wesley, (2006), 614.

[20] Wang, L., A VAR Model for Forecasting Land Market Value in USA, Journal of Mathematics and Statistics, 14(1), (2018), 1-6.

[21] Zhang, G., Patuwo, B.E., Hu, M.Y., Forecasting with artificial neural networks: The state of the art, International Journal of Forecasting, 14, (1998), 35-62.

[22] Zhang, G.P., A neural network ensemble method with jittered training data for time series forecasting, Information Sciences, 177, (2007), 5329–5346.

[23] Zhang, G.P., Time series forecasting using a hybrid ARIMA and neural network model, Neurocomputing, 50, (2003), 159–175.