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Shock-Propagation Down a Non-uniform Tube in Non-ideal Gas

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Abstract

In present paper an attempt is made to study the one dimensional formulation of flow in a tube of varying cross-sectional area for non-ideal gas. Using Whitham rule of characteristic a relation between cross sectional area and Mach number is obtained and result is discussed for different values of internal volume of the gas molecules.

Mathematics Subject Classification: 37E35
Keywords: Shock-Non-ideal Gas

1 Introduction

The assumption that the medium is an ideal gas is no more valid when the flow takes place in the extreme condition. Anisinov and Spiner [1] have studied a problem of point explosion in low density non ideal gas by taking the equation of

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state in a simplified from which describes the behavior of medium satisfactorily. Robert and Wu [4] have studied the gas that obeys a simplified Vander Wall's equation of state. Vishwakarma et-al [1] have investigated the one dimensional unsteady self similar flow behind a strong shock, driven out by a cylindrical or spherical piston in a medium which is assumed to be non ideal and which obey the simplified Vander Wall's equation of state as consider by Robert and Wu[5]. Pandey and Pathak [3] have considered the growth and decay behavior of sonic waves in non ideal gases.

2 Basic-equation and formulation of problem

We consider one dimensional formulation for flow in a tube of a given cross-sectional area, where $A(x) = A_0 =$ constant, in x < 0, and the shock is initially moving in this section with constant Mach number M_0 .We consider that the shock is to be produced by a piston moving with appropriate constant speed far back in the uniform section .The piston is still providing the thrust to keep the shock moving ,but there are no changes due to this and the changes are entirely due to the cross-sectional area. Though the flow is not strictly one dimensional but if the cross-section $A(x)$ does not vary too rapidly, the equations obtained by averaging across the tube will provide a good approximation of one dimensional flow. Thus equations governing the motion are given by

$$\rho_{,t} + u\rho_{,x} + \rho u_{,x} + \rho u \frac{A_{,x}(x)}{A(x)} = 0, \tag{1}$$

$$u_{,t} + uu_{,x} + \frac{1}{\rho} p_{,x} = 0, \tag{2}$$

$$AE_{,t} + \{uA(E + p)\}_{,x} = 0, \tag{3}$$

where ρ, u and p are density, velocity and pressure of gas respectively.

$E = \rho e + \frac{1}{2}\rho u^2$ is the total energy density,

$e = c_v T = \frac{(1-b\rho)}{(\gamma-1)} p$ is the internal energy ,

$b\rho$ being internal volume of the gas molecules and a comma followed by an index implies partial derivative with respect to that index .Substituting value of E in equation (3) it reduces in to following equation

$$p_{,t} + up_{,x} - a^2(\rho_{,t} + u\rho_{,x}) = 0, \tag{4}$$

where $a^2 = \frac{\gamma p}{\rho(1-b\rho)}$.

Thus we see that area change appears only in the continuity equation (1), which may be written as

$$(\rho A)_{,t} + (\rho u A)_{,x} = 0. \tag{5}$$

If we consider the case for propagation into a wedge with apex at x_0, $\frac{A_{,x}(x)}{A(x)} = -\frac{1}{x-x_0}$ and for a cone $\frac{A_{,x}(x)}{A(x)} = -\frac{2}{x-x_0}$, thus we see that if we put $x_0 - x = r$, equation (1) is identical with those for cylindrical and spherical wave .For t<0 ,the flow consists of uniform regions separated by the moving shock. By taking $u = U, p = p_0, \rho = \rho_0$ in the undisturbed state ahead of the shock and $u = u_1, p = p_1, \rho = \rho_1$ in the initial uniform state behind it, we can determine u_1, p_1, ρ_1 in terms of p_0, ρ_0, M_0 by shock conditions, given as

$$u = \frac{2a_0}{(\gamma+1)}\left[M(1-\alpha) - \frac{1}{M}\right], \tag{6}$$

$$p = \rho_0 a_0^{\ 2}\left[\frac{2M^2(1-\alpha)}{(\gamma+1)} - \frac{(\gamma-1)}{\gamma(\gamma+1)}\right], \tag{7}$$

$$\rho = \frac{\rho_0 M^2(\gamma+1)}{M^2(\gamma-1) + 2M^2\alpha + 2}, \tag{8}$$

γ being ratio of specific heat and M is Mach number defined as $M^2 = \frac{u^2}{a^2}$.

Linearizing equations (1), (2), (4) and shock conditions (5 to7) about state u_1, p_1, ρ_1 , we have

$$\rho_{,t} + u_1\rho_{,x} + \rho_1 u_{,x} + \rho_1 u_1 \frac{A_{,x}}{A_0} = 0, \tag{9}$$

$$u_{,t} + u_1 u_{,x} + \frac{1}{\rho_1} p_{,x} = 0, \tag{10}$$

$$p_{,t} + u_1 p_{,x} - a^2(\rho_{,t} + u_1\rho_{,x}) = 0. \tag{11}$$

Combining above three equations we have following differential equations along characteristic $\frac{dx}{dt} = u \pm a$.

The general solution is readily obtained since the equations are linear with constant coefficients; the most significant derivation is via characteristics form of the equations. The characteristic equation for equation (9), (10) and (11) are

$$C_+ : \left\{\frac{\partial}{\partial t} + (u_1 + a_1)\frac{\partial}{\partial x}\right\}(p + \rho_1 a_1 u) + \rho_1 a_1{}^2 u_1 \frac{A^{'}(x)}{A_0} = 0, \tag{12}$$

$$C_+ : \left\{\frac{\partial}{\partial t} + (u_1 - a_1)\frac{\partial}{\partial x}\right\}(p - \rho_1 a_1 u) + \rho_1 a_1{}^2 u_1 \frac{A^{'}(x)}{A_0} = 0, \tag{13}$$

$$p : \left\{\frac{\partial}{\partial t} + u_1 \frac{\partial}{\partial x}\right\}(p - a_1{}^2 \rho) = 0. \tag{14}$$

The general solution taking each in turn is

$$(p - p_1) + \rho_1 a_1 (u - u_1) = -\frac{\rho_1 a^2 u_1}{u_1 + a_1}\frac{A(x) - A_0}{A_0} + F\{x - (u_1 + a_1)t\}, \tag{15}$$

$$(p - p_1) - \rho_1 a_1 (u - u_1) = -\frac{\rho_1 a^2 u_1}{u_1 + a_1}\frac{A(x) - A_0}{A_0} + G\{x - (u_1 - a_1)t\}, \tag{16}$$

$$(p - p_1) - a^2(\rho - \rho_1) = H(x - u_1 t) \ . \tag{17}$$

The three arbitrary functions F, G, H are to be determined from the initial conditions of the problem and boundary conditions at the shock.

The shock conditions give the perturbations $(p-p_1),(u-u_1)$ at the shock in terms of the change in Mach number $(M-M_0)$, given below such as

$$\begin{aligned} u-u_1 &= \frac{2a_0}{(\gamma+1)}\left[1+\frac{1}{M^2(1-\alpha)}\right](M-M_0)(1-\alpha), \\ p-p_1 &= \frac{4\rho_0 a^2(1-\alpha)}{(\gamma+1)}\left[M_0(M-M_0)\right], \end{aligned} \tag{18}$$

where $b\rho_0=\alpha$. Substituting $(p-p_1),(u-u_1)$ from equation (18) into equation (15) with F=0, we have

$$\begin{aligned} &\left[\frac{4}{(\gamma+1)}M_0+\frac{2}{(\gamma+1)}\{1+\frac{1}{M_0^{\,2}(1-\alpha)}\}\frac{\rho_1 a_1}{\rho_0 a_0}\right](M-M_0)(1-\alpha) \\ &\qquad\qquad = -\frac{\rho_1 a_1^{\,2}}{\rho_0 a_0^{\,2}}\frac{u_1}{u_1+a_1}\frac{A-A_0}{A_0}. \end{aligned} \tag{19}$$

Substituting u_1,ρ_1,a_1 in terms of M_0 given by equation (6) to (8) in (19), we have

$$\frac{A-A_0}{A}=-g(M_0)(M-M_0), \tag{20}$$

where

$$g(M)=\frac{M}{\left\{M^2(1-\alpha)-1\right\}}\left[2\mu+(1-\alpha)\left\{1-\frac{1}{M^2(1-\alpha)}\right\}\right]\left[1+\frac{2}{(\gamma+1)}\left\{\frac{1-\mu^2}{\mu}\right\}\right], \tag{21}$$

$$\mu^2=\frac{M^2(\gamma-1)+2M^2\alpha+2}{2\gamma M^2(1-\alpha)-(\gamma-1)}. \tag{22}$$

Applying Characteristic rule for finite area change as suggested by Whitham [8], we have

$$p_{,x}+\rho a u_{,x}+\frac{\rho a^2 u}{u+a}\frac{1}{A}A_{,x}=0, \tag{23}$$

which may be written as ,

$$\left\{\frac{M}{M^2(1-\alpha)-1}\right\}\lambda(M)M_{,x}+\frac{1}{A}A_{,x}=0, \tag{24}$$

where

$$\lambda(M)=\left[2\mu+(1-\alpha)\left\{1-\frac{1}{M^2(1-\alpha)}\right\}\right]\left[1+\frac{2}{(\gamma+1)}\left\{\frac{1-\mu^2}{\mu}\right\}\right]. \tag{25}$$

Solution of equation (24) may be written as

$$Ac=\left\{\frac{M^2(1-\alpha)-1}{\lambda(M)}\right\}^{1/2}. \tag{26}$$

c, being constant of integration. For different cases such as when $M\to 1$,

$$\lambda(M)=\left[(2\mu-\alpha)\right]\left[1+\frac{2}{(\gamma+1)}\left\{\frac{1-\mu^2}{\mu}\right\}\right],$$

where $\mu^2=\dfrac{\gamma+1+2\alpha}{\gamma+1-2\alpha\gamma}$.

Similarly for $M\to\infty$,

$$\lambda(M)=\left[2\mu+(1-\alpha)\right]\left[1+\frac{2}{(\gamma+1)}\left\{\frac{1-\mu^2}{\mu}\right\}\right],$$

where $\mu^2=\dfrac{\gamma-1+2\alpha}{2\gamma(1-\alpha)}$, which shows that in particular cases also internal volume of the gas molecules affects the change in cross-sectional area which is shown through Figures 5a and 5b.

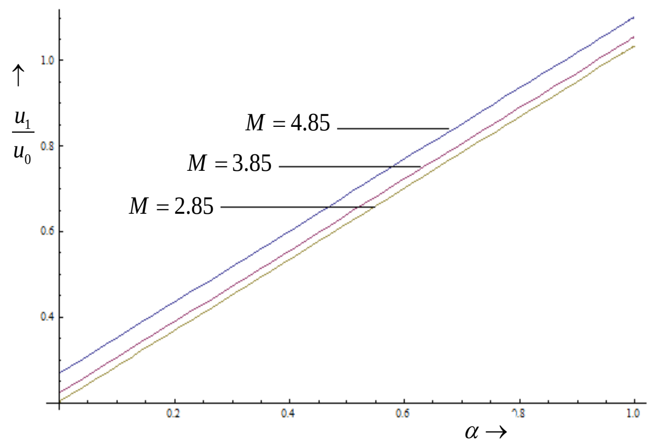

Figure 1: Variation of velocity' (u_1 / u_0) ' for different values of internal volume of the gas molecules α

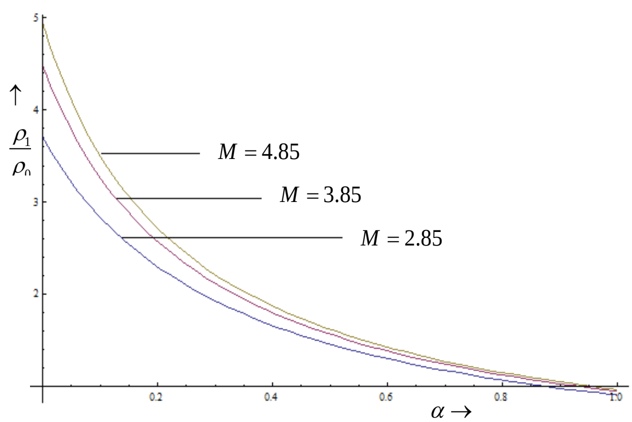

Figure 2: Variation of density' (ρ_1 / ρ_0) ' for different values of internal volume of the gas molecules α

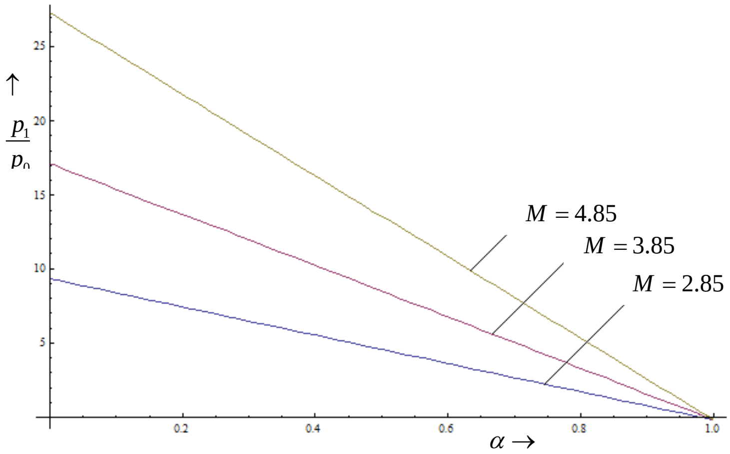

Figure 3: Variation of density' (p_1 / p_0) ' for different values of internal volume of the gas molecules α

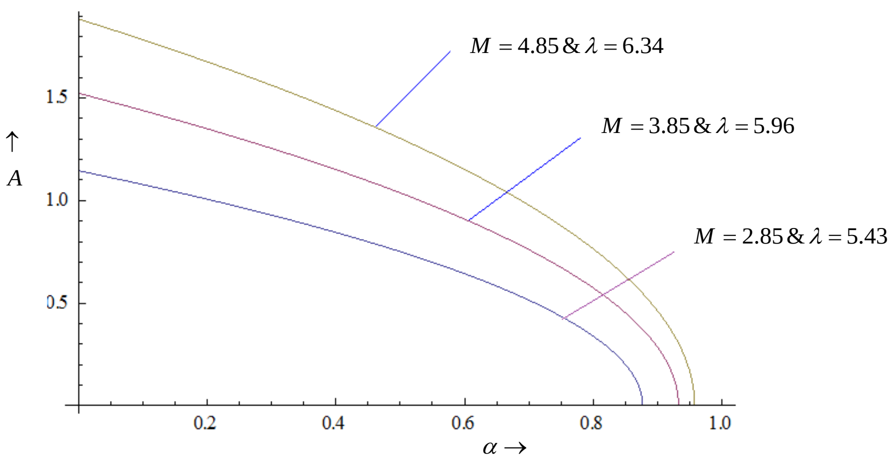

Figure 4: Variation of area of duct' (A) ' for different values of internal volume of the gas molecules α

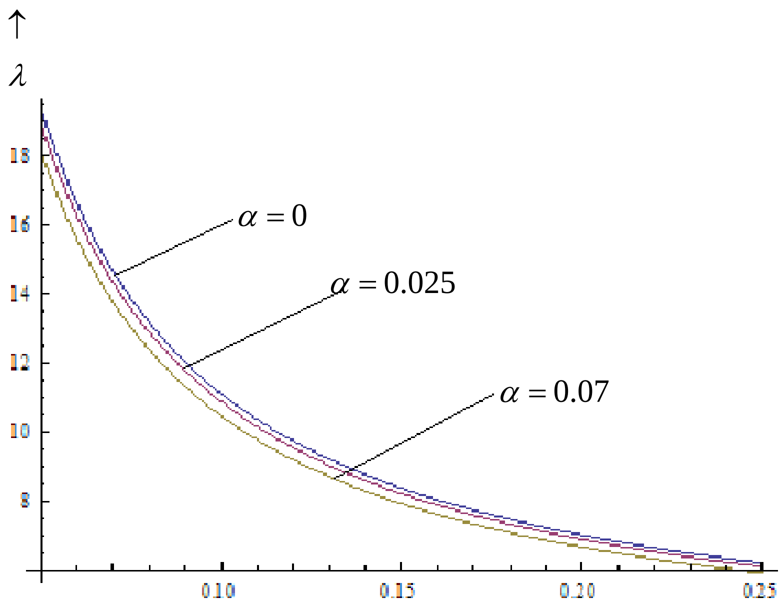

Figure 5a: A graph between the 'λ' and Mach number of the shock relative to the flow 'μ' for different value of 'α'.

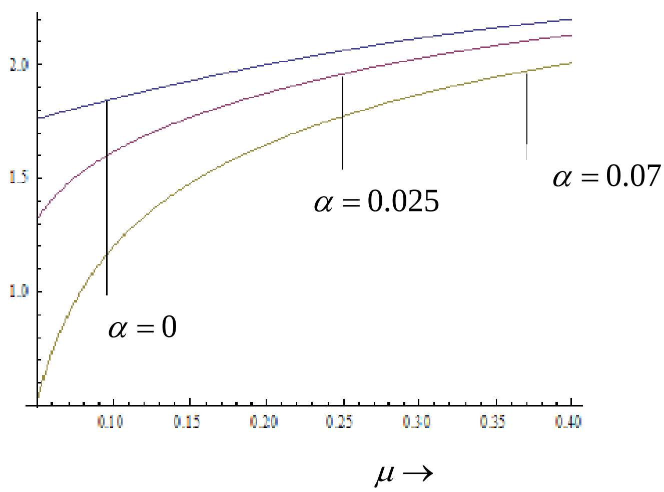

Figure 5b: A graph between the 'λ' and Mach number of the shock relative to the flow 'μ' for different values of 'α'.

3 Conclusion

For various value of α (the internal volume of gas molecules) and three fixed values of Mach Number M =2.85,3.85 and 4.85. Figure 1-4 shows variation of velocity, density, pressure and area of duct. Figure 1 shows that as α is increasing velocity is increasing but for small Mach – Number it is small in comparison to greater values of Mach – Number. Figures 2-4 show that for increasing values of α density, pressure and area of duct have decreasing tendency but remain small for small Mach – Number it is small in comparison to greater values of Mach – Number. Figure 5a and 5b shows that for particular values of $M \to 1 \& \infty$ area change in affected by internal volume of the gas molecules.

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