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A Monte Carlo Simulation Study Assessing the Performance of a Bayesian Approach for Identifying Differences in Change Point Location for Two Time Series

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Abstract

Researchers in a variety of fields work with sequential data, such as measurements made over time. In some instances, one or more of the moments (e.g., mean, variance) of the series may change abruptly at some point in the sequence, yielding what is known as a change point. There is a broad literature describing methods for change point detection. Researchers working with multiple sequential series containing change points may be interested in comparing the locations of these changes. Recently, a method for comparing the locations of 2 or more change points in 2 or more series using a Bayesian estimator has been described in the literature. The purpose of the current Monte Carlo simulation study was to extend this earlier work by assessing the performance of this approach with time series of between 20 and 200 measurements in length, for a normally distributed measurement process. Results of the simulation revealed that the method always controlled the Type I error rate, and had power of 0.75 or higher for series of 50 measurements or longer, when the variance in measurements was relatively low.

Keywords: Change point, Bayesian analysis, Time series
Mathematics Subject Classification: C11, C14, C22

1 Introduction

Across a variety of disciplines, researchers working with sequential data (referred to here as time series, although it is recognized that such data can come from other processes as well) are faced with the need to detect points within their data where one or more of the moments in the measured process changes. For example, climatologists monitoring ocean water temperature may be interested in ascertaining the point(s) at which the mean temperature undergoes a relatively dramatic change. Similarly, an economist may need to detect change points in a nation's mean gross domestic product measured quarterly over a 20 year period. Researchers in

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other fields, including genomics [8], finance [13], computer science [15], and imaging [10] may also encounter situations in which they need to identify change points with respect to the mean, variance, or both aspects of a time series process.

In addition to detecting the location of change points, there may also be situations in which researchers need to compare change point locations for two or more independent series containing measures of the same entity. For example, behavior analysts working with individual children frequently employ an approach involving the administration of a specific intervention, after a period of baseline data collection in which behavior prevalence is measured on a daily basis. This intervention might be designed to reduce the prevalence of some problem behavior, and the baseline data collection period is designed to establish the level of the behavior prior to the introduction of the intervention. The behavior analyst would continue to monitor the behavior of the child daily over an extended period of time in order to determine whether behavior prevalence declined, presumably in response to the intervention. With only one child, standard change point detection methods (to be discussed below) might be employed in order to determine where in the time series the prevalence of the behavior changed, if indeed it did. If this intervention strategy were applied to a second child, we would have access to a second time series dataset to which the change point analysis could also be applied. The behavior analysis may then be interested in determining whether the change points for the two children were at the same locations in the time series. Standard approaches to change point detection do not lend themselves to such comparisons, as they do not provide the mechanism for formally testing the null hypothesis that two series have the same change point. The focus of the current study is on a method described by [3] for making such comparisons using a Bayesian estimator. The manuscript is organized as follows: First, a brief discussion of change point detection methods is provided, followed by a description of the model for comparing change point location that is the focus of the current study. Next, the details of a simulation study that assessed the performance of this model in detecting differences in change point locations for independent time series are outlined. The goals and hypotheses of this study are then described, followed by a discussion of the Monte Carlo simulation methodology used to address these goals. Finally, the results of the Monte Carlo study are presented, and discussed in the context of the prior work in this area.

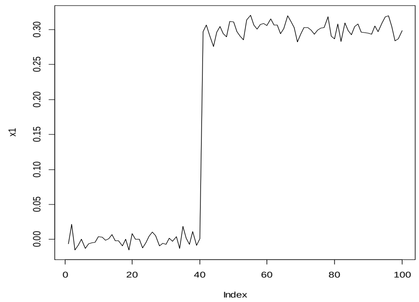
2 Literature Review

2.1 Change point detection

We can formally define a change point as follows. The time series data is defined as

$$y_{1:n} = (y_1, y_2, \dots, y_n) \tag{1}$$

A change point is defined as occurring at point y_T if one or more of the population moments (e.g., mean, variance) in the measurements for y_1 through y_T are different than those for y_{T+1} through y_n. This definition applies to the case in which only a single change point is present. However, a series may have multiple change points, with the definition being expanded to incorporate the presence of multiple segments within the series, each having different population moments from those of the previous segment. Change points are integers between 1 and n-1 thereby creating multiple segments within the time series, and are differentiated by the different population parameter values. Figure 1 displays examples of time series with change points for the mean and variance, respectively.



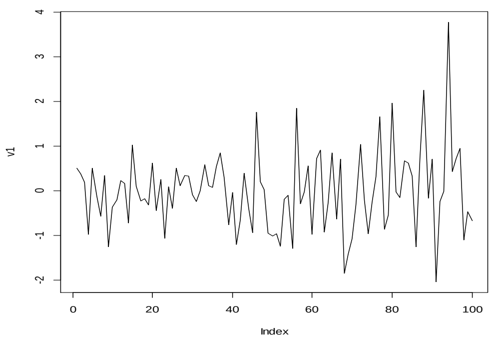


Figure 1: Examples of time series with change points for the mean and variance
Change point

There exist several methods for detecting the number and location of change points within a given time series. Given that the focus of this study is not on these various methods, they will only be reviewed briefly here. For a more detailed discussion of these approaches, a number of useful sources are available [1; 4; 6; 14]. A set of approaches exist for change point detection based on maximum likelihood estimation. Under the null hypothesis of no change point being present in the series, the likelihood function takes the form:

$$ML_0 = ln\left(p\left(y_{1:n}|\hat{\theta}\right)\right) \quad (2)$$

Where
$y_{1:n}$ =Full series from time 1 to n
$\hat{\theta}$=Parameter estimates based on full series
For the alternative hypothesis that there is a change point at T_1, the likelihood function is

$$ML_{T_1} = ln\left(p\left(y_{1:T_1}|\widehat{\theta_1}\right)\right) + ln\left(p\left(y_{T_1+1:n}|\widehat{\theta_2}\right)\right) \quad (3)$$

Where
$y_{1:T_1}$ =Series from time 1 to T_1
$y_{T_1+1:n}$ =Series from time $y_{T_1+1:n}$ to n
$\hat{\theta}_1$ =Parameter estimates based on segment of series from time 1 to T_1
$\hat{\theta}_2$=Parameter estimates based on segment of series from time T_{1+1} to n
Estimates of the parameters are then obtained so as to minimize the two likelihood functions, which are used in turn to construct a test of the null hypothesis of no change points being present in the time series based on the test statistic

$$\lambda = 2\left(ML_{T_1} - ML_0\right) \quad (4)$$

In order to identify the optimal set of change points for a given series, a search algorithm must be employed in conjunction with the likelihood functions described above. This algorithm has an associated cost function that it seeks to minimize, and which is defined as

$$\sum\left[C\left(y_{(T_{i-1}+1):T_i}\right)\right] + \beta f(m) \quad (5)$$

Where
$\left[C\left(y_{(T_{i-1}+1):T_i}\right)\right]$ =Cost to the likelihood function for segmenting series at T_i
$\beta f(m)$ =Penalty to guard against over fitting the data
There exist several algorithms that utilize this cost function in searching for the optimal set of change points for a given dataset, including binary segmentation [5], segment neighborhood search [2], and pruned exact linear time [7]. Given that these are not the focus of the current paper, we refer the interested reader to the references listed above for further discussions regarding their use.

An alternative to the maximum likelihood approach to change point detection is based on the use of a Bayesian estimator. Authors in [12] described a Bayesian approach for detecting multiple change points in a time series. This Bayesian model involves the specification of the following components:

1. Prior distribution for the number of segments (K) in the series, $P(K)$
2. The conditional distribution of partition m, given K, $P(m|K)$
3. Distribution of parameters (θ_k) unique to each segment, $P(\theta_k)$
4. Measured data with distribution $(y_t|m, k \in m, \theta_k, t \in k) \sim \mathcal{g}(\theta_k)$

Given these defining distributions, the probability distribution for the observed data within segment k can then be expressed as

$$P(y_k|\theta_k) = \prod_{t \in k} g(y_t; \theta_k) \quad (6)$$

Where
$g(y_t; \theta_k)$ =Probability distribution function for $\mathcal{g}$.
Given conjugate priors for θ_k, constraints on the distribution of the segmentation process, and the factorability of the function, the Bayesian estimator will provide a posterior distribution for the location of the change points in the series. An example of a constraint on the segmentation process is that its prior be the uniform distribution (Cleynen & Robin, 2016).

2.2 A model for comparing change point locations

Within the Bayesian modeling framework, an approach to comparing the change point locations for two or more independent time series has been proposed [3]. This model builds on the work by [12] with the Bayesian approach to determining the number of change points present in a time series. In this context, it is assumed that there are two or more independent time series measured at the same number of points, with the same length, and the same number of segments. For a problem involving two time series, this approach models the posterior distribution for the difference between the locations of two change points, which [3] refer to as the shift. As an example consider the case where it is of interest to compare the location of change point k for 2 time series. The shift value (difference between the change point locations) is expressed as

$$\Delta_k = \tau_{k1} - \tau_{k2} \quad (7)$$

Where

τ_{k1} =Change point k for time series 1

τ_{k2} =Change point k for time series 2

The posterior distribution for Δ_k is

$$P(\Delta_k = \mathrm{d}_k | y_1, y_2, k_1, k_2) = \sum_t p_{K_1}(t; y_1; k_1) p_{K_2}(t - \mathrm{d}_k; y_2; k_2) \quad (8)$$

where terms are as defined above.

The credibility interval (e.g., 95%) for Δ_k can be used in order to determine whether the change points for 2 series are likely to differ in the population. If 0 falls within the credibility interval, then we would conclude that it is unlikely for the change points to differ from one another in the population, whereas if 0 lies outside of the credibility interval, we would conclude that the change point locations do likely differ from one another. The point estimate of the change point location can be estimated using the mean or median of the posterior distribution in equation (8). Authors in [3] describe an extension to this Bayesian approach that can be used to ascertain whether there exists a set of common change points for more than 2 time series. The interested reader is encouraged to examine their description of this extension.

2.3 Prior research on the Bayesian change point comparison model

The Bayesian estimator for comparing the locations of change points in time series was tested using a Monte Carlo simulation study by Cleynen and Robin (2016). Specifically, their study examined a case with 3 independent time series containing 7 segments each. Two of the series shared a common set of change points, whereas the third series had change points at different locations. Each series was simulated to be 800 units long, and the data came from the negative binomial distribution, which was selected so as to mirror what is seen in gene transcription experiments. A variety of differences in change point locations were simulated, including at points 0 (no change point), 2, 4, 8, 16, and 22. The simulated odds ratios for the responses at the change points were 4, 8, and 16, and were simulated to be the same for each time series. The dispersion parameters for the negative binomial distribution were also manipulated in the study, with a total of 8 different values used.

The authors reported that, as expected, power for detecting differences in change point locations was higher when the difference in change point location was greater. In addition, they found that for higher levels of dispersion, the method had lower power values for detecting differences in change point locations. In other words, the greater the variance in the measured variable, the lower the statistical power. Cleynen and Robin (2016) also found that power for detecting a difference in the change point location was higher when the magnitude of the change within each series, as expressed in the odds ratio, was also higher. Finally, results of this study revealed that the Type I error rate (probability of finding a difference in the change point location when the location was the same) was well below 0.05. Taken together, these results suggest that for time series involving measurements conforming to the negative binomial distribution, power was largest when the difference in change point location was 16 or greater, when the level of dispersion was relatively low, and when the change itself was large.

2.4 Study goals and hypotheses

The primary goal of this study was to investigate the performance of the Bayesian model for estimating the difference in change point location between two time series. This study extends earlier work (Cleynen & Robin, 2016) by examining the accuracy of this approach with normally distributed time series of varying lengths, variances, change magnitudes, and change point location differences. As noted above, prior research with this model focused on count data from the negative binomial distribution, with a length of 800 measurements. Though this earlier simulation study was in keeping with conditions frequently encountered in gene transcription research, from which that work originated, it is not necessarily representative of what researchers in the social sciences and econometrics might expect to encounter. For example, many economic indicators of interest are collected on a quarterly basis, meaning that time series with 40 years' worth of data might be shorter than 200 units long. In addition, quite frequently such measurements follow a continuous distribution, as the entities of interest are sales figures, gross domestic product, corporate profit, and so on. Thus, the earlier work in this area may not be directly relevant to applications in fields outside of genomics. The current study was designed to address this issue, by demonstrating the utility of the Bayesian model for comparing change point location with normally distributed measures in time series consisting of 200 or fewer measurements.

Based upon prior research (Cleynen & Robin, 2016), it was hypothesized that the method under examination here would yield more accurate estimates of change point location difference for longer time series, and when the magnitude of difference in the location was larger. In addition, it was hypothesized here that larger variance in the time series will lead to less accurate detection of differences in change points, due to the increased model error associated with the greater variation in measurements. Finally, it was hypothesized that a greater magnitude of change within each series would be associated with a higher degree of accuracy for detecting differences in change point location, because the change in each series would be more detectable by the Bayesian model.

3 Simulation Methodology

In order to address the research goals of this study, a Monte Carlo simulation design was used. For each combination of simulation conditions, which are described below, 1000 replications were generated and analyzed. Data were generated using the R software system, version 3.3.1 (R Development Core Team, 2017). The EBS library was used to fit the Bayesian change point model, and to test for differences in change point location. Across conditions, 2 time series were generated from the normal distribution, with one having a mean of 0, and the other having one of the values as described in section 3.4, below. In addition, each series had variances as described in section 3.3. The manipulated study factors are described below.

3.1 Length of time series

The time series were generated to be of length 20, 50, 100, and 200. In addition, simulations were attempted for series of length 10, but the MCMC algorithm did not converge, and thus these results are not included in this manuscript. The simulated length conditions were selected to represent relatively short series (20), to long series for the social sciences (200), with intermediate values in between.

3.2 Difference in time series shift location

For each replication, the first time series was treated as the reference such that the location of the shift in the mean of the series was held constant as occurring at the 40% point. Thus, for the 20 unit time series the mean of the first time series shifted after the 8th point (0.4X20), whereas for the 50 unit series the shift occurred after the 20th point, after the 40th point for the 100 unit series, and after the 80th point for the 200 unit series. The difference in shift location between the two time series were then simulated to be 0 times the change point for the first segment (i.e., no difference in change location), 1.5 times the first segment length, and 2 times the first segment length. As an example, if the series was 20 units long, then the change for the first series occurred after point 8. For the 1.5 times condition, the change for series 2 occurred after time 12 (1.5X8), and for the 2 times condition the change for series 2 occurred after time 16.

3.3 Variance of time series

The time series were generated with variances of 0.01, 0.1, 0.5, and 1. These values were selected to represent a range from very small (0.01) to large (1). Both time series were generated to have the same variance.

3.4 Change magnitude

Both time series were simulated to experience the same degree of change in the mean: 0, 0.2, 0.5, and 0.8 standardized units. These values were selected because they correspond to no shift in the mean over time, and shifts that would be categorized as small (0.2), medium (0.5), and large (0.8), based on Cohen's guidelines for interpreting effect sizes for the mean when the data are standard normal. It should be noted that when the variances of the time series were not simulated to be 1, these magnitudes of change could no longer be interpreted by Cohen's guidelines. These conditions were kept at 0, 0.2, 0.5, and 0.8 regardless of the variance magnitude, in order that we could ascertain the impact of different variance conditions on the ability of the Bayesian approach to detect differences in change location for the same set of shift magnitudes.

3.5 Simulation outcomes

The outcomes of interest in this study were the Type I error (incorrectly identifying a difference in change point location when none was present in the population) and power (correctly identifying a difference in change point location when one was present in the population) rates for detecting a statistically significant difference in the location of the change points for the time series. In addition, the accuracy of the difference between the locations of the change points in the two series was also a study outcome of interest.

In order to identify the main effects and interactions of the manipulated study factors that were related to each outcome, analysis of variance (ANOVA) was used. In addition, the partial η^2 effect size was also employed in order to characterize the magnitude of the impact on the outcome variables of the statistically significant terms identified by the ANOVA. For a main effect or interaction to be considered meaningful in the context of this study, it needed to be both statistically significant (□=0.05), and to have $\eta^2_{Partial}$ value of 0.1 or greater, indicating that it accounted for at least 10% of the variation in the study outcome.

4 Main results

4.1 No change point present

When no change in either time series was simulated to be present, results from the ANOVA identified none of the main effects or interactions of them to be statistically significantly related to the Type I error rate. Indeed, in none of the simulated conditions was the test statistic comparing location of the time series' change points statistically significant when neither was simulated to have a change point. Thus, it is possible to conclude that the methodology always correctly indicated that the change points were in the same location, across study conditions.

In terms of the difference in estimated location of the change point locations when no change point was simulated to be present for either series, the ANOVA found none of the manipulated terms to be statistically significant. Table 1 includes the mean distance between the locations of the change points of the two series by series length and variance. In no case was the mean larger than 0.5, or half of a time point, indicating that the Bayesian approach accurately located the two change points as being the same.

Table 1: Mean difference in change point location by length and variance of time series when no change was present

Length	Mean Difference in Change Point
20	-0.02
50	0.09
100	0.01
200	0.13
Variance	
0.01	-0.15
0.10	0.45
0.50	-0.26
1.00	0.05

4.2 Identical change point location for both time series

The second condition of interest in this study was when a change point was present in each time series, and was located at the same location for both. The results of the ANOVA showed that none of the main effects or interactions of the manipulated factors were statistically significantly related to the Type I error rate for a difference in change point location. The overall Type I error rate was 0.0004, which illustrates that it was very unlikely for a statistically significant difference in change point location for the two series to be identified by the Bayesian approach, when no such difference was present in the population.

With regard to the difference in the change point location, only the interaction between series variance and length was found to be statistically significant $(F_{9,18} = 3.023, p =$

$0.022, \eta^2_{Partial} = 0.602)$. Table 2 contains the mean difference in change point location for the two time series by series length and variance.

Table 2: Mean difference in change point location by length and variance of time series when change points were the same for both series

Length	Variance	Mean Difference in Change Point Location Estimate
20	0.01	0.001
	0.10	0.05
	0.50	0.05
	1.00	0.08
50	0.01	0.00
	0.10	0.02
	0.50	0.07
	1.00	0.10
100	0.01	0.004
	0.10	0.03
	0.50	0.09
	1.00	0.14
200	0.01	0.003
	0.10	0.03
	0.50	0.57
	1.00	0.86

The results in Table 2 suggest that when no difference in change point location was simulated, the estimated difference in change point location between the time series was larger when the variances were also larger. In addition, the magnitude of the estimated difference in change points was larger for the large variance condition in conjunction with longer time series. For example, the estimated difference in change point location for a series of length 20 was 0.05 and 0.08, respectively, for variances of 0.50 and 1.00. In contrast, for a series of length 200, the estimated difference in change point location was 0.57 and 0.86, respectively, for variances of 0.5 and 1.00. It is important to note, however, that the largest such difference was less than 1, indicating that regardless of the simulated conditions, the estimated differences in the change points was very small.

4.3 Different change point locations for time series

When differences in change point location were simulated to be present, the ANOVA identified the interactions of time series length by variance by difference in change point location $(F_{9,18} = 5.623, p = 0.001, \eta^2_{Partial} = 0.738)$, and the magnitude of change by difference in change point location $(F_{2,18} = 15.099, p < 0.001, \eta^2_{Partial} = 0.647)$ as statistically significantly related to the power for detecting a difference in the change point locations. Figure 2 displays the power of the method for detecting a difference in change point location by time series length, variance, and the difference in the change point location.

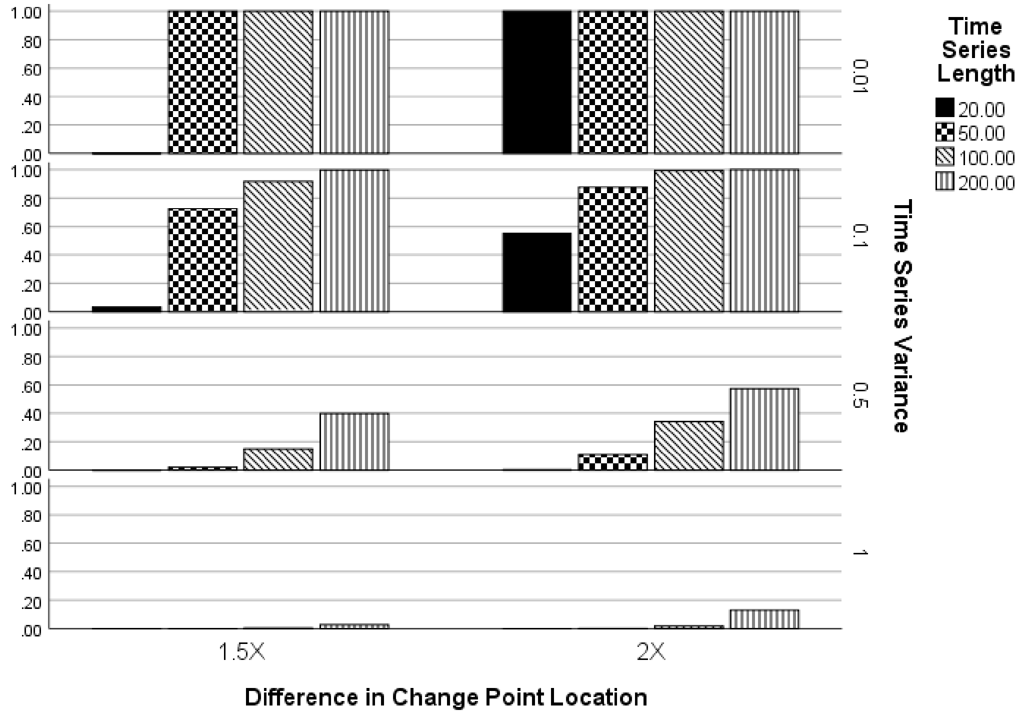


Figure 2: Power for detecting difference in change point location by time series length, time series variance, and difference in change point location

When the variance was 0.01, the power for detecting differences in change point location was 1.00 for lengths of 50, 100, and 200, regardless of the simulated difference. For a time series length of 20, power was very low for the 1.5 times difference in location, but 1.00 for the 2 times difference. As the time series variance increased in value, power declined, with power remaining larger for longer series and a greater difference in the change point location for the two series.

The power for detecting a difference in the change point by the magnitude of change and the difference in change point location appears in Figure 3.

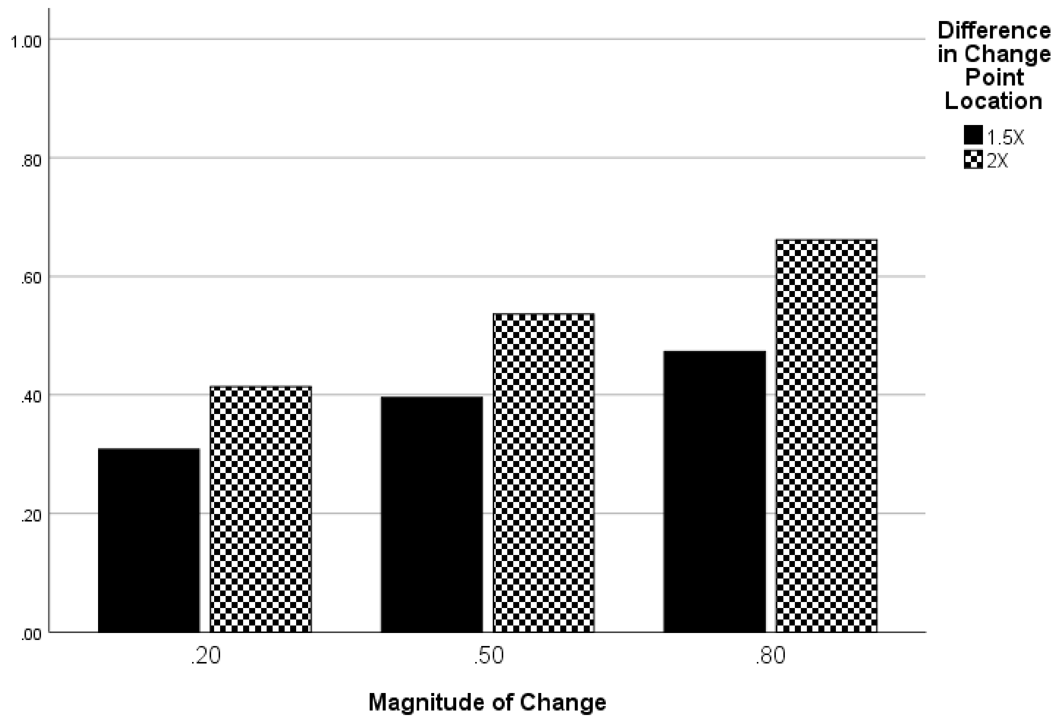


Figure 3: Power for detecting difference in change point location by the magnitude of the change and difference in change point location for the two series.

These results reveal that power for detecting a difference in change point location was higher for a larger magnitude of change in the time series. Furthermore, across the magnitude of change, power was uniformly higher when the difference in the change point was greater, and this effect was larger for greater magnitudes of change.

In regards to the estimate of the difference in change point location, the ANOVA results indicated that the interactions of time series variance by length by difference in change point location $\left(F_{9,18} = 21.795, p < 0.001, \eta^2_{Partial} = 0.923\right)$ and the magnitude of change by length by difference in location $\left(F_{6,18} = 5.786, p = 0.002, \eta^2_{Partial} = 0.659\right)$ were statistically significant. The estimated difference in change point location by time series variance, length, and difference in change point location appears in Table 3.

Table 3: Mean estimated difference in change point location (actual difference in change point location) between the two series by time series variance, length, and difference in change point location

Length	Variance	1.5 times estimated (actual) difference	2 times estimated (actual) difference
20	0.01	1.89 (2)	3.83 (4)
	0.10	1.85 (2)	3.77 (4)
	0.50	0.92 (2)	2.11 (4)
	1.00	0.53 (2)	0.72 (4)
50	0.01	5 (5)	10 (10)
	0.10	4.87 (5)	9.91 (10)
	0.50	2.97 (5)	8.56 (10)
	1.00	1.88 (5)	6.29 (10)
100	0.01	10 (10)	20 (20)
	0.10	9.96 (10)	19.98 (20)
	0.50	8.75 (10)	18.31 (20)
	1.00	6.19 (10)	16.51 (20)
200	0.01	20 (20)	40 (40)
	0.10	20 (20)	39.99 (40)
	0.50	18.46 (20)	37.62 (40)
	1.00	16.75 (20)	35.43 (40)

The results in Table 3 demonstrate that across degrees of difference in change point location and time series lengths, accuracy of the estimated difference in the change point location for the two series decreased concomitantly with increases in time series variance. In addition, estimation accuracy of the difference in location was greater for longer series. Finally, the deleterious impact of increased variance on estimation accuracy was more pronounced for shorter time series. For example, the estimated difference in change point location was nearly perfect for the combination of a time series of length 200 and variances of 0.01 and 0.1, whereas for time series of length 20, the estimated difference was never perfectly accurate, even for the smallest variance condition.

Table 4 contains the estimated difference in change point location by time series length, magnitude of the change, and actual difference in change point location.

Table 4: Mean estimated difference in change point location (actual difference in change point location) between the two series by time series length, magnitude of change, and difference in change point location

Length	Magnitude of change	1.5 times estimated (actual) difference	2 times estimated (actual) difference
20	0.2	0.89 (2)	1.86 (4)
	0.5	1.46 (2)	2.74 (4)
	0.8	1.55 (2)	3.22 (4)
50	0.2	2.64 (5)	5.63 (10)
	0.5	3.98 (5)	7.66 (10)
	0.8	4.43 (5)	9.03 (10)
100	0.2	7.57 (10)	14.58 (20)
	0.5	8.73 (10)	17.57 (20)

	0.8	9.62 (10)	19.11 (20)
200	0.2	16.05 (20)	33.65 (40)
	0.5	17.94 (20)	37.83 (40)
	0.8	19.91 (20)	39.81 (40)

The results in Table 4 reveal that across time series lengths and simulated differences in change point locations, the estimated change point difference was more accurate when the magnitude of change was greater. This effect was particularly pronounced for the shorter time series conditions.

5 Conclusion

The overall goal of this study was to examine the performance of a Bayesian estimation approach to modeling differences in change points for two or more time series. Researchers in fields such as econometrics, climatology, geology, genomics, and health care may be faced with situations in which measurements of two or more time series have been made, change points are clearly present in each, and it is of interest to ascertain whether these change points occurred at different locations. For example, economists may be interested in comparing the impact that a regulatory change might have on the net profits for multiple firms in a given industry. This change may impact each firm in a different way, leading to differences in the point at which net profits change. The model examined here would allow such researchers to formally test the null hypothesis that the change in net profit occurred at the same point in time. Without such a hypothesis test, economists could only use descriptive approaches based on graphs or point estimates of change location, without the ability to make more definitive statements regarding the equality of these locations.

The results of this study revealed that the methodology described by Cleynen and Robin (2016) was able to accurately detect differences in change point location under many conditions simulated here. Specifically, when there was no difference in the location of the time point, the model was very accurate in identifying this to be the case, regardless of series length, variance in the measurements, or the magnitude of change in each series. The Type I error rate in such cases never exceeded 0.01, and the estimated difference in change point location was never greater than 1 time point. Thus, in the worst case the estimated difference in change point when there was no difference in the population was less than 1 time point apart. This result is very similar to the findings from Cleynen and Robin for data simulated from the negative binomial distribution.

When the change point was simulated to differ between the two series, power for detecting this difference was higher when the difference in location was greater, which is in keeping with the hypotheses expressed above, and prior results (Cleynen & Robin, 2016). In other words, the further apart were the change point locations, the higher the power for identifying a difference in change point location, and the greater the accuracy in estimating that difference. In addition, results of the simulation study demonstrated that, as was hypothesized, when the time series had greater variance, the ability of the model to correctly identify differences in change point location, and to estimate the difference was compromised.

Taken together, the results of this simulation study indicate that the model for detecting differences in change points, as fit using the Bayesian estimator, is a promising tool for researchers in a variety of fields in which the time series consists of 200 or fewer measurements. This effectiveness will be greater for researchers who have longer time series. Those working with series of 20 or fewer measurements may have difficulty identifying differences in change points, even when the change point for one series is twice as far down the series as is the one for the other series. However, for series of 50 measurements or longer, and with relatively low variances in the scores, the model featured here will have power of 0.75 or greater, for differences in change point location of 1.5 times. For time series of 100 points or longer, power for detecting differences in the change point locations for two series will be in excess of 0.9 for medium to small variances. With large variances, this model has difficulty detecting differences in change point locations, even when one occurs twice as far down the series as does the other.

5.1 Directions for future research

The current study provides researchers with some initial guidance for when the Bayesian model might be useful for detecting differences in change point locations for two series of 200 or fewer measurements. In this regard, it builds upon earlier work (Cleynen & Robin, 2016), in particular by focusing on shorter time series, and continuous measurements. Future work should further extend this study by including more than two time series combined with relatively short time series, and normally distributed measurements. For example, economists may be interested in comparing the change point locations for net profit from multiple firms after a new regulation is put in place. In addition, future research should also include differences in change values for the different time series. In this study, the amount of change was the same for each series (e.g., if one experienced a mean increase of 0.2, so did the other). However, in actuality such consistent change across time series may not be present. A third direction for future research would be to include more than 2 time segments. As noted above, the purpose of the current study was to investigate how well the Bayesian model can identify differences in change points for two series after a single precipitating event. In some applications, though, there may be multiple such events (e.g., changes in regulatory framework at several points in time), leading to more than one change time point for each series. Finally, future work should also consider a wider array of time series lengths, and differences in change point locations. The current study provides a framework within which researchers can expand, particularly in terms of time series lengths between 100 and 200 measurement occasions long.

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