Advances in Management & Applied Economics, vol. 4, no.1, 2014, 111-121
ISSN: 1792-7544 (print version), 1792-7552(online)
Scienpress Ltd, 2014

Sustainable Energy Management Using Alternative Energy Sources

Duane J. Rosa[1]

Abstract

Sustainable energy management involves the provision of energy supply that meets the needs of the present without compromising the ability of future generation to meet their needs. Sustainable energy strategies must involve energy savings on the demand side, plus on the supply side, efficiency improvements in energy production and the replacement of fossil fuels by various forms of renewable energy. This paper analyzes the issues involving utilization of different energy sources. Formulating a dynamic optimization problem, expressions are derived for optimal energy supply prices from each of three energy sources, wind energy, hydroelectric power, and fossil fuels. Based on the economic characteristics of the energy sources, an optimal solution is derived that involves both periods of specialization in a single energy source as well as the simultaneous utilization of three available sources.

JEL classification numbers: Q01, Q42, C61, L94
Keywords: Alternative Energy Sources, Energy Management, Sustainability, Energy Pricing

1 Introduction

Sustainability implies using, developing and protecting resources in such a manner that enables society to meet current needs and provides that future generations will be able to meet their needs. Sustainable energy management involves the provision of energy that meets the needs of the present without compromising the ability of future generations to meet their energy needs. The concept of sustainability or sustainable development has been discussed by economists and researchers for hundreds of years. Neoclassical economist, Stanley Jevons, back in 1865, was concerned about Britain's energy consumption of coal, its primary energy resource, and if a finite supply of the resource

---

[1]West Texas A&M University, USA.

Article Info: *Received* : December 20, 2013. *Revised* : January 17, 2014.
Published online : February 1, 2014

could be sustained in the future [1]. The first discussion in economic literature of the use of renewable energy sources was during the post-1973 oil shock. Dasgupta and Heal [2] utilized the phrase “backstop technology,” as a technology that would replace exhaustible resources with an energy source continuing forever. William Nordhaus [3] used the same terminology in his book on the efficient allocation of energy resources. Kenneth Boulding [4] and Ralph D’Arge [5] both pointed to the need for inexhaustible low-impact energy sources.

2 Model of Sustainable Energy Management

Sustainable energy management usually involves three major technological changes: efficiency improvements in energy production; energy savings on the demand side; and replacement of fossil fuel sources by various types of renewable energy [6]. Any large-scale renewable energy management plan must include strategies for integrating renewable sources determined by energy savings and increased efficiency. Energy supply may come from a combination of sources including fossil fuels (coal, oil, or natural gas), non-renewable (nuclear), as well as the renewable energy sources of wind, solar, hydropower, biofuel or geothermal. For purposes of this analysis, energy generation will be considered for a region to come from three sources, wind energy, hydroelectric, and fossil fuel. This combination is being used as it allows us to consider the storativity of different energy sources. This is an integral factor relating to sustainable energy management. The variability of using wind, solar and hydroelectric power sources can create a mismatch for the load demand of an energy supplier [7]. Thus, it is important that the supplier utilizing multiple sources of energy consider both available supply (time variant) and the pricing of different energy sources.
In order to evaluate the sustainable management of different energy sources, and to determine an optimal mix of energy sources, the following model is developed. Consider a typical situation for an energy supplier where the energy demand is for both residential and industrial energy purposes for a region with a certain population base. Let the instantaneous demand for energy in the region be given by the inverse demand function:

$$p = D(y,t) \tag{1}$$

where p denotes the demand price and y refers to energy consumption at time t. According to this specification, the function D(y,t) is time variant. We assume (i) that $D(y,t)>0$, (ii) that the function D(y,t) is twice continuously differentiable and (iii) that $D_y(y,t)<0$.

Given this energy demand, a measure of social benefits derived from energy use is provided by the consumer surplus defined by the following expression:

$$B(t) = \int_0^q D(y,t)\,\mathrm{dy} \tag{2}$$

In this case it is mathematically convenient to assume that the function B(t) is concave in q.

Each of the three methods of energy generation involves social costs. Let us for a moment describe these costs at time t by the following cost function:

$$C(t) = C_1(y_{w,}t)+C_2(y_h,t)+C_3(y_f,t) \tag{3}$$

Where y_w, y_h, and y_f refer to the instantaneous consumption of wind energy, hydroelectric, and fossil energy respectively and the functions $C_1(\cdot)$, $C_2(\cdot)$ and $C_3(\cdot)$ represent the corresponding cost functions.

Instantaneous net social benefits may be defined as the difference between social benefits and costs:

$$NB(t) = B(t) - C(t) \tag{4}$$

The problem for the energy supplier is to select the optimal combination of these methods so as to maximize the present value of net social benefits [8]. This problem is equivalent to choosing time paths of y_w, y_h, and y_f to maximize the objective function:

$$V = \int_0^\infty NB(t) \cdot \exp(-rt)dt \tag{5}$$

where r represents the social rate of discount, subject to the constraint that demand is satisfied, i.e. $y= y_w + y_h+ y_f$ and other constraints [9]. Economic optimality requires that demand be satisfied at each point in time, for otherwise prices would convey incorrect signals and could induce less than optimal behavior [10].

The maximization of net social benefits from energy generation subject to satisfying demand is clearly equivalent to minimizing the cost of meeting that demand. This cost minimization approach gives rise to the concept of the marginal social cost of energy supply which is often referred to as the supply price of energy. Thus, a particularly transparent way to approach the problem of optimal energy generation is to proceed in terms of the social supply price of energy from the different energy sources. The following sections will develop a supply price for each power source.

3 Wind Energy Supply Price

Let the cost of electric power generation in the wind energy system be defined by the cost function:

$$C(y),\ C_y >0 \tag{6}$$

This cost function reflects the total costs of operating the wind energy system (i.e. both fixed and variable costs) at a point of time. $C(\cdot)$, however, does not include investment costs (i.e. sunk costs) or costs associated with future capacity changes. For reasons of mathematical convenience we assume that this cost function is convex and twice continuously differentiable.

Production of electricity is constrained by two factors, the installed generating capacity, denoted as Q, and wind speed. Thus,

$$Q \geq y \geq 0 \tag{7}$$

In large scale wind energy farms, capacity usually can only be adjusted in relatively large discrete blocks. Generating capacity is, in other words, highly indivisible. This significantly complicates the analysis and resulting optimal management of the system. We will therefore consider Q fixed for this analysis.

The availability of required minimum wind speed may be taken to be exogenous in the model. A typical 1.5 megawatt wind turbine has a cut-in wind speed of 3.5 meters per second and a cut-out wind speed of 25 meters per second [11]. Referring to the minimum wind speed at time t by x (t) we have:

$$x'(t) = \partial x / \partial t = a(t) - y(t) \tag{8}$$

where a(t) represents the exogenous inflow of wind and y(t) the extraction of wind for energy generation (capacity factor). Wind energy is non-dispatchable in that all available output must be taken when it is available.

The simplifying assumption is used that maximization of social benefits from electricity generation is equivalent to maximizing the present value of future consumer and producer surplus. Thus, assuming for the present that capital is fixed, the problem facing the energy supplier is:

$$Max V = \int_0^\infty [\int_0^q D(y,t)dy - C(y)] \cdot \exp(-rt)dt \tag{9}$$

subject to:

$$Q \geq y \geq 0 \tag{10}$$
$$x' = a - y \tag{11}$$
$$y, x \geq 0 \tag{12}$$

where r>0 denotes the social rate of discount and the term $[\int_0^q D(y,t)dy - C(y)]$ represents the sum of consumer and producer surpluses at time t.

Equation (9) is only dynamic in a limited sense as the stock variable x does not appear explicitly in the objective function. This means that the energy generation remains at all times at the optimal equilibrium level given the exogenous variables. If the stock constraint, x≥0, does not become binding at some point of time, the problem is entirely static.

A Hamiltonian function corresponding to equation (9) may be written as:

$$H = \int_0^q D(y,t)dy - C(y) + \sigma(a-y) + \mu_1 \cdot (Q-y) + \mu_2 \cdot x \quad (13)$$

where σ and μ_1 and μ_2 are the Lagrange multipliers for this problem. Along the optimal solution to equation (9), these variables measure the shadow or, in this case, social values of the respective stock variables. More precisely, σ(t) measures the increase in the present value of net social benefits, from time t onwards, due to a marginal increase in wind speed levels. The variable μ_1 measures the instantaneous change in social benefits due to a marginal increase in electricity generating capacity at time t, and the variable $\mu_2(t)$ measures the instantaneous increase in social benefits due to an increase in wind levels at time t.

The necessary conditions for solving equation (9) are:

$$H_y = D(y,t) - C_y(y) \le \sigma + \mu_1, y \ge 0, H_y \cdot q = 0 \quad (14)$$

$$\sigma_1 - r \cdot \sigma = -\mu_2 \quad (15)$$

$$Q \ \ge y,\ \mu_1 \ge 0,\ (Q-y) \cdot \mu_1 = 0 \quad (16)$$

$$x' = a\text{-}y \quad (17)$$

$$x \ge 0, \mu_2 \ge 0,\ x \cdot \mu_2 = 0 \quad (18)$$

These necessary conditions reveal a set of socially optimal rules for electricity pricing. Provided there is some production of electricity, according to equation (14), the optimal supply price of wind power, p_w, is given by:

$$P_w = D(y,t) = C_y(y) + \sigma + \mu_1 \quad (19)$$

The first term in the supply price is the marginal instantaneous cost of generating electricity. The second, σ, reflects the social cost of using wind for electricity generation. It can be shown that if the wind speed is adequate for future periods, then σ = 0 at all t. The third term, μ_1, measures the social value of a marginal increase in generating capacity. If there is excess capacity at time, $\mu_1=0$, otherwise $\mu_1 \geq 0$. For investment to be optimal, μ_1 must at least equal the marginal cost of investment.
The variables σ and μ_1 in equation (19) reflect the importance of wind availability and investment costs for the optimal supply price of wind power. The movement of σ and μ_1 over time is given by conditions (14) through (18).

4 Wind Supply Constraints

If the wind supply is abundant in the region in the sense that the x ≥ 0 constraint will never become binding, the electricity pricing rules are relatively simple. In that case they are:

$p_w = D(y,t) = C_y(y)$, if Q > y, i.e., excess capacity (20)

$p_w = D(Q,t)$, if Q = y, i.e., full utilization of capacity (21)

It is important to realize that to follow these rules requires knowledge of the current power generation demand facing the utility. In the first phase, from t = 0 to t = t_1, there is excess capacity and equation (20) applies. In the second phase, from t = t_1 onwards, there is full utilization of capacity. Therefore, the supply price must adjust so as to satisfy the demand and equation (21) applies.

At some point during phase 2 it may become optimal to invest in additional capacity. The condition for that is given by:

$$V(Q_1) - V(Q_0) \geq I \qquad (22)$$

where $V(Q_1)$ represents the value of the optimal program under some new capacity, Q_1, and $V(Q_0)$ the value of the optimal program under the old capacity, Q_0. (I) represents the cost of investing in additional capacity.

As suggested by equation (22), investment decisions require knowledge about future conditions since $V(Q_1)$ and $V(Q_0)$ involve the entire future path of electricity generation. More precisely:

$$V(Q_1) = \int_0^{\infty} [\int_0^{q} D(y^*, t)dy - C(y^*)] \cdot \exp(-rt)dt \qquad (23)$$

Where y* represents the optimal path of electricity production and bounded by the constraint
$Q_1 \geq y^*$. A similar expression holds for $V(Q_0)$. Thus, the optimal investment decisions require perfect foresight for the remainder of the program horizon. Only in certain rare circumstances is it possible to base the optimal investment decisions on current data. One such case is when future electricity demand is guaranteed not to fall below the demand at the time of the investment.
Once a discrete addition to electricity generation capacity has taken place, the situation reverts to one described by equations (20) and (21).

5 Hydroelectric Supply Price

Let the cost of electric power generation in the hydroelectric grid system be defined by the cost function:

$$C(y), C_y > 0 \qquad (24)$$

This cost function reflects the total costs of operating the existing hydroelectric system (i.e. both fixed and variable costs) at a point of time. $C(\cdot)$, however, does not include investment costs (i.e. sunk costs) or costs associated with future capacity changes. For reasons of mathematical convenience we assume that this cost function is convex and twice continuously differentiable.

Production of electricity is constrained by two factors, the installed generating capacity, denoted as Q, and the availability of water. Thus,

$$Q \geq y \geq 0 \tag{25}$$

In a typical hydroelectric power system, capacity can only be adjusted in relatively large discrete blocks. Generating capacity is, in other words, highly indivisible. This significantly complicates the analysis and resulting optimal management of the system. We will therefore consider Q fixed for this analysis.

The availability of water depends on the flow rate of the river, which may be taken to be exogenous, and the accumulation of stored water in reservoirs. Referring to the stored water in the reservoirs at time t by x(t) we have:

$$x'(t) = \partial x / \partial t = a(t) - y(t) \tag{26}$$

where a(t) represents the exogenous inflow of water and y(t) the extraction of water for energy generation.

The simplifying assumption is used that maximization of social benefits from electricity generation is equivalent to maximizing the present value of future consumer and producer surplus. Thus, assuming for the present that capital is fixed, the problem facing the utility is:

$$MaxV = \int_0^{\infty} [\int_0^{q} D(y,t)dy - C(y)] \cdot \exp(-rt)dt \tag{27}$$

subject to:

$$Q \geq y \geq 0 \tag{28}$$
$$x' = a - y \tag{29}$$
$$x \geq 0 \tag{30}$$

where r>0 denotes the social rate of discount and the term $[\int_0^{q} D(y,t)dy - C(y)]$ represents the sum of consumer and producer surpluses at time t.

Equation (27) is only dynamic in a limited sense as the stock variable x does not appear explicitly in the objective function. This means that the energy generation remains at all times at the optimal equilibrium level given the exogenous variables. If the stock constraint, x≥0, does not become binding at some point of time, the problem is entirely static.

A Hamiltonian function corresponding to equation (27) may be written as:

$$H = \int_0^{q} D(y,t)dy - C(y) + \sigma(a-y) + \mu_1 \cdot (Q-y) + \mu_2 \cdot x \tag{31}$$

where σ and μ_1 and μ_2 are the Lagrange multipliers for this problem. Along the optimal solution to equation (27), these variables measure the shadow or, in this case, social values of the respective stock variables. More precisely, $\sigma(t)$ measures the increase in the present value of net social benefits, from time t onwards, due to a marginal increase in water reservoir levels. The variable μ_1 measures the instantaneous change in social benefits due to a marginal increase in electricity generating capacity at time t, and the variable $\mu_2(t)$ measures the instantaneous increase in social benefits due to an increase in water levels at time t.

The necessary conditions for solving equation (27) are:

$$H_y = D(y,t) - C_y(y) \leq \sigma + \mu_1, y \geq 0, H_y \cdot q = 0 \tag{32}$$

$$\sigma_1 - r \cdot \sigma = -\mu_2 \tag{33}$$

$$Q \geq y,\ \mu_1 \geq 0,\ (Q - y) \cdot \mu_1 = 0 \tag{34}$$

$$x' = a\text{-}y \tag{35}$$

$$x \geq 0, \mu_2 \geq 0,\ x \cdot \mu_2 = 0 \tag{36}$$

These necessary conditions reveal a set of socially optimal rules for electricity pricing. Provided there is some production of electricity, according to equation (32), the optimal supply price of hydropower, p_h, is given by:

$$P_h = D(y,t) = C_y(y) + \sigma + \mu_1 \tag{37}$$

The first term in the supply price is the marginal instantaneous cost of generating electricity. The second, σ, reflects the social cost of using water for electricity generation. It can be shown that if the water supply is adequate for all future periods, then $\sigma = 0$ at all t. The third term, μ_1, measures the social value of a marginal increase in generating capacity. If there is excess capacity at time, $\mu_1=0$, otherwise $\mu_1 \geq 0$. For investment to be optimal, μ_1 must at least equal the marginal cost of investment.
The variables σ and μ_1 in equation (37) reflect the importance of water availability and investment costs for the optimal supply price of hydropower.

6 Fossil Fuel Energy Supply Price

Let the cost of energy generation by burning fossil fuel (e.g. coal, oil, natural gas) in existing capacity be represented by the twice continuously differentiable and convex cost function:

$$C(y), C_y > 0 \tag{38}$$

Production of fossil fuel energy is constrained by the installed capacity. Thus, referring to the installed capacity by Q:

$$Q \geq y \tag{39}$$

Assume in reality that fossil fuel energy generation capacity can be adjusted upwards and downwards approximately on a continuous basis by changes in investment. It should, however, be recognized that this requires that capacity units are small and that there exists a functioning resale market for fossil fuel capacity. Thus, omitting depreciation, capacity changes according to:

$$Q'(t) = i \tag{40}$$

where i refers to investment.

Finally, let the investment cost per unit of capacity be represented by $s > 0$. Given these specifications, the problem facing fossil-fuel utilities is:

$$MaxV = \int_0^\infty [\int_0^q D(y,t)dy - C(y) - (s \cdot i)] \cdot \exp(-rt)dt \tag{41}$$

subject to:

$$Q \geq y \geq 0 \tag{42}$$

$$Q' = i \tag{43}$$

$$y \geq 0 \tag{44}$$

where, as before, $r > 0$ denotes the social rate of discount and the term $[\int_0^q D(y,t)dy - C(y) - (s \cdot i)]$ represents the sum of consumer and producer surpluses at time t.

It is worth noting that the only stock variable in this problem is the capacity level, Q. However, given the malleability of the fossil fuel capacity, Q will be adjusted to output, y, at each point in time. Given this, the supply price of fossil fuel is given by the following single equation:

$$P_f = C_y(y) + r \cdot s \tag{45}$$

The assumption of perfect malleability of fossil fuel capacity is an approximation. Any changes to this assumption would require some modification of equation (41), but these changes would probably be minor.

7 Multiple Utilization of Energy Sources

The social supply price schedules of wind energy, hydropower and fossil fuel were derived in the previous sections. These schedules, defined in equations (19), (37), and (45) give the respective supply prices as functions of the amount of energy generated as well as other variables. From this, we can examine the conditions necessary for utilization of multiple energy sources.

For purposes of this analysis it is assumed that any energy demand would be met at the lowest available supply price at each point of time. If that were not the case, the present value of social benefits would not be maximized. This condition of lowest available supply price can be formally stated as:

$$D(y_w,y_h,y_f, t) = \text{Min}[p_w(y_w),p_h(y_h),p_f(y_f)], \text{ all } t \tag{46}$$

where $p_w(y_w)$, $p_h(y_h)$, and $p_f(y_f)$ represent the social supply prices of wind, hydroelectric, and fossil fuel respectively.

Thus, the condition for joint utilization of any two energy sources at a point in time is:

$$p_i(y_i) = p_j(y_j)\text{, for } y_i \text{ and } y_i \geq 0 \tag{47}$$

Given the exogenous demand, D(y, t) and the respective supply price functions, it is clear that equations (41) and (42) fully specify utilization of the three energy sources at a given point in time. The conditions are:

$$D(y,t) = p_w(y_w)\text{, or } y_w = 0 \tag{48}$$

$$D(y,t) = p_h(y_h)\text{, or } y_h = 0 \tag{49}$$

$$D(y,t) = p_f(y_f)\text{, or } y_f = 0 \tag{50}$$

$$y = y_w + y_h + y_f \tag{51}$$

7 Conclusion

Any development of sustainable energy strategies must involve energy savings on the demand side, plus on the supply side, efficiency improvements in energy production and the replacement of fossil fuels by various forms of renewable energy. This analysis demonstrates that an optimal energy management system utilizing multiple sources of energy can be achieved. The dynamic optimization model presented determined optimal energy supply prices from each of three energy sources, wind energy, hydroelectric power, and fossil fuels. Based on the economic and technical characteristics of the energy sources, an optimal solution can be derived for the energy supplier that involves both periods of specialization in a single energy source as well as the simultaneous utilization of multiple alternative sources.

References

[1] W. Catton, *Overshoot: The Ecological Basis of Revolutionary Change*, University of Illinois Press, 1982.

[2] P. Dasgupta and G. Heal, The Optimal Depletion of Exhaustible Resources, *Review of Economic Studies* **41**, *Symposium on the Economics of Exhaustible Resources*, (1974) 3-28.

[3] W. Nordhaus, *The Allocation of Energy Resources*, Brookings Papers on Economic Activity **4** (3), (1973), 529-576.
[4] K. Boulding, The Economics of the Coming Spaceship Earth, In *Radical Political Economy: Explorations in Alternative Economic Analysis*, ed. Victor D. Lippit, 1966.
[5] R. D'Arge and K. Kogiku, Economic Growth and the Environment, *Review of Economic Studies* **40** (1), (1973), 61-77.
[6] H. Lund, Renewable Energy Strategies for Sustainable Development, *Energy* **32** (6)**,** (2007), 912-919.
[7] D. Rosa, Sustainability and Infrastructure Resource Allocation, *Journal of Business and Economic Research* **7** (9), (2009), 71-75.
[8] K. Arrow and S. Chang, Optimal Pricing, Use and Exploration of Uncertain Natural Resource Stocks, *Journal of Environmental Economics and Management* **9** (1), 1-10.
[9] L. Pontryagin, *The Mathematical Theory of Optimal Processes*, CRC Press, 1987.
[10] M. Kamien and N. Schwartz, *Dynamic Optimization: The Calculus of Variation and Optimal Control Theory in Economics and Management*, Dover Publication, Second Edition, 2012.
[11] General Electric Energy – 1.5MW Wind Turbine Technical Specifications, (2013), http://site.ge-energy.com/prod_serv/products/wind_turbines/en/15mw/specs.htm