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Improvement of Ridge Estimator When Stochastic Restrictions Are Available in the Linear Regression Model

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Abstract

In this paper we propose another ridge type estimator, namely Stochastic Restricted Ordinary Ridge Estimator (SRORE) in the multiple linear regression model when the stochastic restrictions are available in addition to the sample information and when the explanatory variables are multicollinear. Necessary and sufficient conditions for the superiority of the Stochastic Restricted Ordinary Ridge Estimator over the Mixed Estimator (ME), Ridge Estimator (RE) and Stochastic Mixed Ridge Estimator (SMRE) are obtained by using the Mean Square Error Matrix (MSEM) criterion. Finally the theoretical findings of the proposed estimator are illustrated by using a numerical example and a Monte Carlo simulation.

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1 Introduction

Instead of using the Ordinary Least Square Estimator (OLSE), the biased estimators are considered in the regression analysis in the presence of multicollinearity. Some of these are namely the Ridge Estimator (RE) (Hoerl and Kennard, 1970), Liu Estimator (LE) (Liu, 1993) and Almost Unbiased Liu Estimator (AULE) (Akdeniz and Kaçiranlar, 1995). In the presence of stochastic prior information in addition to the sample information, Theil and Goldberger (1961) proposed the Mixed Estimator (ME). By replacing OLSE by ME in the RE and LE respectively, the Stochastic Mixed Ridge Estimator (SMRE) (Li and Yang, 2010) and Stochastic Restricted Liu Estimator (SRLE) (Hubert and Wijekoon, 2006) are introduced.

Also by replacing OLSE by LE in the ME, Yang and Xu (2007) introduced an Alternative Stochastic Restricted Liu Estimator (ASRLE). In this paper we propose the Stochastic Restricted Ordinary Ridge Estimator (SRORE) by replacing OLSE by RE in the ME. The proposed estimator is a generalization of the ME and RE. Rest of the paper is organized as follows. The model specification and the proposed estimator are given in section 2. In section 3 we see the comparisons among biased estimators. In section 4 a numerical example and a Monte Carlo Simulation are given to illustrate the theoretical findings of the proposed estimator. Finally we state the conclusions in section 5.

2 Model Specification and the Proposed Estimator

We consider the standard multiple linear model

$$y = X\beta + \varepsilon \tag{2.1}$$

where y is an $n \times 1$ vector of observations on the response variable, X is an $n \times p$ full column rank matrix of observations on p non stochastic explanatory regressors variables, β is a $p \times 1$ vector of unknown parameters associated with p regressors and ε is an $n \times 1$ vector of disturbances with $E(\varepsilon) = 0$ and the dispersion matrix $D(\varepsilon) = \sigma^2 I$.

In addition to former model (2.1), related only to sample information, let us be given some prior information about β in the form of a set of j independent stochastic linear restrictions as follows:

$$r = R\beta + v \tag{2.2}$$

where r is an $j \times 1$ stochastic known vector, R is a $j \times p$ random vector of disturbances with $E(v) = 0$ and $D(v) = \sigma^2 \Omega$, and Ω is assumed to be known and positive definite. Further it is assumed that v is stochastically independent of ε .

The Ordinary Least Square Estimator for the model (2.1) and Mixed Estimator (Theil and Goldberger, 1961) due to a stochastic prior restriction (2.2) are given by

$$\hat{\beta}_{OLSE} = S^{-1}X'y \text{ and } \hat{\beta}_{ME} = \left(S + R'\Omega^{-1}R\right)^{-1}\left(X'y + R'\Omega^{-1}r\right) \tag{2.3}$$

respectively, where $S = X'X$.

When different estimators are available for the same parameter vector β in the linear regression model one must solve the problem of their comparison. Usually as a simultaneous measure of covariance and bias, the mean square error matrix is used, and is defined by

$$MSE(\hat{\beta},\beta) = E\left[\left(\hat{\beta}-\beta\right)\left(\hat{\beta}-\beta\right)'\right] = D\left(\hat{\beta}\right) + B\left(\hat{\beta}\right)B'\left(\hat{\beta}\right) \tag{2.4}$$

where $D(\hat{\beta})$ is the dispersion matrix and $B\left(\hat{\beta}\right) = E\left(\hat{\beta}\right) - \beta$ denotes the bias vector. We recall that the Scalar Mean Square Error $SMSE\left(\hat{\beta},\beta\right) = trace\left(MSE\left(\hat{\beta},\beta\right)\right)$.

For any two given estimators $\hat{\beta}_1$ and $\hat{\beta}_2$, the estimator $\hat{\beta}_2$ is said to be superior to $\hat{\beta}_1$ under the MSEM criterion if and only if

$$M\left(\hat{\beta}_1,\hat{\beta}_2\right) = MSE\left(\hat{\beta}_1,\beta\right) - MSE\left(\hat{\beta}_2,\beta\right) \geq 0 \tag{2.5}$$

Since $\left(S + R'\Omega^{-1}R\right)^{-1} = S^{-1} - S^{-1}R'\left(\Omega + RS^{-1}R'\right)^{-1}RS^{-1}$ (see lemma 1 in appendix) the ME can be rewritten as

$$\hat{\beta}_{ME} = \hat{\beta}_{OLSE} + S^{-1}R'\left(\Omega + RS^{-1}R'\right)^{-1}\left(r - R\hat{\beta}_{OLSE}\right). \tag{2.6}$$

To deal with multicollinearity the researchers introduced alternative estimators based shrinkage parameters d and k, where $0 < d < 1$ and $k \geq 0$.

Some of the estimators based on the shrinkage parameter d are Liu Estimator (Liu, 1993), Stochastic Restricted Liu Estimator (Hubert and Wijekoon, 2006) and Alternative Stochastic Restricted Liu Estimator (Yang and Xu, 2007), and given by

$$\hat{\beta}_{LE}(d) = F_d\hat{\beta}_{OLSE}, \tag{2.7}$$

$$\hat{\beta}_{srd} = F_d\hat{\beta}_{ME} \tag{2.8}$$

and

$$\hat{\beta}_{SRLE}(d) = \hat{\beta}_{LE}(d) + S^{-1}R'\left(\Omega + RS^{-1}R'\right)^{-1}\left(r - R\hat{\beta}_{LE}(d)\right) \tag{2.9}$$

respectively, where $F_d = (S+I)^{-1}(S+dI)$ for $0 < d < 1$.

Note that $\hat{\beta}_{SRLE}(d)$ is introduced by replacing OLSE by LE in the ME in (2.6).

Similarly the estimators, Ridge Estimator (Hoerl and Kennard, 1970) and Stochastic Mixed Ridge Estimator (Li and Yang, 2010) are based on the shrinkage parameter k, and defined as

$$\hat{\beta}_{RE}(k) = W\hat{\beta}_{OLSE} \tag{2.10}$$

and

$$\hat{\beta}_{SMRE} = W\hat{\beta}_{ME} \tag{2.11}$$

respectively, where $W = \left(I + kS^{-1}\right)^{-1}$ for $k \geq 0$.

Now we propose the ***Stochastic Restricted Ordinary Ridge Estimator (SRORE)*** by replacing OLSE by RE in the ME in (2.6) and given by

$$\hat{\beta}_{SRORE}(k) = \hat{\beta}_{RE}(k) + S^{-1}R'\left(\Omega + RS^{-1}R'\right)^{-1}\left(r - R\hat{\beta}_{RE}(k)\right). \tag{2.12}$$

Since $WS^{-1} = S^{-1}W$, we can rewrite the SRORE as follows.

$$\begin{aligned}\hat{\beta}_{SRORE}(k) &= S^{-1}WX'y + S^{-1}R'\left(\Omega + RS^{-1}R'\right)^{-1}\left(r - RS^{-1}WX'y\right)\\ &= \left(S^{-1} - S^{-1}R'\left(\Omega + RS^{-1}R'\right)^{-1}RS^{-1}\right)\left(WX'y + R'\Omega^{-1}r\right)\\ &= \left(S + R'\Omega^{-1}R\right)^{-1}\left(WX'y + R'\Omega^{-1}r\right)\end{aligned} \tag{2.13}$$

When $k = 0$, $\hat{\beta}_{SRORE}(0) = \hat{\beta}_{ME}$; When $R = 0$, $\hat{\beta}_{SRORE}(k) = \hat{\beta}_{RE}(k)$

The expectation vector, bias vector, dispersion matrix and Mean Square Error Matrix of SRORE can be shown as follows.

$$E\left[\hat{\beta}_{SRORE}(k)\right] = \beta + A(W-I)S\beta \tag{2.14}$$

$$B\left[\hat{\beta}_{SRORE}(k)\right] = B\left[\hat{\beta}_{SRORE}(k)\right] - \beta = A(W-I)S\beta \tag{2.15}$$

$$D\left[\hat{\beta}_{SRORE}(k)\right] = \sigma^2 A\left(WSW + R'\Omega^{-1}R\right)A \tag{2.16}$$

and

$$MSE\left[\hat{\beta}_{SRORE}(k)\right] = \sigma^2 A\left(WSW + R'\Omega^{-1}R\right)A + A(W-I)S\beta\beta'S(W-I)A \tag{2.17}$$

respectively, where $A = \left(S + R'\Omega^{-1}R\right)^{-1}$.

In this paper we mainly consider the estimators based on shrinkage parameter (k) and ME for comparisons. Therefore the mean square error matrices for the other estimators are not given.

3 Comparisons Among Biased Estimators

Now we compare the Stochastic Restricted Ordinary Ridge Estimator with Ridge Estimator, Mixed Estimator and Stochastic Mixed Ridge Estimator using mean square error matrix criterion.

Since the $\hat{\beta}_{ME}$ is an unbiased estimator, the mean square error matrix of $\hat{\beta}_{ME}$ can be shown as

$$MSE\left(\hat{\beta}_{ME}\right)=\sigma^2 A \tag{3.1}$$

The mean square error matrices of $\hat{\beta}_{RE}\left(k\right)$ and $\hat{\beta}_{SMRE}$ are given by

$$MSE\left[\hat{\beta}_{RE}\left(k\right)\right]=\sigma^2 WS^{-1}W+k^2(S+kI)^{-1}\beta\beta'\left(S+kI\right)^{-1} \tag{3.2}$$

and

$$MSE\left[\hat{\beta}_{SMRE}\right]=\sigma^2 WAW+k^2\left(S+kI\right)^{-1}\beta\beta'\left(S+kI\right)^{-1} \tag{3.3}$$

respectively.

The mean square error matrix differences for the above estimators are given below:

$$\Delta_1=MSEM\left[\hat{\beta}_{ME}\right]-MSEM\left[\hat{\beta}_{SRORE}\left(k\right)\right]=\sigma^2 D_1-b_2b_2' \tag{3.4}$$

$$\Delta_2=MSEM\left[\hat{\beta}_{RE}\left(k\right)\right]-MSEM\left[\hat{\beta}_{SRORE}\left(k\right)\right]=\sigma^2 D_2+b_1b_1'-b_2b_2' \tag{3.5}$$

$$\Delta_3=MSEM\left[\hat{\beta}_{SMRE}\right]-MSEM\left[\hat{\beta}_{SRORE}\left(k\right)\right]=\sigma^2 D_3+b_1b_1'-b_2b_2' \tag{3.6}$$

where $D_1=A-A\left(WSW+R'\Omega^{-1}R\right)A$, $D_2=WS^{-1}W-A\left(WSW+R'\Omega^{-1}R\right)A$,

$D_3=WAW-A\left(WSW+R'\Omega^{-1}R\right)A$, $b_1=-k(S+kI)^{-1}\beta$ and $b_2=A\left(W-I\right)S\beta$.

Now we can state the following theorems.

Theorem 3.1 The Stochastic Restricted Ordinary Ridge Estimator is superior to the Mixed Estimator in the mean square error matrix sense if and only if $b_2'D_1^{-1}b_2\le\sigma^2$.

Proof: The MSEM difference between the SRORE and ME given in (3.4) is $\Delta_1 = \sigma^2 D_1 - b_2 b_2'$. To apply lemma 2 (see appendix) to (3.4) we need to prove that D_1 is a positive definite matrix.

Note that
$$\begin{aligned} D_1 &= A - A\left(WSW + R'\Omega^{-1}R\right)A \\ &= A\left[A^{-1} - \left(WSW + R'\Omega^{-1}R\right)\right]A \\ &= A\left[S + R'\Omega^{-1}R - WSW - R'\Omega^{-1}R\right]A \\ &= AW\left[W^{-1}SW^{-1} - S\right]WA \\ &= kAW\left[kS^{-1} + 2I\right]WA \end{aligned}$$
This implies that D_1 is clearly a positive definite matrix. Hence according to lemma 2, the SRORE is superior to ME if and only if $b_2' D_1^{-1} b_2 \le \sigma^2$. This completes the proof.

Theorem 3.2 When the maximum eigenvalue of $A\left(WSW + R'\Omega^{-1}R\right)A\left(WS^{-1}W\right)^{-1}$ is less than 1, then the SRORE is superior to the RE in the mean square error sense if and only if $b_2'\left(\sigma^2 D_2 + b_1 b_1'\right)b_2 \le 1$.

Proof: The MSEM difference between the SRORE and RE given in (3.5) is $\Delta_2 = \sigma^2 D_2 + b_1 b_1' - b_2 b_2'$.

To apply lemma 3 (see appendix) one required condition is that $D_2 = WS^{-1}W - A\left(WSW + R'\Omega^{-1}R\right)A$ to be a positive definite matrix.

It is obvious that $WS^{-1}W > 0$ and $A\left(WSW + R'\Omega^{-1}R\right)A \ge 0$.

According to lemma 4 (see appendix), $WS^{-1}W > A\left(WSW + R'\Omega^{-1}R\right)A$ if and only if $\lambda_1 < 1$, where λ_1 is the maximum eigenvalue of
$$A\left(WSW + R'\Omega^{-1}R\right)A\left(WS^{-1}W\right)^{-1}.$$
Therefore D_2 is a positive definite matrix. Then according to lemma 3, Δ_2 is a nonnegative definite matrix if and only if $b_2'\left(\sigma^2 D_2 + b_1 b_1'\right)b_2 \le 1$. This completes the proof of the theorem.

Theorem 3.3 When the maximum eigenvalue of $A\left(WSW + R'\Omega^{-1}R\right)A\left(WAW\right)^{-1} < 1$, then the SRORE is superior to the SMRE in the mean square error sense if and only if $b_2'\left(\sigma^2 D_3 + b_1 b_1'\right)b_2 \le 1$.

Proof: The MSEM difference between the SRORE and SMRE given in (3.6) is $\Delta_3 = \sigma^2 D_3 + b_1 b_1' - b_2 b_2'$.

To show that $\Delta_3 \geq 0$, lemma 3 (see appendix) can be used. A requirement to apply lemma 3 is that D_3 to be a positive definite matrix. It is clear that $WAW > 0$ and $A\left(WSW + R'\Omega^{-1}R\right)A \geq 0$.

According to lemma 4 (see appendix), $WAW > A\left(WSW + R'\Omega^{-1}R\right)A$ if and only if $\lambda_2 < 1$, where λ_2 is the maximum eigenvalue of $A\left(WSW + R'\Omega^{-1}R\right)A\left(WAW\right)^{-1}$. Therefore D_3 is a positive definite matrix. Then according to lemma 3, Δ_3 is a nonnegative definite matrix if and only if $b_2'\left(\sigma^2 D_3 + b_1 b_1'\right)b_2 \leq 1$. This completes the proof.

4 Numerical Example and Monte Carlo Simulation

To illustrate our theoretical results, we consider the data set on Total National Research and Development Expenditures as a Percent of Gross National product originally due to Gruber (1998) and later considered by Akdeniz and Erol (2003) and Li and Yang (2011). The data set is given below:

$$X = \begin{pmatrix} 1.9 & 2.2 & 1.9 & 3.7 \\ 1.8 & 2.2 & 2.0 & 3.8 \\ 1.8 & 2.4 & 2.1 & 3.6 \\ 1.8 & 2.4 & 2.2 & 3.8 \\ 2.0 & 2.5 & 2.3 & 3.8 \\ 2.1 & 2.6 & 2.4 & 3.7 \\ 2.1 & 2.6 & 2.6 & 3.8 \\ 2.2 & 2.6 & 2.6 & 4.0 \\ 2.3 & 2.8 & 2.8 & 3.7 \\ 2.3 & 2.7 & 2.8 & 3.8 \end{pmatrix}, y = \begin{pmatrix} 2.3 \\ 2.2 \\ 2.2 \\ 2.3 \\ 2.4 \\ 2.5 \\ 2.6 \\ 2.6 \\ 2.7 \\ 2.7 \end{pmatrix}$$

The four column of the 10×4 matrix X comprise the data on x_1, x_2, x_3 and x_4 respectively, and y is the predictor variable. Note that the eigen values of S are $\lambda_1 = 302.9626$, $\lambda_2 = 0.7283$, $\lambda_3 = 0.0447$ and $\lambda_4 = 0.0345$ and the condition number of X is approximately 8781.53. This implies the existence of multicollinearity in the data set. The OLSE
is given by

$$\hat{\beta}_{OLSE} = S^{-1}X'y = \left(0.6455, 0.0896, 0.1436, 0.1526\right)'$$

with $MSE\left(\hat{\beta}_{OLSE},\beta\right)=0.0808$ and $\hat{\sigma}^2=0.0015$.

Consider the following stochastic restrictions (Li and Yang, 2011)

$$r = R\beta + \nu \, , R = \left(1,-2,-2,-2\right)' \, , \nu \sim N\left(0,\hat{\sigma}^2 = 0.0015\right)$$

Using equations (3.1), (3.2), (3.3) and (2.17) for different shrinkage parameter (*k*) values, the SMSE values for RE, ME, SMRE and SRORE are derived, and given in Table 1.

Table 1:The estimated Scalar Mean Square Error (SMSE) values of RE, ME, SMRE and SRORE for different shrinkage parameter (*k*) values.

k	**RE**	**ME**	**SMRE**	**SRORE**
10	0.2636	0.0451	0.2636	0.0285
5	0.2599	0.0451	0.2599	0.0259
2	0.2456	0.0451	0.2456	0.0180
1	0.2304	0.0451	0.2303	0.0120
0.95	0.2291	0.0451	0.229	0.0116
0.9	0.2276	0.0451	0.2275	0.0111
0.85	0.226	0.0451	0.2259	0.0107
0.8	0.2242	0.0451	0.2241	0.0102
0.75	0.2223	0.0451	0.2222	0.0097
0.7	0.2202	0.0451	0.2201	0.0092
0.65	0.2179	0.0451	0.2177	0.0087
0.6	0.2152	0.0451	0.2151	0.0082
0.55	0.2122	0.0451	0.212	0.0077
0.5	0.2088	0.0451	0.2086	0.0072
0.45	0.2048	0.0451	0.2045	0.0066
0.4	0.2001	0.0451	0.1997	0.0061
0.35	0.1944	0.0451	0.194	0.0056
0.3	0.1873	0.0451	0.1868	0.0051
0.25	0.1784	0.0451	0.1776	0.0047
0.2	0.1664	0.0451	0.1653	0.0045
0.15	0.1497	0.0451	0.1479	0.0046
0.1	0.1247	0.0451	0.1215	0.0055
0.05	0.0858	0.0451	0.0782	0.0096
0	0.0808	0.0451	0.0451	0.0451

From Table 1 we can notice that the proposed estimator has the smallest scalar mean square error values than RE, ME and SMRE for all values of k except 0. When k increases, SMSE value for RE and SMRE increases. However there is no big difference in the SMSE between RE and SMRE for $k > 1$. These results can be graphically explained by drawing Figure 1.

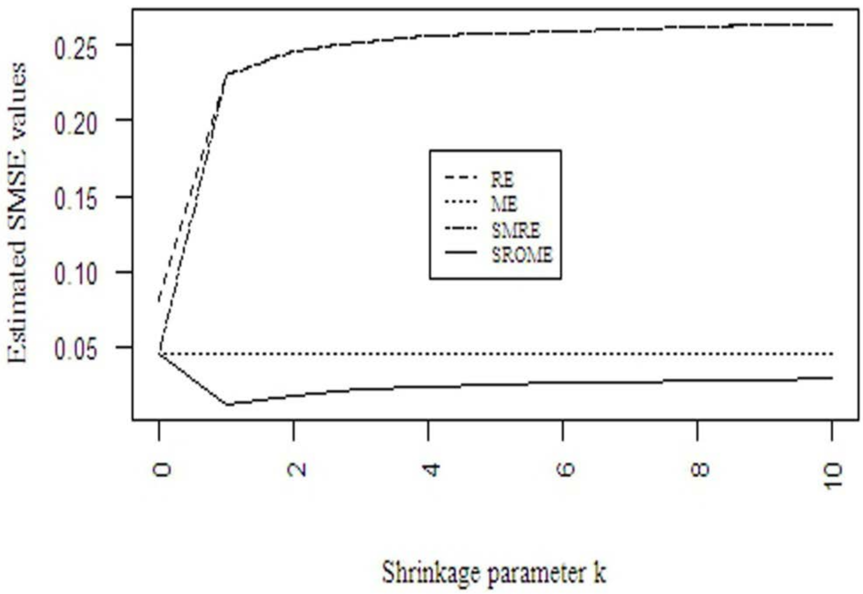

Figure 1: Estimated SMSE values of RE, ME, SMRE and SRORE

For further explanation we perform the Monte Carlo Simulation study by considering different levels of multicollinearity. Following McDonald and Galarneau (1975) we can get explanatory variables as follows:

$$x_{ij} = \left(1-\rho^2\right)^{1/2} z_{ij} + \rho z_{i,p+1}, \; i = 1,2,...,n, \;\; j = 1,2,..., p,$$

where z_{ij} is an independent standard normal pseudo random number, and ρ is specified so that the theoretical correlation between any two explanatory variables is given by ρ^2. A dependent variable is generated by using the equation.

$$y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \varepsilon_i, \; i = 1,2,...,n,$$

where ε_i is a normal pseudo random number with mean zero and variance σ_i^2. In this study we choose $\beta = (\beta_1, \beta_2, \beta_3, \beta_4)' = (1/2, 1/2, 1/2, 1/2)'$ for which

$\beta'\beta = 1$ (see Kibria, 2003), $n = 30$, $p = 4$ and $\sigma_i^2 = 1$. Three different sets of correlations are considered by selecting the value as $\rho = 0.8$, 0.9, 0.99 and 0.999. Using equations (3.1), (3.2), (3.3) and (2.17) for different shrinkage parameter (*k*) values to represent the different levels of multicollinearity, the SMSE values for RE, ME, SMRE and SRORE are derived and given in Table 2 and Table 3.

Table 2: The estimated Scalar Mean Square Error values of RE, ME, SMRE and SRORE for different shrinkage parameter (*k*) values at $\rho = 0.8$ and 0.9.

k	**RE**	**ME**	**SMRE**	**SRORE**	**RE**	**ME**	**SMRE**	**SRORE**
	$\rho = 0.8$				$\rho = 0.9$			
10	0.0966	0.1966	0.0880	0.0994	0.1417	0.3287	0.1303	0.1404
5	0.1271	0.1966	0.1127	0.1214	0.1715	0.3287	0.1476	0.1621
2	0.1717	0.1966	0.1501	0.1551	0.2526	0.3287	0.2076	0.2207
1	0.1955	0.1966	0.1702	0.1731	0.3124	0.3287	0.2535	0.2626
0.95	0.1969	0.1966	0.1714	0.1742	0.3162	0.3287	0.2565	0.2652
0.9	0.1983	0.1966	0.1725	0.1752	0.32	0.3287	0.2595	0.2679
0.85	0.1996	0.1966	0.1737	0.1763	0.324	0.3287	0.2625	0.2707
0.8	0.2011	0.1966	0.1749	0.1773	0.3281	0.3287	0.2657	0.2735
0.75	0.2025	0.1966	0.1762	0.1784	0.3322	0.3287	0.2689	0.2763
0.7	0.204	0.1966	0.1774	0.1795	0.3365	0.3287	0.2722	0.2793
0.65	0.2054	0.1966	0.1786	0.1806	0.3409	0.3287	0.2756	0.2823
0.6	0.2069	0.1966	0.1799	0.1818	0.3454	0.3287	0.2791	0.2854
0.55	0.2084	0.1966	0.1812	0.1829	0.35	0.3287	0.2827	0.2885
0.5	0.21	0.1966	0.1825	0.1841	0.3547	0.3287	0.2863	0.2918
0.45	0.2115	0.1966	0.1838	0.1852	0.3595	0.3287	0.2901	0.2951
0.4	0.2131	0.1966	0.1852	0.1864	0.3645	0.3287	0.294	0.2984
0.35	0.2147	0.1966	0.1865	0.1876	0.3696	0.3287	0.2979	0.3019
0.3	0.2163	0.1966	0.1879	0.1889	0.3748	0.3287	0.302	0.3055
0.25	0.218	0.1966	0.1893	0.1901	0.3801	0.3287	0.3061	0.3091
0.2	0.2196	0.1966	0.1907	0.1914	0.3856	0.3287	0.3104	0.3128
0.15	0.2213	0.1966	0.1921	0.1926	0.3913	0.3287	0.3148	0.3167
0.1	0.223	0.1966	0.1936	0.1939	0.3971	0.3287	0.3193	0.3206
0.05	0.2248	0.1966	0.1951	0.1952	0.403	0.3287	0.324	0.3246
0	0.2265	0.1966	0.1966	0.1966	0.4091	0.3287	0.3287	0.3287

Table 3: The estimated Scalar Mean Square Error values of RE, ME, SMRE and SRORE for different shrinkage parameter (k) values at $\rho = 0.99$ and 0.999.

k	**RE**	**ME**	**SMRE**	**SRORE**	**RE**	**ME**	**SMRE**	**SRORE**
	$\rho = 0.99$				$\rho = 0.999$			
10	1.9195	2.4325	1.9150	0.8768	22.4564	22.939	22.4559	8.0947
5	1.7189	2.4325	1.7029	0.7926	22.1330	22.939	22.1309	7.9633
2	1.4153	2.4325	1.3428	0.7086	21.2289	22.939	21.2161	7.5951
1	1.348	2.4325	1.1611	0.7821	19.9111	22.939	19.863	7.0812
0.95	1.3548	2.4325	1.1561	0.7954	19.7848	22.939	19.7319	7.0339
0.9	1.3641	2.4325	1.1522	0.8105	19.6471	22.939	19.5886	6.9826
0.85	1.3762	2.4325	1.15	0.828	19.4963	22.939	19.4313	6.9271
0.8	1.3918	2.4325	1.1495	0.848	19.3306	22.939	19.2579	6.8667
0.75	1.4113	2.4325	1.1514	0.8711	19.1477	22.939	19.0658	6.8008
0.7	1.4357	2.4325	1.156	0.8976	18.9449	22.939	18.8519	6.7288
0.65	1.4657	2.4325	1.1639	0.9283	18.7187	22.939	18.6123	6.65
0.6	1.5025	2.4325	1.1758	0.9637	18.4652	22.939	18.3423	6.5635
0.55	1.5475	2.4325	1.1927	1.0047	18.1794	22.939	18.0358	6.4685
0.5	1.6024	2.4325	1.2157	1.0523	17.8553	22.939	17.6851	6.3643
0.45	1.6693	2.4325	1.2462	1.1079	17.4852	22.939	17.2806	6.2506
0.4	1.751	2.4325	1.2858	1.1729	17.0604	22.939	16.8096	6.1276
0.35	1.8508	2.4325	1.337	1.2493	16.5705	22.939	16.2559	5.9981
0.3	1.9731	2.4325	1.4025	1.3395	16.0053	22.939	15.599	5.8689
0.25	2.1238	2.4325	1.4863	1.4466	15.3597	22.939	14.8148	5.7585
0.2	2.3105	2.4325	1.5932	1.5747	14.6499	22.939	13.881	5.7133
0.15	2.5434	2.4325	1.7301	1.729	13.9733	22.939	12.8067	5.8586
0.1	2.8366	2.4325	1.9062	1.9165	13.7501	22.939	11.7707	6.5779
0.05	3.2098	2.4325	2.1342	2.1466	16.0081	22.939	11.9179	9.3374
0	3.6912	2.4325	2.4325	2.4325	36.2314	22.939	22.939	22.939

The condition numbers of the data sets when $\rho = 0.8$, 0.9, 0.99 and 0.999 are 13.23, 29.33, 319.46 and 3217.48 respectively. According to Table 2 and 3 when multicollinearity increases the SRORE has the smallest scalar mean square error values than SMRE, RE and ME when k becomes large. Nevertheless the SMRE has smallest scalar mean square values than SROME, RE and ME at $\rho = 0.8$ and $\rho = 0.9$. These results can be graphically explained by drawing Figure 2, Figure 3, Figure 4 and Figure 5.

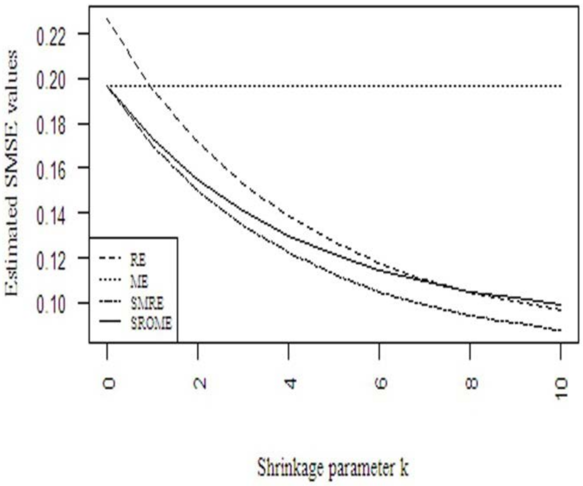

Figure 2: Estimated SMSE values of RE, ME, SMRE and SRORE for ρ =0.8.

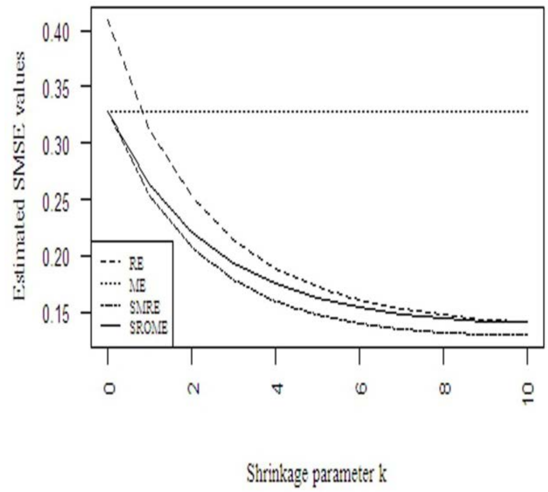

Figure 3: Estimated SMSE values of RE, ME, SMRE and SRORE for ρ =0.9.

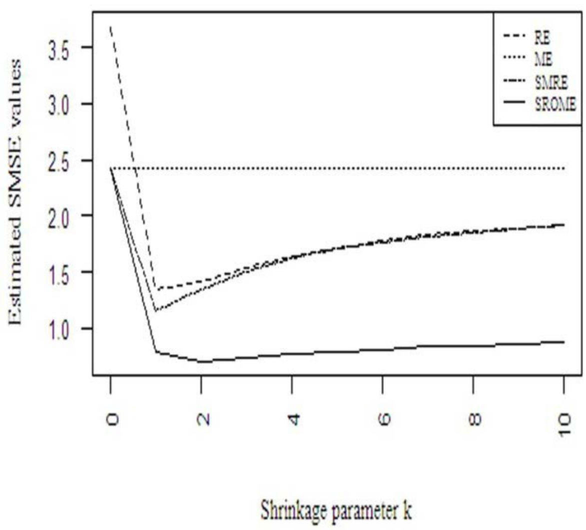

Figure 4: Estimated SMSE values of RE, ME, SMRE and SRORE for ρ =0.99.

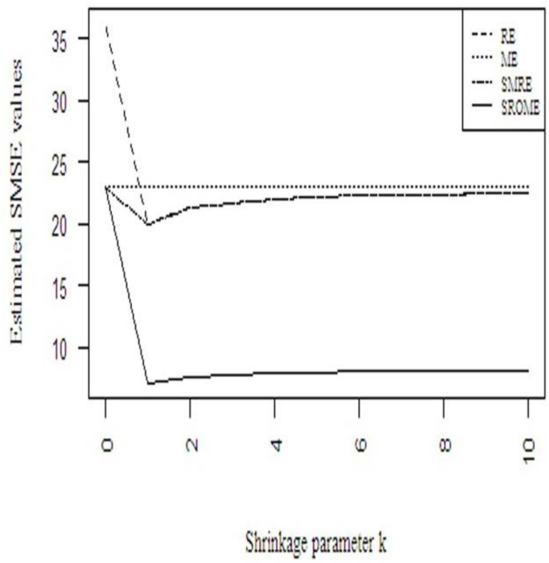

Figure 5: Estimated SMSE values of RE, ME, SMRE and SRORE for ρ =0.999.

5 Conclusion

In this paper we proposed another ridge type estimator, namely Stochastic Restricted Ordinary Ridge Estimator (SRORE) in the multiple linear regression model when the stochastic restrictions are available in addition to the sample information and when the explanatory variables are multicollinear. Necessary and sufficient conditions for the superiority of the Stochastic Restricted Ordinary Ridge Estimator (SROME) over the Mixed Estimator (ME), Ridge Estimator (RE) and Stochastic Mixed Ridge Estimator (SMRE) are obtained using Mean Square Error Matrix (MSEM) criterion. For the numerical example, the proposed estimator has the smallest scalar mean square errors than ME, RE and SMRE for all values of k except 0. When analyzing the simulation results it was noted that the proposed estimator has the smallest scalar mean square error when multicollinearity is large and $k > 0.1$.

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Appendix

Lemma 1 Assume square matrixes A, C are not singular, and B, D are matrixes with proper orders, then $\left(A+BCD\right)^{-1} = A^{-1} - A^{-1}B\left(C^{-1}+DA^{-1}B\right)^{-1} DA^{-1}$.

Proof: see Rao and Touterburg (1995).

Lemma 2 Let M be a positive definite matrix, namely $M > 0$, α be some vector, then $M - \alpha\alpha' \geq 0$ if and only if $\alpha' M^{-1}\alpha \leq 1$.
Proof: see Farebrother (1976).

Lemma 3 Let $\hat{\beta}_j = A_j y$, $j = 1,2$ be two competing linear estimators of β. Suppose that $D = D\left(\hat{\beta}_1\right) - D\left(\hat{\beta}_2\right) > 0$, where $D\left(\hat{\beta}_j\right)$, $j = 1,2$ denotes the dispersion matrix of $\hat{\beta}_j$. Then $\Delta\left(\hat{\beta}_1, \hat{\beta}_2\right) = MSE\left(\hat{\beta}_1, \beta\right) - MSE\left(\hat{\beta}_1, \beta\right) \geq 0$ if and only if $d_2'\left(D + d_1 d_1'\right) d_2 \leq 1$, where $MSE\left(\hat{\beta}_j, \beta\right)$, d_j denote the mean square error matrix and bias vector of $\hat{\beta}_j$, respectively.
Proof: see Trenkler and Toutenburg (1990).

Lemma 4 Let $n \times n$ matrices $M > 0$, $N \geq 0$, then $M > N$ if and only if $\lambda_1\left(NM^{-1}\right) < 1$. where $\lambda_1\left(NM^{-1}\right)$ is the largest eigenvalue of the matrix NM^{-1}.
Proof: see Wang et al. (2006).

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