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Forecasting Volatility in Indian Stock Market using State Space Models

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Abstract

The paper examines and compares forecasting ability of Autoregressive Moving Average (ARMA) and Stochastic Volatility models represented in the state space form and Kalman Filter is used as an estimator for the models.The models are applied in the context of Indian stock market. For estimation purpose, daily values of Sensex from Bombay Stock Exchange (BSE) are used as the inputs. The results of the study confirm the volatility forecasting capabilities of both the models. Finally, we interpreted that which model performs better in the out-of-sample forecast for h-step ahead forecast. Forecast errors of the volatility were found in favour of SV model for a 30-day ahead forecast. This also shows that Kalman filter can be used for better estimates and forecasts of the volatility using state space models.

Mathematics Subject Classification: 60H30
Keywords: ARMA; Kalman Filter; State Space; Stochastic Volatility

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1 Introduction

To fully understand prices, asset returns and risk management, one has to understand the nature and behaviour of the volatility. This is not just explained by simple decision rules, but is the inter-relation of many other market factors, which determine in the end, prices, returns and market volatility. Understanding the volatility of the market is important if one wants to make some prediction for the future for ex. Sensex index.

Sensex is the most followed market index in the Indian stock market and consists of the 30 largest and most actively traded stocks and representative of various sectors on the BSE. The Indian Stock Market has been studied during the last two decades along with financial markets of different countries. In Indian context variants of GARCH models have been considered for study, a large volume of literature focuses on modeling volatility using these models. The present paper tests an alternate modeling technique for the estimation of the volatility in the Indian stock market by using State Space (SS) model.

The State Space models were first introduced by control engineers and physicists for modeling of continuously changing unobserved state variable. The unknown model parameters in such models were estimated by the Kalman filter (KF), popularly named after Kalman (1960). This Kalman Filter algorithm plays a central role in the modeling, estimating and further predicting the states of State Space models.

Several studies have reported applications of SSModel in estimating price volatility [1]. To address the volatility prediction from the data, it has been suggested that such models should be captured by the Kalman Filter Model [2]. Yet, there is also another reason why these models are becoming more popular, financial econometricians began to understand the appeal of State Space models because of their ability to represent complex dynamics equations via a simple structure of matrices. Further, it was found easier to apply the Kalman Filter to SS Models because of the nature of its very powerful and flexible recursive structure and ability to forecast in missing data.

There are two popular approaches to deal with volatility in State Space representation: ARMA and stochastic volatility (SV) approaches. The ARMA model , focuses on capturing the effects of volatility by using price indices directly [3]. On the other hand, the stochastic volatility models the time-varying variance as a stochastic process which can be estimated using Quasi Maximum Likelihood methods [4, 5]. While these studies provide useful modeling approaches of volatility, the predictive ability of competing models needs to be examined for out-of-sample forecast performance. This is of particular importance at least to researchers and investors that require volatility forecasts and interval forecasts to estimate whether an exchange rate will fluctuate within a specified zone.

The bigger problem is narrowed down to model estimation after both the models are formulated in the state space form. It is well known that for linear system with Gaussian innovations the Kalman filter is an optimal filter (in the sense of minimizing mean squared errors)[6]. Some of the important literature which presented and estimated the various models in a state space form are:

Harvey and Shephard(1996)[7] propsed a stochastic volatility model that can be estimated by a quasi-maximum likelihood procedure by transforming to a linear state-space form. The method is extended to handle correlation between the two disturbances in the model and applied to data on stock returns. They conclude that QML method for estimating the parameters in an SV model is relatively simple and has produced plausible empirical results.

Koopman S.J. (1997)[8] presented a new exact solution for the initialization of the Kalman filter for state space models with diffuse initial conditions. He proposed a regression model with stochastic trend, seasonal and other non stationary ARIMA components which requires a (partially) diffuse initial state vector. He proposed an easy to implement and computationally efficient analytical solution and exact solution for smoothing and handled missing observations.

Durbin J.(2004)[9] presents a broad general review of the state space approach to time series analysis by introducing linear Gaussian state space model and Kalman filter and smoother are also described. He also introduces an application to real data which is presented in his work.

Choudhry and Wu(2008)[2] investigates the forecasting ability of four different GARCH models and the Kalman filter method. Forecast errors based on 20 UK company daily stock return forecasts were employed to evaluate out-of-sample forecasting ability of both GARCH models and Kalman method. Measures of forecast errors very much support the Kalman filter approach. Among the GARCH models the GJR model appeared to provide more accurate forecasts.

Tsyplakov A. (2010)[10] aimed to provide a straightforward and sufficiently accessible demonstration of some known procedures for stochastic volatility model. He reviews the important related concepts and has given an informal derivations of the methods. He presented a framework to forecast SV model with QML estimation and also presented a detailed derivations of extended SV models.

The study described in present paper can be justified on following ideas. The evidence from other developing markets provide mixed evidence of forecasting performances in volatility models[11, 12, 13]. However, there has been no comprehensive study of State Space Models of volatility in India, which is one of the fastest developing markets. Hence, the present paper is devoted to compare the volatility forecasting using State Space models in the Indian stock exchange namely, Bombay Stock Exchange (BSE). Additionally, the Indian economy has registered a recession in the recent past and several models have been proposed and are under test to capture the salient stylized facts. This is the first study which examines the issue of forecasting of volatility in the Indian context using State Space Kalman Filter estimator.

The present paper uses the daily closing values of BSE-Sensex for the period 01 January 2006 to 22 August 2013. The daily index values of Sensex are collected from the official websites of BSE[14]. The selected index have enough number of observations to perform time-series analysis on the models to get meaningful results. We have modeled the volatility forecast using a powerful generic state space modeling (SSM) toolbox for Matlab [15]. The models are represented using SSM Toolbox and further estimation methods available in toolbox of MATLAB from Mathworks.

The rest of paper is organized as follows. Section 2 gives a brief overview of the selected volatility models along with the state space representation. We then estimate and analyse the model by presenting the main results of the paper. In next section, out-of-sample forecast of the estimators are discussed. Section 4 explores the comparative performance of the various forecasting models and a 30-day ahead forecast is given with tabular and graphical representation. Section 5 gives the concluding remarks.

2 Models Overview & State Space Representation

State space methods are tools for investigation of state space models, as they allow one to estimate the unknown parameters along with the time varying states. It can also be used to assess the uncertainty of the estimates, to forecast future states and observations. The following sections will review the basic model representation in State Space. The model parameters estimation is briefly discussed in subsequent sections.

2.1 Linear Gaussian State Space Model

This section provides a brief review of linear Gaussian state space model. Let y_t denote an $p \times 1$ observation vector related to an $m \times 1$ vector of unobservable components α_t (states sequences), by the so-called measurement equation eq 1,

$$y_t = Z_t\alpha_t + \epsilon_t, \quad \epsilon_t \sim N(0, H_t) \tag{1}$$

$$\alpha_{t+1} = c_t + T_t\alpha_t + R_t\eta_t, \quad \eta_t \sim N(0, Q_t) \tag{2}$$

The evolution of the states is governed by the process or state equation 2: Thus the matrices Z_t, c_t, T_t, R_t, H_t, Q_t, a_1, P_1 are required to define a linear Gaussian state space model [16, 1]. The matrix Z_t is the state to observation linear transformation matrix, for univariate models it is a row vector $m \times 1$. The matrix c_t is the same size as the state vector, and is the constant in the state update equation, although it can be dynamic or dependent on model parameters. The square matrix T_t $[m \times m]$ defines the time evolution of states. The matrix R_t $[m \times r]$ transforms general disturbance into state space, and exists to allow for more varieties of models. H_t $[p \times p]$ and Q_t $[r \times r]$ are Gaussian variance matrices governing the disturbances, and a_1 and P_1 are the initial conditions[17]. The specification of the state space model is completed by the initial conditions concerning the distribution of $\alpha_1 \sim N(a_1, P_1), \forall t$.

2.2 Autoregressive Moving Average (ARMA) Model

ARMA models are frequently used for the analysis in the form of return series. An ARMA model combines the Auto Regressive and Moving Average models into a compact form so that the number of parameters used is kept small. Here, we have used the ARMA representation to model volatility using the direct observation sequences as shown by [18, 19].

In equation (3) y_t is a scalar time series observations, represented in ARMA*(p, q)* formulation [1]. Here auto regression order is p and moving average order is q. ϕ are auto regressive coefficients and ζ_t are uncorrelated disturbances.

$$y_t = \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \zeta_t + \theta_1 \zeta_{t-1} + \ldots + \theta_q \zeta_{t-q}, \quad \zeta_t \sim N(0, \sigma_\zeta^2), \tag{3}$$

The observation equation of SS representaion of ARMA*(p, q)* is:

$$y_t = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \end{bmatrix} \begin{bmatrix} \alpha_t \\ \alpha_{t-1} \\ \vdots \\ \alpha_{t-r-1} \end{bmatrix} + \epsilon_t \tag{4}$$

The state equation of ARMA*(p, q)* is:

$$\begin{bmatrix} \alpha_{t+1} \\ \alpha_t \\ \vdots \\ \alpha_{t-r} \end{bmatrix} = \begin{bmatrix} \phi_1 & 1 & 0 & \ldots & 0 \\ \phi_2 & 0 & 1 & \ldots & 0 \\ \vdots & & \ddots & \ddots & 0 \\ \phi_{r-1} & 0 & 0 & \ldots & 1 \\ \phi_r & 0 & 0 & \ldots & 0 \end{bmatrix} \begin{bmatrix} \alpha_t \\ \alpha_{t-1} \\ \vdots \\ \alpha_{t-r-1} \end{bmatrix} + \begin{bmatrix} 1 \\ \theta_1 \\ \vdots \\ \theta_{r-1} \end{bmatrix} \zeta_t \tag{5}$$

where,

$$T = \begin{bmatrix} \phi_1 & 1 & 0 & \ldots & 0 \\ \phi_2 & 0 & 1 & \ldots & 0 \\ \vdots & & \ddots & \ddots & 0 \\ \phi_{r-1} & 0 & 0 & \ldots & 1 \\ \phi_r & 0 & 0 & \ldots & 0 \end{bmatrix}, R = \begin{bmatrix} 1 \\ \theta_1 \\ \vdots \\ \theta_{r-1} \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \end{bmatrix}, H_t = 0 \tag{6}$$

The form given is not the only stale space version of an ARMA model but is a convenient one, where r = max(p, q + 1) and for which some coefficients are to be calculated.

2.3 Stochastic Volatility Model

Stochastic volatility modeling is an active research area. Moreover, SV model is a popular modeling technique used in the literature for non-linear/ non-Gaussian state-space models and hidden Markov models. It is probably the important volatility models, with powerful properties similar to ARCH or GARCH.

This subsection briefly introduces the stochastic volatility(SV) model in discrete time for a observation asset log-returns y_t. The Stochastic volatility can be appropriately represented by the unobserved state variable as shown in equation (7) and (8).

$$y_t = \mu + \sigma_* exp\left(\frac{1}{2}h_t\right)\epsilon_t, \qquad \epsilon_t \sim IID(0,1), \tag{7}$$

with

$$h_{t+1} = \phi h_t + \eta_t, \qquad \eta_t \sim IID(0,\sigma_\eta^2), h_1 \sim N(0,\sigma_\eta^2/(1-\phi^2)), \tag{8}$$

for $t = 1,\ldots,T$. The parameter μ denotes the unconditional expectation of the return process y_t . The scaling parameter σ_* is the average standard deviation with $\sigma_* > 0$. The unobserved log-volatility process is denoted by $h_t = log(\sigma_t^2)$ [20].

Here, the logarithms ensures positivity of (σ_t^2). The regression parameter ϕ is $0 < \phi < 1$ [20]and usually is reported to take on values greater than 0.8. The constant μ will be treated as fixed and set to zero as shown by [21]. Although h_t can be modeled by any stationary autoregressive process, it is common to choose a first-order autoregressive process [1]. The disturbances ϵ_t and η_t are Gaussian white noise[22] where, ϵ_t represents the contents of new information (good or bad news); η_t reflects the shocks to the news flow's intensity.

The SV model in (7) and (8) is also commonly referred to as the log-normal SV model. It represents a state space model where the observation equation describes the relationship between the univariate vector of observations, $y = (y_1,\ldots,y_T)'$, and the state vector. The hidden volatility process $\theta = (h_1,\ldots,h_T)'$ is specified in the state equation, which models the dynamic properties of h_t. As ϵ_t and h_t, which both enter the multiplication in the mean equation, are stochastic, the basic SV model is nonlinear, hence linear approaches cannot be used.

The Kalman Filter cannot be applied directly for estimation purposes. This introduces an estimation problem because no closed expression for the likelihood function exists. Now since the parameters of the SV model cannot be estimated by a direct application of standard maximum likelihood techniques, estimation is conducted by approximation of Likelihood or using simulation-based techniques based on observable past information[23]. There are methods proposed [7, 1]to linearize the SV model by squaring the returns and taking logarithms as shown in equation (9) and (10)

$$log y_t^2 = h_t + log \sigma_*^2 + log \epsilon_t^2, \tag{9}$$

$$h_{t+1} = \phi h_t + \eta_t, \tag{10}$$

where the disturbance terms η_t in the transformed model are assumed to be uncorrelated. Taking logarithms leads to a heavily skewed distribution of $\log\epsilon_t^2$ with a long left-hand tail.

Our model-based estimated actual volatility is fitted well, by the Gaussian distribution using Quasi Maximum Likelihood. The estimates of volatility [24] are obtained by

$$\begin{aligned} \sigma_{t|T}^2 &\equiv Var(y_t|Y_T) \\ E(exp(h_t)|Y_T) &= exp(\mu_{t|T} + s_{t|T}^2/2) \end{aligned} \tag{11}$$

with variance

$$\begin{aligned} Var(\sigma_{t|T}^2) &= E(exp(2h_t)|Y_T) - \{E(exp(h_t)|Y_T)\}^2 \\ &= exp(2\mu_{t|T} + 2s_{t|T}^2)\left\{1 - exp(-s_{t|T}^2)\right\} \end{aligned} \tag{12}$$

The smoothing estimates of the square root of volatility are also calculated by

$$\begin{aligned} \sigma_{t|T}^* &\equiv E(exp(h_t/2)|Y_T) \\ &= exp(\mu_{t|T}/2 + s_{t|T}^2/8) \end{aligned} \tag{13}$$

with variance

$$= exp(\mu_{t|T} + s_{t|T}^2/2)\left\{1 - exp(-s_{t|T}^2/4)\right\} \tag{14}$$

2.4 Kalman Filter

The general form of a state space model defines an observation (or measurement) equation and a state transition (or state) equation, similar to linear gaussian model representing the structure and dynamics of system with noise [2]. The measurement equation describes the relation between observed sequences and unobserved (hidden) state variables. State transition equation describes the dynamics of the state variables based on information from the past. The future behavior of the system can be completely described by the knowledge of the present state and the future input.

The Kalman filter method, is an iterative computational algorithm designed to calculate forecasts and forecast variances for time series state space models. The algorithm is as follows, first, each step of the process allows the next observation to be forecasted based on the previous observation and the forecast of the previous observation. Second, each consecutive forecast is found by updating the previous forecast. Finally, the above process is repeated again. Figure 1 explains the recursion process in an easy and convenient flowchart form.

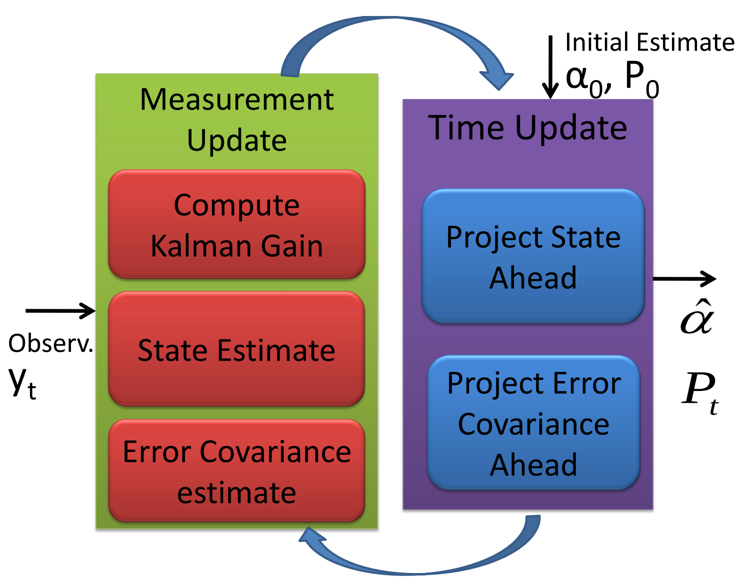

Figure 1: Kalman Filter Recursion

The updates for each forecast are weighted averages of the previous observation and the previous forecast error. The interesting feature of the Kalman filter is that the weights are chosen such that the forecast variances are minimized in least square sense. These weights, are also known as the Kalman gain.

The KF recursively computes the optimal states predictions of y_t which is conditional on past information and also on the variance of their prediction error. The vector v_t is the time t innovation. i.e. the new information in y_t that could not be predicted from knowledge of the past, referred to as the one-step-ahead prediction error. The normal Kalman filter recursion are as follows

Measurement Update:

$$\begin{aligned} v_t &= y_t - Z_t a_t, \\ F_t &= Z_t P_t Z_t^T, \\ K_t &= T_t P_t Z_t^T F_t^{-1}, \\ L_t &= T_t - K_t Z_t, \end{aligned}$$

Time Update:

$$\begin{aligned} a_{t+1} &= c_t + T_t a_t + K_t v_t \\ P_{t+1} &= T_t P_t L_t^T + R_t Q_t R_t^T. \end{aligned}$$

The matrix K_t is sometimes referred to as the Kalman gain. While forecasting it is most desired parameter and all models use it extensively to predict future outcomes. Kalman filter can additionally be exploited to improve upon the measurement of current and past volatility estimates using filtering and state smoothing which are not used in this paper.

3 Estimation and Model Analysis

This study uses closing value of the daily SENSEX during the time period of 01 January 2006 to 22 August 2013. All the stock market index data are collected from the official website of BSE. The daily returns are calculated for each series shown in equation (15).

$$r_t = (log(P_t) - log(P_{t-1})) \tag{15}$$

where r_t is the daily return series, P_t is the current stock price and P_{t-1} is the

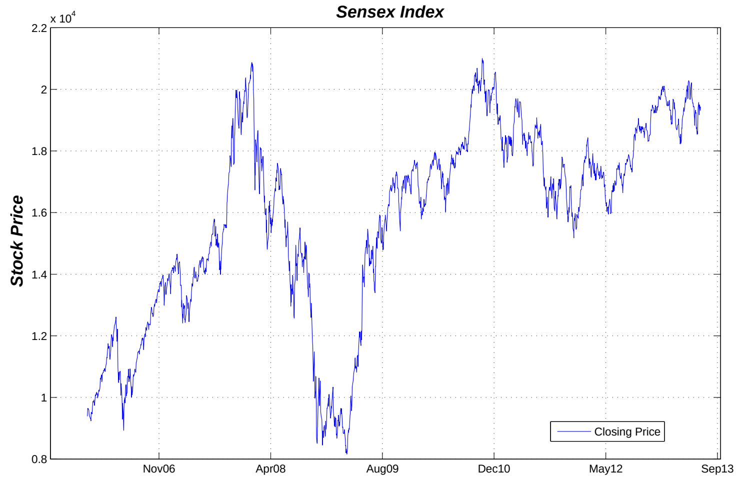

Figure 2: Sensex Closing Index

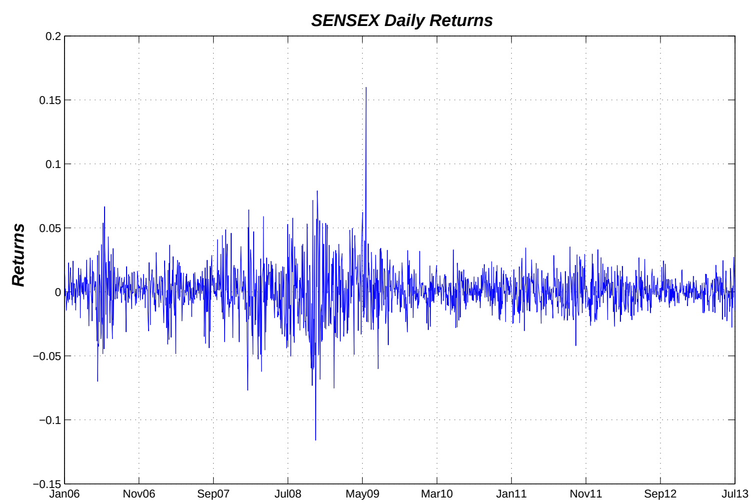

Figure 3: Sensex Return

stock price in the previous period. Figures 2 and 3 plot the price level and the returns on the index over the sample period. Our final working sample consists of 1900 data points for Sensex. In order to make forecasts, the full sample is divided into two parts comprising 1870 in-sample observations from 01 January 2006 to 03 July 2013 and 30 out of sample observations from 04 July 2013 to 22 August 2013 which are used for model performance evaluation. Descriptive

Table 1: Descriptive Statistics(SENSEX)

Period	1/1/2006 to 22/8/2013
Parameter	Value
Mean	0.00039114
Median	0.00097652
Maximum	-0.1160
Minimum	0.1599
Kurtosis	10.1986
Skewness	0.1321
Std. Dev	0.0171

statistics for the Sensex returns series are shown in Table 1. It shows the mean, median, maximum, kurtosis, skewness and standard deviation of the series. As expected for a time series of returns, the mean is close to zero.The mean daily return is 0.00039114. The sample maximum is 0.1599 which happened on 18 May 2009. The volatility (measured as a standard deviation) is 0.0171. The returns are positively skewed (skewness= 0.1321) which indicates that there are more positive than negative outlying returns in Indian Stock Market. The kurtosis coefficient is positive, having high value for the return series(Kurtosis = 10.1986) that is the pointer of leptokurtosis or fat taildness in the underlying distribution.

3.1 Model Order Evaluation

Augmented Dickey Fuller(ADF) test is used to test for the presence of unit root in the returns series. The ADF test statistics is tested for the null hypothesis of unit root at 1% level of significance. A formal application of ADF test on log returns, rejects the null hypothesis of a unit root in the return series. There is rejection because value of ADF statistics is much lower than the critical value for the model *with trend (-2.5691)* and *without trend (-1.9416)* at all the 4-lags shown in Table 2. Hence, the hypothesis that the daily volatility in the Sensex index over the period from January 2006 to July 2013 has a unit root has to be rejected.

Table 2: ADF test on SENSEX

Lags	With Trend stat value	Without Trend stat value	p-value
0	-40.3126	-40.3126	0.001
1	-40.3146	-30.2715	0.001
2	-40.2924	-25.3026	0.001
3	-40.2708	-22.4195	0.001
4	-40.2496	-20.3959	0.001

Several model-selection criteria attempting to overcome the over parameterization problem have been proposed in the literature. The most widely used criteria for ARMA model order selection are Akaike information criterion (AIC) and Bayesian information criterion (BIC). For SV model, this model-selection criteria is not required as we have assumed only AR(1) regression component.

The (p, q)combination that minimizes the AIC should be selected. AIC measures a models goodness of fit in terms of the estimated error variance and penalizes for selecting models with a large number of parameters. The purpose of the penalty term is to avoid over fitting. However, this criterion may give more than one minimum and there is an assumption that the data are normally distributed. ARMA(3, 3) found to be the best model, ARMA(2, 2) is the second best as shown in Table 3.

Table 3: AIC for ARMA (p,q) model

p/q	0	1	2	3	4	5
0	36.4042	31.7016	23.2994	22.0551	21.0147	21.0245
1	25.9341	18.2727	20.0768	16.6292	18.2377	15.5085
2	16.5412	15.5252	14.2032	14.8375	17.3074	14.5653
3	16.7688	14.5645	16.6452	**13.9094**	15.244	21.5604
4	16.7855	15.3837	16.6587	14.8403	16.4298	14.7741
5	16.8792	15.346	16.2668	16.7355	14.894	16.6079

3.2 Model Estimation

Maximum Likelohood estimation is performed for identification of parameters in the both the models. The sufficient conditions for consistency and asymptotic normality of the maximum likelihood estimators and strong consistency of the maximum likelihood estimator is proved in [9]. LogLikelihood is maximised numerically using the kalman covariance, which emerge in computationally convenient forms for the state space model. Details are given by Durbin and Koopman(2001) [1]. Recently, full asymptotic solver based on maximum likelihood estimation of state space models has been provided by Aston and Peng(2011) [17] in SSM Toolbox of Matlab. The *estimate* and *kalman* functions of SSMODEL class were used to perform the estimate and 1-step ahead prediction for ARMA Model.

The model estimates of **ARMA(3,3)** model in state space form after estimation are:

$$T = \begin{bmatrix} 0.004278 & 1 & 0 & 0 \\ 0.9961 & 0 & 1 & 0 \\ -0.0007445 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R = \begin{bmatrix} 1 \\ 1.0714 \\ 0.07902 \\ 0.00003086 \end{bmatrix},$$

$$Z = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, H_t = 0, Q = [59261.9252] \tag{16}$$

The ACF and PACF are used as identification tools, these provide some indication of the broad correlation characteristics of the returns [23]. From the figure 4 the ACF and PACF of squared innovations of ARMA model, there is little indication of correlation, which is clear from the similarity between the graphs. In conclusion, the residuals of the ARMA models are consistent with white noise. This figure shows that ACF die out quickly, indicating that the ARMA process innovations are close to stationary. This is an indication that ARMA models is adequate model class for volatility forecasting as shown in Figure 4.

The QML estimation method was used to estimate the model parameters of SV. This was implemented by combining predefined observation guassian noise with constant and autoregressive model using model concatenation in SSMODEL class in the toolbox . Hence the estimates are obtained by treating ϵ_t and η_t as though they were normal and minimizes the prediction-error via the Kalman filter. To solve the QML estimates similar *estimate* and *kalman*

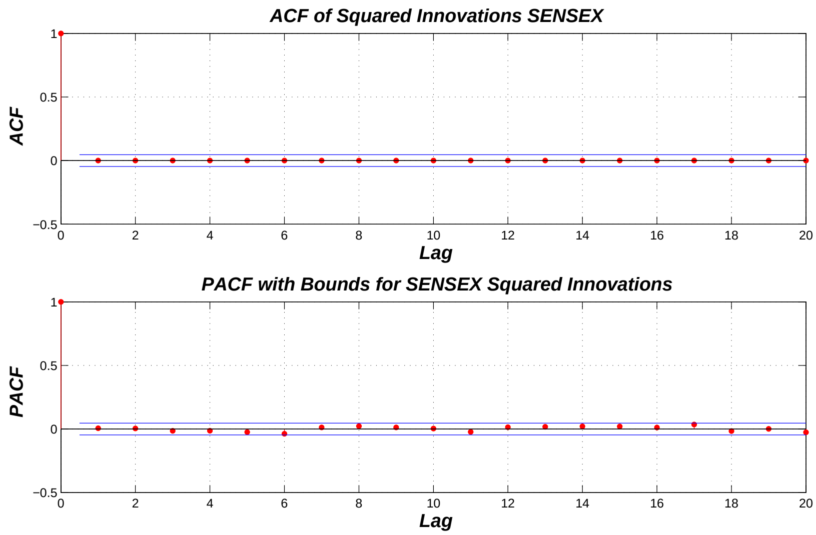

Figure 4: ACF & PCF

function operation was performed on return series with the new updated SV Model parameters. Estimates of **Stochastic Volatility** model in state space form and in Table 4

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 0.9888 \end{bmatrix}, R = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, Z = \begin{bmatrix} 1 & 1 \end{bmatrix}, H_t = [4.8246], Q = [0.0215] \quad (17)$$

Table 4: Parameters Estimates

Variables	**ARMA(3,3)**	**SV Model**
$\phi 1$	0.004278	0.9888
$\phi 2$	0.9961	-
$\phi 3$	-0.0007445	-
$\theta 1$	1.071	-
$\theta 2$	0.07902	-
$\theta 3$	3.086e-05	-
ζ	5.926e+04	0.0215
ϵ	-	4.825

3.3 Evaluation Measures

Four measures are used to evaluate the forecast accuracy ,namely, the mean square error (MSE), the root mean square error (RMSE), the mean absolute error (MAE) and mean absolute percentage error (MAPE). They are defined by

$$
\begin{aligned}
MSE &= \frac{1}{n}\sum_{t=1}^{n}(\hat{\sigma}_t - \sigma_t)^2 \\
RMSE &= \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{\sigma}_t - \sigma_t)^2} \\
MAE &= \frac{1}{n}\sum_{t=1}^{n}|\hat{\sigma}_t - \sigma_t| \\
MAPE &= \frac{1}{n}\sum_{t=1}^{n}(|\hat{\sigma}_t - \sigma_t)/\sigma_t|
\end{aligned}
\tag{18}
$$

where $\hat{\sigma}_t$ is the forecast value and σ_t is the actual value calculated using equation (19). Statistically, actual volatility is often eastimated as the sample standard deviation

$$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \mu)^2} \tag{19}$$

where r_t is the return on day t, and μ is the average return over the T -day period. In this context, the model which has minimum forecast error terms as MSE, RMSE, MAE and MAPE, is the best volatility forecasting model. In Table 5 it clearly shows that both the models has less forecast error values model by using all four evaluation measures.

Table 5: Forecast Error Statistics

	ARMA(3,3)	**SV Model**
MSE	2.5452e-6	2.9666e-8
RMSE	0.0016	1.7224e-4
MAE	0.0013	1.4227e-4
MAPE	0.7760	0.1589

4 Comparison of Out of Sample forecast

After obtaining the daily volatility series, 1-day ahead forecasts are chosen for the forecasting horizon of 30 days. Furthermore, a period has to be chosen for estimating parameters and a period for predicting volatility. The 1/1/2006 to 3/7/2013 of data are used to estimate the models. Thus the first day for which an out-of sample forecast is obtained is 04/07/2013.

Using the estimated models, sequential 1-day ahead forecasts are made. Hence, in total 30 daily volatilities are forecasted. With this setup, the models are required to predict volatility for the above mentioned period. The out of sample forecast for ARMA and Stochastic Volatility models are shown in Figure 5 and Figure 6 respectively. This shows that the volatility forecast varies a lot between both the models. Thus it was more relevant to define a confidence interval of the forecast and plot it along with original observations to give a better idea of the risk in the price index for both the methods.

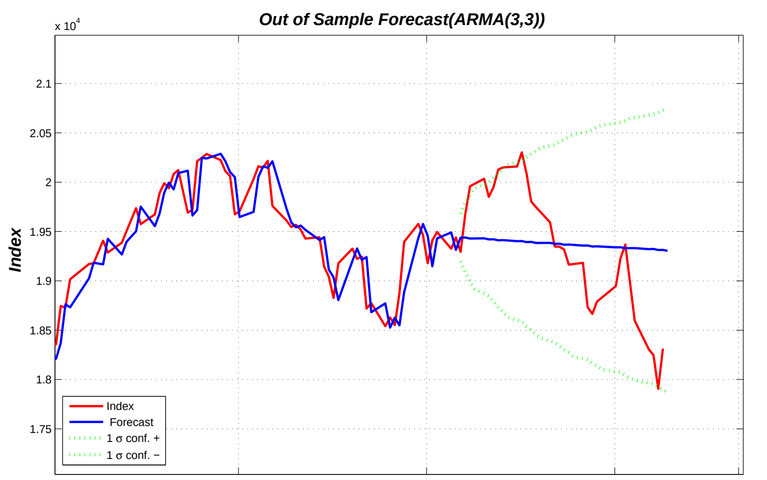

Figure 5: ARMA Out of Sample Forecast

We know that to forecast using the ARMA model, one has to use the price time series. When a price series has been transformed to return series, the conditional distribution of the log return series, will no longer be normal. If logarithms have been taken, the mean and variance of the conditional distribution becomes lognormal[25]. Therefore, to forecast the volatility by ARMA model, we have to convert the price into returns distribution using the following

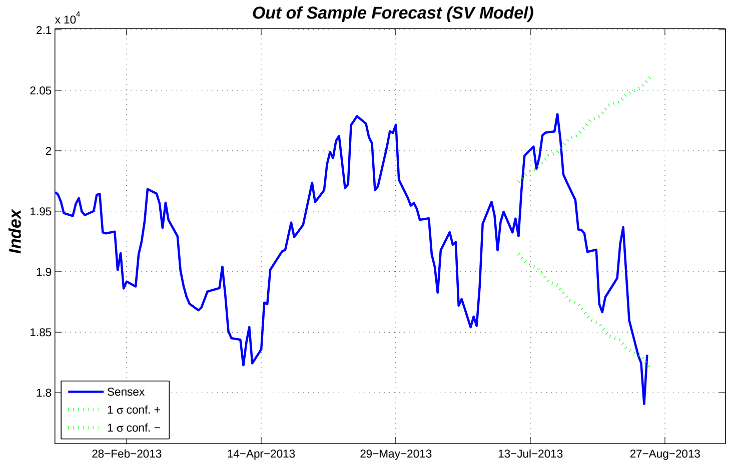

Figure 6: Stochastic Volatility Out of Sample Forecast

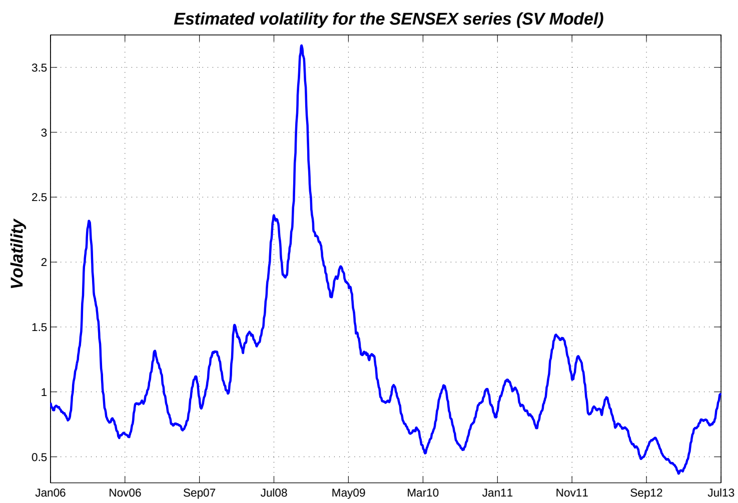

Figure 7: Stochastic Volatility estimate

equation (20).

$$Var(r_t) = log\left(1 + \frac{m2}{(1+m1)^2}\right) \tag{20}$$

Where, $m1$ and $m2$ are mean and variance of forecasted price series of ARMA model respectively. The results of volatility forecasting of SV model is shown in Figure 7. The Figure 8 shows the 30-day ahead point forecast of both the models plotted along with the actual volatility, which is used as benchmark calculated using equation (19). It seems that stochastic volatility has more

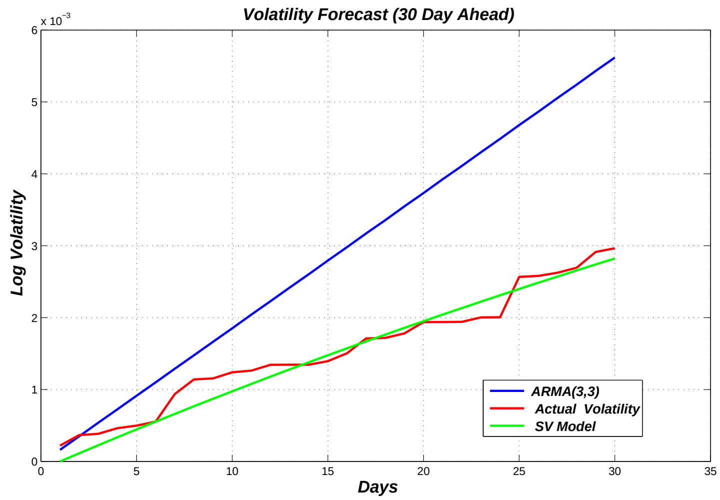

Figure 8: Volatility Forecast Comparison

appropriate forecast as it has lesser residual errors when both models are estimated using maximum-likelihood based technique.

5 Conclusion

This paper examined state space models for forecasting stock market volatility of the Sensex index . The important models considered here are the ARMA and SV models. After estimating the models using SSM toolbox in Matlab, out-of-sample forecasts performance were compared. It was found that the SV model is superior according to the RMSE and other three evaluation measures for 30-day ahead forecast. Unfortunately, the ARMA model has fat tailed residual returns, an empirical study for different noise distribution in this regard would be interesting.

The Kalman filter provides a simple and effective estimation framework for forecasting the volatility. It would be interesting to explore different fat tailed distribution and time varing KF model to fit the class of state space models. The empirical results of this paper provide strong support for the application of the state space model in the Indian stock market. Finally, we have presented a simple forecasting application of the Kalman Filter, which has proved useful to forecast the stock market data.

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