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Soret and Radiation Absorption Effects on the Onset of Magneto-thermosolutal Convection in a Porous Medium

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Abstract

Natural convection with Soret and radiation effects in a binary fluid saturating a horizontal porous layer under the influence of magnetic field is investigated. Adopting the Rosseland approximation for the radiative flux, the onset of instability is studied using the linear stability analysis. The result shows that the radiation absorption parameter delayed the onset of instability with higher values leading to greater stabilization of the system. The Soret parameter has significant effect on convective instability and this is discussed. In the absence of the magnetic field, the effects of the results obtained are more significant. Effects of other governing parameters are quantitatively discussed.

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1 Introduction

Growing interest in the study of double-diffusive convection in porous media has been motivated by its wide range of applications. These include migration of solutes in water-saturated soils, migration of moisture through air contained in fibrous insulations, multi-component melts, storage of nuclear wastes, disposal of waste materials, grain-storage installation and others.

Early studies on the phenomenon of double-diffusive convection are concerned with onset of motion in a horizontal porous layer. On the basis of linear stability theory [1, 2, 3] investigated the onset of double-diffusive convection in a horizontal porous layer. Criteria for the existence of steady and oscillatory convection were derived by these authors. The effect of inclined temperature gradient on thermosolutal instability has been investigated by [4]. Review of convection with inclined temperature gradients can also be found in [5. 6, 7]. Using linear stability analysis [8] considered onset of thermosolutal convection in a horizontal porous layer subject to fixed temperatures and chemical equilibrium on the bounding surfaces when the solubility of the dissolved component depends on temperature. Their result shows that the reactive term may be stabilizing or destabilizing with subtle effects particularly when the thermal gradient is destabilizing but the solutal gradient is stabilizing.

The study of convective flow taking into account Soret effect has also been considered by authors. The fact that Soret effect can give rise to overstable solutions has been demonstrated experimentally by [9]. Soret effect on the linear stability of a fluid mixture in a porous medium in the presence of temperature gradient was investigated in which the temperature gradient was assumed to vary periodically with respect to time by [10] while [11] reported the study on analytical and numerical study of double-diffusive and Soret-induced convection in a horizontal porous layer. [12] have considered the combined effect of thermodiffusion and lateral heating on double-diffusive natural convection in a horizontal porous layer filled with a binary fluid and subjected to uniform fluxes of heat and mass on its long sides. Their results show that the heat transfer is considerably affected by the Soret effect.

Many processes in engineering occur at high temperatures and the knowledge of radiative heat transfer becomes essential. The first study on the mixed free-forced flow of a radiating gas between two vertical plates using the small optical thickness approximation was carried out by [13]; while [14] studied radiation effects on MHD free convection of a gas past a semi-infinite vertical plate. For optical thick fluids where self absorption exists, the Rosseland approximation was used to describe the radiative heat flux in the energy equation.

In this work, we adopt the classical linear stability theory [15] and the Rosseland differential approximation for the radiation absorption [16] to investigate magnets-thermosolutal instability where the buoyancy forces in the binary mixture are driven by Soret and double-diffusive convections.

2 Mathematical Formulation

We consider a laterally infinite horizontal porous layer of height H filled with binary fluid which is electrically conducting and bound between two impermeable parallel plates as shown in Figure 1.

Different temperature and concentration are imposed between the bottom (T_1, C_1) and the top (T_2, C_2) respectively. We apply a magnetic field of magnitude B_0 perpendicular to the plates. The binary fluid is assumed to be Newtonian and incompressible and to satisfy the Boussinesq approximation. Thus the density variation with temperature and concentration is described by the state equation.

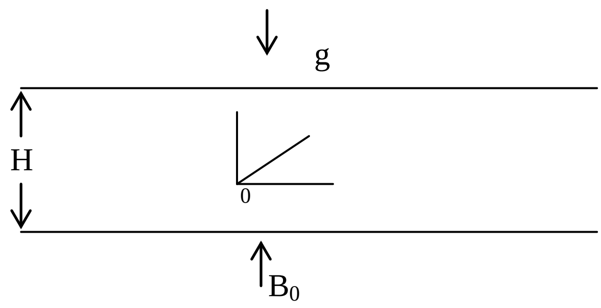

Figure 1: Geometry of the problem.

$$\rho = \rho_0 (1-\beta_T (T'-T_0)+\beta_c (C'-C_0)) \tag{1}$$

where ρ_0 is the density of the fluid mixture at $T = T_0$ and $C = C_0$, and where T_0 and C_0 are the initial temperature and solutal mass concentration inside the fluid layer respectively, β_T *and* β_C are the thermal and concentration expansion coefficients respectively. We consider the grey, absorbing/emitting characteristic of the fluid radiating in a non-scattering medium for which the optically thick approximation is valid. Thus under the consideration above, the governing equation are:

$$\nabla'.\overline{V}' = 0$$

$$\rho_0\left(\frac{\partial \overline{V}'}{\partial t'}+\overline{V}'\nabla'\overline{V}'\right) = -\nabla' P' + \mu\nabla'^2\overline{V}' - g\beta_T\rho_0\left(T'-T_0\right)\hat{e}_z$$
$$+g\beta_C\rho_0\left(C'-C_0'\right)\hat{e}_z - \mu\frac{\overline{V}'}{k} - \sigma B_0^2\overline{V}' \tag{3}$$

$$\rho_0 C_\rho\left(\frac{\partial T'}{\partial t'}+\overline{V}'\nabla' T'\right) = k\nabla'^2 T' - \nabla'.q_r' \tag{4}$$

$$k\left(\frac{\partial C'}{\partial t'}+\overline{V}\ \nabla' C'\right) = \nu\ \left(D\nabla'^2 C' + D^* C'_0 \nabla'^2 T'\right) \tag{5}$$

where κ, D, *and* D^* are thermal conductivity mass diffusivity of species and thermo-diffusion coefficients respectively. Also g is the acceleration due to gravity, $\nu\left(=\frac{\mu}{\rho_0}\right)$ is the kinematic viscosity, $\bar{V}'$ is the velocity vector, ρ' is the pressure, $\hat{e}_z$ is a unit vector in the upward direction, q'_r is the radiative flux, C'_0 *and* T'_0 are dimensional solutal mass concentration and temperature respectively at the centre of the cavity, σ is electrical conductivity.

The boundary conditions are

$$\bar{V}'=0,\ T'=T_1, C'=C_1 \quad \text{at} \quad z=\frac{H}{2} \tag{6}$$

$$\bar{V}'=0,\ T'=T_2, C'=C_2 \quad \text{at} \quad z=\frac{H}{2}$$

For optically thick fluids where self – absorption exists, the Rosseland approximation is adopted for the radiative heat flux in the energy equation as [17]

$$q'_r = -\frac{4\sigma^*\nabla T^4}{3\delta} \tag{7}$$

where σ^* is the Stephen – Boltzmann constant, δ is the mean absorption coefficient. We now assume that the temperature differences within the fluid and the porous medium is sufficiently small such that T^4 may be expressed as a Linear function of temperature about a free stream temperature T_0 using Taylor's expansion and neglecting higher order terms we yields

$$T^4 \approx 4T_0^3\,T - 3T_0^4 \tag{8}$$

so that

$$q_r^1 = -\frac{4\sigma^*}{3\delta}\nabla\ (4T_0^{\ 3}\,T - 3T_0^{\ 4}) = -\frac{16\sigma^*}{3\delta}T_0^{\ 3}\ \nabla T \tag{9}$$

Substituting Equation (9) into Equation (4) the energy equation becomes

$$\rho_0 C_\rho\left(\frac{\partial T'}{\partial t'} + \bar{V}'\nabla' T'\right) = k\nabla'^2 T' + \frac{16\sigma^*}{3\delta}T_0^3\ \nabla T' \tag{10}$$

We introduce the following non-dimensional variables into equations

(2),(3),(5), (6) and (9)

$$t=\frac{t'\nu}{H^2},\ \overline{V}=\frac{H\overline{V}'}{\nu},\ (x,y,z)=\frac{1}{H}(x',y',z'),$$

$$\theta=\frac{T'-T_0}{T_1-T_2},\ C=\frac{C'-C_0}{C_1-C_2},\ P=\frac{P'H^2}{\rho_1\nu^2}$$

$$R^2=\frac{16\sigma^* T_0^{\,3}}{3k\delta},\ \chi^2=\frac{H^2}{\kappa},\ R_T=\frac{g\,\beta_T H^3(T_1-T_2)}{\alpha\nu},$$

$$R_c=\frac{g\beta_c H^3(C_1-C_2)}{\alpha\upsilon},$$

$$\alpha=\frac{k}{\rho_0 c_p},\ \Pr=\frac{\rho_0\nu c_p}{\kappa},\ M^2=\frac{\sigma B_0^{\,2}H^2}{\rho_0\upsilon},\ S=\frac{DC'_0(T_1-T_2)}{D(C_1-C_2)} \tag{11}$$

Under these non-dimensional variables, the governing equations now take the form

$$\nabla.\mathbf{V}=0 \tag{12}$$

$$\frac{\partial\mathbf{V}}{\partial t}=(\mathbf{V}.\nabla)\ \mathbf{V}=-\nabla P+\nabla^2\mathbf{V}-\frac{1}{\Pr}(R_T\theta-R_cC)\hat{e}_z-\chi^2\mathbf{V}-M^2\mathbf{V} \tag{13}$$

$$\frac{\partial\theta}{\partial t}+(\mathbf{V}.\nabla)\ \theta=-\frac{1}{\Pr}(1+R^2)\nabla^2\theta \tag{14}$$

$$\frac{\partial C}{\partial t}+(\mathbf{V}.\nabla)\ C=Le(\nabla^2 C+S\nabla^2\theta) \tag{15}$$

where R_T is the thermal Rayleigh number, R_c the solutal Rayleigh number, Pr the Prandtl number, Le the Lewis number, S, the Soret parameter, M^2, the magnetic parameter, and χ^2, the porosity parameter.

The boundary conditions are

$$\mathbf{V}=0,\ \ \theta=\pm\frac{1}{2},\ C=\pm\frac{1}{2}\ \ at\ \ \ z=\mp\frac{1}{2}. \tag{16}$$

3 Linear Stability Analysis

3.1 Basic state and flow linearization

The basic state of the system is given by the static solution $\mathbf{V} = 0$ of equation (12) - (16). Thus the static temperature, T_s, solutal mass concentration, C_s and pressure, P_s are given by

$$\frac{d^2T_s}{dz^2} = 0, \quad \frac{d^2C_s}{dz^2} + S\frac{d^2T_s}{dz^2} = 0; \quad \frac{dP_s}{dz} - \frac{1}{P_r}\left(R_T\theta_S - R_cC_S\right) \tag{17}$$

subject to

$$T_S = C_S = \pm\frac{1}{2} \; at \; z = \mp\frac{1}{2} \tag{18}$$

The solutions to the equations (17) subject to (18) are

$$T_S = -z, \; C_S = -z, \; P_S = \frac{1}{2\,\mathrm{Pr}} z^2 (R_T - R_C). \tag{19}$$

To access the stability of the steady state, we let the initial solutions described by equations (19) be slightly perturbed. Thus, we define a perturbation of the form [18]

$$V = 0 + (u, v, w), \quad \theta = T_S + \overline{\theta}, \; C = C_S + \overline{C}, \; P = P_S + \overline{P}, \tag{20}$$

Upon substituting these perturbations into the non-dimensional equations (12) – (15) and neglecting the products of disturbances, the linearized perturbation equation are obtained as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{21}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2 + \chi^2 + M^2\right)(u, v, w) = -\nabla\overline{P} - \frac{1}{\mathrm{Pr}}(R_1\overline{\theta} - R_C\overline{C})\hat{e}_z \tag{22}$$

$$\left(\mathrm{Pr}\frac{\partial}{\partial t} - \left(1 + R^2\right)\nabla^2\right)\overline{\theta} = \mathrm{Pr}\, w \tag{23}$$

$$\left(\frac{\partial}{\partial t} - Le\nabla^2\right)\overline{C} = w - Le\,S\nabla^2\,\overline{\theta} \tag{24}$$

now with the boundary condition

$$w = 0, \quad \overline{\theta} = \overline{C} = \pm\frac{1}{2} \quad at \quad z \mp \frac{1}{2}. \tag{25}$$

Proceeding on the analysis, we reduce equation (22) to a scalar equation by taking the double curl of it, using the equation of continuity (equation 21) and keeping only the vertical component of the velocity yields.

$$\left(\frac{\partial}{\partial t} - \nabla^2 + \chi^2 + M^2\right)\frac{\partial^2 w}{\partial z^2} = \frac{1}{p_r}\left(R_T \nabla_h^2 \overline{\theta} - R_C \nabla_h^2 \overline{C}\right) \tag{26}$$

where $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian operator in the horizontal plate.

3.2 The dispersion relation

We next examine the reaction of the system to all possible disturbances. This can be accomplished by expressing an arbitrary disturbance as a superposition of certain modes. Accordingly following [18], we apply the normal mode representation of the form

$$w = W(z)\, f(x,y) e^{\Omega t}, \theta = \Theta(z)\, f(x,y) e^{\Omega t}, \quad C = \phi(z)\, f(x,y) e^{\Omega t} \tag{27}$$

Where $\Omega = \Omega_R + i\Omega_C$ is complex and Ω_R, Ω_C are real numbers. Substituting equation (27) into equation (22), (23), (24) and (25), we obtain

$$\left(D^2 - a^2 - \alpha\,\Omega\right)\Theta = -\alpha W \tag{28}$$

$$\left(D^2 - a^2 - \frac{\Omega}{Le}\right)\varphi = S\,(D^2 - a^2)\Theta - \frac{W}{Le} \tag{29}$$

$$\left(D^2 - a^2\right)\left(D^2 - a^2 - \chi^2 - M^2 - \Omega\right)W - \frac{1}{\Pr}a^2 R_T\,\Theta + \frac{1}{\Pr}R_C\, a^2\,\varphi = 0 \tag{30}$$

subject to

$$W = 0 = \Theta = \phi \quad \text{at} \quad z = \pm\frac{1}{2} \tag{31a}$$

$$D^2W = 0 \quad \text{on a free surface} \tag{31b}$$

where a^2 is a wave number arising from the separation of variables.

Next we reduce the system (28) – (30) to a single scalar equation by eliminating Θ and ϕ to obtain

$$\left(D^2-a^2\right)\left[D^2-a^2-\chi^2-M^2-\Omega\right]\left[D^2-a^2-\chi^2-\alpha\,\Omega\right]\left[D^2-a^2-\frac{\Omega}{Le}\right]W+\frac{a^2\,\alpha\,R_T}{\Pr}\left(D^2-a^2-\frac{\Omega}{Le}\right)W$$
$$-\frac{a^2\,R_C}{\rho_r}\left[\frac{1}{Le}\left(D^2-a^2-\alpha\Omega\right)+S\alpha\left(D^2-a^2\right)\right]W=0 \tag{32}$$

now subject to

$$W = D^2W = D^4W = ... = 0 \quad \text{at} \quad z = \pm\frac{1}{2} \tag{33}$$

where $\frac{d^2}{dx^2}+\frac{u^2}{dy^2}=-a^2, \alpha=\frac{\rho_r}{1+R^2}, \; D=\frac{d}{dz}$.

For the dispersion relation (32) in which the boundary condition (33) holds, we assume the solution of (33) for the lowest state in the form [15]

$$W = w_0\, Sin\,\pi\, z\,, \quad w_0 \quad a \quad \text{constant} \tag{34}$$

Substituting equation (34) into (32) and simplifying for R_T we obtain

$$R_T = \frac{(1+R^2)}{a^2\left(\pi^2+a^2+\frac{\Omega}{Le}\right)}(\pi^2+a^2)(\pi^2+a^2+\chi^2+$$
$$M^2+\Omega)(\pi^2+a^2+\frac{\Pr\Omega}{1+R^2})(\pi^2+a^2+\frac{\Omega}{Le})$$
$$+\frac{a^2\,R_c}{\Pr Le}\left(\pi^2+a^2+\frac{P_r\Omega}{1+R^2}\right)+\frac{a^2\,R_c}{1+R^2}(\pi^2+a^2) \tag{35}$$

The transition from stability to instability occurs via a stationary state. Thus to study the case of marginal stability which corresponds to stationary convection, we set $\Omega = 0$ in Equation (35) to obtain

$$R_T = \frac{(1+R^2)}{a^2}(\pi^2+a^2)^2(\pi^2+a^2+\chi^2+M^2)+\frac{(1+R^2)\,R_c}{\Pr Le}+R_cS \tag{36}$$

4 Results and discussion

Let $a = a_c$ and the corresponding thermal Rayleigh number be R_{Tcri} in Equation (36) then we have

$$R_{Tcri} = \frac{(1+R^2)}{a_c^2}(\pi^2 + a_c^2)^2(\pi^2 + a_c^2 + \chi^2 + M^2) + \frac{(1+R^2)R_c}{\Pr Le} + R_c S \qquad (37)$$

where a_c is the critical wave number and R_{Tcri} is the critical thermal Rayleigh number. The critical wave number for the onset of instability is therefore determined by the condition [15]

$$\frac{\partial R_{T\ cri}}{\partial a_c^a} = 0 \qquad (38)$$

Using condition (38) on Equation (37) and simplifying yields the following sixth order polynomial in a_c^2

$$2\Pr a_c^6 + (3\Pr \pi^2 + \chi^2 + M^2)a_c^4 - \left(\chi^2 \pi^4 + M^2 \pi^2 + P_r \pi^6\right) = 0 \qquad (39)$$

In order to compare our solutions with those in the literature we choose $\chi = 0.12$ and $M = 0.1$ and solve the sixth order polynomial equation (39) using the software *mathematica* [19]. The solution yield six roots of which only one $\left(a_c = 2.22236\right)$ is real and positive. This is in agreement with the Rayleigh-Bernard problem of [15].

In order to understand the physical situation of the problem, we computed the numerical value of the critical wave number for the onset of instability for various values of the radiation parameter, R, the porosity parameter, χ, the magnetic parameter, M, the Lewis number, Le, and the Soret parameter, S.

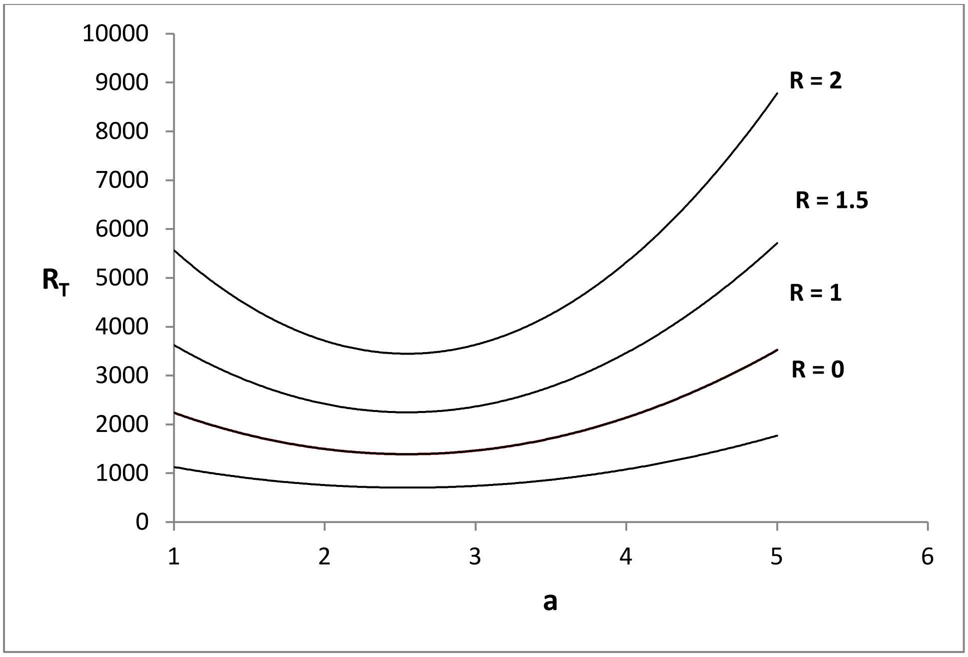

Figure 2: Influence of radiation on the onset of instability for $\chi = 0.2, M = 0.2, Le = 1,\ S = 1,\ Pr = 1.$

Figure 2 shows numerically computed results for $\chi = 0.2$, $M = 0.2$, $Le = 1$ and $S = 1$ for various values of *R*. It is observed that as R increases, the onset of instability is delayed. Higher values of the radiation parameter, *R* leads to greater stabilization of the system. In Figure 3, we, computed the results with $R = 1$, $\chi = 0.2, Le = 1, S = 1$, while M varies. It is noted that as M increases the values of the thermal Rayleigh number increases thereby stabilizing the system. Greater stabilization is achieved at higher values of the magnetic parameter. Thus the onset of instability is delayed by the magnetic parameter. Figure 4 shows the graph of numerically computed results for $R = 1, M = 1,\ S = 1$ for various values of χ.

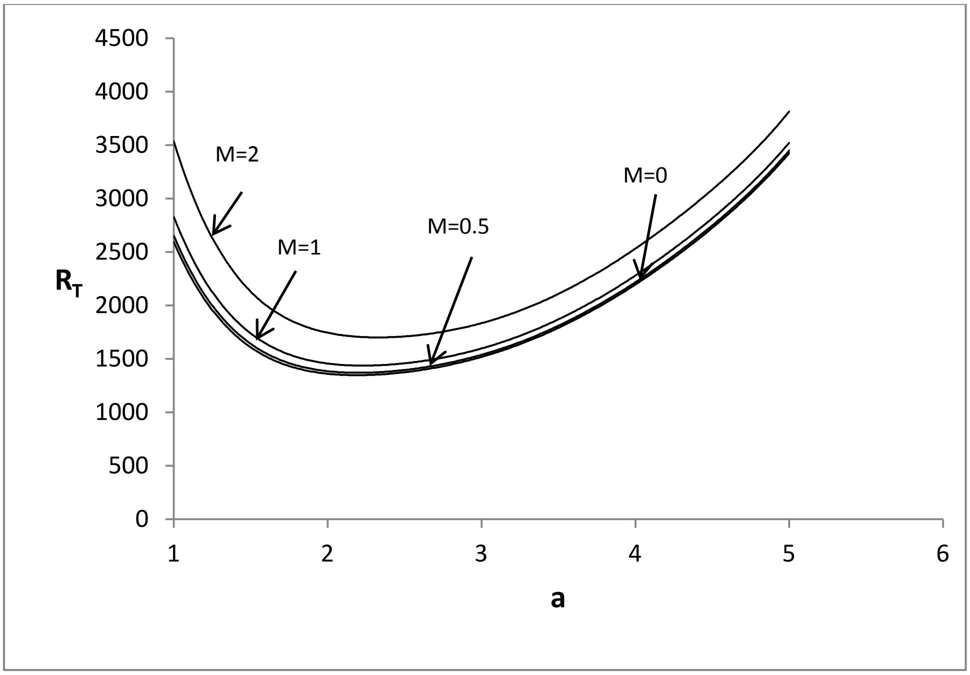

Figure 3: Influence of magnetic field on the onset of instability for $\chi = 0.2,\ R = 1, Le = 1,\ S = 1,\ Pr = 1.$

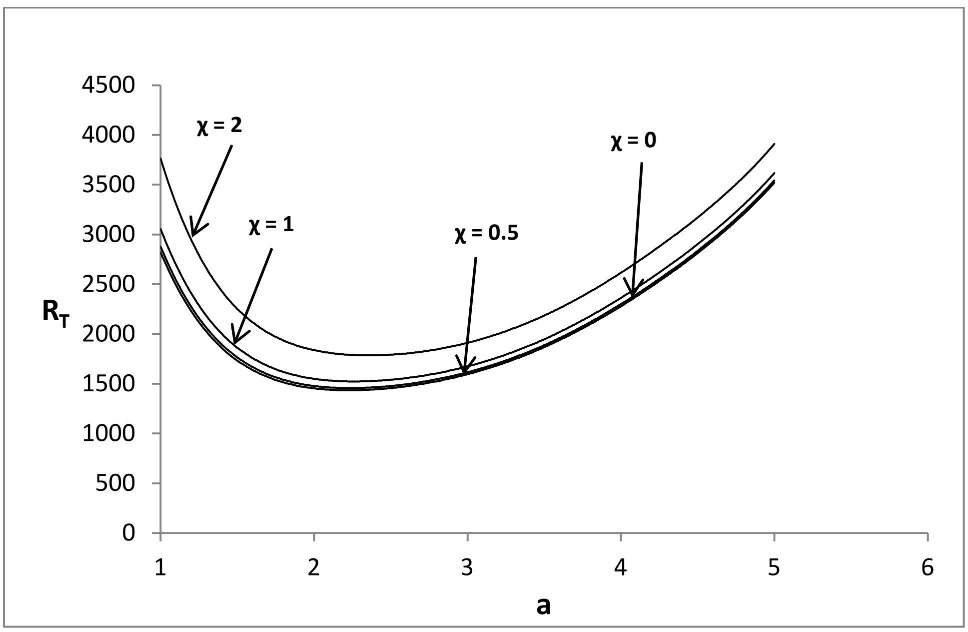

Figure 4: Influence of porosity on the onset of instability for $Le = 1,\ R = 1,\ S = 1,\ Pr = 1.$

Table 1: Influence of the Lewis number on the onset of instability for $\chi = 0.2,\ R = 1, M = 1,\ S = 1$ and $Pr = 1$

a	$Le = 0$	$Le = 1$	$Le = 5$	$Le = 10$
1.0	2839.91	2837.06	2835.34	2834.77
1.2	2219.61	2216.75	2215.04	2214.46
1.4	1863.44	1860.58	1858.86	1858.29
1.6	1651.49	1648.63	1646.91	1646.34
1.8	1526.81	1523.95	1522.24	1521.67
2.0	1459.76	1456.91	1455.19	1454.62
2.2	1433.89	1431.03	1429.32	1428.75
2.4	1439.65	1436.79	1435.08	1434.50
2.6	1471.40	1468.54	1466.83	1466.26
2.8	1525.83	1522.97	1521.26	1520.68
3.0	1601.06	1598.2	1596.49	1595.92
3.2	1696.19	1693.33	1691.62	1691.04
3.4	1810.95	1808.09	1806.38	1805.81
3.6	1945.56	1942.71	1940.99	1940.42
3.8	2100.58	2097.72	2096.01	2095.44
4.0	2276.83	2273.97	2272.25	2271.68
4.2	2475.34	2472.49	2470.77	2470.20
4.4	2697.35	2694.49	2692.78	2692.21
4.6	2944.22	2941.36	2939.65	2939.08
4.8	3217.45	3214.60	3212.88	3212.31
5.0	3518.68	3515.83	3514.11	3513.54

Table 2: Influence of Soret on the onset of instability for
$\chi = 0.2,\ R = 1, M = 1,\ Pr = 1$ and $Le = 1$.

a	$S = 0$	$S = 0.25$	$S = 0.5$	$S = 0.75$	$S = 1$
1.0	2819.91	2824.91	2829.91	2834.91	2839.91
1.2	2199.61	2204.61	2209.61	2214.61	2219.61
1.4	1843.44	1848.44	1853.44	1858.44	1863.44
1.6	1631.49	1636.49	1641.49	1646.49	1651.49
1.8	1506.81	1511.81	1516.81	1521.81	1526.81
2.0	1439.76	1444.76	1449.76	1454.76	1459.76
2.2	1413.89	1418.89	1423.89	1428.89	1433.89
2.4	1419.65	1424.65	1429.65	1434.65	1439.65
2.6	1451.40	1456.40	1461.40	1466.40	1471.40
2.8	1505.83	1510.83	1515.83	1520.83	1525.83
3.0	1581.06	1586.06	1591.06	1596.06	1601.06
3.2	1676.19	1681.19	1686.19	1691.19	1696.19
3.4	1790.95	1795.95	1800.95	1805.95	1810.95
3.6	1925.56	1930.56	1935.56	1940.56	1945.56
3.8	2080.58	2085.58	2090.58	2095.58	2100.58
4.0	2256.83	2261.83	2266.83	2271.83	2276.83
4.2	2455.34	2460.34	2465.34	2470.34	2475.34
4.4	2677.35	2682.35	2687.35	2692.35	2697.35
4.6	2924.22	2929.22	2934.22	2939.22	2944.22
4.8	3197.45	3202.45	3207.45	3212.45	3217.45
5.0	3498.68	3503.68	3508.68	3513.68	3518.68

It is noted that greater stabilization is achieved as χ increases. This again, delayed the onset of instability. We further performed numerical evaluations for the thermal Rayleigh number using $R=1, \chi=0.2, M=1, S=1$ for various values of $Le,$ and the result displayed in Table 1. It is noted that increase in values of $Le,$ increases the onset of instability. Table 2 shows the effect of the Soret parameter on the onset of instability. The numerically computed results were done with $R=1, \chi=0.2, M=1$ and $Le=1$. It is observed that increase in the Soret parameter delay the onset of instability. The variation increases with lower values of the magnetic parameter. In the absence of the magnetic parameter, the stabilizing effect of the Soret parameter is more significant.

5 Conclusion

Natural convection of a binary fluid saturating a horizontal porous medium under the influence of radiation, Soret and magnetic field is investigated. This study show that linear stability analyses successfully captures the processes around the onset of instability and in particular that the critical wave number does not depend on the radiation parameter.

While the results presented here were obtained for a rather idealized geometry, we expect our qualitative findings to be more widely applicable. From a particular point of view, the key implication of our results is that when simulating geothermal system where the onset of convection may substantially increase the heat flux in the system; our result suggest that magnetic stabilization on the process could be very important. The main results show that increases in the radiation, Soret, porosity and magnetic parameters delay the onset of instability with higher values of the parameters leading to greater stabilization of the system while increases in the Lewis number increases the onset of instability.

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