Journal of Applied Mathematics & Bioinformatics, vol.1, no.1, 2011, 151-157
ISSN: 1792-6602 (print), 1792-6939 (online)

On characterizations of slightly *m*-continuous multifunctions

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Abstract

Noiri and Popa introduced upper and lower slightly m-continuous multifunctions in topological spaces and study their properties. The purpose of this paper is to further characterize upper and lower slightly *m*-continuous multifunctions.

Mathematics Subject Classification : 54C60.

Keywords: Multifunctions, lower continuity, upper continuity, topology, minimal structure.

1 Introduction

The multifunctions play a dominant role in topology and in set valued analysis. By a multifunction $F:(X,\tau)\to(Y,\sigma)$, we mean a point to set correspondence from (X,τ) into (Y,σ) with $F(x)\neq\varnothing$ for all $x\in X$. For a multifunction $F:(X,\tau)\to(Y,\sigma)$, let

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Article Info: *Revised* : March 14, 2011. *Published online* : May 31, 2011

$F^+(B)=\{x\in X:F(x)\subseteq B\}$ and $F^-(B)=\{x\in X:F(x)\cap B\neq\emptyset\}$ where $F^+(B)$ and $F^-(B)$ denote the upper and lower inverse of a subset B of Y respectively. In particular $F^+(y)=\{x\in X:F(x)=\{y\}\}$ and $F^-(y)=\{x\in X: y\in F(x)\}$ for each point $y\in Y$. For each $A\subseteq X$, $F(A)=\bigcup_{x\in A}F(x)$. The purpose of this paper is to characterize for upper and lower slightly m-continuous multifunctions.

2 Preliminary Notes

In this section certain definitions and results are recalled. If A is a subset of a topological space X then *cl*(A) and *int*(A) denote the closure and interior of A respectively in X.

Definition 2.1 Let A be a subset of a topological space X. Then A is called

(i) regular open if $A = int(cl(A))$ and regular closed if $A = cl(int(A))$, [7]

(ii) semi open if $A\subseteq cl(int(A))$ and semi-closed if $int(cl(A))\subseteq A$, [3]

(iii) α–open if $A\subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A)))\subseteq A$, [5]

(iv) pre-open if $A\subseteq int(cl(A))$ and pre-closed if $cl(int(A))\subseteq A$, [4]

(v) β-open if $A\subseteq cl(int(cl(A)))$ and β-closed if $int(cl(int(A)))\subseteq A$, [1]

(vi) a p-set if $cl(int(A))\subseteq int(cl(A))$, [8]

(vii) a q-set if $int(cl(A))\subseteq cl(int(A))$. [9]

β-open sets are also called semi-pre-open sets.[2]

Definition 2.2 A sub family m_X of the power set $\wp(X)$ of a non empty set X is called a minimal structure (briefly *m*-structure) on X if $\emptyset\in m_X$ and $X\in m_X$. The pair (X, m_X) is a minimal structure space on X. Each member of m_X is said to be m_X-open and the complement of an m_X-open set is said to be m_X-closed. [6].

Definition 2.3 Let X be a non empty set and m_X an *m*-structure on X. For a subset

A of X , the m_X-closure of A and the m_X-interior of A are defined as follows:

(i) $m_X - cl(A) = \cap \{ F: A \subseteq F, X \setminus F \in m_X \}$,

(ii) $m_X - int(A) = \cup \{U: U \subseteq A, U \in m_X \}$, [6].

Definition 2.4 Let (X, m_X) be a minimal structure space and (Y,σ) be a topological space. A multifunction F:(X, m_X) $\rightarrow$(Y, σ) is said to be

(i) upper slightly m-continuous if for each point $x \in X$ and each clopen set V of Y containing F(x), there exists $U \in m_X$ containing x such that F(U) $\subseteq$V,

(ii) lower slightly m-continuous if for each point $x \in X$ and each clopen set V of Y such that $F(x) \cap V \neq \emptyset$, there exists $U \in m_X$ containing x such that $F(u) \cap V \neq \emptyset$ for each $u \in U$. [6]

Lemma 2.5 For a multifunction F:(X, m_X)$\rightarrow$(Y,σ), the following are equivalent:

(i) F is upper slightly m-continuous,

(ii) $F^+(V) = m_X - int(F^+(V))$ for each $V \in CO(Y, \sigma)$,

(iii) $F^-(V) = m_X - cl(F^-(V))$ for each $V \in CO(Y, \sigma)$. [6]

Lemma 2.6 Let F:(X, m_X) $\rightarrow$ (Y, σ) be a multifunction. Then the following are equivalent.

(i) F is lower slightly m-continuous,

(ii) $F^-(V) = m_X - int(F^-(V))$ for each $V \in CO(Y, \sigma)$,

(iii) $F^+(V) = m_X - cl(F^+(V))$ for each $V \in CO(Y,\sigma)$ where CO(Y,σ), the collection of all clopen sets of (X,τ). [6]

Lemma 2.7 Let B be subset of X . Then the following are equivalent.

(i) B is regular clopen,

(ii) B is clopen,

(iii) B is α-clopen,

(iv) B is pre-clopen and a q-set,

(v) B is semi-clopen and p-set,

(vi) B is β-clopen, p-set and a q-set. [8]

We use the following notations.

αCO(Y,σ) (resp. RCO(Y,σ); PCO(Y,σ); q(Y, σ); SCO(Y, σ) ; βCO(Y, σ))denotes the collection of all α-clopen (resp. regular clopen, pre-clopen, semi-clopen, β-clopen)sets in (Y,σ). p(Y, σ) (resp. q(Y,σ)) denotes the collection of all p-sets (resp. q-sets) in (Y, σ).

3 Slightly *m*-continuity

In this section, upper and lower slightly m-continuous multifunctions are characterized by using regular clopen sets, α-clopen sets, pre-clopen sets, semi-clopen sets, p-sets, q-sets and β-clopen sets.

Theorem 3.1 For a multifunction $F:(X, m_X)\to(Y,\sigma)$, the following are equivalent:

(i) F is upper slightly *m*-continuous,

(ii) $F^+(V) = m_X\text{-}int(F^+(V))$ for each $V\in\alpha CO(Y, \sigma)$,

(iii) $F^+(V) = m_X\text{-}int(F^+(V))$ for each $V\in RCO(Y, \sigma)$,

(iv) $F^+(V) = m_X\text{-}int(F^+(V))$ for each $V\in PCO(Y, \sigma) \cap q(Y, \sigma)$,

(v) $F^+(V) = m_X\text{-}int(F^+(V))$ for each $V\in SCO(Y, \sigma) \cap p(Y, \sigma)$,

(vi) $F^+(V) = m_X\text{-}int(F^+(V))$ for each $V\in \beta CO(Y, \sigma) \cap q(Y, \sigma) \cap p(Y, \sigma)$,

(vii) $F^-(V) = m_X\text{-}cl(F^-(V))$ for each $V\in\alpha CO(Y, \sigma)$,

(viii) $F^-(V) = m_X\text{-}cl(F^-(V))$ for each $V\in RCO(Y, \sigma)$,

(ix) $F^-(V) = m_X\text{-}cl(F^-(V))$ for each $V\in PCO(Y, \sigma) \cap q(Y, \sigma)$,

(x) $F^-(V) = m_X.cl(F^-(V))$ for each $V\in SCO(Y, \sigma) \cap p(Y, \sigma)$,

(xi) $F^-(V) = m_X.cl(F^-(V))$ for each $V\in \beta CO(Y, \sigma) \cap q(Y,\sigma) \cap p(Y, \sigma)$.

Proof

The implications (ii)$\Rightarrow$ (iii) $\Rightarrow$(iv) $\Rightarrow$(v) $\Rightarrow$(vi) $\Rightarrow$(ii) follow from Lemma 2.7.

By using Lemma 2.5 and Lemma 2.7 we see that

F is upper slightly m-continuous $\Leftrightarrow F^+(V) = m_X\text{-}int(F^+(V))$ for each $V \in CO(Y, \sigma)$ $\Leftrightarrow F^+(V) = m_X\text{-}int(F^+(V))$ for each $V \in \alpha CO(Y, \sigma)$. This proves (i)$\Leftrightarrow$ (ii).

Now the implications (vii)$\Rightarrow$ (viii) $\Rightarrow$(ix) $\Rightarrow$(x) $\Rightarrow$(xi) $\Rightarrow$(vii) follow from Lemma 2.7.

Again by using Lemma 2.5 and Lemma 2.7 it follows that

F is upper slightly m-continuous $\Leftrightarrow F^-(V) = m_X\text{-}cl(F^-(V))$ for each $V \in CO(Y, \sigma)$

$\Leftrightarrow F^-(V) = m_X\text{-}cl(F^-(V))$ for each $V \in \alpha CO(Y,\sigma)$.

This proves (i) $\Leftrightarrow$ (vii). □

Theorem 3.2 For a multifunction $F:(X, m_X) \rightarrow (Y,\sigma)$, the following are equivalent:

- **(i)** F is lower slightly *m*-continuous,
- **(ii)** $F^-(V) = m_X\text{-}int(F^-(V))$ for each $V \in \alpha CO(Y, \sigma)$,
- **(iii)** $F^-(V) = m_X\text{-}int(F^-(V))$ for each $V \in RCO(Y, \sigma)$,
- **(iv)** $F^-(V) = m_X\text{-}int(F^-(V))$ for each $V \in PCO(Y, \sigma) \cap q(Y, \sigma)$,
- **(v)** $F^-(V) = m_X\text{-}int(F^-(V))$ for each $V \in SCO(Y,\sigma) \cap p(Y, \sigma)$,
- **(vi)** $F^-(V) = m_X\text{-}int(F^-(V))$ for each $V \in \beta CO(Y, \sigma) \cap q(Y, \sigma) \cap p(Y, \sigma)$,
- **(vii)** $F^+(V) = m_X\text{-}cl(F^+(V))$ for each $V \in \alpha CO(Y, \sigma)$,
- **(viii)** $F^+(V) = m_X\text{-}cl(F^+(V))$ for each $V \in RCO(Y, \sigma)$,
- **(ix)** $F^+(V) = m_X\text{-}cl(F^+(V))$ for each $V \in PCO(Y, \sigma) \cap q(Y, \sigma)$,
- **(x)** $F^+(V) = m_X\text{-}cl(F^+(V))$ for each $V \in SCO(Y, \sigma) \cap p(Y, \sigma)$,
- **(xi)** $F^+(V) = m_X\text{-}cl(F^+(V))$ for each $V \in \beta CO(Y, \sigma) \cap q(Y, \sigma) \cap p(Y, \sigma)$.

Proof

The implications (ii)$\Rightarrow$ (iii) $\Rightarrow$(iv) $\Rightarrow$(v) $\Rightarrow$(vi) $\Rightarrow$(ii) follow from Lemma 2.7.

By using Lemma 2.6 and Lemma 2.7 it is seen that

F is lower slightly m-continuous$\Leftrightarrow F^-(V) = m_X\text{-}int(F^-(V))$ for each $V \in CO(Y, \sigma)$

$\Leftrightarrow F^-(V) = m_X\text{-}int(F^-(V))$ for each $V \in \alpha CO(Y, \sigma)$,

This proves (i)⇔(ii). Now the implications (vii)⇒(viii) ⇒(ix) ⇒(x) ⇒(xi) ⇒(vii) follow from Lemma 2.6 and Lemma 2.7.

Again by using Lemma 2.6 and Lemma 2.7 we see that

F is lower slightly m-continuous$\Leftrightarrow F^+(V) = m_X\text{-}cl(F^+(V))$ for each $V \in CO(Y, \sigma)$,

$\Leftrightarrow F^+(V) = m_X\text{-}cl(F^+(V))$ for each $V \in \alpha CO(Y, \sigma)$.

This proves (i)⇔ (vii). □

Lemma 3.3 For a multifunction $F: (X, m_X) \rightarrow (Y, \sigma)$, the following are equivalent :

(i) F is upper slightly *m*-continuous,

(ii) $F^+(int(cl(V))) = m_X\text{-}int(F^+(int(cl(V)))$ for each $V \in p(Y,\sigma) \cap (Y, \sigma)$,

(iii) $F^+(int(cl(V))) = m_X\text{-}int(F^+(cl(int(V)))$ for each $V \in p(Y,\sigma) \cap q(Y, \sigma)$,

(iv) $F^+(cl(int(V))) = m_X\text{-}int(F^+(int(cl(V)))$ for each $V \in p(Y,\sigma) \cap q(Y, \sigma)$,

(v) $F^+(cl(int(V))) = m_X\text{-}int(F^+(cl(int(V)))$ for each $V \in p(Y,\sigma) \cap q(Y, \sigma)$,

(vi) $F^-(int(cl(V))) = m_X\text{-}cl(F^-(int(cl(V)))$ for each $V \in p(Y, \sigma) \cap q(Y,\sigma)$,

(vii) $F^-(int(cl(V))) = m_X\text{-}cl(F^-(cl(int(V)))$ for each $V \in p(Y,\sigma) \cap q(Y, \sigma)$,

(viii) $F^-(cl(int(V))) = m_X\text{-}cl(F^-(int(cl(V)))$ for each $V \in p(Y,\sigma) \cap q(Y, \sigma)$,

(ix) $F^-(cl(int(V))) = m_X\text{-}cl(F^-(cl(int(V)))$ for each $V \in p(Y,\sigma) \cap q(Y, \sigma)$.

Proof

Suppose (i) holds. Let $V \in p(Y, \sigma) \cap q(Y, \sigma)$. Then $cl(int(V))=int(cl(V))$. Therefore $cl(int(V))$ and $int(cl(V))$ are clopen sets in Y. Then from Lemma 2.5 it follows that $F^+(int(cl(V)))=m_X\text{-}int(F^+(int(cl(V)))$, $F^-(int(cl(V))) = m_X\text{-}cl(F^-(int(cl(V)))$, $F^+(cl(int(V))) = m_X\text{-}int(F^+(cl(int(V)))$ and $F^-(cl(int(V)))= m_X\text{-}cl(F^-(cl(int(V)))$.

This proves (i) ⇒(ii), (i) ⇒(vi), (i) ⇒(v) and (i) ⇒(ix).

Again since $cl(int(V)) = int(cl(V))$, it follows that (i) ⇒(iv), (i) ⇒(viii), (i) ⇒(iii) and (i) ⇒(vii). The reverse implications follow from the fact that every clopen set is both a p-set and a q-set. □

ACKNOWLEDGEMENTS. The authors are thankful to the referees for giving the valuable comments to improve the paper.

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