

The Simple Chaotic Monopoly Price Growth Model and Third-Degree Price Discrimination

Vesna D. Jablanovic¹

Abstract

To maximize profit, the monopolist must first determine its costs and the characteristics of market demand. The monopoly firm can determine different prices to different consumers. This business practice is called price discrimination.

The basic aim of this paper is to construct a relatively simple chaotic monopoly price growth model that is capable of generating stable equilibria, cycles, or chaos. The monopoly firm is practicing third-degree price discrimination.

JEL classification numbers: D4, D42, C2.

Keywords: Market Structure and Pricing, Monopoly, Single Equation Model

1 Introduction

Monopolists often can raise their profits by charging different prices for the same good to different customers. Third – degree price discrimination is a practice of dividing consumers into two or more groups with separate demand curves for each group. Then the monopoly firm charges different prices to each group.

If the monopolist wants to maximize the firm's profit then total output should be divided between the groups of consumers so that marginal revenues for each group should be equal. Also, the marginal revenue for each group of consumers is equal to the marginal cost of production.

Chaos theory attempts to reveal structure in unpredictable dynamic systems. Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (i) extreme sensitivity to initial conditions; (ii) cause and effect are not proportional; and (iii) nonlinearity.

¹University of Belgrade, Faculty of Agriculture, Nemanjina 6, 11081 Belgrade, Serbia.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983), Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), Jablanovic (2011, 2012, 2014), Puu, T. (2003), Zhang (2006), etc.

The basic aim of this paper is to provide a relatively simple chaotic monopoly price growth model that is capable of generating stable equilibria, cycles, or chaos. In this model a monopolist would want to price discriminate.

2 The Model

It is supposed that consumers are divided into two groups, with separate demand curves for each group. The optimal prices and quantities are such that the marginal revenue from each group is the same and equal to marginal cost. It is supposed the quadratic marginal-cost function for a monopoly is

$$MC_t = a + b Q_t + c Q_t^2 \quad (1)$$

MC – marginal cost; Q – monopoly output ; a, b, c – coefficients of the quadratic marginal-cost function.

Marginal revenue of the first group of consumers ($MR_{1,t}$) is

$$MR_{1,t} = P_{1,t} \left[1 + \left(\frac{1}{E_1} \right) \right] \quad (2)$$

Marginal revenue of the second group of consumers ($MR_{2,t}$) is

$$MR_{2,t} = P_{2,t} \left[1 + \left(\frac{1}{E_2} \right) \right] \quad (3)$$

where $P_{1,t}$ – the price charged to the first group of consumers; $P_{2,t}$ - the price charged to the second group of consumers; E_1 and E_2 are the elasticities of demand for the firm's sales in the first and second markets, respectively.

A monopoly firm maximizes profit by producing the quantity at which marginal revenue must be equal across groups of consumers and must equal marginal cost. Thus the profit-maximizing condition is that

$$MR_{1,t} = MR_{2,t} = MC_t \quad (4)$$

where: $MR_{1,t}$ - marginal revenue of the first group of consumers; $MR_{2,t}$ - marginal revenue of the second group of consumers; MC_t - marginal cost.

Thus, $MR_{1,t} = P_{1,t} [1 + (1/E_1)]$ and $MR_{2,t} = P_{2,t} [1 + (1/E_2)]$, where E_1 and E_2 are the

elasticities of demand for the firm's sales in the first and second markets, respectively. Now, equating MR_1 and MR_2 as in equation (4) gives the following relationship that must hold for the prices:

$$\frac{P_{1,t}}{P_{2,t}} = \frac{\left[1 + \left(\frac{1}{E_2}\right)\right]}{\left[1 + \left(\frac{1}{E_1}\right)\right]} \tag{5}$$

The higher price will be charged to consumers with the lower demand elasticity. For example, if the elasticity of demand for consumers in group 1 is -3 and the elasticity for consumers in group 2 is -5 we will have $P_1/P_2 = (1-1/5) / (1-1/3) = (4/5) / (2/3) = 1.2$. In other words, the price charged to the first group of consumers should be 1.2 times as high as the price charged to the second group.

Figure 1. illustrates third-degree price discrimination. The demand curve D_1 for the first group of consumers is less elastic than the curve for the second group: the price charged to the first group is likewise higher. The total quantity produced, $Q_T = Q_1 + Q_2$, is found by summing the marginal revenue curves MR_1 and MR_2 horizontally, which yields the curve MR_T , and finding its intersection with the marginal cost curve. Because MC must equal MR_1 and MR_2 we can find the quantities Q_1 and Q_2 .

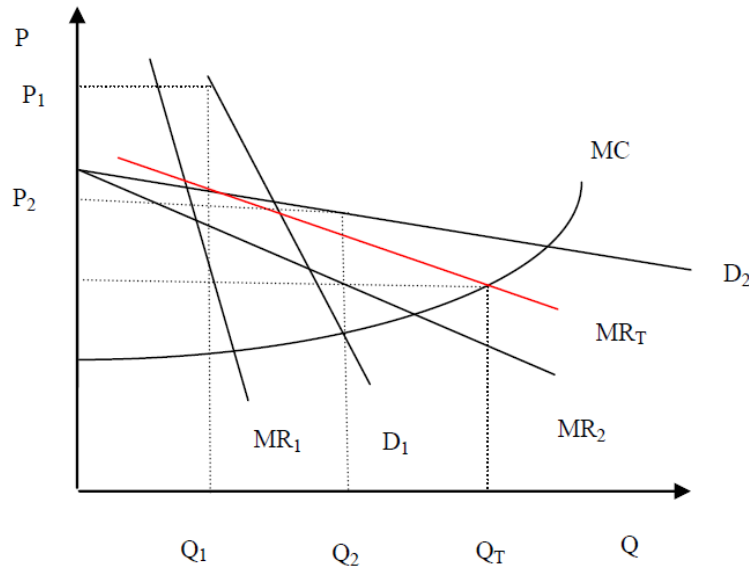


Figure 1: Third-Degree Price Discrimination

Further, in the model of a profit-maximizing monopoly, take the inverse demand function of the first group of consumers

$$P_{1,t} = n - m Q_t \tag{6}$$

Where $P_{1,t}$ – the price charged to the first group of consumers; Q – monopoly output; n ,

m – coefficients of the inverse demand function.

It is supposed that $n=0$. According to (5) and (6) we obtain

$$P_{2,t} = \frac{-m \left[1 + \left(\frac{1}{E_1} \right) \right]}{\left[1 + \left(\frac{1}{E_2} \right) \right]} Q_t \quad (7)$$

Further, it is supposed that

$$MC_{t+1} = MC_t + \Delta MC \quad (8)$$

Or

$$MC_{t+1} = MC_t + \alpha MC_{t+1} \quad (9)$$

i.e.

$$(1-\alpha) MC_{t+1} = MC_t \quad (10)$$

Now, it is supposed that $n = 0$.

By substitution one derives:

$$P_{2,t+1} = \frac{b}{m(\alpha-1) \left[1 + \left(\frac{1}{E_1} \right) \right]} P_{2,t} - \frac{c \left[1 + \left(\frac{1}{E_2} \right) \right]}{m^2 (\alpha-1) \left[1 + \left(\frac{1}{E_1} \right) \right]^2} P_{2,t}^2 \quad (11)$$

Further, it is assumed that the monopoly price of the second group of consumers is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the monopoly price of the second group of consumers growth rate depends on the current size of the monopoly price of the second group of consumers, P_2 , relative to its maximal size in its time series P_2^m . We introduce p_2 as $p_2 = P_2/P_2^m$. Thus p_2 range between 0 and 1. Again we index p_2 by t , i.e., write $p_{2,t}$ to refer to the size at time steps $t = 0,1,2,3,\dots$. Now, growth rate of the monopoly price of the second group of consumers is measured as

$$p_{2,t+1} = \frac{b}{m(\alpha-1)\left[1+\left(\frac{1}{E_1}\right)\right]} p_{2,t} - \frac{c\left[1+\left(\frac{1}{E_2}\right)\right]}{m^2(\alpha-1)\left[1+\left(\frac{1}{E_1}\right)\right]^2} p_{2,t}^2 \tag{12}$$

This model given by equation (12) is called the logistic model. For most choices of α , b , c , m , E_1 and E_2 there is no explicit solution for (12). Namely, knowing α , b , c , m , E_1 and E_2 and measuring $p_{2,0}$ would not suffice to predict $p_{2,t}$ for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (12) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of $p_{2,t}$. This difference equation (12) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point $p_{2,0}$ the solution is highly sensitive to variations of the parameters α , b , c , m , E_1 and E_2 ; secondly, given the parameters α , b , c , m , E_1 and E_2 , the solution is highly sensitive to variations of the initial point $p_{2,0}$. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

3 The Logistic Equation

The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst. It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t) , \quad \pi \in [0,4] , \quad z_t \in [0,1] \tag{13}$$

is equivalent to the iteration of growth model (12) when we use the following identification:

$$z_t = \frac{c\left[1+\left(\frac{1}{E_2}\right)\right]}{b m\left[1+\left(\frac{1}{E_1}\right)\right]} p_{2,t} \tag{14}$$

and

$$\pi = \frac{b}{m(\alpha-1) \left[1 + \left(\frac{1}{E_1} \right) \right]} \quad (15)$$

Using (12) and (14) we obtain

$$\begin{aligned} z_{t+1} &= \frac{c \left[1 + \left(\frac{1}{E_2} \right) \right]}{b m \left[1 + \left(\frac{1}{E_1} \right) \right]} p_{2,t+1} \\ &= \frac{c \left[1 + \left(\frac{1}{E_2} \right) \right]}{b m \left[1 + \left(\frac{1}{E_1} \right) \right]} \left\{ \frac{b}{m(\alpha-1) \left[1 + \left(\frac{1}{E_1} \right) \right]} p_{2,t} - \frac{c \left[1 + \left(\frac{1}{E_2} \right) \right]}{m^2(\alpha-1) \left[1 + \left(\frac{1}{E_1} \right) \right]^2} p_{2,t}^2 \right\} \\ &= \frac{c \left[1 + \left(\frac{1}{E_2} \right) \right]}{m^2(\alpha-1) \left[1 + \left(\frac{1}{E_1} \right) \right]^2} p_{2,t} - \frac{c^2 \left[1 + \left(\frac{1}{E_2} \right) \right]^2}{b m^3(\alpha-1) \left[1 + \left(\frac{1}{E_1} \right) \right]^3} p_{2,t}^2 \end{aligned}$$

On the other hand using (13), (14), and (15) we obtain

$$\begin{aligned} z_{t+1} &= \pi z_t (1 - z_t) = \\ &= \left\{ \frac{b}{m(\alpha-1) \left[1 + \left(\frac{1}{E_1} \right) \right]} \right\} \frac{c \left[1 + \left(\frac{1}{E_2} \right) \right]}{b m \left[1 + \left(\frac{1}{E_1} \right) \right]} \left\{ 1 - \frac{c \left[1 + \left(\frac{1}{E_2} \right) \right]}{b m \left[1 + \left(\frac{1}{E_1} \right) \right]} \right\} = \end{aligned}$$

$$= \frac{c \left[1 + \left(\frac{1}{E_2} \right) \right]}{m^2 (\alpha - 1) \left[1 + \left(\frac{1}{E_1} \right) \right]^2} p_{2,t} - \frac{c^2 \left[1 + \left(\frac{1}{E_2} \right) \right]^2}{b m^3 (\alpha - 1) \left[1 + \left(\frac{1}{E_1} \right) \right]^3} p_{2,t}^2$$

Thus we have that iterating (12) is really the same as iterating (13) using (14) and (15). It is important because the dynamic properties of the logistic equation (13) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that :

- (i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;
- (ii) For $1 < \pi < 3,57$ there exist fixed points the number of which depends on π ;
- (iii) For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1) / \pi$;
- (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;
- (v) For $3 < \pi < 4$ all solutions will continuously fluctuate;
- (vi) For $3,57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

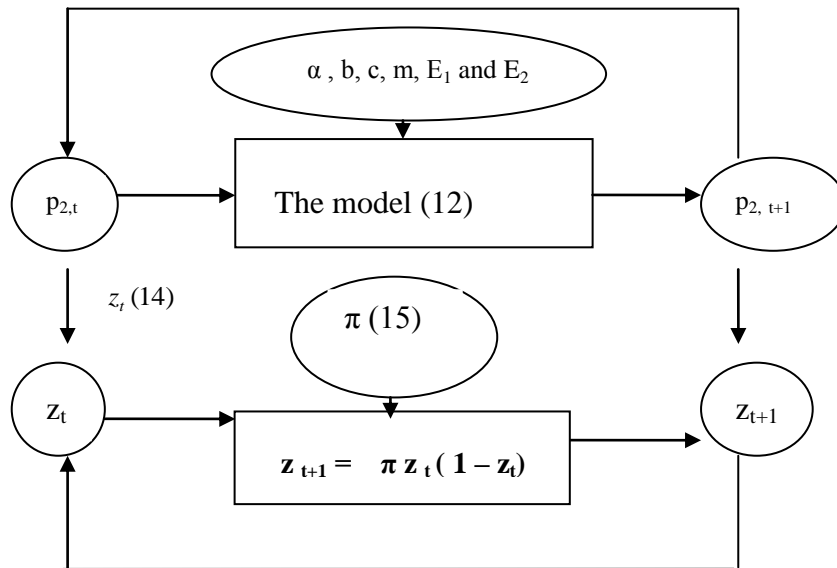


Figure2: Two quadratic iterations running in phase are tightly coupled by the transformations indicated

4 Conclusion

This paper creates the simple chaotic monopoly price model in the situation of the third-degree price discrimination. The model (12) has to rely on specified parameters α , b , c , m , E_1 and E_2 , and initial value of the monopoly price of the second group of consumers, $p_{2,0}$. But even slight deviations from the values of parameters α , b , c , m , E_1 and E_2 and initial value of the monopoly price charged to the second group of customers, show the difficulty of predicting a long-term behavior of the monopoly price charged to the second group of customers $p_{2,0}$.

A key hypothesis of this work is based on the idea that the coefficient

$$\pi = \frac{b}{m(\alpha - 1) \left[1 + \left(\frac{1}{E_1} \right) \right]}$$

plays a crucial role in explaining local stability of the monopoly price charged to the second group of customers, where, b – the coefficient of the marginal cost function of the monopoly firm, m - the coefficient of the inverse demand function of the first group of customers, E_1 - the elasticity of demand for the firm's sales in the first market, α - the marginal cost growth coefficient .

The quadratic form of the marginal cost function of the monopoly firm is important ingredient of the presented chaotic monopoly price charged to the second group of customers growth model (12).

References

- [1] Benhabib, J., & Day, R. H. » Rational choice and erratic behaviour.« *Review of Economic Studies*, **48**, 1981, pp. 459-471.
- [2] Benhabib, J., & Day, R. H. »Characterization of erratic dynamics in the overlapping generation model« *Journal of Economic Dynamics and Control*, **4**, 1982, pp. 37-55.
- [3] Day, R. H. »Irregular growth cycles. *American Economic Review*«, **72**, 1982, pp. 406-414.
- [4] Day, R. H. »The emergence of chaos from classical economic growth«, *Quarterly Journal of Economics*, **98**, 1983, pp. 200-213.
- [5] Goodwin, R. M. »Chaotic economic dynamics«, Oxford: Clarendon Press , 1990.
- [6] Grandmont, J. M. « On endogenous competitive business cycles«, *Econometrica*, **53**, 1985, pp. 994-1045
- [7] Jablanović V. «The Chaotic Monopoly Price Growth Model», *Chinese Business Review*, **10**(11) , 2011. ,pg. 985-990.
- [8] Jablanović, V. »Budget Deficit and Chaotic Economic Growth Models" Aracne editrice S.r.l, Roma , 2012.
- [9] Jablanović V. „Elements of Chaotic Microeconomics“, Aracne editrice S.r.l. , Roma, 2013.
- [10] Li, T., & Yorke, J. »Period three implies chaos«, *American Mathematical Monthly*, **8**, 1975, pp. 985-992.

- [11] Lorenz, E. N. » Deterministic nonperiodic flow«, *Journal of Atmospheric Sciences*, **20**, 1963, pp. 130-141.
- [12] Lorenz, H. W. »Nonlinear dynamical economics and chaotic motion« (2nd ed.). Heidelberg: Springer-Verlag, 1993.
- [13] May, R. M. » Mathematical models with very complicated dynamics«, *Nature*, **261**, 1976, pp 459-467.
- [14] Medio, A. »Chaotic dynamics: Theory and applications to economics«, Cambridge: Cambridge University Press, 1993.
- [15] Puu, T. "Attractors, Bifurcations, and Chaos - Nonlinear Phenomena in Economics", Springer, 2003.
- [16] Zhang W.B. "Discrete Dynamical Systems, Bifurcations and Chaos in Economics", Elsevier B.V, 2012.