Is Skewness Simply Sufficient?: Evidence from Monte Carlo Simulation on Asymmetric Asset Returns

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Abstract

The purpose of this study is to explore the impact of skewness in asset return simulations and the effects of kurtosis on forecast precision. We use 9 years of daily returns of 30 stocks and run a Monte Carlo simulation to identify the forecasted returns based on Gaussian and skew normal distributions. We find that the term and precision do not have a relationship and that the use of the skew normal distribution does not improve the precision of the forecast; it in fact leads the kurtosis to drift to the undesirable direction. Further, persistent negative portfolio forecast errors show that both distribution types lead to significant underestimation of asset returns. The results suggest that simply apply designated skewness to normal distribution do not improve the quality of Monte Carlo simulation, and the fourth moment of realized distribution needs to be incorporated in asset performance forecast.

1   Introduction

This paper first tests the degree of inappropriateness of assuming normality for financial asset returns. We apply the skew normal distribution to the Monte Carlo simulation and examine the improvement of forecast precision relative to the Gaussian distribution. We then explore the relationship of the third and fourth moments of asset distribution in the skew normal distribution to conclude that it is necessary to explicitly designate leptokurtosis in Monte Carlo simulations of asset performance.

The distributions of returns of financial assets are commonly observed to carry non-zero

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skewness and excess kurtosis. Harvey and Siddique (2000) discuss symmetrical skewness of asset returns and recognize the skewness premium. Conrad, Dittmar, and Ghysels (2013) confirm the role of skewness and kurtosis as markers for risk compensation in securities with return dependence controlled. Dong (2014) and Chen, Chen, and Lee (2013) suggest that extreme negative investor sentiment leads to high dependence at the lower tail of the return and this implies the necessity of incorporating skewness in the Monte Carlo simulation.

However, currently there is no quantitative evidence to support the importance of incorporating skewness in simulations. In other words, the degree of forecast precision improvement after skewness is considered needs to be addressed to allow researchers and practitioners to judge the type of distribution that Monte Carlo simulation should be based on. Furthermore, involving skewness in the skew normal distribution will affect the kurtosis of the distribution; therefore it is meaningful to explore whether the drift of kurtosis due to non-zero skewness warrants a more accurate fit to asset returns. In essence, this study attempts to clarify the impact of skewness in asset return simulation and the impact of kurtosis to forecast precision due to the non-zero skewness setting.

2   Data

Using a random selection[3] process, this paper draws 30 stocks from the pool of Russell 3000 index. The index list is according to the latest June 27, 2014 version as of December 22, 2014. Similar to Guo et al. (2015), this study first assigns random values in the range of [0, 1] to all the assets that carry full historical data between December 5, 2005 and December 8, 2014, and then select assets with assigned random values between 0.49 and 0.51. The random values assigned using this procedure are uniformly distributed (Rotz, Falk, Wood, and Mulrow, 2001) and are free of sampling bias and concerns of data mining.

We assume 252 trading days in a year and we use historical returns of 3 years to fit the distribution on which the Monte Carlo simulation is based. In addition we use the future 6 years of stock return data to examine the forecast precision and Monte Carlo term effect. Such in-sample forecast spans a long history of asset prices for a variety of types of equities to ensure that the conclusion is broadly applicable. Specifically, we obtain 2269 daily adjusted closing price levels of the 30 assets from which 2268 daily returns are calculated. These are 252 returns per year for consecutive 9 years. Table 1 summarizes the groups of samples. To covert the prices into continuously compounded returns, this paper applies the following Equation 1.

$$r_t^\gamma = \log_e p_t^\gamma - \log_e p_{t-1}^\gamma \tag{1}$$

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[3]According to Wichman and Hill (1982, 1987), because the fractional part of the sum of three random numbers on [0,1] is still a random number on [0,1]. For integer a, b, and c between 1 and 30000, assign the values to a, b, and c: a←MOD(170*a, 30323); b←MOD(171*a, 30269); c←MOD(172*a, 30307), then the random number is the fractional part of the sum of a, b, and c.

Table 1: Sampling and In-sample Forecast Periods

Group	Year	Start Date	End Date	Number of Observations
In-sample realized return	9	12/10/2013	12/8/2014	252
	8	12/7/2012	12/9/2013	252
	7	12/6/2011	12/6/2012	252
	6	12/7/2010	12/5/2011	252
	5	12/7/2009	12/6/2010	252
	4	12/5/2008	12/4/2009	252
Sample return to fit distribution	3	12/6/2007	12/4/2008	252
	2	12/5/2006	12/5/2007	252
	1	12/5/2005	12/4/2006	252

Table 2 includes the tickers of all the equities with maximum or minimum of the key features of all the assets. The market capitalizations and enterprise values (EV) are based on the market quotes on December 10, 2014. The trailing Price-to-earnings (P/E), Price-to-Sales (P/S), Enterprise Value-to-Revenue, and Enterprise Value-to-EBITDA (EV/EBITDA) are trailing twelve month data, and the Price-to-Book (P/B) uses the most recent quarter data as for December 18, 2014. The forward Price-to-earnings (P/E) in general uses pro forma earnings estimate for the 2015-2016 fiscal year, and the P/E-to-growth (PEG) indicates the 5-year expected growth level. The sample includes small and large stocks, value and growth stocks, and relatively overvalued and undervalued stocks.

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Table 2: The Extreme Key Features of the 30 Assets in the Sample

	Market Cap	EV	Trailing P/E	Forward P/E	PEG	P/S	P/B	EV/Revenue	EV/EBITDA
AHL	2.77B	477.73M	8.72[MIN]	10.73	8.71	1.07	0.83	0.18	0.9
BEL	1.22B	1.70B	N/A	62[MAX]	23.66[MAX]	2.06	1.51	2.85	16.66
HILL	268.74M[MIN]	223.35M[MIN]	222[MAX]	15.88	1.54	1.28	5.05	1.08	43.88[MAX]
EZPW	593.16M	882.93M	N/A	7.09[MIN]	0.63	0.58	0.65[MIN]	0.89	6.26
IM	4.27B	4.83B	16.65	9.01	0.85	0.1[MIN]	1.04	0.11[MIN]	6.65
NUS	2.66B	2.74B	10.25	10.82	-2.26[MIN]	0.92	2.94	0.91	5.49
PX	37.21B[MAX]	46.27B[MAX]	20.3	18.78	2.02	3.04	5.7[MAX]	3.76	11.96
SGMO	948.07M	626.96M	N/A	N/A	N/A	20.89[MAX]	3.75	16.59[MAX]	-21.13[MIN]

For each of the 30 equities in the sample, we fit the returns from year 1 to year 3 for both skew normal distribution and Gaussian distribution. With the fitted parameters for both of the distributions, we then perform Monte Carlo simulations at both the portfolio level and individual asset level.

For the simulated net returns, we set the minimum threshold of -1, consistent with the nature of investment and ruling out the threat of negative account balance. Each observation, $r_{i,t}^\gamma$, is the daily return at the tth day for the ith asset in year γ. The variable γ is a string that can be assigned to any year between year 4 and year 9. Starting from the unit currency initial endowment, the forecasted one-year return from Monte Carlo simulation is:

$$R_i^\gamma = \prod_{t=1}^{n} (1 + r_{i,t}^\gamma)$$

where n=252 and R_i^γ is the simulated one-year return for a specific asset.

Then we perform the simulation for 150 iterations and the forecasted one-year return for a certain distribution method M, which the Monte Carlo is based on. The one-year return is path dependent to the daily realized returns.

$$R_{i,M}^\gamma = \frac{\sum_{j=1}^{n} (\prod_{t=1}^{n} (1 + r_{i,t}^\gamma))}{N}$$

where N=150, and M can be assigned to skew normal (SN) or Gaussian (GAU). The forecast error of the portfolio with 30 assets is the norm of the difference of asset simulation returns and asset realized returns:

$$\varepsilon_M^\gamma = \frac{\sum_{i=1}^{\lambda} \left\| R_M^\gamma - R_i^\gamma \right\|}{\lambda}$$

where R_i^γ is the realized portfolio return. Similarly, the forecast error of a specific asset in the portfolio is the norm of the difference of asset simulation return and asset realized return:

$$\varepsilon_{i,M}^\gamma = \left\| R_{i,M}^\gamma - R_i^\gamma \right\|$$

The portfolio in this paper, which is an equally-weighted portfolio, is different from the Russell 3000 index, which is a market capitalization-weighted portfolio. The reason for this difference is that this paper attempts to avoid forecasting error associated with price-weighted discrimination; the rationale is that the forecasting precision is equally important for every single asset regardless of their price level.

3   Methodology

The skew normal distribution is from the differential equation

$$\omega^4 f''(x) + (\alpha^2 + 2)\omega^2(x - \varsigma)f'(x) + f(x)\big((\alpha^2 + 1)(x - \varsigma)^2 + \omega^2\big) = 0$$

where $f(0) = \dfrac{\exp\left[\left(-\frac{\varsigma^2}{2\omega^2}\right)\right]\frac{2}{\sqrt{\pi}}\int_{\left(\frac{\alpha\varsigma}{\sqrt{2}\omega}\right)}^{\infty} e^{-t^2}\,dt}{\sqrt{2\pi}\,\omega}$,

and $f'(0) = \dfrac{\exp\left[\left(-\frac{(\alpha^2+1)\varsigma^2}{2\omega^2}\right)\right]\left(2\alpha\omega + \sqrt{2\pi}\varsigma\exp\left[\left(\frac{\alpha^2\varsigma^2}{2\omega^2}\right)\right]\frac{2}{\sqrt{\pi}}\int_{\left(\frac{\alpha\varsigma}{\sqrt{2}\omega}\right)}^{\infty} e^{-t^2}\,dt\right)}{\sqrt{2\pi}\,\omega^3}$,

and $\int_{\left(\frac{\alpha\varsigma}{\sqrt{2}\omega}\right)}^{\infty} e^{-t^2}\,dt$ is from the complementary error function $erfc(x) = \frac{2}{\pi}\int_x^{\infty} e^{-t^2}\,dt$

Consistent with Azzalini (2014), if the probability density function of a continuous random variable x is

$$f(x) = 2\phi(x)\Phi(\alpha x)$$

where $\phi(x) = \dfrac{\exp\left[\left(-\frac{x^2}{2}\right)\right]}{\sqrt{2\pi}}$ and $\Phi(\alpha x) = \int_{-\infty}^{\alpha x} \phi(t)\,dt$,

then x follows the standard Gaussian distribution when $\alpha = 0$. The parameter α is a shape argument that increases with skewness. To generalize the standard Gaussian distribution, any random variable Y that is normally distributed can be defined as:

$$Y \sim (\xi, \omega^2)$$

where ξ is the location argument and ω is the scale argument.

To further generalize the normal distribution, any random variable Z that follows skew normal distribution can be defined as:

$$Z \sim (\xi, \omega^2, \alpha)$$

Some characteristic values of the variable Z which follows skew normal distribution are as follows according to Azzalini (2014):

With $\delta = \dfrac{\alpha}{\sqrt{1+\alpha^2}}$,

Mean of skew normal distribution: $E(z) = \xi + \omega\sqrt{\dfrac{2}{\pi}}\,\delta$

Variance of skew normal distribution: $VAR(z) = \omega^2\left(1 - \dfrac{2\delta^2}{\pi}\right)$

Skewness of skew normal distribution: $s = \dfrac{4-\pi}{2}\dfrac{E^3(z)}{VAR^{1.5}(z)}$

Kurtosis of skew normal distribution: $k = 2(\pi - 3)\dfrac{E^4(z)}{VAR^2(z)}$

The paper computes the forecast errors (FE) in Table 3 and Table 4 for the two distributions on which the Monte Carlo simulations are based. The forecast errors are not taken as absolute values, so they carry the direction of the error. This allows us to explore and conclude any systematic bias of the forecast.

4 Discussions and Results

Table 3 and Table 4 present the forecast errors of skew normal distribution-based and Gaussian-based Monte Carlo simulation at the individual asset level, and Table 5 presents the forecast errors of the two distribution simulation at the portfolio level. The results indicate the forecast precision in reference to different terms, different distributions, and the forecast systematic bias.

The term effect of the Monte Carlo simulation is not supported by our results. Intuitively, two arguments are plausible in terms of the validity of Monte Carlo simulation from the forecast term perspective. One argument is that the precision of simulation is better in the short run and gradually declines with the passage of time. In other words, because the future introduces more uncertainty, Monte Carlo analysis should be utilized in the short run. The other argument is that in the short run, the market is very volatile and the price trend is stochastic. Equivalently this argument suggests that Monte Carlo simulation fails to predict the short run chaos but can capture the long run average performance of the asset returns without being significantly affected by the market noise.

The above-mentioned intuitions are inconsistent with our results; we find that at the individual asset level, there is no deterministic relationship between precision and time. The forecast errors in the short run, which is in terms of year 4, and the forecast errors in the long run, which is in terms of year 9, do not unanimously improve or deteriorate. In addition, the average forecast errors in a three-year window do not improve compared with the one-year windows from year 4 to year 9. In a nutshell, skew normal and Gaussian Monte Carlo simulations are not more accurate in the short run compared with the long run, nor vice versa; the simulation is not more appropriate for wide-window averaged data compared to the narrow-window short-run data, nor vice versa.

Table 3: Forecast Error of Skew Normal Distribution-based Monte Carlo Simulation

	AHL	BEAV	BEL	COKE	DBD	LABL	HILL	EZPW	FFIN	FLIC
FE Year 7 to 9	-3.87%	-9.82%	-29.11%	-1.31%	31.01%	8.66%	-38.50%	18.49%	6.39%	2.18%
FE Year 6 to 8	-4.13%	-10.08%	-29.14%	-1.26%	30.31%	8.58%	-38.45%	17.12%	5.94%	1.38%
FE Year 5 to 7	-4.05%	-9.25%	-29.03%	-1.94%	30.22%	9.05%	-37.58%	17.45%	5.92%	1.33%
FE Year 4 to 6	-6.03%	-10.08%	-31.66%	-2.67%	29.94%	7.51%	-35.27%	15.68%	5.16%	1.20%
FE Year 9	-3.36%	-9.79%	-29.28%	-1.59%	31.95%	8.92%	-38.07%	20.45%	7.01%	2.38%
FE Year 8	-3.74%	-10.30%	-28.99%	-0.30%	30.89%	7.98%	-38.40%	16.96%	6.06%	2.01%
FE Year 7	-4.52%	-9.38%	-29.07%	-2.04%	30.20%	9.07%	-39.04%	18.05%	6.10%	2.15%
FE Year 6	-4.12%	-10.56%	-29.35%	-1.46%	29.84%	8.69%	-37.92%	16.35%	5.67%	-0.02%
FE Year 5	-3.52%	-7.80%	-28.68%	-2.32%	30.63%	9.38%	-35.78%	17.94%	5.99%	1.87%
FE Year 4	-10.45%	-11.89%	-36.96%	-4.23%	29.36%	4.45%	-32.11%	12.76%	3.83%	1.75%
	GNCMA	HTLF	HOV	JJSF	IPAR	IM	MKL	MCRI	MOS	NUS
FE Year 7 to 9	-5.51%	-0.34%	-35.58%	-0.82%	-11.45%	-7.81%	-3.22%	-12.24%	-19.48%	-5.36%
FE Year 6 to 8	-6.47%	-1.23%	-35.49%	-0.76%	-11.57%	-8.00%	-3.11%	-12.08%	-19.78%	-5.13%
FE Year 5 to 7	-5.86%	-1.58%	-33.43%	-0.65%	-10.91%	-7.58%	-2.94%	-10.73%	-19.84%	-4.78%
FE Year 4 to 6	-7.15%	-3.50%	-34.41%	-0.53%	-13.82%	-8.31%	-4.06%	-11.97%	-21.30%	-5.69%
FE Year 9	-3.98%	-0.01%	-34.78%	-0.19%	-10.60%	-7.90%	-2.93%	-12.35%	-17.90%	-5.52%
FE Year 8	-6.30%	1.47%	-36.97%	-0.38%	-12.06%	-8.14%	-3.14%	-11.82%	-19.78%	-5.29%
FE Year 7	-6.26%	-2.47%	-34.99%	-1.90%	-11.69%	-7.41%	-3.58%	-12.55%	-20.75%	-5.27%
FE Year 6	-6.85%	-2.69%	-34.49%	0.01%	-10.96%	-8.44%	-2.62%	-11.86%	-18.81%	-4.83%
FE Year 5	-4.46%	0.42%	-30.80%	-0.06%	-10.10%	-6.89%	-2.63%	-7.77%	-19.97%	-4.24%
FE Year 4	-10.15%	-8.22%	-37.93%	-1.54%	-20.40%	-9.60%	-6.95%	-16.28%	-25.13%	-8.01%
	POWL	POM	PX	RPT	ROC	SCHL	SGMO	SWKS	TAP	UFI
FE Year 7 to 9	-1.63%	-2.61%	1.03%	-36.69%	-35.06%	-12.98%	-49.82%	-6.55%	4.33%	7.25%
FE Year 6 to 8	-0.52%	-2.70%	1.12%	-36.90%	-35.57%	-14.00%	-50.37%	-6.03%	4.06%	6.87%
FE Year 5 to 7	-0.30%	-2.71%	1.57%	-36.31%	-35.86%	-14.46%	-46.35%	-6.80%	4.57%	6.44%
FE Year 4 to 6	0.50%	-3.22%	-0.01%	-39.42%	-39.51%	-17.23%	-51.06%	-8.28%	4.26%	5.72%
FE Year 9	1.27%	-1.96%	1.42%	-36.32%	-35.20%	-12.72%	-48.51%	-8.46%	4.99%	6.46%
FE Year 8	0.40%	-3.26%	0.96%	-36.43%	-36.12%	-13.45%	-53.86%	-5.34%	3.86%	6.96%
FE Year 7	-6.55%	-2.62%	0.70%	-37.31%	-33.87%	-12.77%	-47.07%	-5.87%	4.15%	8.35%
FE Year 6	4.60%	-2.22%	1.71%	-36.96%	-36.71%	-15.79%	-50.17%	-6.88%	4.18%	5.32%
FE Year 5	1.05%	-3.30%	2.30%	-34.67%	-37.01%	-14.83%	-41.81%	-7.65%	5.37%	5.65%
FE Year 4	-4.16%	-4.12%	-4.03%	-46.63%	-44.83%	-21.07%	-61.20%	-10.30%	3.25%	6.19%

Table 4: Forecast Error of Gaussian Distribution-based Monte Carlo Simulation

	AHL	BEAV	BEL	COKE	DBD	LABL	HILL	EZPW	FFIN	FLIC
FE Year 7 to 9	-3.42%	-9.73%	-19.21%	-0.73%	-3.94%	-2.15%	-25.16%	14.16%	6.36%	1.11%
FE Year 6 to 8	-3.68%	-9.98%	-19.24%	-0.68%	-4.64%	-2.23%	-25.11%	12.79%	5.91%	0.31%
FE Year 5 to 7	-3.60%	-9.15%	-19.13%	-1.36%	-4.73%	-1.76%	-24.24%	13.12%	5.89%	0.26%
FE Year 4 to 6	-5.58%	-9.99%	-21.76%	-2.09%	-5.01%	-3.30%	-21.93%	11.35%	5.13%	0.13%
FE Year 9	-2.91%	-9.70%	-19.38%	-1.01%	-3.00%	-1.89%	-24.73%	16.12%	6.98%	1.31%
FE Year 8	-3.29%	-10.20%	-19.09%	0.28%	-4.06%	-2.83%	-25.06%	12.63%	6.03%	0.94%
FE Year 7	-4.07%	-9.28%	-19.17%	-1.46%	-4.76%	-1.74%	-25.70%	13.72%	6.07%	1.08%
FE Year 6	-3.67%	-10.47%	-19.45%	-0.88%	-5.11%	-2.12%	-24.58%	12.02%	5.64%	-1.09%
FE Year 5	-3.07%	-7.70%	-18.78%	-1.75%	-4.33%	-1.43%	-22.44%	13.61%	5.96%	0.80%
FE Year 4	-10.00%	-11.79%	-27.06%	-3.65%	-5.59%	-6.36%	-18.77%	8.43%	3.80%	0.68%
	GNCMA	HTLF	HOV	JJSF	IPAR	IM	MKL	MCRI	MOS	NUS
FE Year 7 to 9	-8.20%	-0.65%	-39.38%	-2.39%	-10.76%	-8.88%	-3.35%	-13.23%	14.31%	-6.03%
FE Year 6 to 8	-9.16%	-1.55%	-39.28%	-2.32%	-10.87%	-9.06%	-3.25%	-13.06%	14.01%	-5.80%
FE Year 5 to 7	-8.54%	-1.90%	-37.22%	-2.21%	-10.22%	-8.64%	-3.07%	-11.72%	13.95%	-5.45%
FE Year 4 to 6	-9.84%	-3.81%	-38.20%	-2.09%	-13.13%	-9.37%	-4.20%	-12.96%	12.49%	-6.37%
FE Year 9	-6.67%	-0.32%	-38.57%	-1.75%	-9.91%	-8.96%	-3.07%	-13.34%	15.89%	-6.20%
FE Year 8	-8.99%	1.16%	-40.77%	-1.95%	-11.37%	-9.20%	-3.28%	-12.80%	14.01%	-5.96%
FE Year 7	-8.95%	-2.79%	-38.79%	-3.46%	-10.99%	-8.47%	-3.71%	-13.54%	13.04%	-5.94%
FE Year 6	-9.53%	-3.01%	-38.29%	-1.55%	-10.26%	-9.51%	-2.75%	-12.85%	14.98%	-5.51%
FE Year 5	-7.15%	0.10%	-34.60%	-1.62%	-9.41%	-7.95%	-2.76%	-8.76%	13.82%	-4.91%
FE Year 4	-12.84%	-8.53%	-41.72%	-3.10%	-19.71%	-10.66%	-7.08%	-17.27%	8.66%	-8.68%
	POWL	POM	PX	RPT	ROC	SCHL	SGMO	SWKS	TAP	UFI
FE Year 7 to 9	-2.80%	0.18%	-0.64%	-20.48%	-13.53%	-11.58%	-6.65%	-3.94%	3.62%	9.77%
FE Year 6 to 8	-1.69%	0.09%	-0.54%	-20.69%	-14.03%	-12.61%	-7.20%	-3.41%	3.35%	9.39%
FE Year 5 to 7	-1.47%	0.07%	-0.10%	-20.11%	-14.33%	-13.07%	-3.18%	-4.18%	3.85%	8.95%
FE Year 4 to 6	-0.67%	-0.43%	-1.67%	-23.21%	-17.98%	-15.84%	-7.90%	-5.66%	3.55%	8.23%
FE Year 9	0.10%	0.83%	-0.24%	-20.11%	-13.66%	-11.32%	-5.35%	-5.84%	4.27%	8.97%
FE Year 8	-0.77%	-0.47%	-0.70%	-20.22%	-14.59%	-12.05%	-10.70%	-2.72%	3.15%	9.47%
FE Year 7	-7.72%	0.17%	-0.96%	-21.10%	-12.34%	-11.37%	-3.90%	-3.25%	3.43%	10.86%
FE Year 6	3.43%	0.56%	0.04%	-20.76%	-15.17%	-14.39%	-7.01%	-4.27%	3.46%	7.83%
FE Year 5	-0.12%	-0.51%	0.63%	-18.46%	-15.47%	-13.44%	1.36%	-5.03%	4.65%	8.16%
FE Year 4	-5.33%	-1.34%	-5.70%	-30.42%	-23.29%	-19.68%	-18.04%	-7.68%	2.53%	8.70%

In addition, the prediction precision of using the skew normal distribution is not improved but is actually decreased. This suggests that simply adjusting the skewness of distribution by using the skew normal distribution makes the kurtosis of the distribution drift in an undesirable direction. Such a drift is against the nature of the asset return distribution. The attempt of using the skew normal distribution to remedy the non-normal features of asset return is inappropriate.

Furthermore, according to Table 5, the averaged forecast errors are not improved compared to the single-year forecast errors. This proves that the systematic bias of the Monte Carlo simulation exists. If the systematic bias is absent and the directions of the forecast errors for the single year simulations are random, then the average error is expected to be less in terms of its magnitude. The unanimously negative forecast errors in Table 5 imply that the Monte Carlo analysis based on both of the distributions underestimate asset returns.

Table 5: Portfolio Forecast Error of Skew Normal- and Gaussian-based Simulation

	SN	GAU		SN	GAU
FE Year 7 to 9	-8.35%	-5.58%	FE Year 8	-8.55%	-5.78%
FE Year 6 to 8	-8.58%	-5.81%	FE Year 7	-8.61%	-5.84%
FE Year 5 to 7	-8.21%	-5.44%	FE Year 6	-8.58%	-5.81%
FE Year 4 to 6	-9.51%	-6.74%	FE Year 5	-7.46%	-4.69%
FE Year 9	-7.88%	-5.11%	FE Year 4	-12.49%	-9.72%

5   Conclusion

The first purpose of this study is to test the degree of inappropriateness to assume normal distributions for financial assets. We clarify the role of skewness in asset return simulations and the impact of kurtosis on forecast precision. To test this, we chose 30 stocks from the Russell 6000 index by randomly assigning each stock with relevant historical data a number in the range [0,1] and selecting the assets assigned values between 0.49 and 0.51 to use in the sample. By selecting stocks in this manner, we eliminated risk of bias and the possibility of data mining.

We then fit the year 1 to year 3 data for each asset to a skew normal distribution and a Gaussian distribution. After running both distributions for all 30 assets through a Monte Carlo simulation for 150 iterations, we found the forecast error. Despite intuitive notions regarding the relationship between time and precision, our results show that there is no significant correlation between term and the level of forecast precision. In other words, there is no robust conclusion in terms of the validity of Monte Carlo simulation for either short run or long run forecast.

Further, we found that the use of the skew normal distribution to adjust for skewness leads the kurtosis to move in an undesirable direction. Finally, our results yielded uniformly negative portfolio forecast errors, implying that Monte Carlo simulations underestimate asset returns for both Gaussian and skew normal distributions. The results suggest that simply apply designated skewness to normal distribution do not improve the quality of Monte Carlo simulation, and the fourth moment of realized distribution needs to be incorporated in asset performance forecast in future studies

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