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Using influence function for lag truncation in unit root tests

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Abstract

This paper examined the application of influence function as a criterion for lag truncation in unit root tests. Lag selection in unit root tests has been dominated by standard information criteria and application of influence function as a means of determining truncation lag parameter for unit root tests is a new innovation within the context of augmented Dickey-Fuller (ADF) family of unit root tests and generalized least squares Dickey-Fuller (DF-GLS) test. Influence functions were generated for different lag-lengths and the choice of optimal lag-length was based on the particular lag with the largest influence. This methodology uses autocorrelation of time series data to identify the most influential lag among a set of possible truncation lags which is designated as optimal lag for the purpose of lag truncation in unit root tests. We demonstrated that influence function criterion out-performed the standard information in choosing appropriate lag structure for the unit root tests.

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1 Introduction

An important aspect of unit root testing involves identification of the most influential lag in the unit root test regression. In this paper we consider application of influence function for the identification of optimal truncation lag within the context of ADF family of unit root tests and DF-GLS test introduced by [1] and [2] respectively. The idea of influence function was first mooted by [3] where influence function was used for detecting influential points or outliers. There are various applications of influence function in model selection in time series analysis. [4] developed an ingenious method of outlier detection using a plot of influence function of datum points on the theoretical autocorrelation function. [5] and [6] applied influence function for outlier detection and model order determination in time series data.[7] utilized influence function for model selection in kernel-based regressions, to mention just a few.

The remainder of this paper is organized as follows: Section 2 deals with specification of influence function. Section 3 discusses flowchart for the computation of influence function for lag truncation in unit root tests. Section 4 covers data description and preliminary analysis. Section 5 discusses unit root testing using lag selected by information-based lag selection criteria and influence.

2 Influence function and lag specification

For a general parameter $\theta = T(F)$ expressed as a function of the distribution F; the influence function $I(y;\theta)$ at y is according to [3] in [5] is given by

$$I(y\ ;\theta)=\lim_{\varepsilon\to 0}\left(\frac{\tilde{\theta}-\theta}{\varepsilon}\right) \tag{1}$$

$\tilde{\theta}=T(\tilde{F})$ and $\tilde{F}=(1-\varepsilon)F+\varepsilon\partial y$ is a perturbation of F by ∂y,the distribution function for a point mass of one at y.[8] has demonstrated that the influence function of ρ for any univariate distribution with finite second moment is

$$I(y_1,\mathrm{y}_2;\rho)=-\frac{1}{2}\rho\left(\tilde{y}_1^2+\tilde{y}_2^2\right)+\tilde{y}_1\tilde{y}_2 \tag{2}$$

Where $\tilde{y}_1$ and $\tilde{y}_2$ are standardized forms of variates y_1 and y_2 say. If z_1 and z_2 denote respectively the standardized sum of and difference between $\tilde{y}_1$ and $\tilde{y}_2$.

Also if $V_1=\frac{(z_1+z_2)}{\sqrt{2}}$ and $V_2=\frac{(z_1-z_2)}{\sqrt{2}}$. Then equation (2) may be written as

$$I(y_1,\mathrm{y}_2;\rho)=\left(1-\rho^2\right)V_1V_2 \tag{3}$$

[9] gave a first order approximation to the function and it was noted that a sample analogue of (3) is

$$I(y_{i1},\mathrm{y}_{i2};r)=(n-1)(r-r_{-i})\mathrel{\hat{=}}\left(1-r^2\right)\hat{V}_{i1}\hat{V}_{i2} \tag{4}$$

Where $(y_{i1},y_{i2})(i=1,2,....,n)$ are the n bivariate observations, r_{-i} is the correlation based on all but the $-i\ th$ observation, $\hat{V}_{1i}$ and $\hat{V}_{2i}$ are sample analogues of V_1 and V_2. [4] have considered the influence function for the estimation of time series autocorrelation. Extending the work of [4] and [9], [6] considered the use of influence function to detect the presence of outliers and model order determination for time series data. Equation (4) (see [6]) provides a procedure for model order determination which is particularly useful to check for lag truncation in the ADF and DF-GLS regression models of the form:

$$\left.\begin{aligned}
y_t^{(1)} &= \Delta y_t - \rho y_{t-1} = \sum_{j=0}^{k} \gamma_j \Delta y_{t-j} + \varepsilon_{1t} \\
y_t^{(2)} &= \Delta y_t - \alpha - \rho y_{t-1} = \sum_{j=0}^{k} \gamma_j \Delta y_{t-j} + \varepsilon_{2t} \\
y_t^{(3)} &= \Delta y_t - \alpha - \beta t - \rho y_{t-1} = \sum_{j=0}^{k} \gamma_j \Delta y_{t-j} + \varepsilon_{3t} \\
y_t^{(4)} &= \Delta y_t^d - \alpha - \gamma t - \rho y_{t-1}^d = \sum_{j=0}^{k} \delta_j \Delta y_{t-j}^d + \varepsilon_{4t}
\end{aligned}\right\} \tag{5}$$

Where $y_t^{(1)}$ is ADF model I with no constant and no trend ; $y_t^{(2)}$ is ADF model II with constant but no trend; $y_t^{(3)}$ is ADF model III with both constant and trend and $y_t^{(4)}$ is DF-GLS model with both constant and trend

$\varepsilon_{it}, i = 1,2,3,4$ are white noise error terms, γ_j and δ_j are coefficients of differenced lagged values, k is the truncation lag to be determined empirically using influence function. Suppose that $R(k)$ is autocorrelation function at lag $k = 1, \ldots\ldots, p$ for a p-periodic data; then the influence function (see [5]) is

$$I\left[R(k),(z_i, z_{i+k})\right] = \left[1-\rho^2(k)\right] V_{i,k,1} V_{i,k,2} \tag{6}$$

Where $R(k)$ is the autocorrelation of lag k for any L.H.S $y's$ in equation (5) and

$$\left.\begin{aligned}
V_{i,k,1} &= \frac{\left\{ \dfrac{z_i + z_{i+k}}{\sqrt{1+R(k)}} + \dfrac{z_i - z_{i+k}}{\sqrt{1-R(k)}} \right\}}{2} \\
V_{i,k,2} &= \frac{\left\{ \dfrac{z_i + z_{i+k}}{\sqrt{1+R(k)}} - \dfrac{z_i - z_{i+k}}{\sqrt{1-R(k)}} \right\}}{2}
\end{aligned}\right\} \tag{7}$$

The procedure for lag truncation according to [5] is to construct the influence function matrix $n \times l$ .Where n is the number of observations and l is a fixed number equals to the periodicity of the data (for quarterly data $l = 4$ and for monthly data $l=12$). The construction of this matrix is based on critical value of $\left\{\frac{n-k}{n(n+2)}\right\}^{\frac{1}{2}}$. The influence function estimates exceeding the chosen critical value (in magnitude) are designated plus or minus depending on the sign of the estimates, while others are left blank indicating low influence of particular lag. The $lag(l)$ with the highest number of blanks and for which $\rho(l+1)$ cut-off is the possible order of the model.

3 Flowchart for the computation of influence function for lag truncation in unit root tests

The following steps are used in preparation of R-package code for the computation of influence function for different lags

Step1: Find the first difference of the series under investigation and represent it as Y_t^*

Step 2: Compute the autocorrelation for series Y_t^* at lag k for k =1,2,……,12

Step 3 : Compute $r_k^1=\sqrt{1+R(k)}$ and $r_k^2=\sqrt{1-R(k)}$ for all k =1,2,……,12

Step 4 : Compute for $Z_i=\frac{Y_t^*-\mu_{Y_t^*}}{\sigma_{Y_t^*}}$ where $\mu_{Y_t^*}$ and $\sigma_{Y_t^*}$ are the mean and standard deviation for the series Y_t^* respectively.

Step 5: Compute Z_{i+k} for all k =1,2,……,12

Step 6: Compute $\frac{Z_i+Z_{i+k}}{\sqrt{1+R(k)}}$ and $\frac{Z_i-Z_{i+k}}{\sqrt{1-R(k)}}$

Step7: Compute $V_{i,k,1}=\frac{\left\{\frac{Z_i+Z_{i+k}}{\sqrt{1+R(k)}}+\frac{Z_i-Z_{i+k}}{\sqrt{1-R(k)}}\right\}}{2}$

and $V_{i,k,2}=\frac{\left\{\frac{Z_i+Z_{i+k}}{\sqrt{1+R(k)}}-\frac{Z_i-Z_{i+k}}{\sqrt{1-R(k)}}\right\}}{2}$ for all k =1,2,……,12

Step 8: Compute the critical value $\sqrt{\frac{n-k}{n(n+2)}}$

Step 9: Compute $\left[1-R^2(k)\right]$

Step 10: Compute the influence function denoted by I, where $I=\left[1-R^2(k)\right]V_{i,k,1}V_{i,k,2}$

4 Lag selection by influence function criterion

For empirical illustration, we considered US 10-month Government security (USMGS) for long-term interest rate series and US 3-month Treasury Bills(USMTB) for short-term money market interest rate series. The data cover the period from January, 1962 to February, 2014. For simulated data, we used SPSS random number generator to simulate two different sets of normally distributed series. The first set has $\mu=50$ and $\sigma^2=10$ whilst the second set has $\mu=40$ and $\sigma^2=5$. Using these real and simulated datasets, we apply influence function criterion for the selection of optimal lag-length required for the implementation of unit root test and our results are presented in Tables 1, 2, 3 and 4 below:

Tables 1 and 2 present the summary of lag selection by influence function criterion (IFC) for USMGS and USMTB series respectively. The optimal lags suggested by IFC for USMGS and USMTB series are 5 and 10 respectively being the most influential lags .The critical values for the optimal lags are 0.03996622 and

0.0399610 for USMGS and USMTB respectively.

Table1: Lag selection for USMGS Data

Lag	No of Observations	Critical Value	IFC
1	623	0.03996788	385
2	622	0.03996763	362
3	621	0.03996726	363
4	620	0.03996679	366
5	619	0.03996622	390**
6	618	0.03996553	346
7	617	0.03996474	337
8	616	0.03996384	358
9	615	0.03996282	371
10	614	0.03996170	375
11	613	0.03996046	354
12	612	0.03995910	353

****** indicates optimal lag

Table 2: Lag selection for USMTB data

Lag	No of Observations	Critical Value	IFC
1	623	0.03996788	431
2	622	0.03996763	389
3	621	0.03996726	378
4	620	0.03996679	400
5	619	0.03996622	422
6	618	0.03996553	398
7	617	0.03996474	389

8	616	0.03996384	426
9	615	0.03996282	444
10	614	0.03996170	448**
11	613	0.03996046	425
12	612	0.03995910	375

****** indicates optimal lag

Table 3: Lag selection for simulated data 1

Lag	No of Observations	Critical Value	IFC
1	998	0.03160693	548**
2	997	0.03160685	536
3	996	0.03160674	553
4	995	0.03160659	539
5	994	0.03160642	546
6	993	0.03160621	533
7	992	0.03160597	540
8	991	0.03160569	535
9	990	0.03160538	544
10	989	0.03160504	534
11	988	0.03160467	526
12	987	0.03160426	547

****** indicates optimal lag

Table 4: Lag selection for simulated data 2

Lag	No of Observations	Critical Value	IFC
1	998	0.03160693	552
2	997	0.03160685	568**
3	996	0.03160674	524
4	995	0.03160659	546
5	994	0.03160642	548
6	993	0.03160621	544
7	992	0.03160597	541
8	991	0.03160569	522
9	990	0.03160538	534
10	989	0.03160504	537
11	988	0.03160467	541
12	987	0.03160426	544

****** indicates optimal lag

Tables 3 and 4 present the summary of lag selection by influence function criterion (IFC) for simulated data 1 and 2 respectively. The optimal lags suggested by IFC for simulated data I and II are 1 and 2 respectively being the most influential lags. The critical values for the optimal lags are 0.03160693 and 0.03160685 for simulated data I and 2 respectively.

5 Empirical evaluation of unit root testing using lag selected by information-based lag selection criteria

We run a battery of unit root tests using the various optimal lag lengths suggested by different information-based lag selection criteria such as AIC and FPE proposed by [10] and [11] as well as BIC and HQIC introduced by [12] and [13] respectively. For USMGS series, the optimal lag-lengths suggested by AIC, MAIC, BIC and HQIC are 11, 0, 1 and 1 respectively. Similarly, for USMTB series, the optimal lag-lengths suggested by AIC, MAIC, BIC and HQIC are 12, 0, 12 and 12 respectively. The empirical results are presented in Tables 5, 6, 7 and 8 below:

Table 5 : Unit root test for the level of USMGS and USMTB series using ADF model I

	ADF I	k_{AIC}	ADF I	k_{MAIC}	ADF I	k_{BIC}	ADF I	k_{HQIC}
USMGS	-0.8024	11	-0.7003*	0	-0.8553*	1	-0.8553	1
USMTB	-1.0598*	12	-0.7003*	0	-1.0598*	12	-1.0598	12

*null hypothesis rejected at 0.05 level of significance

Table 6 : Unit root test for the level of USMGS and USMTB series using ADF model II

	ADF II	k_{AIC}	ADF II	k_{MAIC}	ADF II	k_{BIC}	ADF II	k_{HQIC}
USMGS	-0.9913*	11	-0.5683*	0	-1.0509*	1	-1.0509*	1
USMTB	-1.7072*	12	-1.5458*	0	-1.7072*	12	-1.7072*	12

*null hypothesis rejected at 0.05 level of significance

Table 7 : Unit root test for the level of USMGS and USMTB series using ADF model III

	ADF III	k_{AIC}	ADF III	k_{MAIC}	ADF III	k_{BIC}	ADF III	k_{HQIC}
USMGS	-1.7504*	11	-1.3415*	0	-1.7353*	1	-1.7353*	1
USMTB	-2.5131*	12	-2.2999*	0	-1.5131*	12	-2.5131*	12

*null hypothesis rejected at 0.05 level of significance

Table 8 : Unit root test for the level of USMGS and USMTB series using DF- GLS Test

	DF-GLS	k_{AIC}	DF-GLS	k_{MAIC}	DF-GLS	k_{BIC}	DF-GLS	k_{HQIC}
USMGS	-0.8522	10	-0.8522	10	-0.8522	10	-0.8522	10
USMTB	-1.5292*	12	-1.4060*	0	-1.5292*	12	-1.5292*	12

*null hypothesis rejected at 0.05 level of significance

Table 5 through table 8 present the outcome of unit root tests conducted for testing the stationarity properties of the level of USMGS and USMTB series using the optimal lag-lengths suggested by conventional lag selection criteria. The null hypotheses of unit root for the level of USMGS and USMTB cannot be rejected across the various optimal truncation lag-lengths considered since the test statistic is greater than the critical value at 5% level of significance for the three versions of ADF tests considered as well as DF-GLS test. These results indicate that both USMGS and USMTB series are non-stationary at level

Table 9 : Unit root test for the first difference of USMGS and USMTB series using ADF model I

	ADF I	k_{AIC}	ADF I	k_{MAIC}	ADF I	k_{BIC}	ADF I	k_{HQIC}
USMGS	-6.3316	11	-16.7993	0	-17.0449	1	-17.0449	1
USMTB	-6.1472	12	-17.6117	0	-6.1472	12	-6.1472	12

*null hypothesis rejected at 0.05 level of significance

Table 10 : Unit root test for the first difference of USMGS and USMTB series using ADF model II

	ADF II	k_{AIC}	ADF II	k_{MAIC}	ADF II	k_{BIC}	ADF II	k_{HQIC}
USMGS	-6.3377*	11	-16.7919*	0	-17.0384*	1	-17.0384*	1
USMTB	-6.1459*	12	-17.5988	0	-6.1459*	12	-6.1459*	12

*null hypothesis rejected at 0.05 level of significance

Table 11 : Unit root test for the first difference of USMGS and USMTB series using ADF model III

	ADF III	k_{AIC}	ADF III	k_{MAIC}	ADF III	k_{BIC}	ADF III	k_{HQIC}
USMGS	-6.5958*	11	-16.8841*	0	-17.1743*	1	-17.1743*	1
USMTB	-6.2115*	12	-17.6114*	0	-6.2115*	12	-6.2115*	12

*null hypothesis rejected at 0.05 level of significance

Table 12 : Unit root test for the first difference of USMGS and USMTB series using DF-GLS Test

	DF-GLS	k_{AIC}	DF-GLS	k_{MAIC}	DF-GLS	k_{BIC}	DF-GLS	k_{HQIC}
USMGS	-2.0461	10	-2.0461	10	-2.0461	10	-2.0461	10
USMTB	-6.2048*	12	-17.6336*	0	-6.2048*	12	-6.2048*	12

*null hypothesis rejected at 0.05 level of significance

Table 9 through table 12 present the outcome of unit root tests conducted for testing the stationarity properties of the first difference of USMGS and USMTB series using the optimal lag-lengths suggested by conventional lag selection criteria. The null hypotheses of unit root for the first difference of USMGS and USMTB were rejected across the various optimal truncation lag-lengths considered since the test statistic is less than the critical value at 5% level of significance for the three versions of ADF tests considered as well as DF-GLS test. This indicates that both USMGS and USMTB series are stationary after first difference indicating that each series is integrated of order 1.

6 Empirical Evaluation of Unit Root Testing Using Lag Selected by IFC

Based on influence function criterion (IFC), the optimal truncation lag required for the implementation of unit root test for USMGS is lag 5 being the most influential lag among a set of candidate truncation lags. We run unit root test on USMGS series for lag 1 and lag 5 to evaluate possibility of discrepancy in the outcome of unit testing under these two influential lags as shown in table 1 above. Similarly, the optimal truncation lag suggested by IFC for USMTB series is lag 10 being the most influential lag. Consequently we run unit root test on USMTB for lag 1, lag 9 and lag 10 to evaluate the behavior of the test across these three influential lags and the empirical results are presented in Tables 13 and 14 below:

Table 13: Unit root test for the level of USMGS using IFC

ADF I	k_{IFC}	ADF II	k_{IFC}	ADF III	k_{IFC}	DF-GLS	k_{IFC}
-0.8553*	1	-1.0509*	1	-1.7353*	1	-1.0109*	1
-0.7820*	5	-0.8515*	5	-1.5991*	5	-0.8393*	5

*null hypothesis rejected at 0.05 level of significance

Table 14: Unit root test for the level of USMTB using IFC

ADF I	k_{IFC}	ADF II	k_{IFC}	ADF III	k_{IFC}	DF-GLS	k_{IFC}
-1.3901*	1	-2.3995*	1	-3.0979*	1	-2.1273*	1
-1.2015*	9	-2.0511*	9	-2.8207*	9	-1.8112*	9
-1.1525*	10	-1.9260*	10	-2.7011*	10	-1.7106*	10

*null hypothesis rejected at 0.05 level of significance

Tables 13 and 14 present the outcome of unit root tests conducted for testing the stationarity properties of the first difference of USMGS and USMTB series using the optimal lag-lengths suggested by IFC. The null hypotheses of unit root for the level of USMGS and USMTB cannot be rejected series across the various optimal truncation lag-lengths considered since the test statistic is greater than the critical value at 5% level of significance for the three versions of ADF tests considered as well as DF-GLS test. This indicates that both USMGS and USMTB series are non-stationary at level.

Table 15: Unit root test for the first difference of USMGS using IFC

ADF I	k_{IFC}	ADF II	k_{IFC}	ADF III	k_{IFC}	DF-GLS	k_{IFC}
-17.0449*	1	-17.0384*	1	-17.1743*	1	-9.4135*	1
-9.7895*	5	-9.7901*	5	-9.9824*	5	-4.1234*	5

*null hypothesis rejected at 0.05 level of significance

Table 16: Unit root test for the first difference of USMTB using IFC

ADF I	k_{IFC}	ADF II	k_{IFC}	ADF III	k_{IFC}	DF-GLS	k_{IFC}
-17.9198*	1	-17.9072*	1	-17.9345*	1	-17.9538*	1
-7.3429*	9	-7.3403*	9	-7.4022*	9	-7.3982*	9
-6.6370*	10	-6.6348*	10	-6.6964*	10	-6.6909*	10

*null hypothesis rejected at 0.05 level of significance

Tables 15 and 16 present the outcome of unit root tests conducted for testing the stationarity properties of the first difference of USMGS and USMTB series using the optimal lag-lengths suggested by IFC. The null hypotheses of unit root for the first difference of USMGS and USMTB were rejected across the various optimal truncation lag-lengths considered since the test statistic is less than the critical value at 5% level of significance for the three versions of ADF tests considered as well as DF-GLS test.This indicates that both USMGS and USMTB series are stationary after first difference indicating that each series is integrated of order 1.

7 Conclusion

This paper has examined application of influence function as an alternative criterion for lag specification in ADF and DF-GLS unit root tests. This methodology provided a hierarchical order of influence for different candidate optimal lag-lengths and this hierarchical structure could serve as standard guide for applied researchers in avoiding the problem of over-estimation and under-estimation of truncation lag-length that is commonly associated with conventional lag selection criteria.

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Appendix

R-Package Code for the Computation of Influence Function for Lag Truncation in Unit Root Tests

The following R code was used for the computation of influence function for different lags using both real and simulated data :

```
IFC=function(y,k){
            delta_y=diff(y)
            z=(delta_y-mean(delta_y))/sd(delta_y)
            rho=acf(delta_y, lag.max=k, plot=F)$acf[-1]
            r_1=sqrt(1+rho)
            r_2=sqrt(1-rho)
            r [1]    = 1-rho^2
                          n = rep(0,k)
                        c.pt = rep(0,k)
                        IF = rep(0,k)
            for (i in 1:k){
a = (z+lag(z,i))/r_1[i] , b = (z-lag(z,i))/r_2[i]
V_1=(a+b)/2;      V_2=(a-b)/2;
IF=V= r[1]*V_1*V_2
V = c(V)
n[i]=length(V)
c.pt[i]=sqrt(n[i]-i)/(n[i]*(n[i]+2)))
          for (j in 1:n[i]) {
                    if(V[j]<c.pt){V[j]=1}
                   else {V[j]=0}
               }
IF[i] =sum(V)
         }
Critical .pt=c.pt; IFC=IF
```

```
max.IFC=max(IFC)
for(i    in 1:length(IF)){
                        if (IF[i]=max.IFC){IF[i]=”***”}
                                          else{IF[i]=””}
                                          }
result = data.frame(n,critical.pt,IFC,optimal.lag=IF)
return(result)
}
```