Rating models and its Applications: Setting Credit Limits

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Abstract

In regulatory and competitive environments increasingly tight, banks have been forced to constantly improve their internal rating models. Despite the increased supply models and statistical approaches that has been proposed to them, the answer to the question: how much to lend ? remains always at the discretion of the credit risk manager. The aim of this paper is to propose a statistical approach that allows an estimation of the maximum facility bank overdraft (limits) that the credit risk manager may allow coorporate and to identify the determinants of this limits.

Keywords: Rating models; Risk Management; Basel II; Credit limits

1 Introduction

This is the text of the introduction. A rating model is defined as the assign-

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ment of quantitative measure to a potential borrower to provide an estimation of its capacity to repay a loan Feldman [1]. Given that the commercial banks withdraw the major part of their profit business from loans, they are very interested in the statistical rating models. The works of Altman [2] show that the developpment of an internal rating model with high discriminating power is indeed possible. His model was efficient, based on a limited number of financial ratios and provide an imediate response to the credit decision. Henceforth, banks have begun to develop their own rating models.

The accuracy of the model is crucial for the banks that use them (see Figure 1).
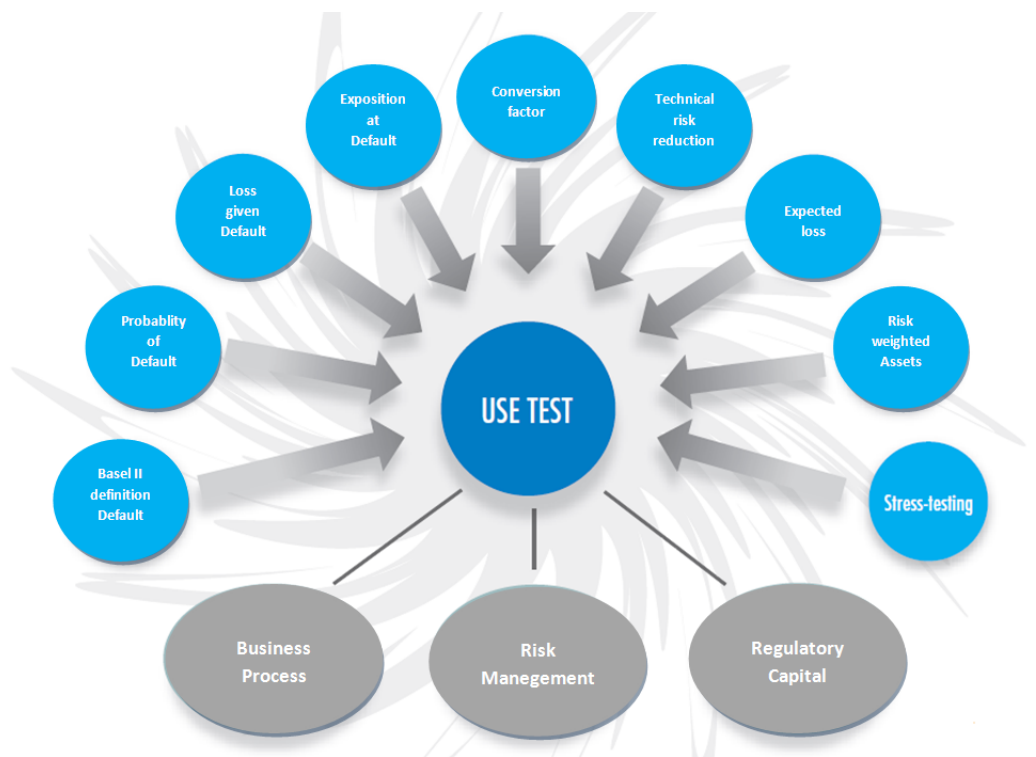

Figure 1: The rating models use test

First, rating model helps the credit risk manager to take a decision on the loan application and proposes a pricing Canabarro[3] which corespond to the real risk of the borrower. Thus, the probability of default (PD) in the bank would depend largely on the accuracy of this model. Second, a good rating model would optimize the regulatory capital and the bank's investment would go into the most profitable projects in total respect of Regulatory constraints

imposed by the Advanced Approach (IRBA) of Basel rules [4]. Finally, a better measure of the risk would allow bank managers to make more informed decisions about the renewal of loans and the use of derivates products for hedging purposes.

One of the largest applications of the rating models is the setting of credit limits which is the main focus of this paper. Credit Limits are threshold that a credit risk manager will allow borrower to owe at any time without having to go back and review their credit file. We modelize this amount on the basis of a credit history requests granted by the senior banker through expert approach.

The second section will give an overview of experts' methods used by credit risk managers to detect the maximum amount that the bank is willing to risk in an account. We also highlight the advantage and the drawbacks of the various approaches. In the section 3 we will propose a statistical approach which gives a direct answer to the question: how much to lend?. The followed section will presents the results of our studies and their intepretation and after that we will finish by the conclusion.

2 Expert Approach of setting Credit Limits

The executive committee authorises the sale management to engage bank funds on a given borrower for a certain amount of loan and not beyond. The risk manager has the mission to determine this amount (limit) according to the credit management process used in the bank. The methods used to set credit limits are many, infinitely varied , using a particular set of ratios, introducing or not qualitative parameters, but they have one point in common: **the lack of theoretical justifications**. In this section we present an overview of three methods : Expected profit method, Profit indicator method and risk point (classes) aproach.

2.1 Expected profit method

In Abtey [5] the author proposes a decision rule in which the amount of loans to be granted to the borrower must be less than or equal to what the bank hopes to win in a given period. Therefore, the optimum exposure is

determined here, over a period, compared to the forcast turnover so that the potential loss is less than the expected profit.

The elements to be considered are:

- t_m the company's margin defined under the analytical accounting

- PD the probability of default of the borrower

- CA_p the forcast turnover to achieve by the the borrower in a given period

The **expected profit** during the period is equal to: CA_p*t_m

The **potential loss** is: E*PD*(1-t_m)   from where $CA_p*t_m >$ E*PD*(1-t_m)
The left side of formula (CA_p*t_m) represents the expected profit: actually,forecast turnover is by definition an uncertain element. Using this formula,the company is betting that the borrower will achieve a turnover of at least equal to forecasts and it will not be on default situation during the period of the loan.

The decision Rule:

$$E \leq \frac{CA_p \, t_m}{PD(1 - t_m)}$$

Example:: PD=0.3 , t_m=0.2 and CA_p= 2 000 kMAD

$$E < \frac{2000 * 0.2}{0.3 * (1 - 0.2)} \implies E < 1 \; 667 \; kMAD$$

The risk is not negligible and the author recommends to apply this method in the case of regular borrowers of the bank on which there are reliable and avaible information.

The author also emphasizes the importance of personal relationships with important clients and consequently it is often difficult to refuse a deferred payment to a well known client. In addition, the author notes that for very low probability of default (close to zero) the optimum amount (limits) becomes very important. Therefore, the amount of loan must be delimited by forcast turnover or another maximum amount proposed by the credit risk manager.

Consequently:

$$E^* < CA_p \text{ or } E^* < \text{Max}$$

However, In the case where the probability of default (PD) is high,this formula allows the setting a cerdit limit, even if the amount is low relative to the

forecast turnover. The company is betting so that the borrower will not go bankrupt before payment of the loan.

2.2 Profitability indicators method

The starting point of the second methodology involves to identify the potentiel profit of the bank on each loan. In Dionne [6] the authors calculated the bank expected profit for one borrower given his probability of default PD:

$$E(profit) = D(r - \mu) - PD\ E_1(c_1) - (1 - PD)\ E_0(c_0) - PD\ L(1 + r)LGD$$

where L is the amount of loan, r the interest rate, μ the alternative placement rate, c_0 is the administrative cost linked to non-payment by a borrower who fails, c_1 is the administrative cost related to non-payment by a customer who did not fail on default and LGD is the loss given default Altman [7]. The maximum amount (Limit) that the bank is willing to risk in an account is the solution L^* of this linear program:

$$L_i^* \in arg\ max_L\ (r - \mu)\ L_i - PD_i\ L_i\ (1 + r)\ LGD_i \quad s.c.\ \ L_i >= 0$$

s.c

$$\begin{cases} L_i * LGD * N(\dfrac{N^{-1}(PD_i) - \sqrt{\rho} * N^{-1}(0.99)}{\sqrt{1 - \rho}}) <= VAR_i \\ \\ L_i > 0 \end{cases}$$

where
L_i is the amount of credit required by the borrower.
N^{-1} The Inverse of the Normal Cumulative Distribution
VAR_i Jorion [8] means the value at risk and it represent the largest loss likely to be suffering regarding the borrower i over a holding period (usually 1 to 10 days) with a given probability (confidence level).

The model elaborated above allows to calculate the optimal amount to lend, which will maximize the profit of the bank without exceding the expected losses (VaR).

However, this model is limited because it relies solely on calculating the optimal amount of each loan to the PD. Indeed, there are other quantitative parameters (Turnover, stock...) and qualitative (sector, Apartenance to a

strong group or not) to take into consideration in the model and which could not be used in the estimation of the PD.

2.3   The risk points method

The method of risk points consists of mixing between the quantitative data as the information in balance sheet and qualitaive information like the quality of management and other criteria about the borrower in order to:

- give a complete picture of the borrowers concerned,

- evaluate more accurately the risk profil.

This determination is particularly important because it sets the credit limits. Thus, the combination of qualitative and quantitative criteria (Figure 2) seems to be the best way to evaluate the borrowers risk.

To use the risk point method the credit risk manager must receive enough information to make both qualitative and quantitative analysis.

There are many ways to use the risk points (classes) method, but the most famous is the method by credit scoring Thomas [9].
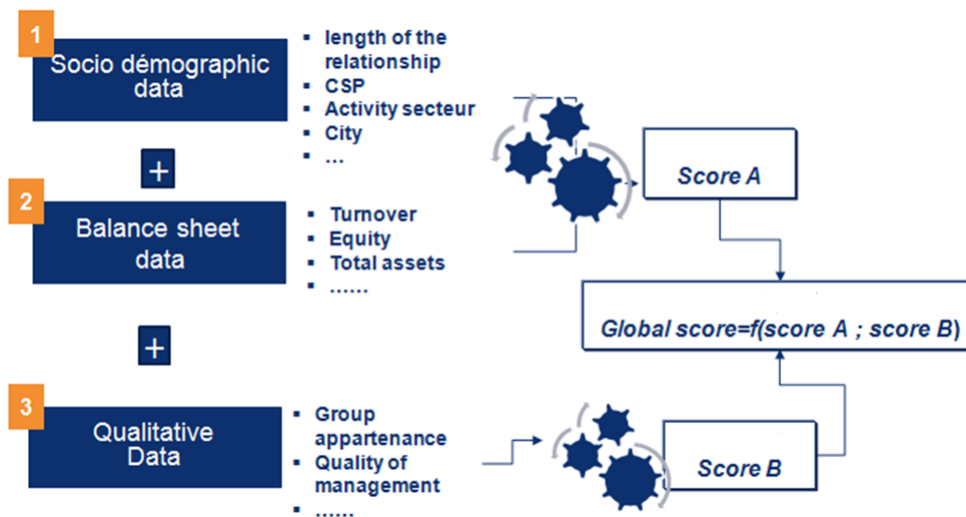

Figure 2: Using quantitative and qualitative data

After calculating the score, the credit risk manager determines a number of cutt-off. Thus, the latter correspond to the decision granting credit limits (on % of the turnover) and other rules like warranty.

Various methods were presented in this section, they depend mainly on the probability of default (PD) and after we deduce indirectly the credit limits. In the following section we propose an empirical method to modelise directly the credit limit.

3   Credit Limits modeling

Despite the proliferation of approaches to build internal rating models they only provide a quantification of the level of risk of the borrower, but they don't provide any indication regarding the amount that this later could possibly repay.

Relying solely on PD to determine the amount to lend would not be a best approach. In Carboni [10] the author showed that the risk score for retail exposures (developped according to a scoring model of a Canadian bank) is more an indicator of delinquency then an indicator of repayment capacity.

Our research is the first statistical study in which a model of a credit limit management that responds to the question: how much to lend? is proposed. Our approach aim is to modelize this amount on the basis of a history of credit applications already confirmed by an expert approach.

3.1   Dataset Description

The database used in this study cames exlusively from a financial institution that has worked with us on this project. It includes data on a corporate samples that have requested facilities between January 1st 2013 and December 2013. Inputs on which Credit risk managers are based themselves to give the agreement is entered one year ago so as to keep an interval of at least 12 months between the request dates and the dates in which the information about the borrower were collected.

We describe in Table below the variables used in the empirical studies in order to setting credit limits. There are three kinds of variables:

- Financial variables

- behavior variables

- Dependent variable or variable to predict.

Financial Ratio	Risk factor
Total Liabilities / Total assets	Leverage
Interest / Turnover	Leverage
Equity / Total assets	Leverage
Long term Debt / Total assets	Leverage
Total Debt / Total assets	Leverage
Total Debt / Current assets	Leverage
Total Debt / (Total assets - Total Debt)	Leverage
Total Debt / Total Liabilities	Leverage
Current assets / Current Liabilities	Liquidity
Working capital / Total assets	Liquidity
Need Working capital / Working capital	Liquidity
Current Liabilities / Total assets	Liquidity
Cash / Current Liability	Liquidity
Working capital / Current Liabilities	Liquidity
(Current Assets - Stocks)/ Current Liability	Liquidity
Net Income / Total Asstes	Profitability
Net Income / Total Asstes	Profitability
Net Income / Equity (ROE)	Profitability
Net Income / Turnover	Profitability
EBIT / Total Assets	Profitability
Stock / (Turnover/ 360)	Activity
Stock / Total Assets	Activity
Acount Receivable / Total Assets	Activity
Acount Receivable / Turnover	Activity
Acount Receivable / Total Liabilities	Activity
Turnover / Total Assets	Activity
Annual average balance	behavior
Annual credit balance	behavior
Annual debit balance	behavior

Dependent variable description

Given the little study on our subject, we were obliged to approach the experts to understand how they do to grant given amount of loan.

Relating to setting credit limit, the credit risk managers reproduce methods and operational processes created by their predecessors, and they try to improve them over time.

After several workshop with the experts, we found that to answer the question how much to lend, the dependent variable can be:

- Autorization [N]: The amount of loan (MAD)request by the borrower the year N

- Autorization[N] / Turnover [N-1] : the part of the authorization in Turnover perfomed by the borrower in the last year.

- Autorization [N] / (Turnover [N-1]/ 360):the number of days turnover perfomed by the borrower in the last year.

The choice of the dependent variable determines the statistical aproach to perfrom. After several simulations, we chosed the second alternative because it presents some properties in line with the assumption of the Beta distribution Ferrari [11] like the support of the distribution (the dependent variable comprised between 0% and 100% of turnover), which can be easly conversed to normally distribution.

To create an approximately normally distributed dependent variable from the raw observations of target (Autorization [N] / Turnover [N- 1]), we first confirmed that this later were approximately Beta distributed (Figure 2).

Beta distributions are described in this case by an upper and lower bound and by two shape parameters, α and β. They are usually bounded between zero and one, where the mean can be any value strictly within this range.

The conversion of the Beta distributed target values to a more normally distribution dependent variable is explicitly defined as follows:

$$Dependent variable = Y = N^{-1}[Betadist(Target, \alpha, \beta, Min, Max)$$

where
Target = Autorization [N] / Turnover[N-1]
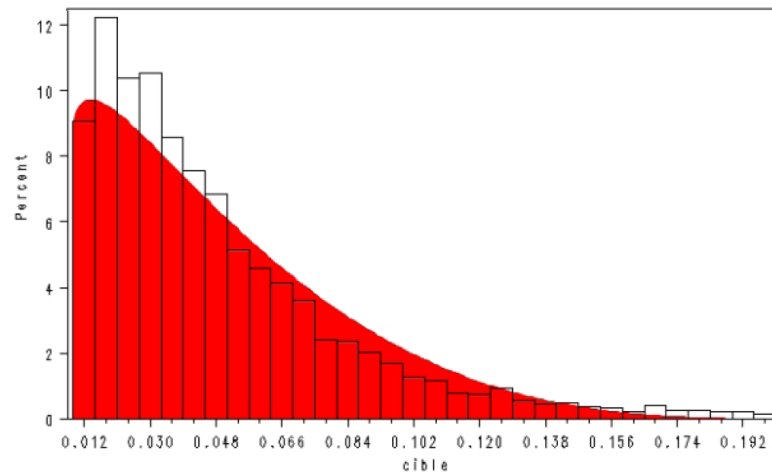N^{-1} The Inverse of the Normal Cumulative Distribution

Figure 3: Fitted Beta Normalised

α= The beta distribution's parameter

β = The beta distribution's shape parameter

Min= Minimum value of the target

Max=Maximum value of the value

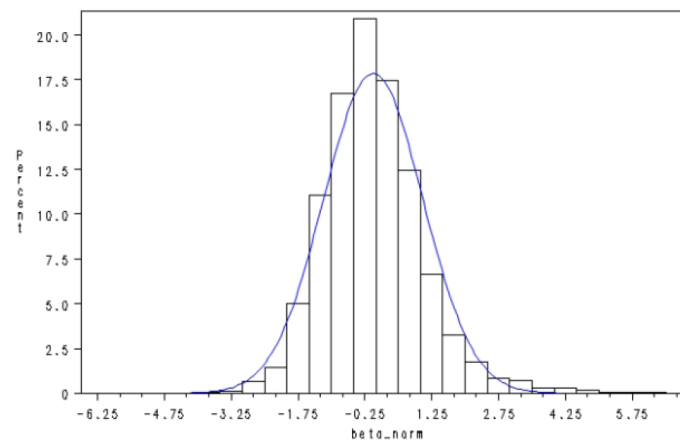The Figure 3 shows the shap of the distribution after conversion:

Figure 4: Normally distributed dependent variable

3.2   Detection of the most discriminating variables

To do our research, we have prepared a wide enough selection variable.

From this selection, we created ratios and interaction terms to expand again (see table above). Our goal was to find a model that meets the following criteria:

- Parsimony: A model should contain a small number of variables. We set 12 variables as maximum.

- Performance: The selected variables should have a strong combined predicting power and satisfy the statistical tests at this level.

- Significance: The selected variables taken individually or the model as a whole had to be statistically significant

- Interpretability: The selected variables have to present different risk factors, little correlation between them and have signs which allow valid economic interpretation.

With over 50 potential variables, the number of possible models exceeded the billiard 10^{15}. In order to meet the selection criteria in the best possible way, we have used our judgment and certain automated selection process (Figure 4).
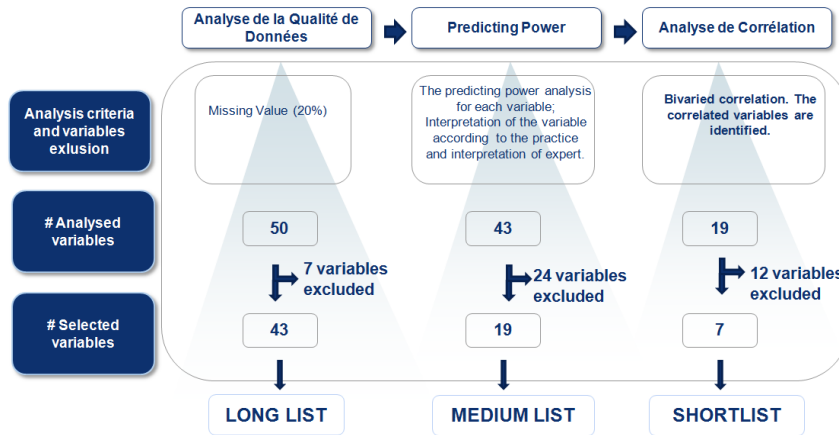

Figure 5: Selection variable process

First, we eliminated the variables which have little explanatory power. The indicator used to measure the predictif power is the coefficient of determination Saporta [12]:

$$R^2 = \frac{\sum_i (\widehat{y_i} - \bar{y_i})^2}{\sum_i (y_i - \bar{y_i})^2} = 1 - \frac{\sum_i (y_i - \widehat{y_i})^2}{\sum_i (y_i - \bar{y_i})^2}$$

with $\bar{y}_i$ the predictif value of y_i and $\bar{y}_i = \frac{\sum_i y_i}{n}$. This represents the part explained by the regression to the sum of squares of the deviations from the mean. More generally, R^2 is the square of the Pearson correlation coefficient of Y with its prediction $\widehat{y}_i$(in other word, its projection on the regression line). In all cases, R^2 ranges from 0 to 1 and the fit improves as R^2 approaches 1.

Once we were able to select a set of variables with a pretty good explanatory power (Medium list), we eliminated those showed colinearity with other variables. The indicator used to measure the degree of correlation between the variables is the coefficient of Spearman Kendall [13]. The Spearman correlation coefficient is a measure of association between two variables, numerical or alphanumeric. It can vary between -1 (when the variables are completely discordant) and 1 (when the variables are completely concordant). The indicator is calculated using the following formula:

$$\rho_s = 1 - \frac{6\sum_j D_i^2}{N(N^2 - 1)}$$

where $D_i = r_i - s_i$ is the difference of ranks (r_i , s_i are respectively the rank of the first and second variable of the i^{iem} observation).

Finally, we have the short list on which we are based to build model. To do this, we use the multiple linear regression with stepwise as methods of selection that ensures compliance with the characteristic that distinguishes a good model:

- parcimony of the variables number

- good explanatory power of the variables that go into the model as measured by the reduction of the residual sum of squares in the inclusion of the variable in the model.

The following equation gives the model selected by the regression.

$$\frac{Autorization}{Turnover}(\%) = 1.35 + 0.06\ R1 - 0.03\ R2 + 0.16\ R3 + 0.32\ R4 + 0.9\ R5.$$

with

R1: Net Income/Total Assets

R2: Interest/ Turnover

R3: Need Working capital / Working capital

R4: Stock * 360 / Turnover

R5: Aberage balance

We can group the five variables selected by the model in order to determine the credit limits in three topic:

- **Profitability** :
  The profitability of the borrower is a major condition for its long term survival. R1 measures how many cents the company can generate each MAD invested in its assets.

- **Leverage** :
  Show the capacity of the borrower to honor its debt. The ratio selected by the model measures the part of financial charge in the turnover. the higher the ratio, the less the bank must risk with the borrower.

- **Liquidity** :
  Liquidity ratios measures the borrower's ability to meet its short-term financial obligations. The model selected three ratios:

  - Need Working capital / Working capital

  - Average balance

- **Activity**:
  Activity ratios are used to measure the relative efficiency of a firm based on its use of its assets. The ratio selected is Stock * 360 / Turnover and measures the stock in days of turnover.

the coefficient of determination (R^2) is 70%. It means that the credit limit predicted by the model described above fit well the amount allowed by the credit risk manager in 70 % of the cases.

How can we improve the result?
After having modeled the amount authorized of facility with statistical approach, we adjust the credit limits by the rating classes obtained by the internal rating model. Thus, the credit limit model will be in line with the credit management process used in the bank.

Rating class	Credit quality	New Client	Old Client
A	Extremely strong	50% of Equity	100% of Expected profit
B	Very strong	50% of Equity	100% of Expected profit
C	strong	40% of Equity	80% of Expected profit
D	strong	30% of Equity	60% of Expected profit
E	Adequate	25% of Equity	40% of Expected profit
F	Medium	25% of Equity	30% of Expected profit
G	bad	25% of Equity	25% of Expected profit
H	Very close to default or in default	25% of Equity	Limit=0

Two ideas are generally accepted:

If the borrower is a new client, it is considered that the bank should never risk more than what the borrower have put themselves in the business. For old borrower, bank must not risk more than the expected profit $(CA_p{}^*t_m)$ achieved last exercise.

4   Conclusion

In first part, various methods to set a credit limits were presented but it is difficult to say that one method is better than another. Indeed, granting of a credit limit by the credit risk manager to a borrower depends on the specific opportunities of the bank, on the risk of the borrowers and on the bank's strategy towards certain sector.

The second part is distinguished by being the first empirical study in which the credit limit is estimated directly and not deduced through the PD.

To achieve our goals, we had to go through certain steps , explore various paths that help us to reveal several empirical evidence along the way.

After cleaning our database, we created a multitude of financial ratios most frequently used in the literature Hayden [14]. The objective here was to have of enough variables in order to develop credit limits model.

After exploring over fifty variables, worning selection techniques Automatic

and using our judgment, we identified model that meet our four objectives: Parsimony, Performance, Significance, Interpretability.

References

[1] Feldman, R., Small Banks and a big change in technology called credit scoring, *Federal Reserve bank of Minneapolis*, **The region** (September), (1997), 19 - 25.

[2] Altman, E., Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *Journal of Finance*, **13**(4), (Sept. 1968), 589 - 609.

[3] Canabarro E, *Pricing and hedging counterparty risk: lessons relearned? In Counterparty Credit Risk*, Risk Books, 2010.

[4] A. Hamerle, T. Liebig and Rosch D., Credit Risk Factor Modeling and the Basel II IRB Approach *Banking and Financial Supervision*, **2**(02/2003), (2003).

[5] Bertrand-H Abtey, *COMMENT EVALUER LES RISQUES LIES AUX INVESTISSEMENTS*, Dunod, 16 fvrier 1993.

[6] Dionne, G., Artis, M., and Guilln, M., Count data models for a credit scoring system, *Journal of Finance*, **journal of Empirical Finance 3**, (1996), 303-325.

[7] Altman E, Resti A, Sironi A (eds.) , *Recovery Risk The next Challenge in Credit Risk Management*, Risk Books, London, 2005.

[8] Jorion, P, *Value at Risk*, McGraw-Hill, 3rd edn, New York, 2006.

[9] LC Thomas & Edelman DB & JN Crook, *Credit Scoring And its Applications*, SIAM Philadelphia, 2000.

[10] Carboni, Alexandre, *Analyse de la prise en compte de la capacit payer dans la cotede risque pour les dfauts des particuliers*, M.Sc. finance, HEC Montral, Juillet 2007.

[11] Ferrari SLP, Cribari-Neto F, Beta Regression for Modelling Rates and Proportions, *Journal of Applied Statistics*, **31**(7), (2004), 799815.

[12] Gilbert Saporta, *Probabilits, Analyse des Donnees et Statistique*, Editions Technip, 2nd edn, 2006.

[13] Kendall M.G., *Rank correlation methods*, Griffin, Londres, 1962.

[14] Hayden E., *Modelling an Accounting-Based Rating Model for Austrian Firms*, unpublished PhD dissertation, University of Vienna, 2002.