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Reliability in the estimates and compliance to invertibility condition of stationary and nonstationary time series models

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Abstract

In this paper, we fit models to stationary and non-stationary series for comparison of the estimates of the data, considering invertibility condition for the models. The condition requires that every parameter of a time series model should lie between -1 and 1 exclusive. The distribution of autocorrelation and partial autocorrelation functions as shown Appendixes 1A, 1B, 2A and 2B, suggested AR(1) model for the non-stationary series and ARIMA(2,1,2) for the stationary series. The two models have given good estimates for the series, with residuals which are independently and identically distributed. This paper has established the fact that not until a series is stationary, it becomes invertible. This is affirmation of assertion by Box and Jenkins (1976) that invertibility is independent of stationarity. The models of non-stationary series that are not invertible are those whose data series are absolutely explosive in nature.

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1 Introduction

In time series analysis, there are two processes which explain the nature and distribution of time series data. There are autoregressive and moving average processes. The processes are identified on the basis of the distribution of autocorrelation and partial autocorrelation functions. Box and Jenkins (1976) described a process to be autoregressive, if it exhibits exponential decay or sine wave pattern in autocorrelation function and a cut off at a certain lag in partial autocorrelation function. While, moving moving average process is described by the exhibition of exponential decay or sine wave pattern in partial autocorrelation function and cut off at certain lag in the autocorrelation function. It is a popular practice in time series that stability of data has to be ensured before a suitable model is suggested to the time series data. This is so because parameter(s) of the fitted time series model is expected to have values that will give room for invertibility. The assumption of stationarity means the mean and variance of the series are constant over time and that the structure of the series depends only upon the relative position in time of the observations, Kendell and Ord (1993). Box and Cox(1964) introduced the class of variance stability transformation. The condition of stationarity is clearly fundamental to the statistical analysis of time series, but it is not an assumption that can be made automatically. For the assumption of stationarity, condition of weak, second order or covariance stationary should be satisfied at least to a reasonable degree. This fact does not negate fitting time series models to non-stationary series so as to ascertain if stability is required in every non-stationary series. Usoro and Omekara (2008) fitted Bilinear Autoregressive Vector models to non-stationary revenue data. The fitted models gave good estimates with uncorrelated error term. Multivariate time series models were fitted to non-stationary series, with a response and two predictor vectors. Estimates obtained from the models were good and autocorrelation functions were uncorrelated, Usoro and Omekara (2007). The motivation behind this work is to fit time series models to both stationary and non-stationary series for comparison of estimates and checking if the parameters of both models give room for invertibility.

2 Stationary and non-stationary models

Kendall and Ord (1973) stated the general autoregressive time series model as,

$$\varphi(B)Y_t = \epsilon_t \quad (2.1)$$

By expansion, the model becomes,

$$(1 - \varphi_1 B - \varphi_2 B^2 - \varphi_3 B^3 - \dots - \varphi_p B^p)Y_t = \epsilon_t$$

$$\Rightarrow \quad Y_t - \varphi_1 BY_t - \varphi_2 B^2 Y_t - \varphi_3 B^3 Y_t - \dots - \varphi_p B^p Y_t = \epsilon_t$$

$$\Rightarrow \quad Y_t - \varphi_1 Y_{t-1} - \varphi_2 Y_{t-2} - \varphi_3 Y_{t-3} - \dots - \varphi_p Y_{t-p} = \epsilon_t$$

$$\Rightarrow \quad Y_t = \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \varphi_3 Y_{t-3} + \dots + \varphi_p Y_{t-p} + \epsilon_t \quad (2.2)$$

where Y_t is the time series process, φ_1 φ_2, …,φ_p are the parameters of the model and B, B^2, …, B^p are the backward shift operators.

The general autoregressive moving average model is given by,

$$\varphi(B)Y_t = \Theta(B)\epsilon_t \quad (2.3)$$

By expansion, the model becomes

$$(1 - \varphi_1 B - \varphi_2 B^2 - \varphi_3 B^3 - \dots - \varphi_p B^p)Y_t = (1 - \Theta_1 B - \Theta_2 B^2 - \dots - \Theta_q B^q)\epsilon_t$$

$$\Rightarrow \quad Y_t - \varphi_1 BY_t - \varphi_2 B^2 Y_t - \varphi_3 B^3 Y_t - \dots - \varphi_p B^p Y_t = \epsilon_t - \Theta_1 B - \Theta_2 B^2 - \dots - \Theta_q B^q$$

$$\Rightarrow \quad Y_t - \varphi_1 Y_{t-1} - \varphi_2 Y_{t-2} - \varphi_3 Y_{t-3} - \dots - \varphi_p Y_{t-p} = \epsilon_t - \Theta_1 \epsilon_{t-1} - \Theta_2 \epsilon_{t-2} - \dots - \Theta_q \epsilon_{t-q}$$

$$\Rightarrow Y_t = \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \varphi_3 Y_{t-3} + \dots + \varphi_p Y_{t-p} + \epsilon_t - \Theta_1 \epsilon_{t-1} - \Theta_2 \epsilon_{t-2} - \dots - \Theta_q \epsilon_{t-q} \quad (2.4)$$

Model '1.4' is ARMA model for non-difference series, Johnston and Dinardo (1997). If a series is differenced, model 1.1 and becomes,

$$\varphi(B)(1-B)Y_t = \epsilon_t$$

$$\Rightarrow \quad (1 - \varphi_1 B - \varphi_2 B^2 - \varphi_3 B^3 - \dots - \varphi_p B^p)(1-B)Y_t = \epsilon_t \qquad (2.5)$$

While (1.3) becomes,

$\varphi(B)Y_t = \Theta(B)\epsilon_t$

$$\Rightarrow (1 - \varphi_1 B - \varphi_2 B^2 - \varphi_3 B^3 - \dots - \varphi_p B^p)(1-B)Y_t = (1 - \Theta_1 B - \Theta_2 B^2 - \dots - \Theta_q B^q)\epsilon_t \quad (2.6)$$

Where (1-B) is the difference operator

3 Estimation of parameters of non-stationary and stationary series

Before the parameters estimated, there must be a choice of a model through the distribution of correlogram. From Appendix 1a, 1b, 2a and 2b, the distribution of autocorrelation and partial autocorrelation functions have suggested AR (1) model for the non-stationary series and ARIMA (2, 1, 2) for the stationary series.

3.1 The AR (1) model

The AR (1) model is given by,

$$Y_t = \varphi_1 Y_{t-1} + \epsilon_t \qquad (3.1)$$

where φ_1 is the parameter of the model, ϵ_t is the error term assumed to independently and identically distributed with zero mean and constant variance. The fitted model is $Y_t = 0.9989Y_{t-1}$. The graph of original with estimated values is shown in Figure 1. The estimates from the model are in Appendix 3.

3.2 The ARIMA (2,1,2) model

The ARIMA (2, 1, 2) model for the stationary series is given by,

$$(1 - \varphi_1 B - \varphi_2 B^2)(1-B)Y_t = (1- \Theta_1 B - \Theta_2 B^2)\epsilon_t$$
$$=> (1 - B - \varphi_1 B - \varphi_1 B^2 - \varphi_2 B^2 + \varphi_2 B^3)Y_t = \epsilon_t - \Theta_1\epsilon_{t-1} - \Theta_2\epsilon_{t-2}$$
$$=> Y_t - Y_{t-1} - \varphi_1 Y_{t-1} + \varphi_1 Y_{t-2} - \varphi_2 Y_{t-2} + \varphi_2 Y_{t-3} = \epsilon_t - \Theta_1\epsilon_{t-1} - \Theta_2\epsilon_{t-2}$$
$$=> Y_t - Y_{t-1} = \varphi_1(Y_{t-1} - Y_{t-2}) + \varphi_2(Y_{t-2} - Y_{t-3}) + \epsilon_t - \Theta_1\epsilon_{t-1} - \Theta_2\epsilon_t \qquad (3.2)$$

If $Y_t - Y_{t-1} = y_t$, $Y_{t-1} - Y_{t-2} = y_{t-1}$, $Y_{t-2} - Y_{t-3} = y_{t-2}$, in (3.2), becomes

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \epsilon_t - \Theta_1\epsilon_{t-1} - \Theta_2\epsilon_{t-2} \qquad (3.3)$$

where y_t is the difference series.

Therefore $\hat{Y}_t$(estimate Y_t) can be obtained in either of the following ways:

(1) fitting ARMA (2,0,2) to y_t, so that $\hat{y}_t + Y_{t-1} = \hat{Y}$. (2) fitting ARIMA (2,1,2) to Y_t.

The fitted ARMA (2,0,2) to y_t yields,

$\hat{y}_t = 0.5244y_{t-1} + 0.1991y_{t-2} + \epsilon_t - 0.4294\epsilon_{t-1} - 0.0516\epsilon_{t-2}$.

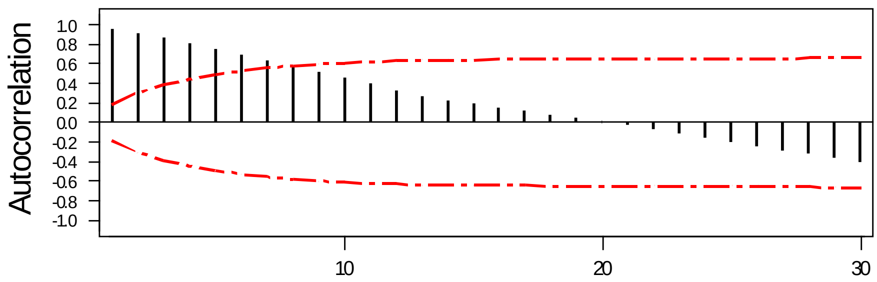
The graph of original with estimated values is shown in Figure 2. The estimates from the model are in Appendix 3.

4 Conclusion

There is no gainsaying the fact that stationarity of time series data is very expedient in building autoregressive moving average model. This is due to the condition of invertibility and of course duality between the autoregressive and moving average processes. The invertibility condition provides that the parameter of a model, say AR(1) should neither be less than -1 nor greater than 1. It is the fear of the unknown explosive or evolutionary behavior of non-stationary series that motivates stationarity of a series before model building. However, in this paper, we have been able to show that non-stationary series can be invertible (that is the roots of $\varphi(B) = 0$ lie outside the unit circle, as the parameters lie within the unit circle). It is an indisputable fact that any non-stationary series that is absolutely explosive in nature must have a parameter lying outside the unit circle.

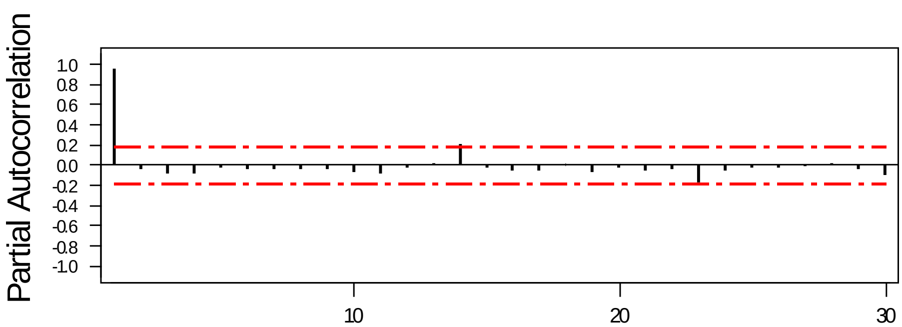
That is a clear case of violation of invertibility condition. The exhibition of such explosive behavior calls for differencing for stability of the process. Therefore, it is not every non-stationary series that violet invertibility condition.

Appendix 1A: ACF of original data



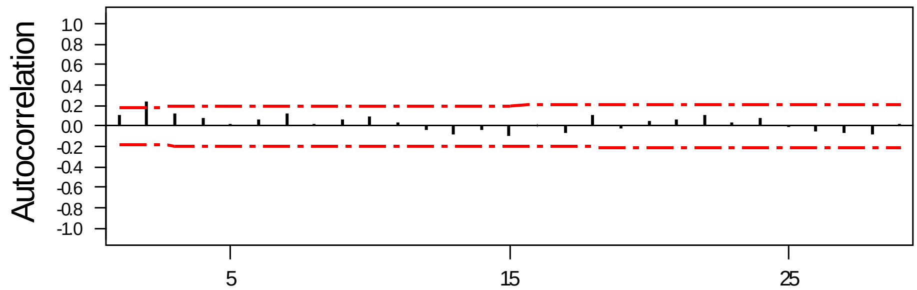
Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.96	10.49	112.86	10	0.46	1.48	685.93	19	0.04	0.12	748.22	28	-0.33	-0.99	801.53
2	0.91	5.95	216.46	11	0.39	1.24	706.39	20	0.00	0.01	748.22	29	-0.37	-1.10	823.27
3	0.86	4.46	309.85	12	0.33	1.02	720.80	21	-0.03	-0.11	748.40	30	-0.41	-1.21	850.39
4	0.81	3.62	392.38	13	0.27	0.83	730.46	22	-0.07	-0.22	749.18				
5	0.75	3.05	464.51	14	0.23	0.70	737.47	23	-0.12	-0.36	751.26				
6	0.70	2.62	526.57	15	0.19	0.57	742.30	24	-0.16	-0.49	755.21				
7	0.64	2.28	579.14	16	0.15	0.46	745.38	25	-0.20	-0.62	761.55				
8	0.58	1.98	622.78	17	0.11	0.34	747.14	26	-0.25	-0.75	770.97				
9	0.52	1.72	658.21	18	0.08	0.24	747.99	27	-0.29	-0.88	784.25				

Appendix 1B: PACF of original data



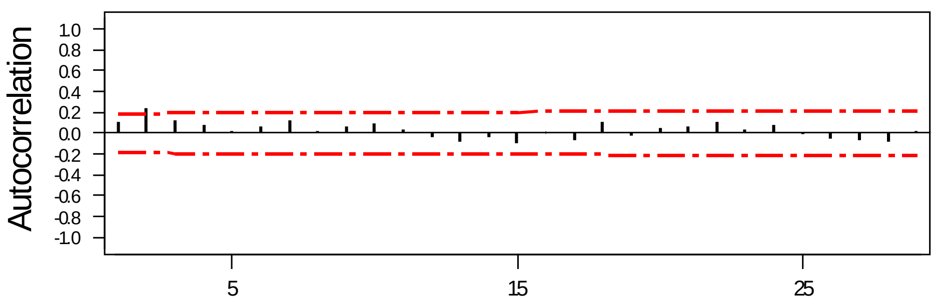
Lag	PAC	T	Lag	PAC	T	Lag	PAC	T	Lag	PAC	T
1	0.96	10.49	10	-0.07	-0.74	19	-0.08	-0.86	28	0.01	0.09
2	-0.04	-0.48	11	-0.09	-1.01	20	-0.03	-0.38	29	-0.04	-0.44
3	-0.09	-1.02	12	-0.02	-0.25	21	-0.06	-0.70	30	-0.11	-1.18
4	-0.09	-0.99	13	0.01	0.09	22	-0.04	-0.45			
5	-0.03	-0.35	14	0.21	2.30	23	-0.17	-1.86			
6	-0.04	-0.49	15	-0.03	-0.35	24	-0.06	-0.60			
7	-0.04	-0.41	16	-0.05	-0.60	25	-0.02	-0.26			
8	-0.04	-0.49	17	-0.05	-0.60	26	-0.03	-0.37			
9	-0.04	-0.44	18	-0.00	-0.05	27	-0.02	-0.20			

Appendix 2A: ACF of difference data

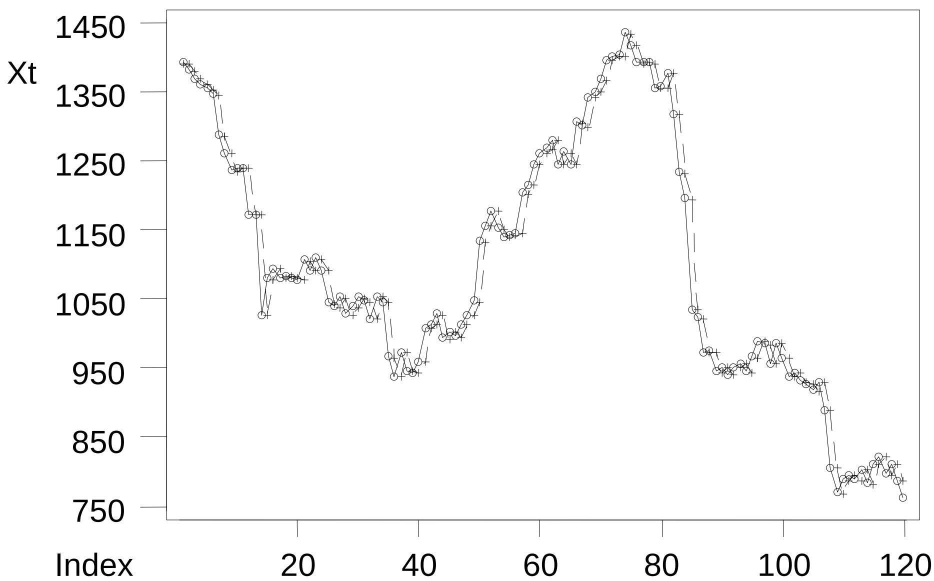


Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.11	1.19	1.46	10	0.09	0.85	14.16	19	-0.03	-0.33	20.13	28	-0.08	-0.78	26.17
2	0.23	2.53	8.23	11	0.03	0.31	14.28	20	0.05	0.44	20.43	29	0.02	0.15	26.22
3	0.12	1.19	9.92	12	-0.04	-0.40	14.50	21	0.07	0.62	21.06				
4	0.07	0.76	10.62	13	-0.09	-0.84	15.49	22	0.10	0.98	22.66				
5	0.02	0.21	10.67	14	-0.05	-0.48	15.82	23	0.03	0.28	22.80				
6	0.06	0.56	11.06	15	-0.11	-1.06	17.45	24	0.07	0.70	23.64				
7	0.11	1.15	12.75	16	0.01	0.05	17.45	25	-0.02	-0.17	23.69				
8	0.01	0.11	12.77	17	-0.08	-0.77	18.35	26	-0.06	-0.55	24.22				
9	0.06	0.56	13.19	18	0.11	1.02	19.95	27	-0.07	-0.68	25.06				

Appendix 2B: PACF of difference data

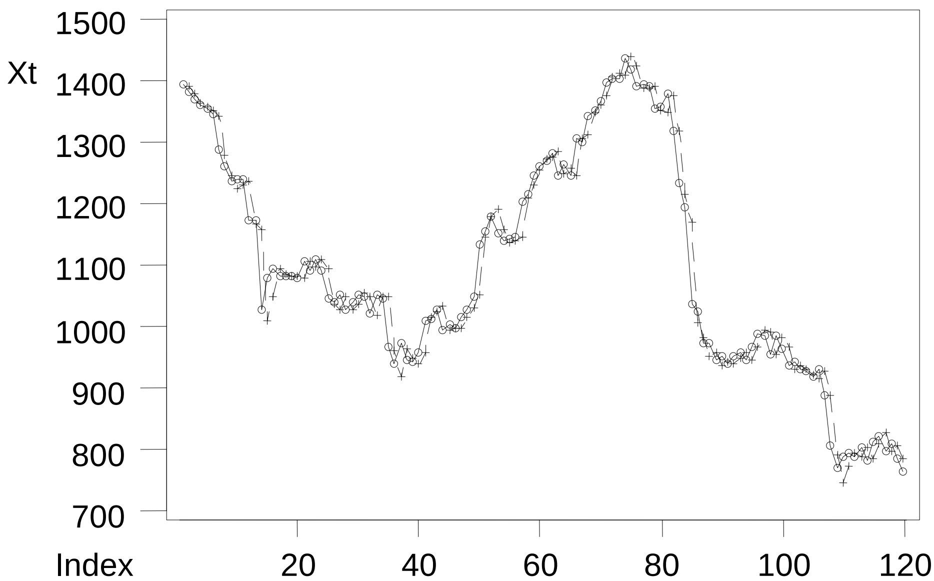


Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ	Lag	Corr	T	LBQ
1	0.11	1.19	1.46	10	0.09	0.85	14.16	19	-0.03	-0.33	20.13	28	-0.08	-0.78	26.17
2	0.23	2.53	8.23	11	0.03	0.31	14.28	20	0.05	0.44	20.43	29	0.02	0.15	26.22
3	0.12	1.19	9.92	12	-0.04	-0.40	14.50	21	0.07	0.62	21.06				
4	0.07	0.76	10.62	13	-0.09	-0.84	15.49	22	0.10	0.98	22.66				
5	0.02	0.21	10.67	14	-0.05	-0.48	15.82	23	0.03	0.28	22.80				
6	0.06	0.56	11.06	15	-0.11	-1.06	17.45	24	0.07	0.70	23.64				
7	0.11	1.15	12.75	16	0.01	0.05	17.45	25	-0.02	-0.17	23.69				
8	0.01	0.11	12.77	17	-0.08	-0.77	18.35	26	-0.06	-0.55	24.22				
9	0.06	0.56	13.19	18	0.11	1.02	19.95	27	-0.07	-0.68	25.06				



Plot of Original in circle
Plot of Estimates in plus

Figure1: Graph of Original with Estimates of Non-Stationary Data



Plot of Original in circle
Plot of Estimates in plus

Figure2: Graph of Original with Estimates of Stationary Data

Appendix 3: Original and Estimates From Stationary and Non-Stationary Models

S/N	Y_t	$S\hat{Y}_t$	$NS\hat{Y}_t$	S/N	Y_t	$S\hat{Y}_t$	$NS\hat{Y}_t$
1	1393	1389.99	-	61	1269	1234.66	1242.53
2	1382	1391.49	1390.99	62	1281	1238.66	1245.37
3	1369	1380.51	1379.16	63	1246	1259.64	1266.23
4	1362	1367.52	1364.82	64	1263	1278.62	1287.43
5	1355	1360.53	1357.48	65	1246	1289.60	1298.31
6	1346	1353.54	1351.14	66	1306	1296.60	1303.81
7	1287	1344.54	1342.22	67	1301	1304.59	1310.66
8	1261	1285.61	1278.24	68	1342	1299.59	1304.01
9	1236	1259.64	1245.87	69	1351	1340.55	1346.69
10	1240	1234.66	1222.84	70	1368	1349.54	1360.07
11	1240	1238.66	1230.26	71	1397	1366.52	1375.08
12	1173	1238.66	1235.73	72	1402	1395.49	1405.77
13	1173	1171.73	1159.16	73	1404	1400.48	1410.88
14	1027	1025.89	1006.73	74	1435	1402.48	1409.20
15	1079	1077.83	1052.99	75	1418	1410.47	1415.74
16	1094	1092.82	1090.88	76	1392	1416.47	1421.63
17	1081	1079.83	1079.29	77	1394	1390.50	1392.16
18	1083	1081.83	1080.38	78	1392	1392.49	1390.61
19	1081	1079.83	1079.89	79	1356	1390.50	1390.66
20	1078	1076.83	1079.89	80	1357	1354.53	1351.53
21	1106	1076.83	1076.81	81	1378	1355.53	1349.80
22	1091	1104.80	1107.65	82	1318	1376.51	1376.82
23	1109	1089.82	1094.35	83	1233	1316.58	1314.52
24	1092	1107.80	1110.02	84	1195	1231.67	1214.52

25	1044	1090.82	1093.65	85	1035	1193.71	1170.74
26	1039	1042.87	1037.69	86	1023	1033.88	1002.82
27	1052	1037.88	1028.82	87	972	1021.89	983.19
28	1028	1050.86	1047.80	88	974	970.95	946.63
29	1039	1026.89	1025.31	89	944	972.95	953.72
30	1052	1037.88	1035.13	90	951	942.98	931.42
31	1047	1050.86	1053.06	91	940	949.97	940.79
32	1021	1045.87	1048.70	92	951	938.98	934.96
33	1053	1019.90	1018.57	93	957	949.97	947.73
34	1045	1051.86	1051.25	94	944	955.97	957.53
35	966	1043.87	1048.08	95	966	942.98	943.71
36	938	964.96	958.55	96	989	964.96	966.08
37	973	936.99	920.65	97	985	987.93	994.45
38	946	971.95	964.36	98	956	983.94	990.36
39	943	944.98	943.99	99	986	954.97	955.23
40	959	941.98	937.42	100	965	984.93	984.52
41	1008	957.96	957.58	101	937	963.96	966.75
42	1013	1006.91	1014.12	102	943	935.99	931.92
43	1028	1011.90	1023.26	103	931	941.98	937.35
44	993	1026.89	1034.88	104	926	929.99	928.06
45	1003	991.93	995.37	105	917	925.00	922.20
46	996	1001.92	1000.16	106	929	916.01	913.62
47	1014	994.92	995.71	107	889	928.00	927.17
48	1027	1012.90	1014.41	108	806	888.04	886.01
49	1047	1025.89	1031.05	109	769	805.13	790.83
50	1133	1045.87	1052.58	110	788	768.17	746.58
51	1156	1131.78	1146.73	111	794	787.15	773.94
52	1178	1154.75	1177.05	112	788	793.14	790.18

53	1152	1176.73	1193.23	113	803	787.15	785.95
54	1140	1150.75	1160.40	114	782	802.13	802.46
55	1143	1138.77	1139.42	115	811	781.15	781.88
56	1146	1141.76	1141.70	116	822	810.12	810.58
57	1204	1144.76	1146.14	117	796	821.11	827.14
58	1215	1202.70	1209.95	118	810	795.14	797.33
59	1245	1213.69	1227.16	119	786	809.12	808.33
60	1261	1222.68	1232.01	120	763	785.15	785.14

Key:

Y_t = Original Series

$S\hat{Y}_t$ = Estimates from Stationary Model

$NS\hat{Y}_t$ = Esimates from Non-Stationary Model

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