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Loss Given Default: Estimating by analyzing the distribution of credit assets and Validation

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Abstract

The Basel II Accord offers banks the opportunity to estimate Loss Given Default (LGD) if they wish to calculate their own value for the capital required to cover credit losses in extreme circumstances. This paper will analyze the various methods of modeling LGD and will provide an alternative estimate of LGD using Merton's model for the valuation of assets. Four components will be developed in this document: estimation of the minimum value that could have a financial asset, estimation of the loss given default LGD, development of a practical component, and finally validation of the proposed model.

JEL classification numbers: G17, G24, G32
Keywords: Credit Risk Modeling, Loss Given Default, Rating Model, Basel 2, Merton's model, Backtesting.

1 Introduction

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Loss Given Default (LGD) is one of the most crucial key parameters needed to evaluate the expected and unexpected credit losses necessary for credit pricing as well as for calculation of the regulatory Basel requirement. While the credit rating and probability of default (PD) techniques have been advancing in recent decades.

A lot of focus has been devoted to the estimation of PD while LGD has received less attention and has at times been treated as a constant. Das and Hanouna noted in 2008 that using constant loss estimates could be misleading inasmuch as losses vary a great deal. According to Moody's 2005 findings; average recovery rates, defined as 1- LGD, can vary between 8% and 74% depending on the year and the bond type. For sophisticated risk management, LGD undoubtedly needs to be assessed in greater detail.

If a bank uses the Advanced IRB approach, the Basel II Accord allows it to use internal models to estimate the LGD. While initially a standard LGD allocation may be used for The Foundation Approach, institutions that have adopted the IRB approach for probability of default are being encouraged to use the IRB approach for LGD because it gives a more accurate assessment of loss. In many cases, this added precision changes capital requirements.

This paper is formulated into two sections:

The theoretical section, which has highlighted the overall LGD estimation models in recent decades as well as a theoretical model proposed by way of:

- Calculating the minimum value that could be an asset for T based on the Merton model.
- Elaborating a mathematical development to estimate LGD calculated using the minimum value.
- A detail will be provided in the model developed to specify the LGD formula in the case of a single asset then again in the case of several assets.

The Practical Section, which includes:

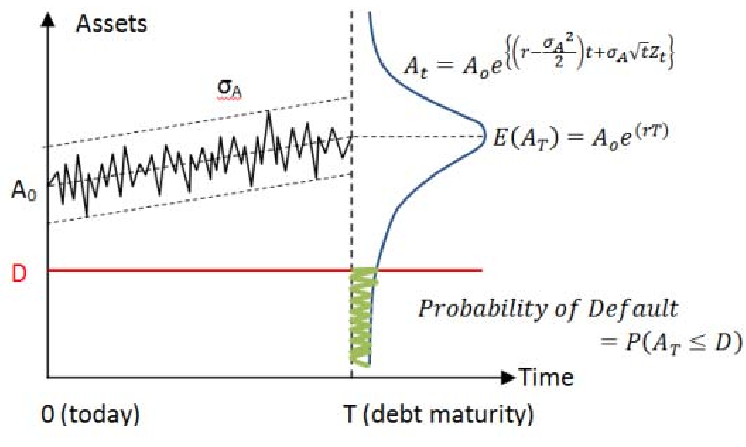
- An application made according to the proposed model using actual data from a Moroccan bank. This application will be done in two cases: single asset then again in several assets to highlight the effect of the correlation of assets that could minimize LGD rates.
- A Backtesting program will be conducted to check the estimated power of the proposed model.

2 Literature Review of LGD Estimation Models

LGD has attracted little attention before the 21st century; one of the first papers on the subject written by Schuermann 2004 provides an overview of what was known about LGD at that time. Since the first Basel II consultative papers were published there has been an increasing amount of research on LGD estimation techniques (Altman – Resti – Sironi, 2004; Frye, 2003; Gupton, 2005; Huang – Oosterlee, 2008; etc.).
One of the last models produced to estimate the LGD is the LossCalc model introduced by Moody's KMV3 The general idea for estimating the recovery rate is to apply a multivariate linear regression model including certain risk factors, e.g., industry factors, macroeconomic factors, and transformed risk factors resulting from "mini-models".
Another estimation model proposed by Steinbauer and Ivanova (2006)4, consists of two steps, namely a scoring and a calibration step. The scoring step includes the estimation of a score using collateralization, haircuts, and expected exposure at default of the loan and recovery rates of the uncollateralized exposure. The score itself can be interpreted as a recovery rate of the total loan but is only used for relative ordering in this case.

2.1 Theoretical Framework for Estimating Expected Loss Given Default

Merton (1974) and Black and Scholes (1973) proposed a simple model of the firm that provides a way of relating credit risk to the capital structure of the firm. In this model the value of the firm's assets is assumed to follow a lognormal diffusion process with a constant volatility.



[3] Losscalc v2: dynamic prediction of LGD, modeling methodology, Gupton and Sttein (2005)
[4] Internal LGD Estimation in Practice

$$A_{i,t} = A_{i,0} e^{\left(\left(\mu i - \frac{\sigma i^2}{2}\right).t + \sigma i.X_{i,t}\right)} \quad (1)$$

$$X_{i,t} \sim N\left(0, \sqrt{t}\right) \quad (2)$$

$X_{i,t}$ is a Wiener process with an expectation of 0 and variance t

$$(1) \text{ et } (2) \Rightarrow ln\,(A_{i,t}) = \ln\,(A_{i,0}) + \left(\left(\mu i - \frac{\sigma i^2}{2}\right).t + \sigma i.X_{i,t}\right)$$

$$\Rightarrow ln\,(A_{i,t}) \sim N\left(\ln\,(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right).t, \sigma i\sqrt{t}\right)$$

So $A_{i,t}$ follows a lognormal distribution with parameters $\ln\,(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right).t$ and $\sigma i\sqrt{t}$ with a density function $g(x) = \frac{1}{x} \cdot \frac{1}{b\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln(x)-a}{b}\right)^2}$

$$a = \ln\,(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right).t \text{ et } b = \sigma i\sqrt{t}$$

It is possible to calculate expectancy of $A_{i,t}$ according to the log normal distribution $\mu_{A_{i,t}} = E\left(A_{i,t}\right) = e^{\ln\,(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right).t + \sigma i^2.\frac{t}{2}}$

So $\mu_{A_{i,t}} = A_{i,0}.\,e^{\mu i.t}$

As the variance

$$\sigma^2_{A_{i,t}} = e^{2\left(a+b^2\right)}.\left(\frac{e^{b^2} - 1}{e^{b^2}}\right)$$

$$= e^{2\left(\ln\,(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right).t + \sigma i^2.t\right)}.\left(\frac{e^{\sigma i^2.t} - 1}{e^{\sigma i^2.t}}\right)$$

$$= e^{2\ln\,(A_{i,0}) + 2\mu i.t + \sigma i^2.t}.\left(\frac{e^{\sigma i^2.t} - 1}{e^{\sigma i^2.t}}\right)$$

$$\sigma^2_{A_{i,t}} = A^2_{i,0} e^{2\mu i.t}\left(e^{\sigma i^2.t} - 1\right)$$

Calculation of the minimum value of the asset A_i

We have $\ln(A_{i,t}) \sim N\left(\ln(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right).t, \sigma i\sqrt{t}\right)$

For a fixed probability α, we define Min_α by $P(\ln(A_{i,t}) < Min_\alpha) = \alpha$

$$=> P\left(\frac{\ln(A_{i,t}) - \ln(A_{i,0}) - \left(\mu i - \frac{\sigma i^2}{2}\right).t}{\sigma i\sqrt{t}} < \frac{Min_\alpha - \ln(A_{i,0}) - \left(\mu i - \frac{\sigma i^2}{2}\right).t}{\sigma i\sqrt{t}}\right) = \alpha$$

$$=> Min_{\ln(A_{i,t}),\alpha} = \ln(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right).t + \sigma i\sqrt{t}.N^{-1}(\alpha)$$

$$=> Min_{A_{i,t},\alpha} = e^{\ln(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right).t + \sigma i\sqrt{t}.N^{-1}(\alpha)}$$

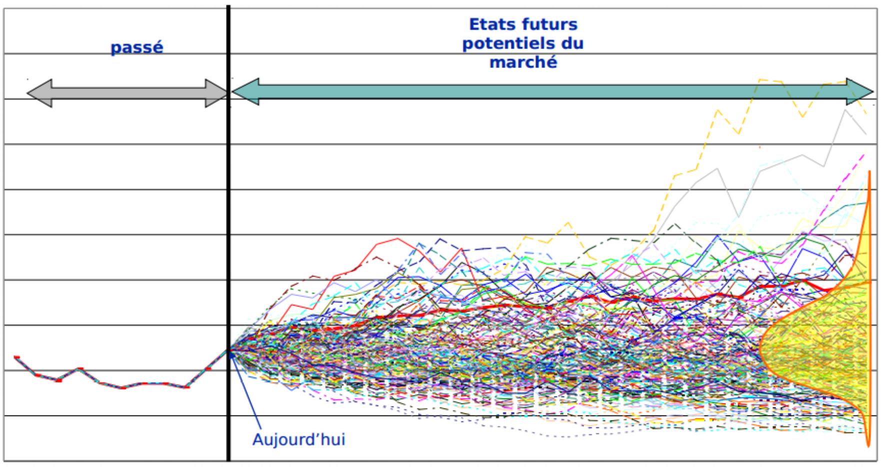
$$=> Min_{A_{i,t},\alpha} = e^{\ln(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right).t + \sigma i\sqrt{t}.N^{-1}(\alpha)}$$

In the fact that

$t = \frac{n}{T}$ And $n = 1 \ldots . T$

In the maturity $n = T$

$$Min_{A_{i,T},\alpha} = e^{\ln(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right) + \sigma i N^{-1}(\alpha)} \quad (3)$$



The formula (3) is very useful for financial calculations under the minimum value that could reach the asset Ai at any time t, specifically at maturity T, which can be regarded as a VaR according to a previously specified risk level.

2.2 Estimated loss rate (LGD)

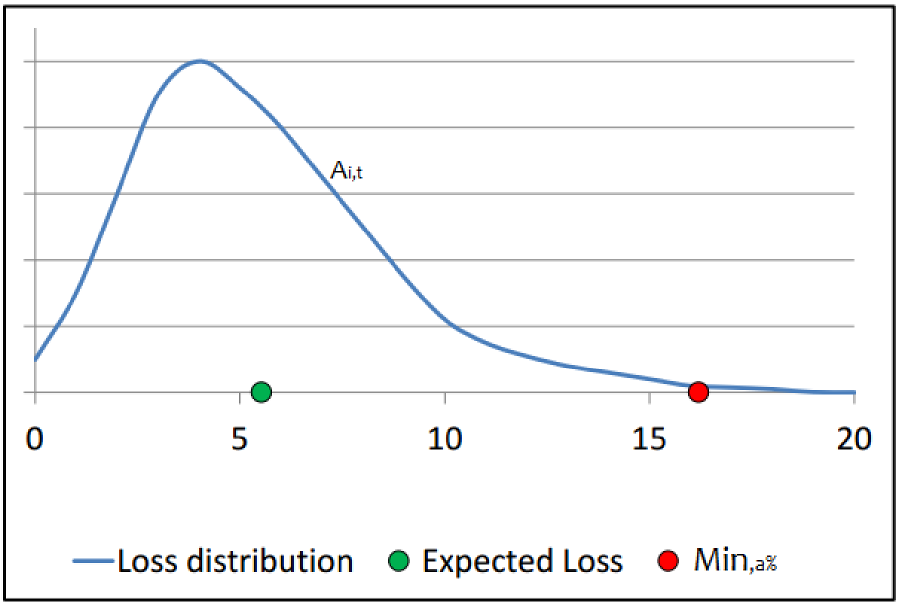
LGD is calculated in various ways, but the most popular is 'Gross' LGD, where total losses are divided by exposure at default (EAD). An alternate method is to divide losses by the unsecured portion of a credit line (where security covers a portion of EAD. This is known as 'Blanco' LGD. If the collateral value is zero in the last case then Blanco LGD is equivalent to Gross LGD. A variety of statistical methods may be applied.

In this article, the rate of LGD will be calculated according to the minimum value

With the formula (3), we can already get an idea of the impairment of financial assets over time (t), which is essential to calculate the rate of percentage loss of the initial value of a financial asset.

In this section, a development of the formula (3) will be established by calculating loss rates (LGD) that could represent a financial asset.

The chart below revealed two losses of asset $A_{i,t}$, an average loss and other unexpected with a level of risk α.



With α lower level of risk, it is possible to calculate an unexpected loss as in the previous section. This loss will be used to determine the unexpected loss rate with the use of the initial value of the asset A as:

$$LGD_{A_{i,t},\alpha} = \frac{A_{i,0} - Min_{A_{i,t}\,\alpha}}{A_{i,0}}$$

$$LGD_{A_{i,t},\alpha} = 1 - \frac{e^{\ln(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right).t + \sigma i\sqrt{t}.N^{-1}(\alpha)}}{A_{i,0}}$$

$$LGD_{A_{i,t},\alpha} = 1 - e^{\left(\mu i - \frac{\sigma i^2}{2}\right).t + \sigma i.\sqrt{t}.N^{-1}(\alpha)}$$

With $t = \frac{n}{T}$ And $n = 1 \ldots . T$

En n = T => t = 1

$$LGD_{A_{i,T},\alpha} = 1 - e^{\left(\mu i - \frac{\sigma i^2}{2}\right) + \sigma i.N^{-1}(\alpha)} \quad (4)$$

a. Case of a Single Asset A_i

When t = T $\quad LGD_{\alpha} = 1 - e^{N^{-1}\left(\left(\mu i - \frac{\sigma i^2}{2}\right),\ \sigma i,\ \alpha\right)}$

$$LGD_{\alpha} = 1 - e^{\left(\mu i - \frac{\sigma i^2}{2}\right) + \sigma i.\varepsilon_{\alpha}}$$

ε_{α} is the risk taken on assets (standard normal distribution law)

b. Case of two Assets

$$Min_{A_{i,t} + A_{j,t},\alpha} = e^{N^{-1}\left(\ln(A_{i,0} + A_{j,0}) + \left(w_i.\mu i + w_j.\mu j - \frac{\sigma ij^2}{2}\right).t,\ \sigma ij\sqrt{t},\ \alpha\right)}$$

w_i , w_j are weights of the assets i, j

$\sigma ij^2 = w_i{}^2.\sigma i^2 + 2 * \rho.w_i.w_j\sigma i.\sigma j + w_j{}^2\sigma j^2$ And ρ is the correlation between $A_{i,t}$ and $A_{j,t}$

$$LGD_{A_i + A_j,\alpha} = 1 - e^{\left(w_i.\mu i + w_j.\mu j - \frac{\sigma ij^2}{2}\right) + \sigma ij.\varepsilon_{\alpha}} \quad (5)$$

c. Case of several credit portfolio as well

$$LGD_{\sum_{i=1}^{p} A_{i,T},\ \alpha} = 1 - e^{\sum_{i=1}^{p}\left(w_i\mu i - \frac{R}{2}\right) + \sqrt{R}.\varepsilon_{\alpha}} \quad (6)$$

Such as $R = t_W.\sum w$ and $\sum i$ is the variance covariance matrix of the assets and w_i is the weight of the asset i

With the presence of several assets (credits) in the bank's portfolio, it could establish the correlation of assets to minimize LGD shown with this correlation. The average of LGD is less than the calculated LGD overall portfolio (diversification principle).

Main Results

2.3 Illustration of the calculation of the minimum value and the LGD

2.3.1 Case of a single Asset

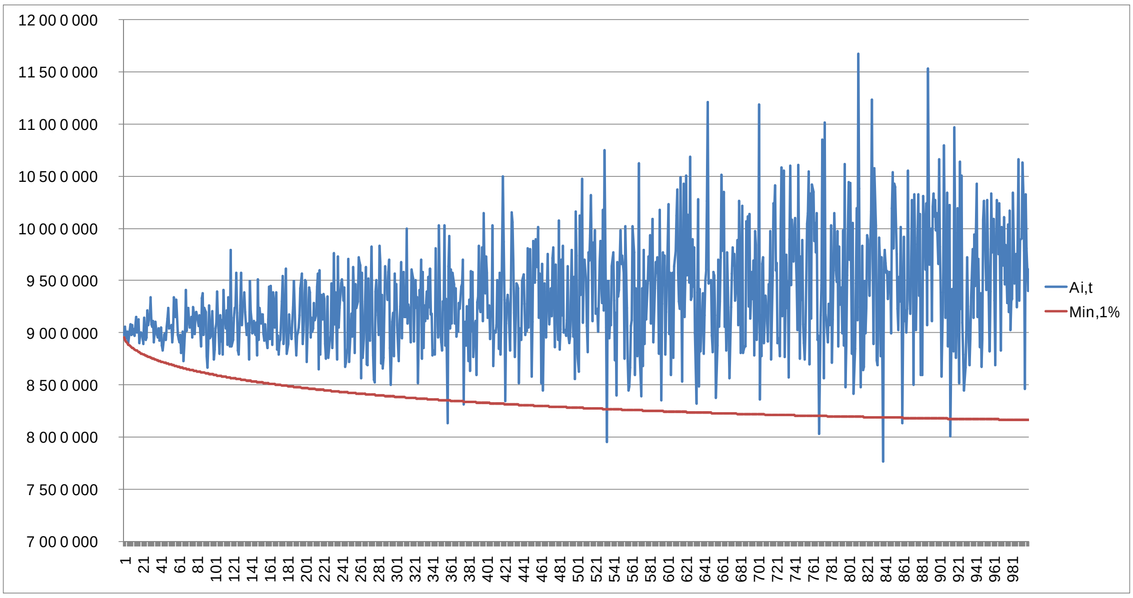
Taking the formula: $\text{Min}_{A_{i,t},\alpha} = e^{N^{-1}\left(\ln(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2}\right).t \ , \ \sigma i\sqrt{t} \ , \ \alpha\right)}$

Company	Year	Sales (MAD)	Assets (MAD)	Rate of return
C1	1	17 500 000	7 000 000	
	2	16 250 000	6 500 000	-7,1%
	3	20 000 000	8 000 000	23,1%
	4	18 750 000	7 500 000	-6,3%
	5	22 500 000	9 000 000	20,0%
Average return	7,42%			
Volatility	16,35%			

With $\sigma i^2 = \frac{\sum_{j=1}^{n}(\mu_{i,j} - \bar{\mu}_i)^2}{n}$ and $\bar{\mu}_i = \frac{\sum_{j=1}^{n}\mu_{i,j}}{n}$

$\sigma i = 16{,}35\%$ and $\bar{\mu}_i = 7{,}42\%$

We would calculate $\text{Min}_{A_{i,t},\alpha}$ with $\alpha = 1\%$ as a risk level from the fifth year, posing $A_{i,0} = 9.000.000$ Dhs



The chart above shows the distribution of asset Ai,t versus t, with T = 1.000 according to a number of simulations, the final value of Min $_{A_{i,T},1\%}$= 6,538,538 MAD with $LGD_{1\%}$ = 27.35% which is equivalent to the A_i loss percentage.

2.3.2 Case of two Assets A_i and A_j

Company	Year	Sales (MAD)	Assets (MAD)	Rate of return	Average return	Volatility	Assets correlation
C1	1	17 500 000	7 000 000		7,42%	16,35%	-59%
	2	16 250 000	6 500 000	-7%			
	3	20 000 000	8 000 000	23%			
	4	18 750 000	7 500 000	-6%			
	5	22 500 000	9 000 000	20%			
C2	1	22 500 000	9 000 000		15,84%	26,94%	
	2	23 750 000	9 500 000	6%			
	3	21 250 000	8 500 000	-11%			
	4	32 500 000	13 000 000	53%			
	5	37 500 000	15 000 000	15%			

In this case, we have:

$$LGD_{A_i + A_j,1\%} = 1 - e^{\left(w_i.\mu i + w_j.\mu j - \frac{\sigma ij^2}{2}\right) + \sigma ij.\varepsilon_{1\%}}$$

With $\sigma ij^2 = w_i^2.\sigma i^2 + 2 * \rho.w_i.w_j\sigma i.\sigma j + w_j^2\sigma j^2$

$A_{i,0}$ = 9.000.000 μi = 7,42 % and σi = 16,35% w_i = 0,38

$A_{j,0}$ = 15.000.000 µi = 15,84 % and σi = 26,94% w_j = 0,62

Asset correlation ρ = − 59%

σij = 14,14 %

So : $Min_{A_{i,T},1\%}$ = 6.538.453 Dhs $Min_{A_{j,T},1\%}$ = 9.056.148 Dhs

$LGD_{A_{i,T},1\%}$ = 26.37 % $LGD_{A_{j,T},1\%}$ = 37.39 %

$Min_{A_{i,T},1\%} + Min_{A_{j,T},1\%}$ = 15.594.601

$LGD_{A_{i,T}\ et\ A_{j,T},1\%}$ (separated calculation of $LGD_{A_{i,T}}$ and $LGD_{A_{j,T}}$) = 35,02%

And $Min_{A_{i,T}+A_{j,T},1\%}$ = 19.080.004 Dhs

$LGD_{A_{i,T}+A_{j,T},1\%}$ (calculated according to the formula (5)) = 20,50 %

2.3.3 Calculations over the two separated Assets A_i and A_j

In this section an illustration was executed according to the developed model to demonstrate its utility in predicting risk related to depreciation in the value of assets of companies that could represent a risk to the bank.

It should be noted that with the developed model, a simulation was performed on 1.000 daily variations to calculate the minimum value for the two assets Ai and Aj The loss rate LGD was calculated using the formula (5).

Among the results of this section:

The minimum value of the two assets separately calculated is less than the diversification hypothesis to show that the developed model takes into consideration the correlation of assets which makes the difference in the value of LGD;

It is observed that the $LGD_{A_{i,T}+A_{j,T},1\%}$ of the two assets is less than both $LGD_{A_{i,T},1\%}$ and $LGD_{A_{j,T},1\%}$ separated, this is due to the diversification effect and primarily to the negative correlation between the two assets.

2.4 Backtesting of the calculated minimum value

The two graphs below show two simulations of the assets distribution in two Ai risk levels 1% and 5%, T = 1000

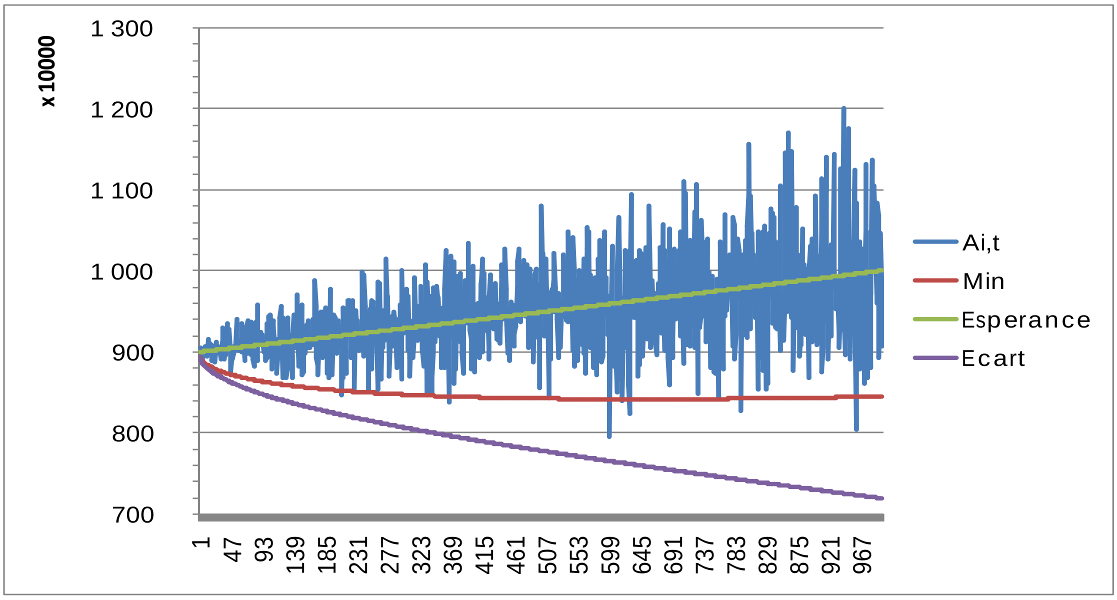


Chart 1: calculation of Min (Ai), α = 1%

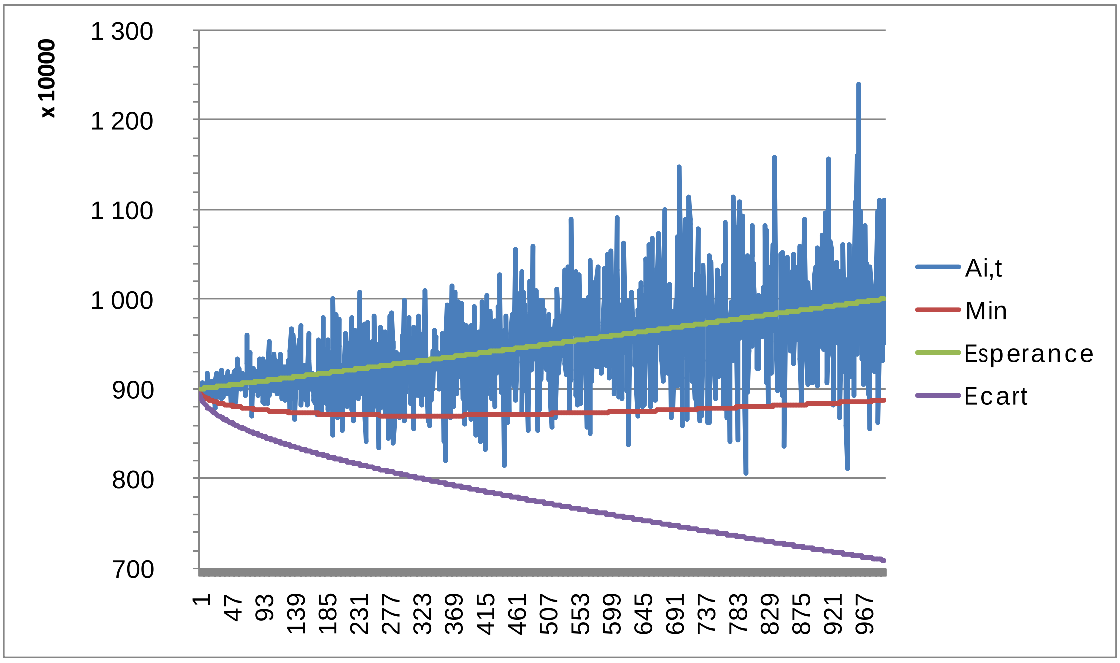
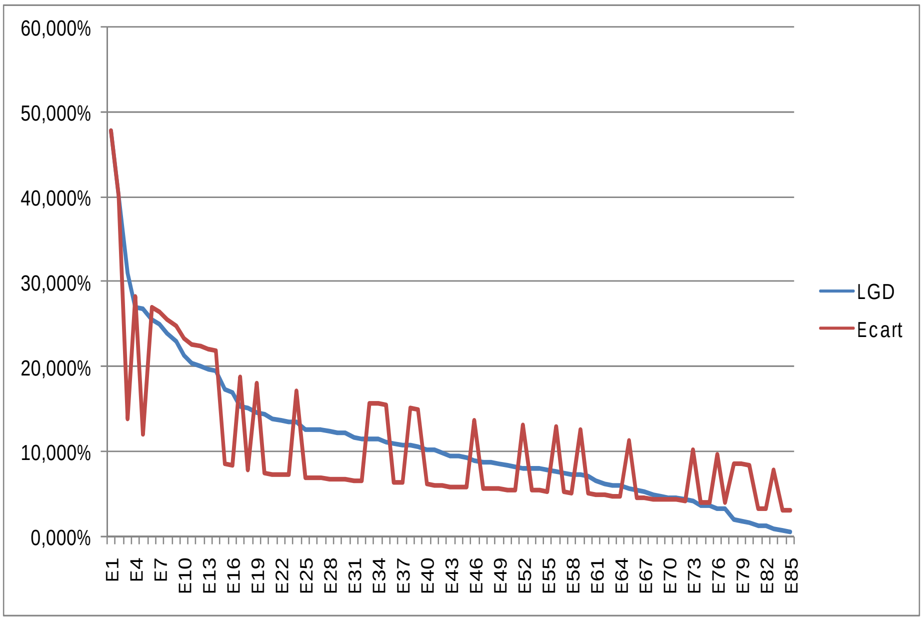


Chart 2: Calculation of Min (Ai), *α* = 5%

	Number of simulation					
	100		1000		10000	
Confidence level	5%	1%	5%	1%	5%	1%
The overrun percentage	6,20%	1,35%	5,60%	1,15%	5,04%	0,99%
Quality of significance	76%	65%	88%	85%	99,20%	99%
	70,50%		86,50%		99,10%	



The objective of this section is to develop a backtesting program for the developed model. It is shown that the greater the number of simulations the greater the importance of estimated power.

For 100 simulations, the exceedance rate is 6.20% for a level of risk of 5%, which is a quality of 76% significance.

For 10.000 simulations, the model becomes more significant with a quality of 99.10%, the exceedance is 5.04% for a risk of 5% and 0.99% for a 1% risk.

2.5 Development of a score of LGD

From the formula:

$$LGD_{\alpha} = 1 - e^{\left(\mu i - \frac{\sigma i^2}{2}\right) + \sigma i.\varepsilon_{\alpha}}$$

LGD rate is between 0% and 100% in the case of total loss of assets $A_{i,0}$
The scoring system we want to develop is giving a score between 0 and 100 according to the rate of loss:

$$LGD_{A_i} = 0\% \quad => \quad Score_{A_i} = 100 \; LGD_{A_i} = 100\% \quad => \quad Score_{A_i} = 0$$

$$Score_{A_i} = 100.\left(1 - LGD_{A_i}\right) = 100.e^{\left(\mu i - \frac{\sigma i^2}{2}\right) + \sigma i.\varepsilon_{\alpha}}$$

And the goal is to build 5 score classes with 8 notations by score:

Classe	Score
A	80-100
B	60-80
C	40-60
D	20-40
E	0-20

2.6 Illustration

Entreprise	Actif	Rent N	Volatilité historique	Min,1%	LGD,1%	Score
E1	2 301 000	57,897%	47,897%	1 201 253	47,794%	52
E2	5 309 000	33,475%	23,475%	4 180 745	21,252%	79
E3	6 979 000	32,745%	22,745%	5 558 677	20,351%	80
E4	17 846 000	32,437%	22,437%	14 282 259	19,969%	80
E5	138 630 000	32,226%	22,226%	111 309 603	19,707%	80
E6	21 721 000	32,002%	22,002%	17 500 925	19,429%	81
E7	644 353 000	1,412%	8,588%	533 201 706	17,250%	83
E8	3 098 000	1,510%	8,490%	2 572 128	16,975%	83
E9	6 367 000	28,782%	18,782%	5 389 115	15,359%	85
E10	31 938 000	2,188%	7,812%	27 136 712	15,033%	85
E11	2 544 000	28,105%	18,105%	2 175 389	14,489%	86
E12	8 750 082	50,197%	40,197%	5 233 715	40,187%	60
E13	2 581 590	2,427%	7,573%	2 211 396	14,340%	86
E14	6 747 095	2,600%	7,400%	5 813 721	13,834%	86
E15	6 635 000	2,645%	7,355%	5 725 920	13,701%	86
E16	14 169 000	2,727%	7,273%	12 261 873	13,460%	87
E17	3 850 000	27,257%	17,257%	3 334 416	13,392%	87
E18	8 778 580	3,002%	6,998%	7 668 134	12,649%	87
E19	8 778 580	3,002%	6,998%	7 668 134	12,649%	87
E20	13 626 132	3,025%	6,975%	11 912 043	12,579%	87

3 Conclusion

In this paper, a mathematical development of the Merton formula was made to calculate the LGD rates resulting in: development of a theoretical framework for measuring LGD loss rate directly related to the Merton model by using the value minimum that could have this asset to maturity at a α risk level.
Among the results of this article: In the first theoretical section, a mathematical development was conducted to determine the minimum value that could have a financial asset; thereafter a second mathematical development has been performed in order to find the results concerning the loss given default LGD rate in the case of one and in addition several assets.
Note that a Backtesting program is necessary to test the estimated level of the model developed, which has shown a positive level of estimation given that the number of simulations was set at 1.000
The limitations of this article are the limited number of searches that have been done in the development of calculating the LGD and the lack of a real database to develop classes of scoring for the LGD.
Among the perspectives of this article: The developed model for the calculation of the LGD was based primarily on the principle of VaR, but VaR was always criticized, however; can demonstrate an idea of developing an LGD calculation based on CVaR mean losses beyond the VaR, as well as compare the results of both models, notably in terms of the significance of estimated power.

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Appendices

Various models for LGD Calculation:

Model	Implied LGD Function
Frye-Jacobs	$\Phi[\Phi^{-1}[cDR] - k]/cDR$ k = LGD risk index = $(\Phi^{-1}[PD] - \Phi^{-1}[EL])/\sqrt{1-\rho}$
Frye (2000)	$1-(\mu + \sigma q(\sqrt{1-\rho}\,\Phi^{-1}[cDR] - \Phi^{-1}[PD])/\sqrt{\rho})$ μ = recovery mean, σ = recovery SD, q = recovery sensitivity
Pykhtin	$\Phi\left[\frac{-\mu/\sigma - \beta Y}{\sqrt{1-\beta^2}}\right] - Exp\left[\mu + \sigma\beta Y + \frac{\sigma^2}{2}(1-\beta^2)\right]\Phi\left[\frac{-\mu/\sigma - \beta Y}{\sqrt{1-\beta^2}} - \sigma\sqrt{1-\beta^2}\right];$ $Y = (\Phi^{-1}[PD] - \sqrt{1-\rho}\,\Phi^{-1}[cDR])/\sqrt{\rho}$ μ = log recovery mean, σ = log recovery SD, β = recovery correlation
Tasche	$\int_{-\Phi^{-1}[cDR]}^{\infty} \phi[z]\, BetaCDF^{-1}[\frac{\Phi[\sqrt{1-\rho}\,\Phi^{-1}[cDR] - \Phi^{-1}[PD] + \sqrt{1-\rho}\,z] - 1 + PD}{PD},$ $a = \frac{ELGD(1-v)}{v}, b = \frac{(1-ELGD)(1-v)}{v}]\, dz\, /\, cDR$ ELGD = expected LGD; v = fraction of maximum variance of Beta distribution
Giese	$1 - a_0(1 - PD^{a_1})^{a_2}$ a_1, a_2, a_3 = values to be determined
Hillebrand	$\int_{-\infty}^{\infty} \Phi[a - \frac{bdc}{e} + \frac{bd}{e}\,\Phi^{-1}[cDR] - b\sqrt{1-d^2}\,x]\,\phi[x]\,dx$ a, b = parameters of cLGD in second factor; d = correlation of latent factors; $c = \Phi^{-1}[PD]/\sqrt{1-\rho}$; $e = \sqrt{\rho}/\sqrt{1-\rho}$

LGD Rating companies

Entreprise	Actif	Rent N	Volatilité historique	Min,1%	LGD,1%	Score
E1	2 301 000	57,897%	47,897%	1 201 253	47,794%	52
E2	5 309 000	33,475%	23,475%	4 180 745	21,252%	79
E3	6 979 000	32,745%	22,745%	5 558 677	20,351%	80
E4	17 846 000	32,437%	22,437%	14 282 259	19,969%	80
E5	138 630 000	32,226%	22,226%	111 309 603	19,707%	80
E6	21 721 000	32,002%	22,002%	17 500 925	19,429%	81
E7	644 353 000	1,412%	8,588%	533 201 706	17,250%	83
E8	3 098 000	1,510%	8,490%	2 572 128	16,975%	83
E9	6 367 000	28,782%	18,782%	5 389 115	15,359%	85
E10	31 938 000	2,188%	7,812%	27 136 712	15,033%	85
E11	2 544 000	28,105%	18,105%	2 175 389	14,489%	86
E12	8 750 082	50,197%	40,197%	5 233 715	40,187%	60
E13	2 581 590	2,427%	7,573%	2 211 396	14,340%	86
E14	6 747 095	2,600%	7,400%	5 813 721	13,834%	86
E15	6 635 000	2,645%	7,355%	5 725 920	13,701%	86
E16	14 169 000	2,727%	7,273%	12 261 873	13,460%	87
E17	3 850 000	27,257%	17,257%	3 334 416	13,392%	87
E18	8 778 580	3,002%	6,998%	7 668 134	12,649%	87
E19	8 778 580	3,002%	6,998%	7 668 134	12,649%	87
E20	13 626 132	3,025%	6,975%	11 912 043	12,579%	87
E21	2 649 086	3,117%	6,883%	2 323 106	12,305%	88
E22	92 981 000	3,149%	6,851%	81 627 562	12,210%	88
E23	9 688 671	-3,888%	13,888%	6 681 477	31,038%	69
E24	92 981 000	3,149%	6,851%	81 627 562	12,210%	88
E25	26 178 000	3,326%	6,674%	23 120 108	11,681%	88
E26	27 252 000	3,388%	6,612%	24 118 812	11,497%	89
E27	4 265 000	25,781%	15,781%	3 776 070	11,464%	89
E28	1 004 000	25,766%	15,766%	889 101	11,444%	89
E29	2 568 000	25,552%	15,552%	2 281 344	11,163%	89
E30	169 045 366	3,604%	6,396%	150 709 130	10,847%	89
E31	2 640 783	3,627%	6,373%	2 356 205	10,776%	89
E32	9 234 376	25,225%	15,225%	8 243 294	10,733%	89
E33	2 240 000	25,042%	15,042%	2 005 007	10,491%	90
E34	5 321 000	38,304%	28,304%	3 881 427	27,055%	73
E35	100 712 000	3,791%	6,209%	90 360 692	10,278%	90
E36	36 554 332	3,847%	6,153%	32 859 397	10,108%	90
E37	2 109 000	3,914%	6,086%	1 900 138	9,903%	90
E38	6 448 193	4,033%	5,967%	5 832 965	9,541%	90
E39	41 881 036	4,067%	5,933%	37 929 680	9,435%	91
E40	10 957 000	4,115%	5,885%	9 939 229	9,289%	91

E41	11 320 703	23,830%	13,830%	10 315 420	8,880%	91
E42	13 666 000	4,279%	5,721%	12 465 664	8,783%	91
E43	9 011 776	4,301%	5,699%	8 226 271	8,716%	91
E44	89 048 000	4,353%	5,647%	81 428 903	8,556%	91
E45	16 057 487 000	-2,161%	12,161%	11 754 786 295	26,796%	73
E46	12 786 000	4,434%	5,566%	11 724 324	8,303%	92
E47	5 716 189	4,490%	5,510%	5 251 450	8,130%	92
E48	10 599 000	23,238%	13,238%	9 741 719	8,088%	92
E49	97 449 010	4,516%	5,484%	89 603 638	8,051%	92
E50	49 310 000	4,557%	5,443%	45 403 628	7,922%	92
E51	254 666 973	4,608%	5,392%	234 893 221	7,765%	92
E52	1 993 000	22,953%	12,953%	1 839 411	7,706%	92
E53	20 430 000	4,683%	5,317%	18 892 016	7,528%	92
E54	506 768 659	4,766%	5,234%	469 934 536	7,268%	93
E55	2 231 000	22,606%	12,606%	2 069 488	7,239%	93
E56	17 852 000	37,024%	27,024%	13 292 161	25,542%	74
E57	3 588 000	4,799%	5,201%	3 330 903	7,165%	93
E58	46 817 000	5,002%	4,998%	43 761 125	6,527%	93
E59	43 535 355	5,088%	4,912%	40 811 639	6,256%	94
E60	95 145 000	5,146%	4,854%	89 368 714	6,071%	94
E61	8 265 000	5,152%	4,848%	7 764 706	6,053%	94
E62	10 302 000	21,369%	11,369%	9 728 723	5,565%	94
E63	3 993 377	5,369%	4,631%	3 779 207	5,363%	95
E64	48 130 396	5,403%	4,597%	45 601 205	5,255%	95
E65	271 178 710	5,504%	4,496%	257 805 847	4,931%	95
E66	28 805 000	5,554%	4,446%	27 431 359	4,769%	95
E67	3 328 000	36,531%	26,531%	2 497 494	24,955%	75
E68	3 440 396	5,606%	4,394%	3 282 018	4,603%	95
E69	213 435 000	5,644%	4,356%	203 868 986	4,482%	96
E70	23 964 142	5,671%	4,329%	22 911 486	4,393%	96
E71	12 422 000	20,318%	10,318%	11 908 923	4,130%	96
E72	4 442 000	5,897%	4,103%	4 279 289	3,663%	96
E73	214 841 846	5,932%	4,068%	207 216 203	3,549%	96
E74	30 375 000	19,750%	9,750%	29 357 409	3,350%	97
E75	9 395 000	6,022%	3,978%	9 088 904	3,258%	97
E76	4 622 000	18,702%	8,702%	4 534 091	1,902%	98
E77	12 729 000	18,641%	8,641%	12 497 625	1,818%	98
E78	3 607 000	35,651%	25,651%	2 744 928	23,900%	76
E79	2 990 363	18,441%	8,441%	2 944 295	1,541%	98
E80	17 802 000	6,606%	3,394%	17 563 737	1,338%	99
E81	68 367 000	6,609%	3,391%	67 459 868	1,327%	99
E82	2 760 000	17,946%	7,946%	2 736 520	0,851%	99
E83	1 177 777 000	6,798%	3,202%	1 169 534 421	0,700%	99
E84	37 922 000	6,851%	3,149%	37 724 283	0,521%	99
E85	5 191 000	34,888%	24,888%	3 998 234	22,978%	77