

# **Dynamic Speculative Behaviors and Mortgage Bubbles in the Real Estate Market of Mainland China**

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## **Abstract**

It is systemic to expound how that the speculative behaviors are driven by the number of a series housing policies promulgated by China's government for the real estate market. The market is resulted in full of speculative behaviors, and has been booming since the beginning of this decade, even though the market has only a very short history, How to rationally measure the speculative behaviors still remains open in the academe and practice. Based on the level of housing prices, this paper establishes some innovative dynamic models, proposes a concept of net mortgage loan to net speculative gain ratios to investigate the speculative behaviors and mortgage bubbles, and finally concludes some interesting results: The necessary condition of speculative behaviors' existence is that the growth rate of housing prices must have a positive minimum boundary, and, the net mortgage loan to net speculative gain ratios subsequently exist, which approximately depict the potential risks banks would face. Furthermore, investor's behaviors may

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impact the housing prices' trend through changing the expectation or variance of housing returns in sale or resale massively. The last finding indicates that how investor's behaviors change market' behaviors, which evidently testifies that the efficient market hypothesis would be invalid.

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**Keywords:** Dynamic speculative behavior, housing price, net mortgage loan to net speculative gain ratio, real estate market

## 1 Introduction

The local government in China<sup>2</sup> has always believed in the real estate industry, for the chief executive officer of it pertinaciously perceives that the promotion of the development of real estates could improve the level of the local Gross Domestic Product (GDP) and easily gains a good performance of its administration. However, the real estate industry has been publicly being mixed with boosters and objectors for at least ten years, and it is now up to a current hot topic in controversy throughout the country.

One of focal problems is whether the real estate market exists with speculative behaviors?

China Department of State issued two documents in policy to adjust the market's behaviors on January 7, 2010 and on April 17, 2010 respectively. The documents firmly advocate that the government (including all local governments) should gauge the reasonable consumption for houses instead of the speculative demands in houses<sup>3</sup>, and uncompromisingly control the irrational demands in

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<sup>2</sup> Here, China only points the China-Mainland, excluding Taiwan, Hong Kong and Macao.

<sup>3</sup> China Department of State's acknowledgements on firmly keep within limits of the fast

houses<sup>4</sup>. In consequence, the Central bank adjusted the reserve rates from 15.5% to 16.0% on January 18, 2010, from 16.0% to 16.5% on February 25, 2010, from 16.5% to 17.0% on May 2, 2010. A strong sign is released with no doubt from those government's activities in adjustment: The speculative behaviors in the real estate market are inescapable.

It may be absorbing that how the speculative behaviors are born and how large magnitude in quantity the speculative behaviors could affect the mortgage loans. Its scenario is gonna be discovered below in general.

### **1.1 The Real Estate Market Up To Early 1990's**

From the beginning of 1949 to the end of 1970's, it is well known that China's residential system was of dual-structure, that is, the government supplied public houses for non-rural residents' lease with low rent, and rural residents built their own houses scattered in the broad rural areas around. Thereafter, the government had massively started to reform the residential system during the 1980's-1990's, and sold all public houses to those who occupied them through leases, but the rural residents were obviously excluded from those social welfares. The rural residents would become some more impotent potential demanders in the new real estate market.

The real estate market is made up of the commercial housing market, housing lease market, mortgage market for house, and investment loans for house, or, the market is composed of housing for rent, for sale, for exchange and for mortgage

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growth of real estates' price in some metropolises, April 17, 2010. See website: [www.mohurd.gov.cn](http://www.mohurd.gov.cn), throughout the following footnotes in this paper, except for special note(s).

<sup>4</sup> The Office of China Department of State's acknowledgements on boosting the real estates' market in stability and health, January 7, 2010.

loan<sup>5</sup>. Commercial real estate market only occupies a small part of the market, and the housing market absolutely predominates the market (Wang, 2009). As a matter of fact, the market is of strong characteristics of policies, not of a pure character of market economy in the sense of the linkage of the supply of and demand in houses. The government's policies create the social supply of and demand in houses. In deed, the real estate market is oriented by polices, which leads the market into the disequilibrium in the supply of and demand in houses.

The government has proposed a thought to develop unique commercial house market since the reforming and opening up policy was initiated in 1978, and a series of policies on housing were promulgated by the government. These polices vary from tax policies to trade off policies in house. Even though the government had no experience in the housing market before, the government still has a consuming dream to establish a more mature commercial housing market relying on an environment of equity. And the related tax imposed might be an important basis of this equity<sup>6</sup>. But, in order to develop the housing market, access to the reformation of rents, and eliminate a big holdback for the emerging real estate market, the holdback is 3% income of the house sale had been the house property tax for all sales of commercial houses since December 1985 based on finance and tax documnets or Caishuiyingzi [1985] No. 050, the General Tax Bureau of Ministry of Finance has decided to waiver in collecting housing property tax depending upon the residents' rent of the lease from the housing administration

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<sup>5</sup> The Office of Department of State forward the decision by the Department of State's leading group of residential system reform on boosting the national urban housing system reform, Guobanfa [1991] No.73

The market include commercial real estate market and housing market, but the commercial market only occupies a small fraction, for example, (Wang, 2009). Hence, in this paper, the real estate market mainly point the housing market(s).

<sup>6</sup> The Real Estate Tax's Temporary Byelaw of PRC, Guofa [1986]No.90

since January 1, 1988<sup>7</sup>, and made a determination to temporarily deduct sale tax of the sale of a commercial house built by public or collectivity house enterprises<sup>8</sup>. These tax policies provide a broad real or potential space for housing supply. But, the temporary policies could confuse the market behaviors subsequently. Hence, it is very necessary to regulate the buying-selling activities in the housing market.

In order to standardize the trade off of the urban houses, it is ordered at every level as low as to town to set up regular housing exchanges as soon as possible<sup>9</sup>. The trade off of urban houses, including housing buy, leasehold, transfer, mortgage loan must be publicly done in the Housing Exchange Centers. The government strongly supervises the exchanging process and levy any possible extra profits derived from the transaction<sup>10</sup>. The Office of Town Construction Administration (OTCA) and the Office of Urban Housing Administration (OUHA) are the two professional offices: OTCA is responsible for managing town lands for developing, OUHA is authorized to administrate the lands access to transfer<sup>11</sup>. As the government's representative, OTCA and OUHA toughly regulate the real estate market. As a result, OUHA is clearly asked to book all houses with their own responding property rights, and make the house property right be accessible

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<sup>7</sup> Acknowledgments issued by The General Tax Bureau of Ministry of Finance about deferment in collection of the house property tax derived from the residents' rent based on the lease from the housing administration, Caishuidizi [1987] No.030

<sup>8</sup> Acknowledgement issued by The General Tax Bureau of Ministry of Finance about preventing from the collection of sale tax from the sale of commercial houses, Caishuiyingzi [1987] No.046

<sup>9</sup> Acknowledgements issued by the Ministry of Construction, about set up the regular house exchanges, Jianfangzi [1988] No.148

<sup>10</sup> Acknowledgements issued by the Ministry of Construction, the National Price Bureau, and the National Industry and Commerce Administrative Authorities, about the strengthening administration on the business of the real estate market, Jianfangzi [1988] No.170

<sup>11</sup> Acknowledgements issued by Ministry of Construction about the remising urban lands, Jianfangzi [1988] No.250

to confirm and transfer in the transaction<sup>12</sup>.

The supply of houses in the real estate markets was contributed by new houses built by professional companies, by public houses provided by the government, and by used houses from residents. The demand in houses was simultaneously limited by reason of the low income-flow of households, and so, the housing prices were at a low level. The Aggregate Supply (AS) of houses was not less than the Aggregate Demand (AD) in houses during this period. Before the early of 1990's, the original structure of the real estate market discovers that the real estate market was at the initial stages of development, the efficient demand in houses was not franchised, and the market were of low level of prices.

## 1.2 Creating Demands in House

It might be strange that the government did not like the market with the low level of prices, because the government believed in that the low level of prices strategy was attributed to be the disadvantages in the development of the housing industry and in promoting the level of GDP in consequence. The government wants to develop the market to substitute its initial status under the instruction of high price level strategy. This course of the market modified is called an evolution of the market. The significant change with the evolution of the market is that the market is running under the control of AS of houses, and the AS gap created keeps effectively. The vacancy rates become a common measurement for AS gap. This phenomenon engenders a paradox of high vacancy rates and shortage in house (Ramo, 1998). This AS gap keeps the market speculatively boosting. The government creates the speculative market through decreeing a series of laws,

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<sup>12</sup> Acknowledgements issued by the Ministry of Construction, about strengthening the town planning, housing reform and housing industry administration under the Documents [1990] No. 31 issued by the Department of State, Jianfangzi [1990] No.279

administrative regulations and special policies. Let's have a retrospect below.

All urban residential houses must have been gradually commercialized and socialized since 1978, because urban residential system reform has been being pointed to be an important part of economic system reform,. For the sake of developing real estate industry, the adjustable real estate market should be established, and the rents for residential house must be promoted to improve the houses for sales<sup>13</sup>. The leasehold housing market has become a kind of instrument for pushing the housing market forward, that is, the promoting level of rents lessees to buy houses instead of ones for rent through housing finance, which forms the public fund of housing reform. The financial operation of housing reform is undertaken by the professional banks, including deposits and loans for house in transaction and other operation correlated with residential consumptions<sup>14</sup>. The operations of the real estate market, including the financial operation, are new, and the related administrative regulations or laws are needed to be constituted.

In fact, the decree of *the Regulation on the Administration of the Urban Real Estates* implies that the real estate market have to be run in order under the laws<sup>15</sup>. The regulation says that all commercial houses under construction may be presold by the real estate management corporation or the developer, and the developer collects the deposits, earnest money or money in prices. As a requirement, the developer must invest the money collected into the remaining construction of

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<sup>13</sup> The Office of Department of State forward the decision by the Department of State's leading group of residential system reform on boosting the national urban housing system reform, Guobanfa [1991] No.73

<sup>14</sup> Acknowledgements issued by Ministry of Finance about housing reform financial operation concerned the policy of finance and tax, Caizongzi [1992] No.148

<sup>15</sup> Regulation on the Administration of the Urban Real Estate of the People's Republic of China, Jianfang [1994] No.435, revised in 2007

houses<sup>16</sup>. And furthermore, the developer could obtain a loan from the public fund to offset the budget balance in the investment. The housing public fund system is a deep-set basis of the urban residential housing reformation system, and the fund could provide a large and strong support in the investment and transaction of houses around the country<sup>17</sup>. How do the professional banks operate the fund? People's Construction Bank of China (PCBC), Industrial and Commercial Bank of China (ICBC), Agricultural Bank of China (ABC) are authorized to be responsible for the policy of the residential credit operation. The People's Bank of China (PBC) mainly decides the deposit rates, loan rates or mortgage loan rates, and bond interest rates for the support of the resident housing operation as policy<sup>18</sup>. Banks play crucial roles in boosting the real estate market (Allen and Carletti, 2010), since there is no transaction in houses without being forcefully supported by banks.

The transaction in houses is permitted through the way of sale, of donation, or other legal channels, and the government only collects its related tax and fees based on the strike price. The strike price must be clearly written down in the memorandum, but it can not be significantly lower than the current normal market prices in business<sup>19</sup>. After having prepaid a small fraction of the total amount of the house's values in trade off, the investor should pay the rest through loans backed by the banks, and the house is temporally as a mortgage for the loan, even

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<sup>16</sup> Regulations on the Administration of Urban Commercial Houses' Presell, Jianshebuling [1994] No.40

<sup>17</sup> The Temporal Regulation on the Establishment of Housing Accumulation Fund System, Caizongzi [1994] No.126

<sup>18</sup> Temporal Regulations of Administration on the Policy of the Resident Credit Operation, Yinfa [1994] No.313

<sup>19</sup> Regulation on the Administration of Urban Real Estates Transfer, Jianshebuling [1995] No.45



though the house is a future one<sup>20</sup>.

Investment as the aim of the transaction is legitimate. According to the Building Realty Market (Jianzhufangshi) [1998] No.017, the investment in houses is encouraged and supported by banks<sup>21</sup>. Mortgager can obtain the loans for buying a house from ICBC on condition of bringing forth the Certification of Homeowner or the Housing Sale Contract. The total loans are at most up to seventy percent of the total amount of the house's market values or values estimated under the strike price, the rest thirty percent as down payment must be prepaid by the mortgager, and the ownership of the house is temporally occupied by the mortgage lender or the bank<sup>22</sup>. GDP is a close positive relation to the housing boom in short term (Wang, 2009), and the housing industry is illusionary to be considered as a long term instrument to promote the level of GDP. So as to keep the economic up growth trend, PBC informs all commercial banks including PCBC, ICBC, ABC, and the Communications Bank of China should provide investment loans for the realty as much as fifteen percent growth more than the total amount loans expected in the same fiscal year, from the investment loans for resident house to investing house demands<sup>23</sup>, and the loans should be backed by their houses as mortgages<sup>24</sup>. The loan policy may be a little different among the

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<sup>20</sup>Regulation on the Administration of Urban Real Estate Mortgage, Jianshebuling [1997] No. 56

<sup>21</sup> Acknowledgements on the distribution of the prints of "Summaries on the conference of the second and third orders of the real estate market'administration", Jianzhufangshi [1998] No.017

<sup>22</sup> Regulation of the Administration of Individual Housing Loan by ICBC, Gongyinfu [1998] No.94

<sup>23</sup> People's Bank of China's acknowledgements on increasing housing credits to support the construction and consumption of houses, Yinfu [1998] No.169

<sup>24</sup> Regulation on the Administration of Individual Housing Loans by the People' Bank of China, Yinfu [1998] No.190

banks. Comparing with ICBC, PCBC and its branches provide a loan to anyone who buys a house, and the loan, under the mortgage of the house bought in whole, is at most up to eighty percent of total amount of the housing market values with maturity time as long as thirty years<sup>25</sup>. The developer could share the loan policy with investors. As rules, the developer has the right to apply for a loan up to eighty percent of its total amount market values in investment from banks depending on the rest prepaid by its capital fund, and the developer may publicly pre-sell the houses under construction, or may sell or rent the houses constructed through ads for refinancing<sup>26</sup>. So, in order for supporting the development of real estates, as well for simplifying the process of the real estate transaction and house property registration, and for regulating the housing mortgage registration, it urges to strengthen the efficient administration on the cointegration of the real estate transaction and house property registration, including the registration of the housing mortgage contract<sup>27</sup>. For the realization of the cointegration of the administration of real estate transaction and housing property right registration, it is significant to standardize the service within a package including receipts of application for transaction in houses and registration of houses (re)sold, dealing with it, receiving the related fees and enacting the certification of the houses bought<sup>28</sup>. The standardization of the adjustment and regulation on market's behaviors just reveals the government's considering the crucial roles that the real

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<sup>25</sup> Construction Bank of China's Regulation on Individual Housing Loan, Jianzongfa [2000] No.1

<sup>26</sup> Byelaw on the Administration of Urban Realty's Development and Operation, Guowuyuanling [1998] No.248

<sup>27</sup> Instruction opinions on simplifying the real estate trade off and house property registration, Jianzhufang[2000] No.201

<sup>28</sup> Acknowledgements on printing and distribution of "Check Criterion on the Normal Administration of Real Estate Trade off and House Property Registration" and "Check Methods on the Normal Administration of Real Estate Trade off and House Property Registration", Jianzhufang[2002] No.251

estate market plays.

As the case stands, the government defines that the real estate industry is always a predominated industry in the national economic system with high linkage to up and down industries, or the dynamic industries. Furthermore, in order to activate the second order real estate market, any investment in houses should be encouraged, and banks are forcefully required to provide credits in support<sup>29</sup>, and as well, the tax bureau has not to levy upon the tax before the houses are sold<sup>30</sup>. Furthermore, for a stimulating policy in the exchange market, anyone sells his/her house bought one year ago, he/she doesn't have to file a sale tax based on this selling income, and any vacant house built before 2000 can be sold without filing a sale tax and contract tax in a makeshift<sup>31</sup>. The adjustment instruments are frequently produced so as to form a modest policy market. From June 1, 2006 on, anyone who has bought and kept a house at least for two years resold the house without being levied upon a sale tax<sup>32</sup>. These policies only encourage the investors' speculation through resale or changing hands in transaction in the real estate market.

All of the above laws enacted, administrative regulations and policies set down, are converged to a point, that is, to create a hungered demand market in houses.

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<sup>29</sup> The Department of State's acknowledgements on boosting the real estate market to be sustainable in health, Guofa [2003] No.18

<sup>30</sup> The General National Tax Bureau's acknowledgements on the policy of the house property tax and the urban land usage tax, Guoshuifa [2003] No.89

<sup>31</sup> Ministry of Finance and National Tax Bureau's Acknowledgements on the adjustment of the tax policy for the real estate market, Caishuizi [1999] No.210

<sup>32</sup> The Office of the Department of State forward the acknowledgements issued by the Ministry of Construction et al on the job of stabilizing the housing prices, Guobanfa [2005] No.26

### 1.3 The Current Real Estate Market of AS Gap

The government has gradually established the real estate market and speeded the development of urbanization across the China-mainland since the end of 1990's. The earlier real estate market has the significant profile of AS no less than AD, and the market were running at a low price level. But, the government has firmly thought that the low price level strategy is contradicted to the healthy development of construction industry or of the real estate market, and continuously produced potential demand in houses through enacting a series of laws, administrative regulations, and policies in loosening liquidities. The market prices have been gradually pushed as high as multiplies times the initial ones. For example, as observation, the price, in RMB, was about 8,000 Yuan per a square meter in Xicheng District, Beijing, in 2003. But the price is soaring up to the average price 50,000 Yuan per a square meter in August 2010. In this district, the prices have a clear fluctuation, for example, average 25,000 Yuan per a square meter from 04/24/2007 to 10/11/2007, average 28,000 Yuan per a square meter from 10/11/2007 to 10/26/2007, average 40,000 Yuan per a square meter from 10/26/2007 to 01/12/2009, average 37,000 Yuan per a square meter from 01/12/2009 to 11/19/2009, average 50,000 Yuan per a square meter from 11/19/2009 to the second half year of 2010<sup>33</sup>.

So the government has accepted a high price level strategy in the development of real estate market. And the market must be subsequently oriented by the AS gap. This market has its special feature, i.e. all houses entering the market are built by developers only, and the supply of houses is factitiously under control by providers: developers, investors, and residents. These facilities form a beckoning imagination: AS of houses would be always lacking, and AD in houses would be always driving.

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<sup>33</sup> Sources: <http://house.focus.cn/votehouse/1708.html#price-his>

Thereafter, anyone who would like to obtain his/her own house has to buy it in the real estate market, because the government takes the housing industry as the predominated industry in China, and forbids any non-rural resident to build house of his/her own by himself/herself in laws or in administrative regulations. In addition, anyone or his/her organization affiliations, without being permitted, are not allowed to invest in realty industry based on the collecting funds<sup>34</sup>. From the beginning of 2000 to present, there are more and more residents who have bought houses through mortgages loans from commercial or investment banks, and the real estate market has been being boomed for years, which is driven by the excess AD in houses. The excess AD in houses is factitiously made by AS gap, which is produced by the control AS and implies that AS lacks elasticity, this case is consistent with findings of Mayer and Quigley (2003). The relationship between AS of and AD in houses is being created by at least three factors including the control AS or high vacancy rates, the extension of urbanization, and housebreaking engineering. But, the main causality or the vacancy rates are continuously being homemade, and the last two factors are not the main causations, since all potential residents released by these two factors could be completely resided in the current vacant houses existed through the country.

#### **1.4 A Summary and Organization of Remains**

Irrational exuberances are seriously obvious in the real estate market of China. But, the speculative behaviors are sustainable to be stimulated depending upon some illusion or predicted perceptions from the residents, investors, and developers: there are more and more non-rural residents created from the speedup of the urbanization program; that the lands for construction become fewer and

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<sup>34</sup> Acknowledgements on prohibiting build houses through collecting funds without being permitted, Jianzhufang [2006] No.196

fewer would certainly lead to higher prices of house; the expectation of inflation would be greater and the devaluation would be hardly escaped; and a house is always appreciated with no doubt.

Of course, with strong expectation to swoop irrational profits, the speculative behaviors have significantly impacted the uncertainty of China's economy and its structures (Wang, 2009). How to describe the irrational behaviors and measure their effects on the uncertainty of mortgage loans is a great challenge in the academe and the administrative authorities. According to the differential characteristics of the real estate market of China, I have established notable dynamical models for describing the speculative behaviors to evaluate net mortgage loan to net speculative gain ratio later, which approximately depict the potential risks Banks would face.

The rest of the paper is organized by the dynamic models of speculative behaviors in Section 2, application of investor behaviors in Section 3, net mortgage loan to net speculative gain ratio in Section 4, simulation of the speculative behaviors and net mortgage loan to net speculative gain ratio in Section 5, conclusion and remarks in Section 6, and the appendix following in order.

## **2 Dynamic Speculative Behaviors**

### **2.1 Definition and Measurement of Speculative Behaviors**

China's real estate market has a very short history with three decades or more, unlike the United State of America's market, which has a longer history over one hundred years, and is equipped by various investment tools such as its second order market (Weiss, 1989). China's market is only in the primary status of the development, and lack of experience in administration and legal regulations. The market has been booming for about the last ten years. This situation could be

interpreted by the discovering that speculative considerations were a prime motive for buying homes in boom cities in a survey made by Shiller (1990). So, irrational behaviors in the housing market could be inevitable.

**Definition 2.1** *A speculative behavior is defined as an action in trade off only depending upon it that a speculative profit is derived from a fluctuant price of the asset invested in the market rather than the asset's appreciation based on its fundamental value.*

If an investor cannot earn a speculative profit in short term, his/her behavior is not speculative<sup>35</sup>. Hereby,

**Definition 2.2** *Dynamic speculative behaviors mean that such sequent actions defined by Definition 2.1 are iteratively effective, and the asset prices have a strong upside trend in motion.*

Definition 2.2 implies that the value of a house contains all construction costs and benefits derived from its environment advantages such as the facilities around, dwelling, location, sitting direction, etc. The housing value, including the rigid costs, may be assumed to maintain a stable status during the transaction of it, and so do the related tax and fees, and mortgage loan rate. This kind of assumption implies that the housing prices could reflect the housing value, and the value is the compact fundamental value of the house. The value measures the function of the house, and is not affected by the variety of housing prices. This measurement of housing value is different from the measurement of those by Case and Shiller (2003), which includes many variables like market factors: resale index, price-to-income ratio, etc. In this sense, the increase of the prices must be driven by other factors, not the fundamental value except for the rigid costs here.

Being differ from the housing value, housing prices are heterogeneous and

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<sup>35</sup> If an agent can gain in a long term, its behaviors may be called the investment behaviors rather than the speculative ones.

are difficult to measure in precise (Case, 2008). Price to rent ratio (Brunnermeier and Julliard, 2008) or price to income ratio (Case and Shiller, 2003) are ubiquitously introduced to measure housing prices. As for China's situation, the price to rent ratio does not fit, for the leasehold housing market is not access to being mature. That is, the lease housing market has never accrued the AS of house so far. In China, all non-rural resident houses are allowed to be leased out<sup>36</sup>. But, the administration of the lease houses is an important essential job of the administration of the social and public security, for variable criminal actions may happen based on the houses rented by the outlaws<sup>37</sup>. The problems of leasing houses are the very problems of the administration, not the problems of the market. By the way, there isn't a systemic statistic data for leasehold market in current China. It is incredible for one to study China's real estate market in term of price-to-rent ratio. On the other hand, the item of income is not a proper instrument to measure the distribution of Chinese incomes, since the polarization between the rich and the poor has been becoming larger. These reasons adequately deny the price-rent ratio and price-income ratio as the effective measurement of the housing prices in China.

According as the degree of market's maturity, the level of the housing prices actually reflects the situation of the development of China's real estate market, which will be studied in this paper. If one wants to investigate diversified price levels all together, it is needed to integrate them based on housing prices modeled in individual here, and one can realize this research by picking out the sum of varied prices for discrete cases or the integration of varied prices for continuous

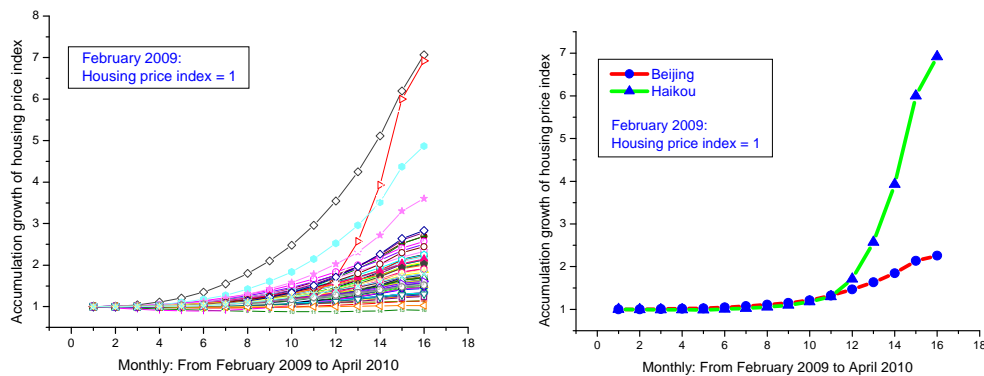
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<sup>36</sup> Regulation on the Administration of the Urban House for Lease out, Jianshebuling [1995] No.42

<sup>37</sup> Acknowledgements on the strengthening and improving on the administration of the lease houses, Gongtongzi [2004] No.83



cases. The methodology may seem complex, but the theory is consistent with those studied in this paper. The housing price index introduced is an all-pervading measurement for the speculative behaviors in real estate market, see Buttimer Jr., Kau, Slawson, Jr. (1997), Case, Glaeser and Parker (2000). The housing price index has a predominance to eliminate heterogeneity of housing prices. The reality of the housing prices might be buried in the process of indexed prices, but the housing price index may conveniently discover the market's behaviors including speculative behaviors in an easy way. China's real estate market does evidently have speculative characteristics in the sense of housing price index. For example, consider the housing price index growth of seventy cities of China, calculate the accumulation distribution of the price index from February 2009 to April 2010 respectively, and plot them below:



(a) Accumulation distribution of housing price-index growth of seventy cities

(b) Accumulation distribution of housing price-index growth of Beijing and Haikou

Resources: The data are from the web site, namely <http://www.stats.gov.cn/tjsj/>

Figure 1: Accumulation distribution of housing price-index

Figure 1(a) shows: Set the housing price index equal to 1 in the February of 2009, all seventy cities' housing price indexes shows that the housing price levels have strongly been increasing over the past year. In fact, this is an epitome of the development of China's real estate market within current decade or more. The

housing price index has well plotted the market's behaviors, but unfortunately, the housing price index can not really identify the difference of the housing prices among all different places. For instance, considering two cities: Beijing and Haikou, their housing price indexes are compared in Figure 1(b).

Figure 1(b) demonstrates that Haikou's housing prices shall begin to have a steep running up than Beijing's in one year. However, it is well known that Beijing's housing price level is ranking at the front in China, and Haikou's housing price level only occupies a small fraction of Beijing's. One couldn't read the real different levels of housing prices between Beijing and Haikou from the Figure 1(b). I think it is more convenient to investigate speculative behaviors based on the real housing price levels rather than based on the housing price index in China. For this reason and the research profile of the paper, only housing price levels are accepted as the research objects, except for a special mention.

## 2.2 A Measure of Dynamic Speculative Behaviors

In order to simplify the discussing thereafter, I give out a hypothesis as the precondition for the remaining investigation.

**Hypothesis 2.3** An investor's fund for buying a house is completely from self-financing or mortgage lending.

The real estate market cannot exist without the support of financial systems such as banks in finance. Banks play very key roles in the market, since the market can be sustainably booming through mortgage lending backed by banks, and absorb liquidity as much as possible which are released from the loose monetary policy (Allen and Carletti, 2010). This is very reason why the hypothesis is well explanatory.

Let  $S_t$  be a price of a house for sale per unit at time  $t$ ,  $E_t S_{t+1}$  the

expectation of the price at time  $t+1$ . According to the regulation of administration in the real estate market, anyone need to buy a house due to his/her own endowment like currency as a down payment  $\alpha$  ( $0 \leq \alpha \leq 1$ ) of its total value in advance or  $\alpha$  denotes the part of down payment by the investor or buyer, and a mortgage loan from banks for covering the remaining of its total market value, he/she should pay back to the banks at mortgage loan rate  $r$  ( $0 \leq r < 1$ ) monthly in the future, and pay tax and fees related to the government's administration at tax-fee rate  $\tau$  ( $0 \leq \tau < 1$ ) at the same time. The investor may speculatively earn a positive profit thru the fluctuation of house prices in resale, subject to his/her previous earnings from the volatility of the house prices, for "Case and Shiller suggest instead that an asset bubble appears when current prices depend upon expectations of future price increases" (Mayer and Quigley, 2003), and the bubble is driven by speculative behaviors. As a speculative goal, the investor just wants to use loans from banks instead of its self-financing to buy a house. Hereby, the definition is stated as follows:

**Definition 2.4** *A single dynamic speculative behavior in real estate market may be reasonably measured in the expression of:*

$$\begin{aligned} & \max_{S_{t+1}} E_t S_{t+1} - (1 + \tau + (1 - \alpha)r)S_t & (1) \\ \text{s.t. } & S_t - (1 + \tau + (1 - \alpha)r)S_{t-1} - \alpha E_t S_{t+1} \geq 0 \end{aligned}$$

The investor wants to gain a positive profit based on the fundamental Model 1, he/she must expect the house prices to be growing at some rate such as  $\eta$  ( $-\infty < \eta < +\infty$ ), or

$$E_t S_{t+1} = (1 + \eta)S_t \quad (2)$$

Model 2 describes the dynamical system of the housing prices. For simplicity, it might as well assume that the parameter  $\eta$  is fixed. A necessary condition on speculative behavior can be proved as follows.

**Theorem 2.5** *Suppose that a speculative behavior follows Model 1 and the*

housing prices follow Model 2. Then the investor can obtain a positive speculative profit only if the necessary condition satisfies:

$$\eta \geq \begin{cases} \frac{\alpha + (1 + \alpha)\tau + (1 - \alpha^2)r}{1 - \alpha - \alpha\tau - (1 - \alpha)\alpha r}, & 0 < \alpha < 1 \\ \tau + r, & \alpha = 0 \end{cases}.$$

**Proof:** See the proof of the Theorem 2.5 in the Appendix.  $\square$

If  $\alpha$  is equal to 1, then the investor mustn't obtain a positive profit from Model 1. This implies that there is no intent to gain a speculative benefit in the real estate market on condition of  $\alpha = 1$ . See the proof of Theorem 2.5 in the Appendix for details.

The coefficient  $1 + \tau + (1 - \alpha)r$  consequentially occurs once a house is (re)sold or bought. This coefficient may be defined as a measure of the housing fundamental value, or the house can be appreciated at the growth rate:  $\tau + (1 - \alpha)r$ , since  $\tau + (1 - \alpha)r$  measures the rigid costs in the transaction, which is outright contained in the fundamental value of the house. This definition implies that the house is under valued if  $\eta < \tau + (1 - \alpha)r$ , over valued if  $\eta > \tau + (1 - \alpha)r$ , exactly valued if  $\eta = \tau + (1 - \alpha)r$ .

**Definition 2.6** A model without speculative behavior may be defined as

$$E_t S_{t+1} = (1 + \tau + (1 - \alpha)r)S_t \quad (3)$$

Model 3 is also called a rational model of the investment behavior, and the relation  $\eta = \tau + (1 - \alpha)r$  is called **the rational investment curve**. The area belonging to the district over the rational investment curve is called the area where a speculative behavior may happen, and the area belonging to the district under the rational investment curve is called the area in which non speculative behavior would happen.

Model 3 means that the process of the prices' varying is submartingale, and the prices expand at the rational or natural rate  $\tau + (1 - \alpha)r$ . The housing bubbles

could exist under the possible condition:  $\eta > \tau + (1 - \alpha)r$ .

If housing prices are not vulnerably affected by investors' behaviors, and are contingently oscillated, the price  $S_{t+1}$  may be less than  $S_t$ , and  $S_{t+2}$  may be bigger than  $S_t$ , Model 1 needs to be changed in homologous according to the behaviors preferred options. That is to say, the coefficients may be changed correspondently.  $\alpha$  and  $\tau$  keep maintaining, and  $r$  is counted accumulation along with the fleeing time. There is a sample as reference for the modification of Model 1, see Brunnermeier and Julliated (2008) in detail. Model 2 may formally remain, and only needs to revise the subscript of the housing prices correspondingly. The processing may be more complex, but the principle is clear as it should be. So, it is not necessary to consider this complicated form in this paper here.

### 2.3 Dynamic Multiple Speculative Behaviors

Model 1 gives the expression of an investor's behavior at some time. If we consider  $t$  investors whose speculative behaviors are successively iterated, and what will happen aftermath?

For a briefness, the first investor buys a house at price  $S_1$ , and the second investor buys the house from the first one at price  $S_2$ , and so on, the  $t^{\text{th}}$  investor buys the house from the  $t-1^{\text{th}}$  one at price  $S_t$ . That the process describes the dynamic multiple speculative behaviors can be characterized by the following model:

$$\begin{aligned} & \max_{\substack{S_{j+1} \\ j=1, \dots, t}} \left\{ \sum_{j=1}^t \left[ E_j S_{j+1} - (1 + \tau + (1 - \alpha)r) S_j \right] \right\} & (4) \\ & s.t. \quad \sum_{j=2}^t \left[ S_j - (1 + \tau + (1 - \alpha)r) S_{j-1} - \alpha E_j S_{j+1} \right] \geq 0 \end{aligned}$$

In order to keep the effect of the speculative behaviors in the consequences, every investor must earn a positive profit thru the trade off or resale of the house. This strategy, called **the  $t$  investors' speculative trade strategy**, is valid. So, the main result may be stated below.

**Theorem 2.7** *Under the assumption of Model 2 and Model 4, if  $t$  investors' speculative trade strategy is valid, only if the necessary condition satisfies*

$$\eta \geq \begin{cases} \frac{2\alpha(1+\tau+(1-\alpha)r)-1}{(1-\alpha)(1-\alpha r)-\alpha\tau}, & 0 < \alpha < 1 \\ \tau+r, & \alpha = 0 \end{cases}.$$

**Proof:** Please see the proof of the Theorem 2.7 in the Appendix for details.  $\square$

If  $\alpha=1$ , the  $t$  investors' speculative trade strategy is invalid. The real estate market may have housing price bubbles if the condition holds:  $\eta > \tau+(1-\alpha)r$ . The housing bubbles are pushed by investors' speculative behaviors. That the speculative behaviors go together is called **the cluster of the speculative behaviors**. According to Theorem 2.5 and Theorem 2.7, a corollary is deduced:

**Corollary 2.8** *The denser the cluster of the dynamic speculative behaviors is in the real estate market, the easier it is to earn an extra investment profit as long as the lesser growing housing price bubbles exist.*

**Proof:** From Theorem 2.5 and Theorem 2.7, we can obtain

$$\begin{aligned} x &= \frac{\alpha+(1+\alpha)\tau+(1-\alpha^2)r}{1-\alpha-\alpha\tau-(1-\alpha)\alpha r} - \frac{2\alpha(1+\tau+(1-\alpha)r)-1}{(1-\alpha)(1-\alpha r)-\alpha\tau} \\ &= \frac{(1-\alpha)(1+\tau+r-\alpha r)}{(1-\alpha)(1-\alpha r)-\alpha\tau} \end{aligned}$$

From the proof of Theorem 2.5, it is known that  $(1-\alpha)(1-\alpha r)-\alpha\tau > 0$ , and then  $x > 0$ .  $\square$

Corollary 2.8 clarifies that why China's government has been advocating to maintain the real estate market's growing in a little small pace, since the real

estate market is full of speculative behaviors, and any small pace in growing housing pricing bubbles can release a large profit which could cover as much as the exposed black hole of the mortgage loans from banks. However, the black hole would expand larger in trend. How to measure the exposed black hole? We shall discuss this problem thru the introduction of the net mortgage loan to net speculative gain ratio.

**Corollary 2.9** *If an investor is very difficult to earn an extra investment profit from the fluctuant real estate market, then the market must keep prone away from the speculation.*

Corollary 2.9 means: if the market is out of the speculation, no one could simply earn an extra return from the market without based on its fundamental value appreciation. It is self-evident from the Corollary 2.8.  $\square$

If a real estate market is full of speculative behaviors, anyone involved in this market could easily earn an extra profit by (re)selling houses. The current market of China is the very market like. When you face a different market, you may choose a different investment strategy. If the market is rational, the best strategy may be to pay a house as much as you could and to loan as little as possible for covering the rest. This strategy could help you earn as much as possible. On the other hand, if the market is irrational, the best strategy may be to pay a house as little as you could and to loan as much as possible for covering the rest. This strategy could help you earn as much as possible, which forms a useful investment principle as follows:

**Investment Principle 2.10** In order to earn as much as possible in real estate markets, if the real estate market is mature, it would maximize your benefits for buying a house paid by your self-financing as much as possible; if the market is immature, it would maximize your benefits for buying a house paid by the mortgage lending as much as possible.

In empirical model, the real estate market with responding to a bigger  $\alpha$  is

more mature than one with responding to a lesser  $\alpha$ .

The above investment principle is just erected from the angle of investors. From the administrative angle, financial institutions like banks should adopt the reversed strategy, that is to say:

**Mortgage Loan Principle 2.11** In order to avoid risks in mortgage lending in the real estate market, Banks should decrease the mortgage rates as much as possible under the full-grown real estate market, and increase the mortgage rates as much as possible under the part-grown or less-blown market.

The mortgage loan principle 2.11 could keep banks or the financial systems away from credit crunch or capital crunch (Bernanke, Lown, and Friedman, 1991). China's current situation in real estate market is just inverse of Mortgage Loan Principle 2.11.

### 3 An Application of Dynamic Speculative Behaviors

Following Buttimer Jr. et al (1997), a housing price follows a geometric Brownian motion with a drift and volatility, under the assumption of risk neutral, the price follows the process as follows:

$$d \ln S = \mu dt + \sigma dW \quad (5)$$

where  $\mu$  denotes the instantaneous expectation of the returns, and  $\sigma$  the standard deviation of the returns when the house is resold,  $S$  follows a logarithmic normal distribution,  $dS$  the infinitesimally small change in price,  $dt$  the infinitesimally small increment of times,  $dW$  the infinitesimally small increment, a Winner process, see also (Hull, 2006). If the house has a return from lease before it resale,  $\mu$  must deduct the return expectation from lease, and also denote it by  $\mu$  for convenience. In general,  $\mu S dt$  is the drift or mean of the price under the assumption of risk neutral. According to Itô lemma, it is easy to



get

$$S_t = S_0 e^{(\frac{\mu - \sigma^2}{2})t + \sigma \Delta W_t} \quad (6)$$

where  $\Delta W_t$  denotes the integration of the Wiener process,  $S_0$  the original price of the house,  $S_t$  the price of the house at time  $t$ .

In order to investigate a speculative behavior under the constrained condition shown by Model 5, it is needed to design a special trading strategy, that is, one buys a house with price  $S_t$  at time  $t$ , and resells it with price  $S_{t+1}$  satisfying the following trading strategy at time  $t+1$ :

$$\begin{aligned} & \max_{S_{t+1}} S_{t+1} - \gamma(t+1)S_t \\ & s.t. \quad S_t - \gamma(t)S_{t-1} - \delta(t+1)S_{t+1} \geq 0 \end{aligned} \quad (7)$$

where  $\gamma(t) \geq 1$  and  $\delta(t) > 0$ , and their special cases as shown in Model 1, respectively. It is needed to note that the interval between  $t$  and  $t+1$  may be very different from those shown in Model 1, since Model 1 describes a speculative behavior in the real estate market of China, and Model 7 is just created to describe the same or similar behavior in a more general market with the character of Model 5. In order to make the problem clear, I introduce some positive integer index  $n$  such as  $n_t < n_{t+1}$ , and Model 7 may be rewritten as the trading strategy:

$$\begin{aligned} & \max_{S_{n_{t+1}}} S_{n_{t+1}} - \gamma(t+1)S_{n_t} \\ & s.t. \quad S_{n_t} - \gamma(t)S_{n_{t-1}} - \delta(t+1)S_{n_{t+1}} \geq 0 \end{aligned} \quad (8)$$

This trading behavior following Model 8 is called an investor's dynamic speculative behavior, provided that there is no income flow from the house before it resale, for the sake of succinctness. If someone would like to consider the case including its income flow of rent, he/she might follow Brunnermeier and Julliard (2008) to reset Model 8 similarly, but the analysis process remains valid. Based on the trading strategy of Model 8, we can deduce the following result:

**Theorem 3.1** *Suppose the housing prices are defined by Model 5. If a investor*

*buys a house by the trading strategy or by Model 8, then the price can be affected by the speculative behaviors thru changing the expected returns or variance of returns derived from the house for resale.*

**Proof:** See the proof of Theorem 3.1 in Appendix for details. □

If many investors buy assets following the same trading strategy, then their behaviors must change the expectation or variance of the returns of the assets, and consequently change the asset price level, for example, via Model 6. This means that the investors' behaviors affect the trend of asset prices. Theorem 3.1 supports the assertion of behavioral finance in stock market. De Bondt and Thaler (1985) demonstrated that the deviation of the returns of assets from the Effective Market Hypothesis (EMH), and pointed out that EMH is denied by the investors' behaviors, which are due to the investors' overreaction to the market's volatility, and due to the investors' psychology. The question is how the investors' overreaction destroys EMH? So far, no clear answer for this problem. However, by Theorem 3.1, the answer is that the investors' overreaction changes the expectation or variance of the market returns, and the level of the prices is affected in the consequence. So, we obtain the following conclusion:

**Corollary 3.2** *Investors' behaviors affect the level of prices of the market by changing the shift of the expectation or variance of the assets' returns.*

## 4 The Net Mortgage Loan to Net Speculative Gain Ratio

What is a mortgage bubble? It would be defined in the following:

**Definition 4.1** *When a financing, refinancing are only backed by the same asset as the mortgage, and the growth rate of prices of the asset is bigger than the natural rate, then the remortgage is called **a mortgage bubble**.*

In the literature, the mortgage bubble is used to be a practicable concept more

than a theoretic one, which could be defined by loan to value ratio (Genesove and Mayer, 2001). In this paper, based on the Definition 4.1, a new expression of the definition will be characterized instead of the loan-value ratio<sup>38</sup>. In the spurring real estate market of China, net mortgage loan to net speculative gain ratio is very popular, since a house may be shifted from one investor to another, and every investor could apply for a loan from banks based on the same house as the mortgage, and the loan would be larger than the house's original market value or the housing fundamental value, by the current laws, regulations. In this paper, a formula for counting net mortgage loan to net speculative gain ratio is explicitly characterized at first, as my known so far. For buying a house, most loans are given by banks. In the discussion forward, under the hypothesis described above, the trading strategy is assumed as follows:

**Speculative Trading Strategy 4.2** An investor buys a house at price  $S_1$ , prepays  $\alpha$  times the total market value of the house, and the remaining is covered by the loan from bank with mortgage rate  $r$ ; the second investor buys the house from the first one at price  $S_2$ , prepays  $\alpha$  times the total market value of the house, and the remaining is covered by the loan from bank with mortgage rate  $r$ ; and so on, the  $t^{\text{th}}$  investor buys the house from the  $t-1^{\text{th}}$  investor at price  $S_t$ , prepays  $\alpha$  times the total market value of the house, and the remaining is covered by the loan from bank with mortgage rate  $r$ . Each investor could gain incomes by (re)sale the house(s) bought earlier, and each transaction must include the house property tax by the buyer and other related tax and fees with tax-fee rate  $\tau$  altogether, and the prices follow Model 2.

I am going to calculate the net mortgage loan to net speculative gain ratio below.

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<sup>38</sup> In this paper, a mortgage bubble is defined as banks' mortgage loan at some time, or net mortgage lending-net speculative gain ratio instead of loan-value ratio.

By the Speculative Trading Strategy 4.2, every time an investor finishes the transaction,  $t$  investors have finished the trading chain until the time  $t+1$ . The accumulation of  $t$  investors' net profits earned through the trading chain is<sup>39</sup>:

$$\sum_{t+1}^a = \sum_{j=1}^t S_{j+1} - (1-2\alpha - \tau - (1-\alpha)r) \sum_{j=1}^t S_j \quad (9)$$

And until the time  $t+1$ , how much the banks have utterly lent out through mortgage loans as expressed:

$$\sum_{t+1}^b = (1-\alpha)S_{t+1} - (1-\alpha)rS_{t+1} \quad (10)$$

Model 9 and Model 10 are established based on the Speculative Trade Strategy 4.2. Let's give a processing skeleton of the models as an explanation. An investor cannot gain a benefit in capital from buying a house with price  $S_t$  through mortgage loans at first time  $t$ , but the investor must pay the prepaid  $\alpha$ , tax-fee rate  $\tau$ , and interest accounting as mortgage rates  $r$ . So, the investor could earn the homologous profits from the proclitic investor, who buys the house from the former one with price  $S_{t+1}$  at time  $t+1$ . For this process, the former investor could gain profits as<sup>40</sup>:

$$S_{t+1} - (1-2\alpha - \tau - (1-\alpha)r)S_t > 0.$$

This forms the basis of Model 9. On the other hand, banks, who provide mortgage loans to the investors, have taken back principals and interests from the investors as shown:

$$(1-\alpha)S_t + (1-\alpha)rS_{t+1}.$$

But, the part  $(1-\alpha)S_t$  had been borrowed by investors until the time  $t+1$ . And now, it is returned to the banks. For this reason, this part will be excluded from the

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<sup>39</sup> Investors' net gains are completely direct or indirect from the banks' previous mortgage loans under the assumption of the existence of speculative behaviors.

<sup>40</sup> If the profit is not positive, then the speculative behavior does not exist in the sense of rationality.

future incomes by loans, and Model 10 illustrates how much the banks expect taking back in the maturity.

**Definition 4.3** *The net mortgage loan to net speculative gain ratio  $MB_{t+1}$  up to the time  $t+1$  as:*

$$MB_{t+1} = \frac{\sum_{t+1}^b}{\sum_{t+1}^a}$$

Obviously,  $0 \leq MB_{t+1} \leq \infty$ .

**Definition 4.4** *If  $MB_{t+1} = 0$ , then net mortgage loan to net speculative gain ratio is defined as nonexistence.*

The net mortgage loan to net speculative gain ratio means that an agent gains per capital from its speculative behavior or from (re)sale the house(s), how much the bank should lend out by mortgage in advance.

**Definition 4.5** *If  $MB_{t+1} = \infty$ , then the net mortgage loan to net speculative gain ratio is also defined as nonexistence.*

If  $MB_{t+1} = \infty$ , the investor's net profit from the house is 0, and this implies no speculative behaviors happen. Hence, according to the Definitions 4.4-4.5, the following result is self-evident.

**Theorem 4.6** *The necessary condition of the existence of a dynamic speculative behavior must satisfy:  $0 < MB_{t+1} < \infty$ .*

From Models 2, 9 and 10, it gets

$$\begin{aligned} \sum_{t+1}^i &= \sum_{j=1}^t (1+\eta)^j S_1 - (1-2\alpha - \tau - (1-\alpha)r) \sum_{j=1}^t (1+\eta)^{j-1} S_1 \\ &= \frac{S_1}{\eta} \left\{ [1+\eta - (1-2\alpha - \tau - (1-\alpha)r)] (1+\eta)^t - 1 - \eta + (1-2\alpha - \tau - (1-\alpha)r) \right\} \\ \sum_{t+1}^b &= (1-\alpha)S_1(1-r)(1+\eta)^t \end{aligned}$$

And then

$$MB_{t+1} = \frac{(1-\alpha)\eta(1-r)(1+\eta)^t}{\left[1+\eta-(1-2\alpha-\tau-(1-\alpha)r)\right](1+\eta)^t - 1 - \eta + (1-2\alpha-\tau-(1-\alpha)r)}$$

As a speculative goal, the investors must strongly require more loans from the bank(s) rather than from their own self-financing. If this case comes into reality, the banks have to suffer in a heavy burden in mortgage loans. That is, each capital the banks lend to the investors for buying house(s), the more loans the banks have to give to the sequelae so as to cover the existed black hole of liquidity in the real estate market. The free-lunch could be realized in this situation, which seems to be consistent with China's reality in the real estate market.

**Definition 4.7** *The potential net mortgage loan to net speculative gain ratio MB is defined as:*

$$MB = \lim_{t \rightarrow \infty} MB_{t+1}$$

MB measures the speculative behavior chain has been being valid until the infinity. And then

$$MB = \frac{(1-\alpha)\eta(1-r)}{\eta + 2\alpha + \tau + (1-\alpha)r}$$

$MB_{t+1}$  points out that the net mortgage loan to net speculative gain ratio is just calculated up to the time  $t+1$ , the status at the time  $t+2$  is excluded. When the investor refuses to maintain the house effect or decides to let the mortgage default at time  $t+2$ , and then the house has to be foreclosed by a mortgage delinquency, according to the contract of the loans, the bank has the priority to take over the house. In order to redeem the loss as much as possible, the bank has to put an auction on the house. Let  $\rho$  ( $\rho < 1$ ) be a losing coefficient, the bank regains the earnings from the auction of the house at the price  $(1-\rho)S_{t+2}$ . The new earnings could reduce the net mortgage loan to net speculative gain ratio, and the potential net mortgage loan to net speculative gain ratio could become small like that:

$$MB^\rho = \frac{(1-\alpha)\eta(1-r) - \text{sgn}(\alpha)(1-\rho)(1+\eta)\eta}{\eta + 2\alpha + \tau + (1-\alpha)r - \text{sgn}(\alpha)(1-\rho)(1+\eta)\eta}$$

where, the signal function is defined throughout as

$$\text{sgn}(\alpha) = \begin{cases} 1, & 0 \leq \alpha < 1 \\ 0, & \alpha = 1 \end{cases}$$

The limit of  $MB^\rho$  is MB, i.e.

$$\lim_{\rho \rightarrow 1} MB^\rho = MB$$

If the bank resells the house to a terminal consumer with  $\alpha = 1$  at the price  $S_{t+2}$ , or exercise an auction with  $\rho \leq 0$ , without re-providing mortgage lending based on the same house to the terminal consumer, the bank totally earns the multiple of the original values of the house:

$$((1+\eta) - (1-\alpha)(1-r))(1+\eta)^t S_1 \quad \text{or} \quad (\text{sgn}(\alpha)(1-\rho)(1+\eta) - (1-\alpha)(1-r))(1+\eta)^t S_1$$

The above relations imply that banks have had nothing to be regained from the previous mortgage lending hereafter.

On another hand, under the conditions  $\alpha = 1$  or  $\rho \leq 0$ , no investor has an intent to buy the house, for no benefit could be obtained from this transaction in the sense of speculative expectation. So, in the above situations, the net sum of the investors' or the terminal consumer's profits in capital are negative, as shown below:

$$(1+\eta)^{t+1} S_1 - \sum_{t+1}^a > 0, \quad (1-\rho)(1+\eta)^{t+1} S_1 - \sum_{t+1}^a > 0$$

Hence,  $MB^\rho = 0$ . If  $\alpha < 1$  and  $\rho > 0$ , and banks can not withdraw all mortgage loans, then no speculator could take over the house(s) without a expectation of speculative gains at time  $t+1$ , and the result is  $MB^\rho > 0$ . This statement means that there is no speculative behavior if the action in resale is forbidden.

Therefore, any house foreclosure could lead the banks to a loss in the mortgage lending for the houses bought. So far, it could conclude the following Lemma.

**Lemma 4.8** *A dynamic speculative behavior does exist unless one of the following conditions satisfies:*

- (a) *Any investor is prohibited reselling the house(s) bought;*
- (b)  $\alpha = 1$ ;
- (c)  $\rho \leq 0$ .

**Proof:** The above discussion clarifies the cases (b) and (c) are valid. The remaining is to prove the case (a) is also valid. Whether under the lease or not, the investor must pay off the money  $(\tau + \alpha + (1 - \alpha)r)S$  for the house, and the mortgage lender could earn the returns from loans as  $(1 - \alpha)rS$  with mortgage rate  $r$  at every follow-up time. The investor could not earn any extra return based on the house in capital. In this extreme case, the MB must be the infinity. Therefore, by the Definitions 4.3-4.7 and Theorem 4.6, the dynamic speculative behaviors are nonexistent.  $\square$

A speculator has not an intent to buy a house under the conditions of resale banned,  $\alpha = 1$  or  $\rho \leq 0$ . So, the following theorem can be directly deduced from Lemma 4.8:

**Theorem 4.9** *The dynamic speculative behaviors would exist if the condition holds:  $0 < MB < 1$ .*  $\square$

Our simple model for calculating the net mortgage loan to net speculative gain ratio may be generalized to fit lots of the real reaction of investors in the real estate market. And the above discussion is under the assumption of the fixed  $\alpha$ ,  $\tau$ ,  $r$  and  $r_f$ . If the four parameters are flexible, or floatable, the discussion is similar too, except that the statement may be more complex. Omission is avoided here.

The net mortgage loan to net speculative gain ratio measures the loss of mortgage lenders or banks under the existence of speculative behaviors in literal. An interesting question is how much the banks could lose in case a mortgage



delinquency occurs? Consider that the investors' speculative incomes are mainly from the mortgage lending, as the precondition of speculative behaviors' happening. These arguments self-evidently prove the following:

**Theorem 4.10** *Under the mortgage delinquency at time  $t+1$ , the following holds:*

$$\text{The banks' total loss} = MB_{t+1}^{\rho} \times \text{the investors' total gains}$$

or

$$\text{The banks' total loss} = MB_{t+1}^1 \times \text{the investors' total gains}$$

where,

$$MB_{t+1}^{\rho} = \frac{((1-\alpha)\eta(1-r) + \text{sgn}(\alpha)(1-\rho)(1+\eta))(1+\eta)^t}{\left[1+\eta-(1-2\alpha-\tau-(1-\alpha)r) - \text{sgn}(\alpha)(1-\rho)(1+\eta)\right](1+\eta)^t - 1 - \eta + (1-2\alpha-\tau-(1-\alpha)r)}$$

and

$$MB_{t+1}^1 = -\frac{((1-\alpha)\eta(1-r) + \text{sgn}(\alpha)(1+\eta))(1+\eta)^t}{((\text{sgn}(\alpha)-1)(1+\eta) + 1 - 2\alpha - \tau - (1-\alpha)r)(1+\eta)^t + 1 + \eta - (1-2\alpha-\tau-(1-\alpha)r)}$$

□

Theorem 4.10 demonstrates the method for calculating the potential risks of Banks or financial institutions based on mortgage bubbles.

## 5 A Simulation of the Dynamic Speculative Behaviors and MB

The growth rates of housing prices may be defined as

$$\eta = \frac{S_{t+1} - S_t}{S_t} \quad (11)$$

The main problem would be encountered when Model 11 is applied for calculating the growth rates, because real estate market is heterogeneous and the prices are difficult to measure (Case, 2008). Even for the record of price-level series of resale of the same house, no one could collect the data efficiently. As an operation

of the speculative activity, the level of housing prices would be better than the housing price-index in practices. But, as calculating the growth rates of housing prices, the housing prices index, instead of the level of housing prices, would have more advantages in many aspects. For example, the housing price index is easier to collect. As the measurement of speculative behaviors, the level of housing prices and housing price index are comparably significant and important. Hereby, that the growth rates of housing prices are calculated through the housing price index, instead of the level of housing prices, is reasonable and flexible. Beside the parameter of the growth rates, other parameters are explained bellow.

Table 1: The values of three parameters

$t$	$t+1$	$\tau$	$r$	$\eta$
02/2009	03/ 2009	0.05	0.005	0.002
02/ 2009	04/ 2009	0.05	0.010	0.008
02/ 2009	05/ 2009	0.05	0.015	0.020
02/ 2009	06/ 2009	0.05	0.020	0.041
02/ 2009	07/ 2009	0.05	0.025	0.071
02/ 2009	08/ 2009	0.05	0.030	0.113
02/ 2009	09/ 2009	0.05	0.035	0.164
02/ 2009	10/ 2009	0.05	0.040	0.225
02/ 2009	11/ 2009	0.05	0.045	0.306
02/ 2009	12/ 2009	0.05	0.050	0.413
02/ 2009	01/ 2010	0.05	0.055	0.548
02/ 2009	02/ 2010	0.05	0.060	0.712
02/ 2009	03/ 2010	0.05	0.065	0.914
02/ 2009	04/ 2010	0.05	0.070	1.170

Resources: The growth rate of housing prices is calculated thru housing price index, and the data of housing price index of China from the website, i.e. <http://www.stats.gov.cn/tjsj/>

As shown before,  $\tau$  denotes the property tax and other related taxes and fees in buying houses, and the highest property tax was accounted up to 3% of the total market value of the house in buying. According to the government's regulation, 5% may include all related taxes and fees in buying a house altogether, that is, that  $\tau$  equals 5% is appropriate.  $r$  denotes the mortgage rate, and about 0.5% per month is carried into execution in buying a house. All of these parameters are shown in the Table 1. Table 1 shows the time windows varies from  $t$  to  $t+1$  dividing into 14 groups, and each group has its responding values of parameters, i.e.  $(t, \tau, r, \eta)$ . These parameters are the basis of the simulation of net mortgage loan to net speculative gain ratio (MB) and speculative behaviors. In order to calculate MB and speculative behaviors, the two formulas are defined respectively as follows:

$$MB_{t+1} = \frac{(1-\alpha)(1-r)(1+\eta)}{\eta + 2\alpha + \tau + (1-\alpha)r}$$

and

$$\tilde{\eta} = \begin{cases} \frac{\alpha + (1+\alpha)\tau + (1-\alpha^2)r}{1-\alpha - \alpha\tau - (1-\alpha)\alpha r}, & 0 < \alpha < 1 \\ \tau + r, & \alpha = 0 \end{cases}$$

where  $\tilde{\eta}$  denotes the minimum growth rate of housing prices on condition of the existence of dynamic speculative behaviors. That is, if the growth rate of housing market is less than the minimum growth rate or  $\eta < \tilde{\eta}$ , the speculative behaviors cannot occur. When the net mortgage loan to net speculative gain ratio is positive or  $MB_{t+1} > 0$ , and the speculative behaviors don't exist or  $\eta - \tilde{\eta} < 0$ , the mortgage default or mortgage delinquency could happen; if  $MB_{t+1} > 0$  and  $\eta - \tilde{\eta} > 0$ , the speculative behaviors could happen and the mortgage default could not occur. This assertion could explain why the government sustainably advocates controlling the up spurring trend of the housing prices in motion. In an irrational market, by Theorem 2.7 and its Corollary 2.8, a slow increasing trend of the

housing prices could be a setback against the potential crisis of mortgage defaults.

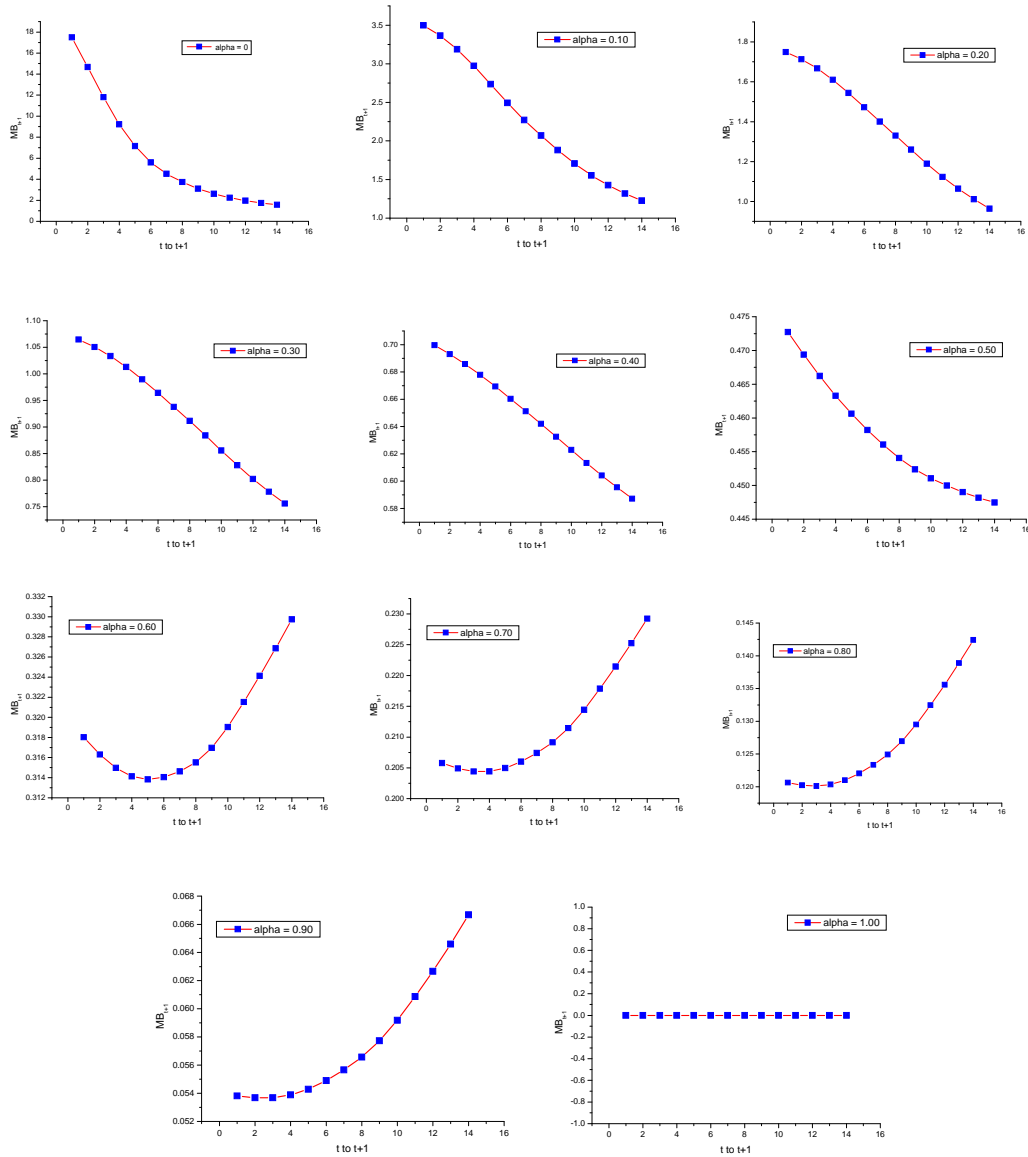


Figure 2: Net mortgage loans to net speculative gains ratios varying based on the different rates of down payment

Notes:  $t$  = Feb. 2009, and  $t+1$  = Mar. 2009, Apr. 2009, ..., Apr. 2010 respectively. Each point on the horizontal axis represents the time from Feb. 2009 to some month for example Jan. 2010.

Resources: Data from Tables 2-12 in the Appendix

This is a living mirror of China's current real estate market. The above discussion discovers that the irrational market has positively responded to a low value of  $\alpha$ .  $\alpha$  is exogenous and is a pivotal factor to determine the levels of the net mortgage loan to net speculative gain ratio and speculative behaviors. Let's start to analyze the relationship among them. All of the parameters can be determined based on Table 1, and their values are listed in Tables 2-12 respectively.

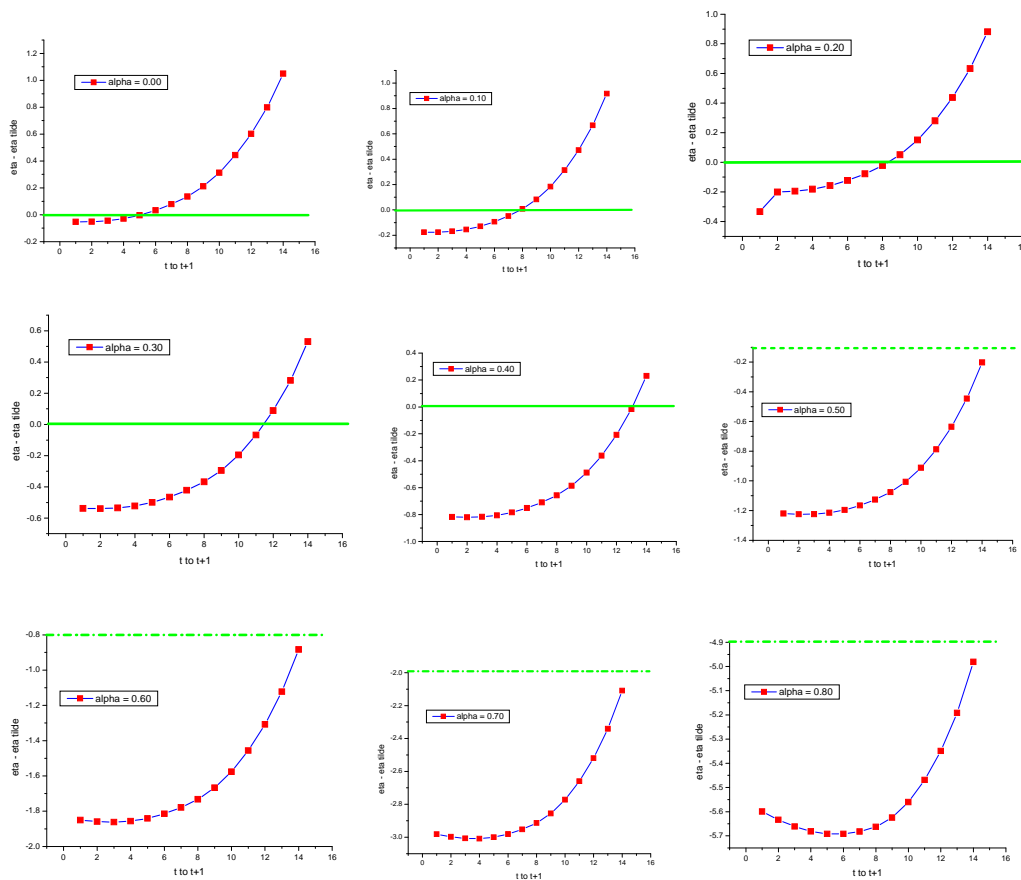


Figure 3: (Non) existence of Speculative behaviors

Resources: Data from Tables 2-12 in the Appendix.  $t$  = Feb. 2009, and  $t+1$  = Mar. 2009, Apr. 2009, ..., Apr. 2010 respectively. Each point on the horizontal axis represents the time from Feb. 2009 to some month for example Jan. 2010.

Let  $\alpha$  vary from 0.00 to 1.00 with an isometry or equal spacing 0.10, and consider how the net mortgage loan to net speculative gain ratio and speculative behaviors happen subsequently.

Whether the follow-up speculative behaviors happen or not, the net mortgage loan to net speculative gain ratio is prone in decrease when  $\alpha$  varies from 0.00 to 1.00 with equal spacing 0.10 adjustment up to some degree for example  $\alpha \leq 0.50$ , and is prone in moving up along convex curves when  $\alpha$  is surpassing some degree for example  $\alpha = 0.50$ . See Figure 3 for visual aid.

When  $\alpha \leq 0.50$  as shown in Figure 2, the net mortgage loan to net speculative gain ratio decreases as the interval of time between  $t$  and  $t+1$  increases, that is, from Feb. 2009 up to Apr. 2010, for fixed  $\alpha$ . But, when  $\alpha > 0.50$ , the track of net mortgage loan to net speculative gain ratio becomes a convex curve as the interval of time between  $t$  and  $t+1$  increases for fixed  $\alpha$ .

The position of green line is seriously affected by the level of  $\alpha$ . In Figure 3, the part of curve under the green line means the speculative behaviors' disappearing, and the speculative behaviors could exist at the part of the curve over the green line. If  $\alpha \geq 50\%$ , any investment behavior in housing market could be hard to earn an extra profit by resale house in a short term. But, the situation might be inverse on the condition of  $\alpha \leq 40\%$ . Figure 2 illustrates that  $\alpha$  is lesser, the speculative opportunity is more, and the speculative behaviors easily happen.

## 5.1 Conclusions in the Simulation

The existence of speculative behaviors may decrease the degree of net mortgage loan to net speculative gain ratio, but the speculative behaviors defaults must increase the degree of net mortgage loan to net speculative gain ratio, on the condition of  $\alpha < 1$ . Speculative behaviors affect the track form of net mortgage loan to net speculative gain ratio.

This conclusion is derived from the relationship between speculative

behaviors and net mortgage loan to net speculative gain ratio, that is, the negative relationship between them when the time is fleeing on condition of the fixed  $\alpha$ , Figure 2 and Figure 3 are more intuitionistic for the argument. Why is the result so strange? It is actually reasonable. When the investment behaviors are under the green line, the investor could not obtain any profit based on the investment activity, and this subsequently improve the crisis of mortgage default or house foreclosure, and the net mortgage loan to net speculative gain ratio could burst. However, when the speculative behaviors continue, that is, the investment activity is over the green line, the net mortgage loan to net speculative gain ratio could decrease, because the growth rate of real estate market remains a positive boundary or  $\eta - \tilde{\eta} > 0$ .  $\eta - \tilde{\eta}$  measures the level of speculative behaviors. In a short term or within one year or so, when  $\alpha \geq 50\%$ , the speculative behaviors couldn't happen, and the net mortgage loan to net speculative gain ratio could increase for the crisis of house foreclosure soaring. Under this case, the disappearing speculative behaviors are positively related to the net mortgage loan to net speculative gain ratio.

If house foreclosure happens at time  $t+2$ , there are two cases to be analyzed. First case, the banks take over the house, and sell it to an investor without mortgage lending through auction. As stated above, the bank could drawback  $(1-\rho)S_{t+2}$  from the auction, and the net mortgage loan to net speculative gain ratio is expressed as

$$MB_{t+1}^{\rho} = \frac{(1-\alpha)(1-r)(1+\eta) - \text{sgn}(\alpha)(1-\rho)(1+\eta)^2}{\eta + 2\alpha + \tau + (1-\alpha)r - \text{sgn}(\alpha)(1-\rho)(1+\eta)^2}$$

Second case, the bank directly sells the house to an investor at price  $S_{t+2}$ , without back by mortgage lending, and the net mortgage loan to net speculative gain ratios are expressed as

$$MB_{t+1}^{t+2} = \frac{(1-\alpha)(1-r)(1+\eta) - \text{sgn}(\alpha)(1+\eta)^2}{\eta + 2\alpha + \tau + (1-\alpha)r - \text{sgn}(\alpha)(1+\eta)^2}$$

Under the condition of foreclosure, the net mortgage loan to net speculative gain

ratio could become complex and their sign means different senses.  $MB_{t+1}^{\rho}$  and  $MB_{t+1}^{t+2}$  are positive, that don't mean the net mortgage loan to net speculative gain ratio does exist. For example,  $MB_{t+1}^{0.2}$  and  $MB_{t+1}^{t+2}$  are positive, see Tables 2-11. But, that  $MB_{t+1}^{\rho}$  and  $MB_{t+1}^{t+2}$  are positive means no speculative behaviors happened and the mortgage lenders earned net positive profits. If  $\rho$  is large enough, the positive of  $MB_{t+1}^{\rho}$  and  $MB_{t+1}^{t+2}$  could mean the net mortgage loan to net speculative gain ratio would exist and burst. If  $MB_{t+1}^{\rho}$  and  $MB_{t+1}^{t+2}$  are negative, that means the investors could lose in speculative activities and the mortgage lenders or banks could lose in loans, one could analyze the cases described through Tables 2-11.

Therefore, any foreclosure could break the nonnegative property of the net mortgage loan to net speculative gain ratios and make them singularly.

## 6 Remarks and Conclusions

From 1978 to present, China's real estate market is still unfledged, therefore, it is not reasonable to give the category on the boom-bust cycles of China's real estate market (Zhang and Sun, 2006). Zhang and Sun continuously argued that the market was driven by three factors, which are the economic growth, macroeconomic environment and institutional establishment. GDP could synthesize the three factors altogether. As a matter of fact, the real estate investment does improve the growth of GDP in a short term, but not significantly affect GDP in a long run, in the sense of Granger causality test (Wang, 2009). In fact, Eschenbach, Schuknecht, Thum and Franco (2004) point out that "boom-bust cycles and financial crises, is highly complex". This evidently argues that boom-bust cycles do not exist in China, since China hasn't had a financial crisis so far.



As aforementioned, China's real estate market has been developed for more than decades, and driven by a consequence of the national residential policies. China's real estate market is strongly characterized by the government's policies, regulations and laws on the real estate industry. The administrative behaviors have great effects to the real estate market. The government's function is self-evidently bigger than the pure market power in the effects on the real estate market, and the conclusion is the unbalance between aggregate supply of and demand in houses, which is aroused by the supply gap in houses, and the supply gap is artificially controlled by the hybrid of the government's regulations, laws and administrative policies. The unbalance of AD and AS is outwardly characterized by speculative behaviors in the real estate market.

Appreciation illusion: Many people are anxious about the inflation which could beget their own assets devaluated, and absolutely believe in that any investment of real estate must maintain or increase its value. This panic emotion drives people to buy houses for deposit or arbitrage. This incentive motive causes the real estate market full of speculative behaviors.

Net mortgage loan to net speculative gain ratio push the speculative behaviors ahead, and the speculative behaviors allure loan-lenders for example banks to optimistically expand the loan magnitude to the borrowers like house buyers or investors. This forms a price pushing loop, or a swirling speculative cycle. Investors want to shortly earn extra profits from buying and selling houses backed by the banks via mortgage loans, and the banks expect to gain super returns of loans deriving from the investors. In a logical view, this game would eventually make the banks become the biggest losers if the banks couldn't stop playing, and the investors were absolutely the biggest winners whereas.

There are some interesting conclusions are stated below:

1. Any dynamic speculative behavior is subject to the growth rate of the real estate market. If the growth rate is lower than some minimum boundary,

there is no speculative profit obtained through the dynamic speculative strategy in the market.

2. The denser the cluster of the speculative behaviors is, the easier it is to earn an extra investment profit as long as the lesser growing bubbles exist. This assertion may be described as another way: If an investor is very difficult to earn an extra investment profit from the fluctuant real estate market, then the market must keep away from the speculation.
3. That the investors' behaviors impact the market behaviors may be realized thru changing the expected returns or variance of returns derived from the house for resale. This conclusion is effect in the financial markets, which provides a new angle to support the behavioral finance hypothesis.
4. Net mortgage loan to net speculative gain ratio accurately measure the variance of dynamic speculative behaviors in theory. As an example, one can read their relationship from the simulation test under foreclosure.

There are two direct policy implications:

1. Try to prolong the interval of a house for resale if the house was bought through mortgage loan. This interval is larger, the net mortgage loans to net speculative gains ratio is less.
2. The levels of  $\alpha$  are negative related to the investors' dynamic speculative behaviors.

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## Appendix

### A. A supplement of Models

In the establishment of Model 1, I did not consider the inflation's impact. In fact, the inflation i.e.  $\pi$  must affect the property returns when a house or home is sold. But, I only discuss the speculative behaviors in the housing market, as the factor of inflation does little thing with our intents in this paper. If one would like to consider the factor of inflation in the models through the paper, only the coefficients need to be responsively modified, for example, for Model 1, it could be rearranged below. Other models could be similarly revised, omitted here.

$$\begin{aligned} & \max (1 - E_t \pi_{t+1}) E_t S_{t+1} - (1 + \tau + (1 - \alpha)r) S_t & (1)^* \\ & s.t. \quad (1 - \pi_t) S_t - (1 + \tau + (1 - \alpha)r) S_{t-1} - \alpha(1 - E_t \pi_{t+1}) E_t S_{t+1} \geq 0 \end{aligned}$$

In addition, the housing prices are only considered in the same unit. If one wants to consider a composite index on the prices, or the local prices, the prices need to be identified by labeling subscript such as  $S_{i,t}$  denoting the housing prices in the place  $i$  at time  $t$ .

## B. Proof of Theorem 2.5

Let

$$L = E_t S_{t+1} - (1 + \tau + (1 - \alpha)r)S_t + \lambda(S_t - (1 + \tau + (1 - \alpha)r)S_{t-1} - \alpha E_t S_{t+1})$$

where  $\lambda$  is a Lagrange multiplier. According to the first order condition, we have

$$E_t S_{t+1} = \frac{1}{\alpha}(S_t - (1 + \tau + (1 - \alpha)r)S_{t-1}), \alpha > 0$$

And then

$$L(E_t S_{t+1}) = \left(\frac{1}{\alpha} - (1 + \tau + (1 - \alpha)r)\right)S_t - \frac{1}{\alpha}(1 + \tau + (1 - \alpha)r)S_{t-1}$$

Substitute Model 2 into the above, we obtain

$$L((1 + \eta)S_t) = \left[\left(\frac{1}{\alpha} - (1 + \tau + (1 - \alpha)r)\right)(1 + \eta) - \frac{1}{\alpha}(1 + \tau + (1 - \alpha)r)\right]S_{t-1}$$

In order to continue the process of investment, especially to maintain the speculative behaviors, the nonnegative profit gained must be the lowest boundary of the action. So, the following holds

$$\left(\frac{1}{\alpha} - (1 + \tau + (1 - \alpha)r)\right)(1 + \eta) - \frac{1}{\alpha}(1 + \tau + (1 - \alpha)r) \geq 0$$

or

$$(1 - \alpha)(1 - \alpha r) - \alpha \tau > 0$$

Therefore,

$$\eta \geq \frac{\alpha + (1 + \alpha)\tau + (1 - \alpha^2)r}{1 - \alpha - \alpha\tau - (1 - \alpha)\alpha r}$$

If  $\alpha = 0$ , then Model 1 becomes

$$\begin{aligned} & \max E_t S_{t+1} - (1 + \tau + r)S_t \\ & \text{s.t. } S_t - (1 + \tau + r)S_{t-1} \geq 0 \end{aligned}$$

Similarly, we can deduce that

$$\eta \geq \tau + r$$

The proof is complete so far.  $\square$

### C. Proof of Theorem 2.7

Let

$$\wp = \sum_{j=1}^t (E_j S_{j+1} - (1 + \tau + (1 - \alpha)r)S_j) + \lambda \sum_{j=2}^t (S_j - (1 + \tau + (1 - \alpha)r)S_{j-1} - \alpha E_j S_{j+1})$$

By the first order condition, it gets

$$\sum_{j=1}^t E_j S_{j+1} = \frac{1}{\alpha} \sum_{j=2}^t (S_j - (1 + \tau + (1 - \alpha)r)S_{j-1}), \alpha > 0$$

And then, combining Model 2, we have

$$\wp \Big|_{\sum_{j=1}^t E_j S_{j+1}} = \Delta S_1$$

where

$$\Delta = 1 + \eta + \beta + \frac{(1 - \alpha\beta)(1 + \eta)}{\alpha} \frac{(1 + \eta)^{t+1} - 1}{\eta} - \beta \frac{(1 + \eta)^{t+1} - 1}{\eta}$$

and

$$\beta = 1 + \tau + (1 - \alpha)r$$

In order to maintain the speculative activity,  $\Delta$  must be nonnegative. Hence, for a sufficiently large  $t$ , we must obtain that

$$\frac{(1 - \alpha\beta)(1 + \eta)}{\alpha} - \beta \geq 0$$

This result is expected when substitute  $\beta$  into it.

If  $\alpha = 0$ , the rest can be similarly resolved like the proof of Theorem 2.5. Omit in detail.  $\square$

### D. Proof of Theorem 3.1

Let

$$\tilde{h} = S_{n_{t+1}} - \gamma(t+1)S_{n_t} + \lambda(S_{n_t} - \gamma(t)S_{n_{t-1}} - \delta(t+1)S_{n_{t+1}})$$

By the first order condition, it deduces that

$$\tilde{h}(S_{n_{t+1}}) = \left( \frac{1}{\delta(t+1)} - \gamma(t+1) \right) S_{n_t} - \frac{\gamma(t)}{\delta(t+1)} S_{n_{t-1}}$$

Substitute Model 6 into above function, and have

$$\tilde{h} = \left( \frac{1}{\delta(t+1)} - \gamma(t+1) \right) e^{\left( \mu - \frac{\sigma^2}{2} \right) n_t + \sigma \Delta W_{n_t}} S_0 - \frac{\gamma(t)}{\delta(t+1)} e^{\left( \mu - \frac{\sigma^2}{2} \right) n_{t-1} + \sigma \Delta W_{n_{t-1}}} S_0$$

Let

$$\frac{\partial \tilde{h}}{\partial \mu} = 0$$

We have

$$\left( \frac{1}{\delta(t+1)} - \gamma(t+1) \right) e^{\left( \mu - \frac{\sigma^2}{2} \right) n_t + \sigma \Delta W_{n_t}} n_t S_0 - \frac{\gamma(t)}{\delta(t+1)} e^{\left( \mu - \frac{\sigma^2}{2} \right) n_{t-1} + \sigma \Delta W_{n_{t-1}}} n_{t-1} S_0 = 0$$

where

$$\frac{1}{\delta(t+1)} - \gamma(t+1) > 0$$

Therefore

$$\mu = \frac{\sigma^2}{2} + \frac{1}{n_t - n_{t-1}} \left\{ \ln \frac{n_{t-1}}{n_t} + \ln \frac{\gamma(t)}{\delta(t+1)} - \ln \left( \frac{1}{\delta(t+1)} - \gamma(t+1) \right) - \sigma \left( \Delta W_{n_t} - \Delta W_{n_{t-1}} \right) \right\}$$

The above relationship means that the expectation or variance of the returns can be changed by the investors' behaviors, and the changing expectation or variance transfer to the prices of the house via Model 6.  $\square$

**E. Tables**Table 2: Net mortgage loans to net speculative gains ratios and speculative under  $\alpha = 0$ 

$t$	$t+1$	$\eta$	$MB_{t+1}$	$\tilde{\eta}$	$\eta - \tilde{\eta}$	$MB_{t+1}^{0.2}$	$MB_{t+1}^{t+2}$
2/2009	03/2009	0.002	17.491	0.055	-0.053	-0.259	0.007
02/2009	04/2009	0.008	14.671	0.060	-0.052	-0.195	0.019
02/2009	05/2009	0.020	11.800	0.065	-0.045	-0.180	0.038
02/2009	06/2009	0.041	9.213	0.070	-0.029	-0.158	0.065
02/2009	07/2009	0.071	7.143	0.075	-0.004	-0.126	0.103
02/2009	08/2009	0.113	5.604	0.080	0.033	-0.085	0.152
02/2009	09/2009	0.164	4.517	0.085	0.079	-0.034	0.209
02/2009	10/2009	0.225	3.729	0.090	0.135	0.021	0.274
02/2009	11/2009	0.306	3.110	0.095	0.211	0.090	0.351
02/2009	12/2009	0.413	2.617	0.100	0.313	0.172	0.441
02/2009	01/2010	0.548	2.239	0.105	0.443	0.261	0.536
02/2009	02/2010	0.712	1.958	0.110	0.602	0.349	0.627
02/2009	03/2010	0.914	1.739	0.115	0.799	0.433	0.711
02/2009	04/2010	1.170	1.565	0.120	1.050	0.511	0.787

Table 3: Net mortgage loans to net speculative gains ratios and speculative under  $\alpha = 0.10$ 

$t$	$t+1$	$\eta$	$MB_{t+1}$	$\tilde{\eta}$	$\eta - \tilde{\eta}$	$MB_{t+1}^{0.2}$	$MB_{t+1}^{t+2}$
2/2009	03/2009	0.002	3.498	0.179	-0.177	-0.172	0.143
02/2009	04/2009	0.008	3.364	0.184	-0.176	-0.090	0.157
02/2009	05/2009	0.020	3.188	0.190	-0.170	-0.075	0.180
02/2009	06/2009	0.041	2.973	0.196	-0.155	-0.053	0.213
02/2009	07/2009	0.071	2.735	0.201	-0.130	-0.022	0.258
02/2009	08/2009	0.113	2.493	0.207	-0.094	0.018	0.314



02/2009	09/2009	0.164	2.271	0.213	-0.049	0.066	0.378
02/2009	10/2009	0.225	2.070	0.218	0.007	0.120	0.447
02/2009	11/2009	0.306	1.882	0.224	0.082	0.186	0.526
02/2009	12/2009	0.413	1.706	0.230	0.183	0.262	0.612
02/2009	01/2010	0.548	1.553	0.235	0.313	0.345	0.698
02/2009	02/2010	0.712	1.425	0.241	0.471	0.425	0.774
02/2009	03/2010	0.914	1.318	0.247	0.667	0.501	0.841
02/2009	04/2010	1.170	1.225	0.252	0.917	0.571	0.897

Table 4: Net mortgage loans to net speculative gains ratios and speculative under  $\alpha = 0.20$

$t$	$t+1$	$\eta$	$MB_{t+1}$	$\tilde{\eta}$	$\eta - \tilde{\eta}$	$MB_{t+1}^{0.2}$	$MB_{t+1}^{t+2}$
2/2009	03/2009	0.002	1.749	0.336	-0.334	0.016	0.377
02/2009	04/2009	0.008	1.713	0.209	-0.201	0.0419	0.396
02/2009	05/2009	0.020	1.667	0.216	-0.196	0.0819	0.424
02/2009	06/2009	0.041	1.610	0.222	-0.181	0.140	0.464
02/2009	07/2009	0.071	1.544	0.229	-0.157	0.219	0.515
02/2009	08/2009	0.113	1.472	0.235	-0.123	0.314	0.575
02/2009	09/2009	0.164	1.400	0.242	-0.078	0.419	0.640
02/2009	10/2009	0.225	1.330	0.248	-0.023	0.527	0.706
02/2009	11/2009	0.306	1.260	0.255	0.051	0.641	0.775
02/2009	12/2009	0.413	1.189	0.262	0.151	0.754	0.844
02/2009	01/2010	0.548	1.123	0.268	0.280	0.854	0.905
02/2009	02/2010	0.712	1.064	0.275	0.437	0.932	0.955
02/2009	03/2010	0.914	1.011	0.281	0.633	0.990	0.993
02/2009	04/2010	1.170	0.963	0.288	0.882	1.029	1.020

Table 5: Net mortgage loans to net speculative gains ratios and speculative under  $\alpha = 0.30$ 

$t$	$t+1$	$\eta$	$MB_{t+1}$	$\tilde{\eta}$	$\eta - \tilde{\eta}$	$MB_{t+1}^{0.2}$	$MB_{t+1}^{t+2}$
2/2009	03/2009	0.002	1.065	0.540	-0.538	0.713	0.878
02/2009	04/2009	0.008	1.050	0.548	-0.540	0.773	0.904
02/2009	05/2009	0.020	1.033	0.555	-0.535	0.850	0.937
02/2009	06/2009	0.041	1.013	0.563	-0.522	0.943	0.976
02/2009	07/2009	0.071	0.990	0.570	-0.499	1.042	1.019
02/2009	08/2009	0.113	0.964	0.578	-0.465	1.136	1.062
02/2009	09/2009	0.164	0.938	0.586	-0.422	1.213	1.101
02/2009	10/2009	0.225	0.912	0.593	-0.368	1.268	1.134
02/2009	11/2009	0.306	0.884	0.601	-0.295	1.304	1.159
02/2009	12/2009	0.413	0.856	0.609	-0.196	1.317	1.176
02/2009	01/2010	0.548	0.828	0.616	-0.070	1.312	1.183
02/2009	02/2010	0.712	0.802	0.624	0.088	1.295	1.182
02/2009	03/2010	0.914	0.778	0.632	0.282	1.270	1.174
02/2009	04/2010	1.170	0.756	0.640	0.530	1.240	1.161

Table 6: Net mortgage loans to net speculative gains ratios and speculative under  $\alpha = 0.40$ 

$t$	$t+1$	$\eta$	$MB_{t+1}$	$\tilde{\eta}$	$\eta - \tilde{\eta}$	$MB_{t+1}^{0.2}$	$MB_{t+1}^{t+2}$
2/2009	03/2009	0.002	0.697	0.819	-0.817	-3.956	2.723
02/2009	04/2009	0.008	0.693	0.828	-0.820	-4.187	2.744
02/2009	05/2009	0.020	0.686	0.837	-0.817	-4.931	2.710
02/2009	06/2009	0.041	0.678	0.846	-0.806	-7.021	2.612
02/2009	07/2009	0.071	0.6694	0.855	-0.784	-16.000	2.465
02/2009	08/2009	0.113	0.660	0.865	-0.752	35.446	2.293
02/2009	09/2009	0.164	0.651	0.874	-0.710	8.432	2.130

02/2009	10/2009	0.225	0.642	0.883	-0.657	4.861	1.978
02/2009	11/2009	0.306	0.633	0.892	-0.586	3.393	1.831
02/2009	12/2009	0.413	0.623	0.901	-0.488	2.603	1.693
02/2009	01/2010	0.548	0.613	0.911	-0.362	2.137	1.573
02/2009	02/2010	0.712	0.604	0.920	-0.208	1.847	1.474
02/2009	03/2010	0.914	0.596	0.929	-0.016	1.647	1.392
02/2009	04/2010	1.170	0.587	0.939	0.231	1.499	1.322

Table 7: Net mortgage loans to net speculative gains ratios and speculative under  $\alpha = 0.50$

$t$	$t+1$	$\eta$	$MB_{t+1}$	$\tilde{\eta}$	$\eta - \tilde{\eta}$	$MB_{t+1}^{0.2}$	$MB_{t+1}^{t+2}$
2/2009	03/2009	0.002	0.473	1.222	-1.220	-1.213	-10.011
02/2009	04/2009	0.008	0.4694	1.233	-1.225	-1.255	-11.023
02/2009	05/2009	0.020	0.466	1.244	-1.224	-1.347	-14.574
02/2009	06/2009	0.041	0.463	1.255	-1.215	-1.522	-32.493
02/2009	07/2009	0.071	0.461	1.267	-1.195	-1.835	45.344
02/2009	08/2009	0.113	0.458	1.278	-1.166	-2.406	11.593
02/2009	09/2009	0.164	0.456	1.290	-1.126	-3.526	6.452
02/2009	10/2009	0.225	0.454	1.301	-1.076	-6.519	4.428
02/2009	11/2009	0.306	0.452	1.313	-1.007	-53.403	3.306
02/2009	12/2009	0.413	0.451	1.324	-0.911	8.481	2.606
02/2009	01/2010	0.548	0.450	1.336	-0.788	4.061	2.159
02/2009	02/2010	0.712	0.449	1.348	-0.636	2.786	1.867
02/2009	03/2010	0.914	0.448	1.360	-0.446	2.179	1.661
02/2009	04/2010	1.170	0.447	1.372	-0.202	1.824	1.508

Table 8: Net mortgage loans to net speculative gains ratios and speculative under  $\alpha = 0.60$ 

$t$	$t+1$	$\eta$	$MB_{t+1}$	$\tilde{\eta}$	$\eta - \tilde{\eta}$	$MB_{t+1}^{0.2}$	$MB_{t+1}^{t+2}$
2/2009	03/2009	0.002	0.318	1.852	-1.851	-0.897	-2.421
02/2009	04/2009	0.008	0.316	1.867	-1.859	-0.921	-2.509
02/2009	05/2009	0.020	0.315	1.882	-1.862	-0.971	-2.713
02/2009	06/2009	0.041	0.314	1.897	-1.856	-1.061	-3.131
02/2009	07/2009	0.071	0.3139	1.912	-1.841	-1.211	-3.972
02/2009	08/2009	0.113	0.314	1.927	-1.815	-1.453	-5.894
02/2009	09/2009	0.164	0.315	1.942	-1.779	-1.841	-12.282
02/2009	10/2009	0.225	0.316	1.958	-1.732	-2.520	100.232
02/2009	11/2009	0.306	0.317	1.973	-1.667	-4.135	9.157
02/2009	12/2009	0.413	0.319	1.989	-1.576	-12.355	4.656
02/2009	01/2010	0.548	0.322	2.004	-1.456	13.650	3.140
02/2009	02/2010	0.712	0.324	2.020	-1.308	4.740	2.420
02/2009	03/2010	0.914	0.327	2.036	-1.122	2.990	2.001
02/2009	04/2010	1.170	0.330	2.052	-0.883	2.245	1.726

Table 9: Net mortgage loans to net speculative gains ratios and speculative under  $\alpha = 0.70$ 

$t$	$t+1$	$\eta$	$MB_{t+1}$	$\tilde{\eta}$	$\eta - \tilde{\eta}$	$MB_{t+1}^{0.2}$	$MB_{t+1}^{t+2}$
2/2009	03/2009	0.002	0.206	2.984	-2.982	-0.775	-1.568
02/2009	04/2009	0.008	0.205	3.005	-2.997	-0.792	-1.611
02/2009	05/2009	0.020	0.204	3.027	-3.007	-0.827	-1.704
02/2009	06/2009	0.041	0.204	3.049	-3.008	-0.889	-1.879
02/2009	07/2009	0.071	0.205	3.071	-3.000	-0.990	-2.188
02/2009	08/2009	0.113	0.206	3.094	-2.981	-1.146	-2.738
02/2009	09/2009	0.164	0.207	3.116	-2.952	-1.380	-3.765
02/2009	10/2009	0.225	0.209	3.139	-2.913	-1.746	-6.186
02/2009	11/2009	0.306	0.211	3.162	-2.856	-2.447	-20.908

02/2009	12/2009	0.413	0.214	3.185	-2.772	-4.253	13.453
02/2009	01/2010	0.548	0.218	3.208	-2.660	-15.270	5.118
02/2009	02/2010	0.712	0.221	3.231	-2.519	11.289	3.259
02/2009	03/2010	0.914	0.225	3.255	-2.341	4.375	2.443
02/2009	04/2010	1.170	0.229	3.279	-2.109	2.809	1.983

Table 10: Net mortgage loans to net speculative gains ratios and speculative under  $\alpha = 0.80$

$t$	$t+1$	$\eta$	$MB_{t+1}$	$\tilde{\eta}$	$\eta - \tilde{\eta}$	$MB_{t+1}^{0.2}$	$MB_{t+1}^{t+2}$
2/2009	03/2009	0.002	0.121	5.602	-5.600	-0.711	-1.240
02/2009	04/2009	0.008	0.120	5.641	-5.633	-0.724	-1.268
02/2009	05/2009	0.020	0.120	5.681	-5.661	-0.751	-1.328
02/2009	06/2009	0.041	0.120	5.722	-5.681	-0.800	-1.437
02/2009	07/2009	0.071	0.121	5.763	-5.692	-0.878	-1.623
02/2009	08/2009	0.113	0.122	5.804	-5.692	-0.995	-1.925
02/2009	09/2009	0.164	0.123	5.846	-5.682	-1.164	-2.420
02/2009	10/2009	0.225	0.125	5.888	-5.663	-1.416	-3.318
02/2009	11/2009	0.306	0.127	5.931	-5.625	-1.858	-5.619
02/2009	12/2009	0.413	0.130	5.974	-5.561	-2.792	-22.578
02/2009	01/2010	0.548	0.132	6.017	-5.469	-5.579	11.187
02/2009	02/2010	0.712	0.1356	6.061	-5.349	-69.662	4.683
02/2009	03/2010	0.914	0.139	6.106	-5.192	7.275	3.043
02/2009	04/2010	1.170	0.142	6.151	-4.981	3.607	2.297

Table 11: Net mortgage loans to net speculative gains ratios and speculative under  $\alpha = 0.90$

$t$	$t+1$	$\eta$	$MB_{t+1}$	$\tilde{\eta}$	$\eta - \tilde{\eta}$	$MB_{t+1}^{0.2}$	$MB_{t+1}^{t+2}$
2/2009	03/2009	0.002	0.054	18.26	-18.256	-0.670	-1.066
02/2009	04/2009	0.008	0.054	18.427	-18.419	-0.682	-1.087

02/2009	05/2009	0.020	0.054	18.600	-18.579	-0.705	-1.132
02/2009	06/2009	0.041	0.054	18.774	-18.734	-0.745	-1.212
02/2009	07/2009	0.071	0.054	18.953	-18.881	-0.809	-1.344
02/2009	08/2009	0.113	0.0549	19.134	-19.021	-0.905	-1.552
02/2009	09/2009	0.164	0.056	19.318	-19.155	-1.040	-1.872
02/2009	10/2009	0.225	0.057	19.506	-19.280	-1.234	-2.396
02/2009	11/2009	0.306	0.058	19.697	-19.391	-1.558	-3.477
02/2009	12/2009	0.413	0.059	19.891	-19.478	-2.181	-6.858
02/2009	01/2010	0.548	0.061	20.089	-19.541	-3.647	-354.460
02/2009	02/2010	0.712	0.063	20.290	-19.578	-9.792	7.627
02/2009	03/2010	0.914	0.065	20.495	-19.582	17.184	3.903
02/2009	04/2010	1.170	0.067	20.704	-19.535	4.822	2.681

Table 12: Net mortgage loans to net speculative gains ratios and speculative under  $\alpha = 1.00$ 

$t$	$t+1$	$\eta$	$MB_{t+1}$	$\tilde{\eta}$	$\eta - \tilde{\eta}$	$MB_{t+1}^{0.2}$	$MB_{t+1}^{t+2}$
2/2009	03/2009	0.002	0	-22	—	0	0
02/2009	04/2009	0.008	0	-22	—	0	0
02/2009	05/2009	0.020	0	-22	—	0	0
02/2009	06/2009	0.041	0	-22	—	0	0
02/2009	07/2009	0.071	0	-22	—	0	0
02/2009	08/2009	0.113	0	-22	—	0	0
02/2009	09/2009	0.164	0	-22	—	0	0
02/2009	10/2009	0.225	0	-22	—	0	0
02/2009	11/2009	0.306	0	-22	—	0	0
02/2009	12/2009	0.413	0	-22	—	0	0
02/2009	01/2010	0.548	0	-22	—	0	0
02/2009	02/2010	0.712	0	-22	—	0	0
02/2009	03/2010	0.914	0	-22	—	0	0
02/2009	04/2010	1.170	0	-22	—	0	0