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Have bull and bear markets changed over time? Empirical evidence from the US-stock market

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Abstract

This contribution analyzes bull and bear markets from 1954:1-2011:2 in the US-stock index S&P 500. Thereby, a 2-State-Markov-Switching model is applied to figure out bull and bear market regimes within the latter period, whereby the estimated state probabilities are used to estimate a dummy variable model by employing operational criteria. A sample-split analysis, where the data set is divided into two samples of equal length, gives evidence for a structural break in the expectation of returns being associated with bull market regimes whereas no structural break can be ascertained concerning bear market regimes. This outcome has strong implications for modern asset allocation theory which takes the presence of regime switching into account as investors who expect a significant increase in stock returns would allocate a higher weight to stocks even though they would face bull market regimes at the time point when deciding on asset allocations.

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1 Introduction

In recent years, studying financial cycles has become an important issue in the academic financial literature and attracts more and more attention. According to Gonzalez et al [6] academics agree on that bull markets are associated with persistently rising share prices, strong investor interest, and raised financial well-being. Aroa and Buza [2] conclude that bull markets are usually associated with a period of prosperity, when the future seems bright and investors have easy access to money. They argue furthermore that the same mass psychology evoking the expectation that every dot-com company will be profitable and, hence, created the boom in the stock market during 1995-1999, was accountable for the crash in the US-stock market in January-March 2000. The U.S economy began to slow down during the end of the year 2000, and the rest of the world followed, resulting in a worldwide recession as pointed out by Aroa and Buza [2].

While bull markets involve an enhancement of the investors' financial well being, the opposite takes place when the direction of the equity price changes alters and the investors face bear markets. Motivated by the global scale of the October 1987 stock market crash and the subsequent Asian and Russian crises of 1997–1998, the understanding of how trends in stock market returns change over time has led to new implications concerning the asset allocation problem. Claessens, Koese and Torrones [5] highlight that asset prices influence consumption through the their effect on household wealth, and can especially effect investment by changing companies' net worth as well as the market value of the capital stock relative to its replacement value. They conclude that in particular the financial crisis that started in 2007 in the United States have been accompanied by an intensive debate over its impact on the broader economy. In their studies they analyze linkages between key macroeconomic and financial variables around business cycles where 21 OECD countries over the period 1960-2007 are taken into account. Their findings give evidence that the interactions between macroeconomic and financial variables can play a role in determining the magnitude and length of a recession.

Guidolin and Timmermann [8] explore the asset allocation and utility cost implications of return predictability from a small, expected utility maximizing investor with a multiple-period horizon. Thereby, the investor optimizes the asset allocation problem while facing three assets - namely stocks, bonds and cash. In contrast to the traditional investment advice recommending an increased exposure of stocks the longer the investment horizon, Guidolin and Timmermann [8] conclude that the optimal allocation to stocks declines as a function of the investment horizons given the investors face bull markets. The optimal asset allocations vary considerably over time being dependent on whether the investors face bull- or bear markets.

However, Guidolin and Timmermann's [8] findings do not account for structural changes in the underlying distributions of bull- and, respectively, bear markets. Have the properties of bull- and bear markets changed over time? If the properties of financial cycles significantly change over time, the latter has to be imbedded in the asset allocation problem and, thus, the existing asset allocation literature ought to be revised. Even though Gonzalez et al [6] conclude that the distributions of bull- and bear markets have changed between the last two centuries, their results can be challenged due to the data set being employed, for instance. The stock index being employed for the period 1885-1925 rests upon Schwert's [15] index of stock prices and is mainly composed of railroad, insurance and banking stocks. Their studies suggest that capital market returns increased by almost 7.20% p.a. in the 20th century, while bear mean returns increased in terms of absolute amount by13.32% p.a. It is worth noting that they report that their results are statistically not significant (i.e. significant at the 10% level only) when the earliest part of the data set prior to 1835 is excluded. Moreover, potential differences which may be assessed between different centuries may imply neglecting impact on the investment decision concerning the time horizon which an investor usually faces.

There are no studies available that analyze structural changes in bull- and bear markets over time while figuring out potential implications for investors who maximize their utilities. The following contribution fills this gap in the literature and is organized as follows. First, a literature review is presented where different approaches will be considered being employed to estimate low frequency trends in the data such as bull and bear markets. Secondly, a convenient model is suggested which captures bull and bear market returns and allows for testing potential structural breaks while accounting for an unknown form of heteroskedasticity. Afterwards the results will be presented and discussed. The last section concludes.

2 Literature Review

In order to estimate bull and bear markets, Gonzalez et al [6] and Claessens, Koese and Torrones [5] employ a model resting upon the dating algorithm as introduced by Bry and Broschan [4]. Gonzalez et al [6] mention that Bry and Broschan’s [4] algorithm basically replicates the NBER business cycle turning points and hence generates formal dating rules in order to determine local peaks and troughs in stochastic time series. Thereby, the minimum length of a cycle is defined to last 15 months from the peak to the trough and back to the peak, whereas one phase (expansion or contraction) is supposed to last at least 5 months. Pagan and Sossounov [14] extend the required minimum duration of a financial cycle to be 16 months rather than 15 months, whereby each phase is defined to last at least 4 months. Furthermore, sharp stock price movements are accounted for by disregarding the minimum phase length if the stock index falls by more than 20% in a single month as mentioned by Gonzalez et al [6].

Furthermore, Claessens, Koese and Torrones [5] employ quarterly data in order to analyze recessions, credit contractions, house price declines and equity price declines. Thereby, the dating algorithm introduced by Harding and Pagan [10] is employed extending the Bry and Broschan's [4] algorithm to identify also turning points in financial time series. The algorithm suggested by Harding and Pagan [10] determines local maxima and minima that meet defined censoring rules, requiring a certain minimum length, respectively, duration for financial cycles. Claessens, Koese and Torrones [5] restrict financial cycles to last at least 20 months while a sole expansion, respectively, contraction is supposed to last at least 8 months. Their studies suggest an equity price declines' (i.e. bear markets) average duration of 26.56 months, whereby the average duration of equity price busts is estimated at 47.16 months. The latter is defined as equity price decline which is in duration that is from peak-to-trough within the top quartile of all price declines.

Lunde and Timmermann [12] who investigate the duration dependence in bull and bear markets estimate the average duration of bull and bear markets to be 21.25 and 9 months respectively. However, they report that the shortest bull and bear markets lasted one week only, whereas the longest durations are estimated to be 113.50 and 34 months respectively. Their statistical model treats the variable driving the stock market as observable while the probability that a bull or bear market lasts for a certain period of time is derived from hazard rates which is estimated via discrete survivor functions. Lunde and Timmermann [12] employ daily stock price data in the US from 2/17/1885-12/31/1997. Hence, Lunde and Timmermann [12] studies involve the same drawback as the studies of Gonzalez et al [6] as they also employ Schwert's [15] index.

In contrast to Lunde and Timmermann [12], Guidolin and Timmermann [8] investigate the asset allocation problem in the presence of regime switching while distinguishing between four different stock market regimes where the first regime is a low return, high volatility crash/bear state, regimes 2 and 3 are bullish states with low-volatility and regime 4 is a recovery state with high volatility which tends to follow crash regimes. Guidolin and Timmermann [8] employ a 4-State-Markov-Switching model in order to account for the latent nature of stock market regimes while employing monthly stock market data. While exhibiting persistency the stock markets regimes are seen to change frequently over time. If the investor faces a slow growth state (i.e. regime 2) which is seen to be the most persistent regime, Guidolin and Timmermann [8] conclude that investors who are assumed to invest on longer horizons will hold the less stocks the longer the horizons.

In contrast to formal dating algorithms, Markov-Switching models account for the latent nature of low frequency trends as the variable driving the returns is assumed to unobservable, whereby transitions between states are governed by a Markov chain. The Markov-Switching model rests upon the pioneering work of Hamilton [9] who has according to Alexander and Dimitriu [1] provided the first formal statistical representation of the idea that economic recessions and expansions influence the behaviour of economic variables. In the financial literature, Markov-Switching models are widely employed. Furthermore, employing formal dating algorithms while accounting for restrictions such as a minimum length will ignore events such as the stock markets crash of October 1987 where according to Gonzalez et al [6] only three months separated the peak and the trough. In the following bull and bear market regimes are estimated by employing a 2-State-Markov-Switching model which will not be restricted. Afterwards an operational criteria is incorporated to translate the probability estimates of the Markov-Switching model into a dummy-variable model being used to test the hypothesis whether bull and bear markets show a structural break in their distributions over time.

3 Econometric Methodology

In the following the statistical methodology is described. First, an ordinary 2-State-Markov-Switching model is employed accounting for a constant transition probability matrix. Afterwards, the estimated smoothed state probabilities are employed to construct a dummy-variable model. Then, different hypotheses are tested to assess whether statistical properties of the stock market under consideration (i.e. S&P 500 stock index) have changed over time. Thereby, an ordinary sample-split analysis is performed.

3.1 Applying the 2-State-Markov-Switching Model to empirical stock market data

In the following stock market cycles are assumed to be the outcome of different regimes. Following Ang and Baekert [3] and Alexander and Dimitriu [1] stock market returns are modeled while accounting for two regimes, bull- and bear market regimes. During bull market regimes the investor expects positive returns on average while the bear market regime is expected to be associated with negative returns. Then the model can be written as

$$y_t = \mu_{S,t} + \varepsilon_{S,t} \tag{1}$$

where

y_t = vector of log-returns

$\mu_{S,t}$ = vector of expected returns depending on the regime S_t

$\varepsilon_{S,t}$ = vector of disturbances, assumed normal with time-varying state dependent variance $\sigma^2_{S,t}$

Since stock markets returns of developed countries do basically not exhibit serial correlation equation (1) does not involve any lagged variables of y_t. Furthermore, the transition probabilities for the two states are in line with Hamilton [9] assumed to follow a first order Markov chain which is constant over time and given by

$$P\left(S_t = j \middle| S_{t-1} = i, S_{t-2} = i-1, ...\right) = P\left(S_t = j \middle| S_{t-1} = i\right) = p_{ij} \tag{2}$$

The matrix of transition probabilities S is in line with Alexander and Dimitriu [1] given by

$$S = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{pmatrix} = \left(p_{ij}\right).$$

Let a Markov chain be given by η_t with $\eta_t = (1,0)'$ when $S_t = 1$ and $\eta_t = (0,1)'$ when $S_t = 2$ then the conditional expectation of η_{t+1} given for $S_t = i$ may be given by

$$E\left[\eta_{t+1} \middle| S_t = i\right] = \begin{pmatrix} p_{i1} \\ p_{i2} \end{pmatrix} = S\eta_t .$$

The conditional probability density function of y_t are assumed to be normal and collected in a 2x1 vector $\vartheta_t = \left(\vartheta_{1t}, \vartheta_{2t}\right)$ where $\vartheta_{1t} = f\left(y_t \middle| S_t = i, \theta\right)$ is the normal density function where the parameter vector θ is conditional on the state so that $\vartheta_{it} = \left[(2\pi)^{1/2}\sigma_i\right]^{-1} \cdot \exp\left(-\left(y_t - \mu_i\right)^2 / 2\sigma_i^2\right)$. The conditional state probabilities are in line with Alexander and Dimitriu [1] obtained recursively by

$$\hat{\eta}_{t|t} = \frac{\hat{\eta}_{t|t-1} \otimes \vartheta_t}{\mathbf{1}'\left(\hat{\eta}_{t|t-1} \otimes \vartheta_t\right)} \qquad \text{with} \qquad \hat{\eta}_{t+1|t} = S\hat{\vartheta}_{t|t} \tag{3}$$

where

$\hat{\eta}_{t|t}$ = vector of conditional probabilities for each state estimated at time t

$\hat{\eta}_{t+1|t}$ =prediction of the same conditional probabilities for time $t+1$

$\mathbf{1}$ = vector of ones

In equation (3) the symbol $\otimes$ denotes the element by element multiplication. Furthermore, the ith element of the product $\hat{\eta}_{t|t-1} \otimes \vartheta_t$ can be considered as the conditional joint distribution of the vector y_t and $S_t = i$ whereas the numerator in equation (3) denotes the density of the observed vector y_t conditional on the current information set. The conditional density of the error vector is then given by

$$L(\theta,S)=\sum_{t=1}^{T}\log f\left(y_t\left|\theta,S\right.\right)=\sum_{t=1}^{T}\log \mathbf{1}'\left(\hat{\eta}_{t|t-1}\otimes\vartheta_t\right).$$

3.2 Using the estimated smoothed state probabilities to set up a dummy-variable model

The estimated state probabilities of the Markov-Switching model as given by equation (1)-(3) can be used to define operational criteria to transfer the model into one which accounts for dummy variables as following. Let the dummy variable d_1 be equal to one if a probability threshold indicates that $PT_t(S_1)>\omega$ with $\omega\in[0,1]$ and analogously the variable $d_2=1$ indicating state $S_t = 2$ if $PT_t(S_1)\leq\omega = PT_t(S_1) > (1-\omega)$ and zero otherwise. Then the following model can be estimated:

$$y_t = \mu_1 d_{1,s_t} + \mu_2 d_{2,s_t} + u_t \tag{4}$$

where u_t is assumed to be a white noise error term. The parameter μ_1 and μ_2 denote the expectation of the log-returns y_t for bull- and, respectively, bear markets concerning the overall sample. As bull- and bear markets are assumed to be persistent, the model will be restricted as following:

$$d_{1,s_t}\overset{!}{=}1 \quad \text{if} \quad PT_t(S_1)\leq\omega,\ PT_{t-1}(S_1)>\omega \ \text{ and } \ PT_{t+1}(S_1)>\omega$$

and analogously

$$d_{2,s_t}\overset{!}{=}1 \ \text{ if } \ PT_t(S_1)>\omega,\ PT_{t-1}(S_1)\leq\omega \ \text{ and } \ PT_{t+1}(S_1)\leq\omega,$$

which means that outliers (i.e. if the regime switch has a duration of one month only) will not be considered as a switch in the regime. As a consequence, a regime is assumed to have a minimum duration of at least three months.

Then the sample is divided into two subsamples of equal length where $T_1 = 1,...,T*$ and $T_2 = T*+1,...,T$. In equation (5) the dummy variables d_3 and d_4 account for the same operational criteria as in equation (4) but take the value zero if $t \in T_1$:

$$y_t = \mu_1 d_{1,s_t} + \mu_2 d_{2,s_t} + \mu_3 d_{3,s_t} + \mu_4 d_{4,s_t} + u_t \quad \text{with} \tag{5}$$

$$d_{3,s_t} = d_{4,s_t} = 0 \quad \forall\ d_{3,s_t},\ d_{4,s_t} \in T_1.$$

3.2 Hypotheses testing: Have first or second order moments of bull- and bear markets changed over time?

If bull- and bear markets have not changed over time, the parameters d_{3,s_t} and d_{4,s_t} should not be significant. Consequently, three pairs of hypothesis will be tested:

H1: The parameters in bull and bear markets have not changed over time
H2: The parameter in bull markets has not changed over time
H3: The parameter in bear markets has not changed over time

The corresponding pairs of hypotheses are given by

H1: $H_0:\ d_{3,s_t} = d_{4,s_t} = 0$ versus $H_1:\ d_{3,s_t} = d_{4,s_t} \neq 0$ (6)

H2: $H_0:\ d_{3,s_t} = 0$ versus $H_1:\ d_{3,s_t} \neq 0$ (7)

H3: $H_0:\ d_{4,s_t} = 0$ versus $H_1:\ d_{4,s_t} \neq 0$ (8)

where the test statistic λ_1 concerning equation (6) is under the null hypothesis asymptotically chi-square distributed with two degrees of freedom whereas the test statistics λ_2 and λ_3 concerning equations (7) and (8) are under the null hypothesis chi-square distributed with one degree of freedom. Since the error term of stock market returns is due to empirical studies not normally distributed the test statistics are bootstrapped while accounting for q bootstrapping samples. Thereby the wild bootstrapping methodology is applied to the data as described in detail in Godfrey [7]. Under the assumption of the errors' independency, the wild bootstrapping methodology exhibits in accordance to Liu [11] and Mammen [13] and asymptoticially valid p-values and accounts moreover for the presence of an unknown form of heteroskedasticity. For the resampling procedure the picking distribution as suggested by Liu [11] is employed where $e_i = g_1 \cdot g_2 - E[g_1] \cdot E[g_2]$ and $g_1 \sim N\left(1/2\left(\sqrt{17/6}+\sqrt{1/6}\right),1/2\right)$ as well as $g_2 \sim N\left(1/2\left(\sqrt{17/6}-\sqrt{1/6}\right),1/2\right)$. According to Godfrey [7] a typical observation for a wild bootstrapping scheme may be given by

$$y*_t = \mu_1 d_{1,s_t} + \mu_2 d_{2,s_t} + \mu_3 d_{3,s_t} + \mu_4 d_{4,s_t} + u_t e_i . \tag{9}$$

where $y*_t$ denotes the stock market returns under the null hypothesis. Godfrey [7] argues that the randomness of $y*_t$ about its conditional mean depends only on the specification of the picking distribution.

4 Empirical Results of the US-stock market

In the Analysis the US-stock index S&P 500 is taken into account. Following Guidolin and Timmermann [8] monthly stock market data is employed accounting for data from 1954:1-2011:2 corresponding to 686 observations. The Markov-Switching model which employs the log-returns of the time series gives the following estimates (standard errors are given in parenthesis):

$$\begin{pmatrix} \mu_{S_1} \\ \mu_{S_2} \end{pmatrix} = \begin{pmatrix} \underset{(0.0007)}{0.0046} \\ \underset{(0.0030)}{-0.0071} \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} \sigma_{S_1}^2 \\ \sigma_{S_2}^2 \end{pmatrix} = \begin{pmatrix} \underset{(0.0000)}{2.19e-04} \\ \underset{(0.0001)}{8.05e-04} \end{pmatrix} \tag{10}$$

$$S_t = \begin{pmatrix} \underset{(0.04)}{0.96} & \underset{(0.08)}{0.18} \\ \underset{(0.02)}{0.04} & \underset{(0.07)}{0.82} \end{pmatrix} \tag{11}$$

Bull- and bear markets are highly persistent. The average duration of bull- and bear-markets are estimated at 50.92 months and 7.64 months respectively. That is the expected duration of bear markets is only 15% of the bull market's duration. The probability threshold $PT_t(.)$ which is employed to assess the exact time point of a change in the regime is used with $\omega = 0.50$. Consequently all observations where the time varying state probability of bull-state exceeds 0.50 are classified as bull-regimes whereas all other observations are counted among bear market regimes. This approach shows that the longest bull market is between 1975:3-1986:6 corresponding to 137 months whereas the longest bear market is estimated from 1973:10 - 1975:2 and corresponds to 15 months. Table 2 shows that the overall time window (i.e. 1954:1-2011:2) accounts for eleven bear- and bull markets where the median length of bear- and bull markets is estimated at seven and 30 months, respectively.[2] Figure 1 shows the smoothed state probabilities from 1954:1-2011:2. Since the estimated state probabilities for bull states are 0.58 and 0.66 for 1980:3 and 1980:5, respectively, and 0.27 for 1980:4, the observation at 1980:4 is treated as outlier and hence assumed to be a part of the bull market from 1973:10 - 1975:2. The dummy variable model (see equation

[2] The 25%- and 75%-quantiles for bear- and bull-markets are estimated at four and ten month for bear markets and seven and 64 months for bull markets.

4) that is employed to estimate low frequency trend being involved in the data estimates the parameter of the log-returns as following (t-statistics are given in parenthesis).

$$y_t = \underset{(6.3562)}{0.0046} \cdot d_{1,s_t} - \underset{(-6.5255)}{0.0127} \cdot d_{2,s_t} + u_t \tag{12}$$

The parameter estimates show that the low frequency trends being estimated by the Markov-Switching model are statistically significant. The amount of the parameter μ_2 that indicates the bear market regime is 2.76 times larger in comparison to the bull market parameter μ_1. In order to figure out a change in the distributions' expectations the overall sample is divided into two subsamples where the first sample corresponds to data from 1954:2-1982:7 (i.e. 342 observations) and the second sample corresponds to monthly stock market data from 1982:8-2011:2 (i.e. 343 observations). The dummy variables d_{3,s_t} and d_{4,s_t} indicate if the expectations of the log-returns have changed during bull- and bear-market regimes in the second sample.

$$y_t = \underset{(3.0641)}{0.0031} \cdot d_{1,s_t} - \underset{(-3.8090)}{0.0130} \cdot d_{2,s_t} + \underset{(2.2438)}{0.0033} \cdot d_{3,s_t} + \underset{(0.1228)}{0.0005} \cdot d_{4,s_t} + u_t \tag{13}$$

Table 1 shows the results from testing the pairs of hypothesis (i.e. H1-H3). Only H2 can be rejected on a common significance level of α =5%. Even though testing the bear-market parameter (H3) suggests that the structural change in the second sample is not significant (bootstrapped p-value is 0.9066) a simultaneous test of both, bull- and bear parameter rejects the null hypothesis on a 10% significance level. In order to support H2 equation (5) is re-estimated under the restriction that $d_{4,s_t} \overset{!}{=} 0$. Then the restricted model which is given by equation (14)

$$y_t = \underset{(3.0663)}{0.0031} \cdot d_{1,s_t} - \underset{(-6.5448)}{0.0127} \cdot d_{2,s_t} + \underset{(2.2454)}{0.0033} \cdot d_{3,s_t} + u_t \tag{14}$$

is tested whether the change in the bull market parameter is significant concerning the second sample (H4). Applying the F-test presents a test statistic of $\lambda_4 = 5.0418$ (p-value is 0.0251). Wild bootstrapping of the test statistic results in a p-value of 0.0262 whereby the number of bootstrapped samples is $q = 5000$. Since H2 and H4 clearly reject the null hypothesis on a common significance level of 5% there is strong evidence for a structural change concerning the expected returns during bull markets. Equation (14) suggests that the mean during bear markets has increased by 106.45% within the second sample (i.e. 1982:8-2011:2). Furthermore, the absolute amount of the negative expected returns in bear markets $\left| d_{2,s_t} \right|$ is clearly larger in comparison to bull markets expectations concerning all estimated models. In terms of ordinary returns the bull- market returns in the first sample are estimated at 9.26% p.a. with an annual volatility of 12.44% whereas the corresponding figures for the second sample are estimated at 18.24% p.a. with volatility of 11.74% p.a. In bear markets between 1954:2-1982:7 investors faced a mean return of -32.85% p.a. with volatility 23.18% whereas the corresponding figures for the period 1982:8-2011:2 represent -30.93% and 24.65%. Interestingly applying the same probability threshold to the Markov-Switching model suggest a total bear market time of 27 months from 1954:2-1982:7 whereas from 1982:8-2011:2 the investors stayed 57 months in bear market regimes during the second sample which is an increase of 211%. However, bull markets summed up duration are estimated at 315 month in the first sample and 286 months in the second sample corresponding to a slight decrease of 9.02%. That means the investor faced 7.89% of time bear markets concerning sample 1, whereas the corresponding figure for sample 2 is 16.62%.

Table 2 shows that the ordinary returns of both regimes are normally distributed as the Jarque-Bera test statistics are close to three and consequently the null hypothesis of normality cannot be rejected. The estimated skewness in bull markets is slightly positive in the second sample and slightly negative within the first sample. The volatility in bear markets is about twice as much as the volatility in bull markets, whereas the latter has only marginally decreased within the second sample concerning bull markets.

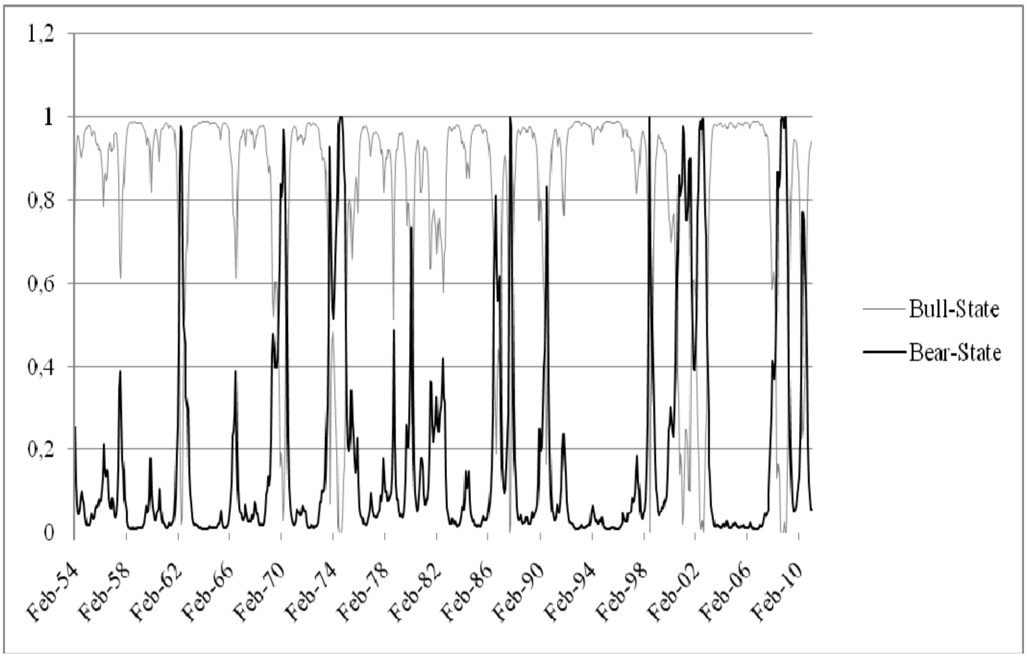

Figure 1: Smoothed state-probabilities of the Markov-Switching Model

Table 1: Hypotheses testing

Hypothesis	Test statistic λ_i	p-value	Bootstrapped p-value*
H1	5.0500	0.0801	0.0788
H2	5.0345	0.0248	0.0244
H3	0.0151	0.9023	0.9066

*The number of bootstrapped samples is $q = 5000$.

5 Discussion

Bull market regimes have statistically changed in a significant way over time and the study here gives strong evidence for a structural break in bull markets as the expected return given bull markets has strongly increased during the second sample.

Table 2: Summary statistics

Period	1954:2-1982:7	1982:8-2011:2	1954:2-2011:2
Type	Bull	Bull	Bear
Mean*	0.7813	1.5201	-2.6292
Observations	315	286	84
Median*	0.9012	1.3756	-3.0920
Minimum*	-10.1795	-6.8817	-21.7630
Maximum*	11.8306	11.5977	16.3047
Std. Dev.*	3.5930	3.3882	6.9420
Skewness	-0.0902	0.2551	0.2941
Kurtosis	3.0520	3.0214	3.1774
Jarque-Bera	3.0520	3.1083	3.1774
p-value	0.7940	0.2114	0.5166

*Note: In monthly terms

The expected return given a bull market regime is 94.56% higher during 1982:8-2011:2 in comparison to the previous sample (i.e. 1954:2-1982:7), whereas the bull market volatility has not changed over time.[3]

The expected return in bear markets though is clearly negative and from a larger magnitude in terms of amount. The volatility in bear markets is with 24.05% p.a. about twice as high as the bull market volatility which is estimated at 12.45% p.a. (i.e. based on sample 1). In contrast to bull markets' expected returns, expected return in bear markets have not changed over time. The previous outcome may have strong implications concerning the asset allocation problem as discussed by Guidolin and Timmermann [8] who conclude that investors who are assumed to invest over longer periods of time will hold fewer stocks the longer the investment period. Since the investors expect according to the results of this study higher returns in the future, they would allocate a higher weight to stocks even though they may face a current bull market since bull markets in the future exhibit higher returns than current bull markets.

In contrast to Lunde and Timmermann [12] and Claessens, Koese and Torrones [5] who employ daily and quarterly data in order to estimate financial cycles, the study suggested here employs monthly data which is also in line with Gonzalez et al [6] and Guidolin and Timmermann [8]. This data frequency is adequate as Gonzalez et al [6] highlight that bull and bear markets are broad market movements and would be best captured by employing low-frequency data. As the Markov-Switching model provide estimates of the probabilities of being in a bull or bear markets rather than providing peak to trough dates, the regimes being estimated by the 2-Markov-Switching model do not exactly match the peak to trough dates as provided by Gonzalez et al [6]. For instance, the well known stock market crash in October 1987 and the associated bear market is according to

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[3] Estimating a 95%-confidence interval for the bull market variance concerning sample 1 results in $VAR(y_t) \in$ [11.1058; 15.1935] while taking into account the chi-square distribution with (315-1) degrees of freedom. As the bull market variance concerning sample 2 is 11.4799 (i.e. on a monthly base), the null hypothesis of volatility equality cannot be rejected on a significance level of 5%.

Gonzalez et al [6] from 1987:8-1987:11 corresponds to four months. The corresponding bear market probabilities for the latter period are 0.2854, 0.5885, 1.0000 and 0.9787. Employing operational criteria as suggested in the study here, results in an estimated corresponding bear market period from 1987:9-1987:12 as the estimated probabilities of being in a bear market for these months are higher than 0.50 (i.e. 0.5885, 1.0000, 0.9787, 0.6127).

Furthermore, Gonzalez et al [6] report bull and bear markets kurtosis at 15.8886 and 5.9952 respectively. As the estimated kurtosis of the bull market return distribution is two to three times higher than the bear market kurtosis, Gonzalez et al [6] argue that bull markets are more likely to show larger movements as they are seen to be prone to outliers. However, the 2-State-Markov-Switching model being employed in this study estimates both distributions, the distribution of the ordinary bull and bear market returns to be close to normality (see table 2). Since the absolute amount of the mean and the volatility are 72.96% and 204.89% higher in bear markets compared to bull markets (i.e. based on the sample 2), bear markets are more likely to involve outliers rather than bull markets. Moreover, Gonzalez et al [6] report that bull market capital returns increased by 7.20% p.a. when comparing the last two centuries. However, their results become insignificant on a 5% significance level when the earliest part of the data set prior to 1835 is excluded. This study suggests an increase of 8.87% within the last 60 years, whereas there is no evidence given for a change in the bear market distribution.

The model being employed here is a 2-State-Markov Switching model, whereas Guidolin and Timmermann [8] estimate a 4-State-Markov Switching model where the bull market regime is parted in three sub-regimes exhibiting different properties. However, Guidolin and Timmermann’s [8] approach involves two drawbacks. First, one runs the risk of overfitting which means the model estimates states the data actually does not contain. The second point is that the states become the less persistent the more states will be included. Therefore, the choice of a 2-State-Markov-Switchung model is preferred in this study even though sub-regimes which are often referred to as market-corrections (i.e. due to their short-run character) may exist.

6 Concluding Remarks

Bull markets have significantly changed within the last three decades. This is the outcome of a statistical analysis. However, there still remains a need for reasonable economic explanations. Future research may consider potential linkages between changes in macroeconmic variables and changes in the financial markets. If changes in the stock markets cannot be explained economically other sciences may be considered that explain empirical facts from a behavioral or psychological point of view, for instance. The outcome of this study has strong implications concerning the asset allocation problem because investments in stocks become the more attractive the higher the expected returns in the future. A significant increase in the return distribution corresponds to a drift in the integrated time series from a statistical point of view. There may be also need for future research to figure out the underlying stochastic processes of stock markets against the background of structural breaks. Furthermore, this methodology can be also applied to other stock markets in order to investigate whether the outcome of this study is supported in an international context, too. This study here gives evidence for a drift term being incorporated in the integrated time series. Therefore another avenue of future research may aim at figuring out the time point when the break occurred or whether the break is rather a potential proxy for a linear or non-linear trend in the data.

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