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Using Surface and Volume Integrals for the Determination of Surface Area and Volume of Potholes on a given length of Road

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Abstract

Potholes are very important factors to be considered in taking decisions on road repairs and management. Roads that require little maintenance to bring it back to its normal shape are left unattended to because of inadequate knowledge of their surface area and volume. Therefore stakeholders on road, particularly Government and site engineers, are always concerned with the surface area and volume of these potholes so that they can determine the amount of materials to be used for the repair and the amount of money to budget for it. Scientific estimation of the surface area and volume of potholes on a given length of road is a major problem to stakeholders. This paper therefore presents the use of surface and volume

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integrals, coupled with the idea of mathematical expectation, in finding the surface area and volume of potholes on a given length of road. We assumed that all the potholes have the same shape of circular paraboloid and considered some case studies to illustrate the significance of the model. The model is found to be appropriate for stakeholders on road to determine the amount of materials and money to budget for road repairs.

Mathematics Subject Classification: 45F10

Keywords: Potholes, Surface Area, Volume, Circular Paraboloid, Surface Integral, Volume Integral

1 Introduction

Road transport is one of the most popular means of transportation in Nigeria. It is an essential part of land transportation. Stakeholders on road, particularly Government, are always interested on the condition of the road in which vehicles travel. Many of our roads in Nigeria have potholes and they require maintenance to restore them back to normal condition. Having an idea of the surface area and volume of potholes on a given length of road will enable the stakeholders to have good plan on the amount of materials and money to budget for the maintenance and the time to spend on road repairs.

According to Baley (1997) and Ajayi (2005), Potholes may be caused by weak soil under top layer. High infiltration of water into the foundation of the road now dissolves the soil under foundation. The affected portion of the road now deflects under its own weight. Vehicle tires also rub the surface of the road as they move and this is another cause of potholes. Timoshenko (1953) and Baley (1997) showed that weights of vehicles also have effects on the roads particularly in areas where we have weak soils. Ajayi (2005) showed that erosion also has serious effects on our roads in this part of the country. It normally removes bitumen in areas where there are cracks and depressions. These cracks, according to Baley (1997) and Ajayi (2005) are now widened by the activities of erosion. In this paper, we assume that all our potholes have the same shape of circular paraboloid. Surface and volume integrals are now used to model the surface area and volume of a pothole. We now applied mathematical expectation to get the surface area and volume of potholes on the entire length of road.

2 Formulation of the Problem

2.1 Mathematical model for the surface area of potholes

Let us assume that all the potholes have the same shape of circular paraboloid and they are represented by the formula:

$$f(x, y) = x^2 + y^2 \quad (1)$$

In the application of surface integral, as we can see in Kreyszig (1987), Lyons (2000), Goldstein et.al. (2007), Simon (1982) and Oke (2001), the surface area of a function is given by:

$$SA = \iint_R \sqrt{1 + (\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}\, dxdy \quad (2)$$

The surface area for a pothole represented by the circular paraboloid in (1) above is therefore given by:

$$SA = \iint_R \sqrt{1 + 4x^2 + 4y^2}\, dxdy \quad (3)$$

Putting the surface area in polar form, we have:

$$SA = \iint_R \sqrt{1 + 4r^2(\cos^2\theta + \sin^2\theta)}\, rdrd\theta \quad (4)$$

Let $0 \le r \le r_*$ and $0 \le \theta \le 2\pi$ for the given paraboloid so that:

$$SA = \int_0^{2\pi} \int_0^{r_*} \sqrt{1 + 4r^2}\, rdrd\theta \quad (5)$$

$$SA = \frac{\pi}{6}\sqrt{(1+4r_*^2)^3} \tag{6}$$

Equation (6) represents the surface area of a pothole on the road. To get the surface area for all the potholes on the entire length of road, we will use the idea of mathematical expectation. According to Moran (1984) and Harper (1984), the expectation of a random variable Y denoted by $E(Y)$ is given by:

$$E(Y) = \sum_{i=1}^{n} Y(E_i)P(E_i) \tag{7}$$

where $P(E_i)$ is the probability of an event, which in this case is the probability of a pothole occurring on one kilometer length of road.

Assuming that the average radius for all the potholes on the road is r_* , then the surface area for all the potholes on the road is the expected number of potholes on the road multiplied by the surface area of one pothole. That is:

Surface area for all potholes on the road is: $SA_* = E(Y)\frac{\pi}{6}\sqrt{(1+4r_*^2)^3}$

$$SA_* = \sum_{i=1}^{n} Y(E_i)P(E_i)\frac{\pi}{6}\sqrt{(1+4r_*^2)^3} \tag{8}$$

where SA_* represents the surface area for all potholes on the road.

2.2 Mathematical model for volume of potholes

Let *T* be a closed region bounded by a surface *S* and let *F* be a vector field defined at each point of *T* and on its boundary surface *S*. Then according to Kreyszig (1987), Lyons (2000), Thomas et.al. (1984), Oke (2001), and Swokowski (1984),

$$\iiint_T F dv \tag{9}$$

is the volume integral of *F* through the bounded region *T*. The volume of the region is given by:

$$V = \iiint_T dv = \iiint_T dxdydz \tag{10}$$

Converting (10) from Cartesian coordinate to polar coordinate we have

$$V = \iiint_T rdrd\theta dz \tag{11}$$

Let us assume that all the potholes we have on our roads have the same shape of circular paraboloid given by

$$z = f(x, y) = x^2 + y^2 \tag{12}$$

Let $0 \le r \le r_*$ and $0 \le \theta \le 2\pi$ for the given paraboloid so that (11) becomes

$$V = \int_0^{2\pi} \int_0^{r_*} \int_0^{z} dzrdrd\theta \tag{13}$$

$$= \int_0^{2\pi} \int_0^{r_*} zrdrd\theta \tag{14}$$

$$= \int_0^{2\pi} \int_0^{r_*} r^3 drd\theta \tag{15}$$

$$= \frac{\pi r_*^4}{2} \tag{16}$$

Equation (2.16) represents the volume of a pothole on the road. Let r_* be the average radius for all potholes on the road. Then the volume of all potholes on the road is the expected number of potholes on road multiplied by the calculated volume of a pothole. That is

Volume of all potholes on the road is given by the formula

$$V_* = E(Y)\frac{\pi r_*^4}{2} = [\sum_{i=1}^{n} Y(E_i)P(E_i)]\frac{\pi r_*^4}{2} \tag{17}$$

3 Main Results and Case Study

3.1 Case study 1

Let us consider Ado-Akure Expressway. This is the road that links the Ekiti State Capital (Ado) with the Ondo State Capital (Akure) in Nigeria. The length of the road is 45km. The average radius of potholes on the road is 0.2m. We divide the road into ten equal parts of length 4.5km each. Table 1 shows the number of potholes on each portion of the road together with their probability of occurrence. The surface area for one pothole on the road is given by the formula:

$$SA = \frac{\pi}{6}\sqrt{(1+4r_*^2)^3} = \frac{22}{42}\sqrt{(1+4\cdot(0.2)^2)^3} = 0.654425742\ m^2$$

The surface area for all the potholes on Ado - Akure road is given by:

$$\begin{aligned} SA_* &= \sum_{i=1}^{n} Y(E_i)P(E_i)\frac{\pi}{6}\sqrt{(1+4r_*^2)^3} \\ &= [1\cdot(0.22) + 2\cdot(0.44) + 2\cdot(0.44) + 1\cdot(0.22) + 1\cdot(0.22) + 1\cdot(0.22) \\ &\quad + 2\cdot(0.44) + 2\cdot(0.44) + 3\cdot(0.66) + 1\cdot(0.22)]\cdot 0.654425742\ m^2 \\ &= 4.319209897\, m^2 \end{aligned}$$

The volume of a pothole on the road is given by:

$$V = \frac{\pi r_*^4}{2} = \frac{22}{14}(0.2)^4 = 2.514285714\cdot 10^{-3} m^3$$

Volume for all the potholes on Ado –Akure road is given by:

$$\begin{aligned} V_* &= [\sum_{i=1}^{n} Y(E_i)P(E_i)]\frac{\pi r_*^4}{2} \\ &= [1\cdot(0.22) + 2\cdot(0.44) + 2\cdot(0.44) + 1\cdot(0.22) + 1\cdot(0.22) + 1\cdot(0.22) \\ &\quad + 2\cdot(0.44) + 2\cdot(0.44) + 3\cdot(0.66) + 1\cdot(0.22)]\cdot 2.514285714\cdot 10^{-3} m^3 \\ &= 0.016594285\, m^3 \end{aligned}$$

3.2 Case study 2

Let us now consider Ado - Ilorin Expressway. This is another major road that links the Ekiti State Capital (Ado) with the Kwara State Capital (Ilorin) in Nigeria. The length of the road is 146km. The average radius of potholes on this road is 0.22m. We divide the road into ten equal parts of length14.6km each. Table 2 shows the number of potholes on each portion of the road together with their probability of occurrence.

The surface area for one pothole on the road is given by the formula:

$$SA = \frac{\pi}{6}\sqrt{(1+4r_*^2)^3} = \frac{22}{42}\sqrt{(1+4\cdot(0.22)^2)^3} = 0.683064328\,m^2$$

The surface area for all the potholes on Ado – Ilorin road is given by the formula:

$$\begin{aligned} SA_* &= \sum_{i=1}^{n} Y(E_i)P(E_i)\frac{\pi}{6}\sqrt{(1+4r_*^2)^3} \\ &= [4\cdot(0.27)+5\cdot(0.34)+1\cdot(0.07)+0\cdot(0.00)+5\cdot(0.34)+3\cdot(0.21) \\ &\quad +2\cdot(0.14)+3\cdot(0.21)+6\cdot(0.41)+4\cdot(0.27)]\cdot 0.683064328\ m^2 \\ &= 6.577909479\,m^2 \end{aligned}$$

The volume of a pothole on the road is given by:

$$V = \frac{\pi r_*^4}{2} = \frac{22}{14}(0.22)^4 = 3.681165714\cdot 10^{-3}\,m^3$$

Volume of all potholes on Ado –Ilorin road is given by the formula:

$$\begin{aligned} V_* &= [\sum_{i=1}^{n} Y(E_i)P(E_i)]\frac{\pi r_*^4}{2} \\ &= [4\cdot(0.27)+5\cdot(0.34)+1\cdot(0.07)+0\cdot(0.00)+5\cdot(0.34)+3\cdot(0.21) \\ &\quad +2\cdot(0.14)+3\cdot(0.21)+6\cdot(0.41)+4\cdot(0.27)]\cdot 3.681165714\cdot 10^{-3}\,m^3 \\ &= 0.035449625\,m^3 \end{aligned}$$

4 Labels of figures and tables

Table 1: Potholes on Ado-Akure road

Length of road from Ado to Akure	Number of potholes	Probability of pothole on one kilometer length of road
4.5KM	1	0.22
4.5KM	2	0.44
4.5KM	2	0.44
4.5KM	1	0.22
4.5KM	1	0.22
4.5KM	1	0.22

4.5KM	2	0.44
4.5KM	2	0.44
4.5KM	3	0.66
4.5KM	1	0.22

Table 2: Potholes on Ado-Ilorin road

Length of road from Ado to Ilorin	Number of potholes	Probability of pothole on one kilometer length of road
14.6KM	4	0.27
14.6KM	5	0.34
14.6KM	1	0.07
14.6KM	0	0.00
14.6KM	5	0.34
14.6KM	3	0.21
14.6KM	*2*	0.14
14.6KM	*3*	0.21
14.6KM	6	0.41
14.6KM	4	0.27

5 Conclusion

Scientific estimation of the surface area and volume of potholes on a given length of road is a major problem to road stakeholders. This has prevented them from having proper planning and budgeting. Hence prompt repair of potholes on roads which needs light maintenance has been limited. An attempt to solve this major problem has motivated the current study in which we developed models to find the surface area and volume of potholes on a given length of road. It is believed that the model would benefit the stakeholders on road management.

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