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The dynamic of a SIV epidemic disease model with vertical infection and pulse Vaccination

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Abstract

In this paper, we conside a SIV epidemic disease model with two delays and vertical infection, and the dynamic behaviors of the model under pulse vaccination are analyzed. Using a new modeling method, we obtain sufficient condition for the permanence of the epidemic model with pulse vaccination.

Mathematics Subject Classification: 92B05

Keywords: Pulse vaccination, Vertical infection, Time delay, Permanence

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1 Introduction

Many infectious diseases in nature transmit through both horizontal and vertical modes [2, 3]. These diseases include such human diseases as rubella, herpes simplex Hepatitis B, AIDS, and so on. For human and animal diseases, horizontal transmission typically occur through direct or indirect physical contact with infectious host, or through disease vector such as mosquitoes, tick, or other biting insects. Vertical transmission can be accomplished through transplacental transfer of disease agents. Busenberg and Cooke [1] discussed a variety of diseases that transmit both vertically and horizontally and gave a comprehensive survey of the formulation and the mathematical analysis of compartment models that incorporate vertical transmission. In our paper, we assume a fraction of the offspring of infected hosts is infected at birth, and hence the infected birth flux will enter class I. Besides a susceptible individual goes through an infectious period. Since time delay has important biologic meaning in epidemic models.

Therefore, in our paper, we consider two time delays, that is, the number of the susceptible individuals during an infectious period and temporary immunity period should be considered, denoted by τ, ω, respectively. Now we create a new delay SIV epidemic model with vertical infection, pulse vaccination into epidemic model and to obtain some important qualitative properties with delays and valid pulse vaccination strategy.

2 Main Results

In the following, we consider the SIV epidemic model with vaccination.

$$\begin{cases} \left.\begin{aligned} \frac{dS}{dt} &= (1-\alpha)(bN - pbI) + \lambda I + \theta V - \frac{\beta S(t-\tau)I(t-\tau)}{N} - bS, \\ \frac{dI}{dt} &= pbI + \frac{\beta S(t-\tau)I(t-\tau)}{N} + \frac{\zeta\beta V(t-\omega)I(t)}{N} - (b+\lambda)I, \\ \frac{dV}{dt} &= \alpha(bN - pbI) - \frac{\zeta\beta V(t-\omega)I(t)}{N} - (b+\theta)V, \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} S(t^+) &= (1-\delta)S(t), \\ I(t^+) &= I(t), \\ V(t^+) &= V(t) + \delta S(t), \end{aligned}\right\} t = nT. \end{cases} \tag{1}$$

Since the natural birth rate and death rate are the same and the disease is assumed not to inflict death on the infected host, so the total population is constant, without loss of generality, let $N(t) = 1$, thus, the system (1) can be reduced as following:

$$\begin{cases} \left.\begin{aligned} \frac{dS}{dt} &= (1-\alpha)(b - pbI) + \lambda I + \theta V - \beta S(t-\tau)I(t-\tau) - bS, \\ \frac{dI}{dt} &= pbI + \beta S(t-\tau)I(t-\tau) + \zeta\beta V(t-\omega)I(t) - (b+\lambda)I, \\ \frac{dV}{dt} &= \alpha(b - pbI) - \zeta\beta V(t-\omega)I(t) - (b+\theta)V, \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} S(t^+) &= (1-\delta)S(t), \\ I(t^+) &= I(t), \\ V(t^+) &= V(t) + \delta S(t), \end{aligned}\right\} t = nT. \end{cases} \tag{2}$$

Since $V(t) = 1 - S(t) - I(t)$, therefore we may just discuss $S(t), I(t)$. Let

$$C_h^+ = \{\phi = (\phi_1(s)\ \phi_2(s)) \in C_h : \quad \phi_i(0) > 0\},$$

where ϕ_i is positive, bounded and continuous function for s $s \in [-\theta, 0]$, where $\theta = \max\{\tau, \omega\}$.

Denote

$$R_2 = \frac{\hbar((1-\alpha)b+\theta)(1-\delta)(1-e^{-(b+\theta)T})}{(b+\theta)[1-(1-\delta)e^{-(b+\theta)T}]}, \qquad m_2^* = \frac{((1-\alpha)b+\theta)(R_2-1)}{((1-\alpha)pb+\theta+\beta)R_2},$$

where

$$\hbar = \frac{\beta}{(1-p)b+\lambda}.$$

Theorem 2.1 Suppose $\lambda+(1-p)b-\zeta\beta>0$. If $R_2>1$, then there exists a positive constant m_2 such that $I(t)\geq m_2$ for t large enough.

Proof: Suppose that $x(t)=(S(t),I(t))$ is any positive solution of system (2). The second equation of system (2) may be rewritten as follow:

$$\frac{dI}{dt}=\beta S(t-\tau)I(t-\tau)-(b+\lambda-pb-\zeta\beta V(t-\omega))I(t).$$

Define

$$R(t)=I(t)+\beta\int_{t-\tau}^{t}S(\theta)I(\theta)d\theta \tag{3}$$

calculating the derivative of $R(t)$ along the solution (2), it follows from (3)

$$\begin{aligned}\frac{dR(t)}{dt}&=\frac{dI(t)}{dt}+\beta S(t)I(t)-\beta S(t-\tau)I(t-\tau)\\&\geq(\lambda+(1-p)b-\zeta\beta)(\hbar S(t)-1)I(t).\end{aligned} \tag{4}$$

since $R_2>1$, then $m_2^*>0$ and there exists a positive constant ε_1 small enough such that $\hbar\varpi>1$. Where

$$\varpi=\frac{(1-\alpha)(b-pbm_2^*)+(1-m_2^*)\theta-m_2^*\beta}{b+\theta}\cdot[1-\frac{\delta}{1-(1-\delta)\exp(-(b+\theta)T)}]-\varepsilon_1>0,$$

for any positive constant t_0, we claim that the inequality $I(t)<m_2^*$ can not hold for $t\geq t_0$ otherwise, there is a positive constant t_0, such that $I(t)<m_2^*$ for all $t\geq t_0$. From the first and the fourth equations of system (2), we have

$$\begin{cases}\dfrac{dS}{dt}\geq(1-\alpha)(b-pbm_2^*)+(1-m_2^*)\theta-\beta m_2^*-(b+\theta)S, & t\neq nT,\ n\in N.\\ S(t^+)=(1-\delta)S(t), & t=nT,\ n\in N.\end{cases} \tag{5}$$

we know that there exists such $T_1\geq t_0+\tau$ for $t\geq T_1$, that

$$S(t) > \frac{(1-\alpha)(b-pbm_2^*)+(1-m_2^*)\theta-\beta m_2^*}{b+\theta}\cdot[1-\frac{\delta}{1-(1-\delta)\exp(-(b+\theta)T)}]-\varepsilon_1=\varpi \tag{6}$$

We have that $S(t)>\varpi$ for $t \geq T_1$.

By (4) and (6), we see that

$$\frac{dR(t)}{dt} > (\lambda+(1-p)b-\zeta\beta)(\hbar\varpi-1)I(t), \qquad t \geq T_1. \tag{7}$$

Let

$$I^l = \min_{t\in[T_1,T_1+\tau]} I(t)$$

We show that $I(t) \geq I^l$ for all $t \geq T_1$, otherwise, there exists a nonnegative constant T_2 such that $I(t) \geq I^l$ for $t \in [T_1,\ T_1+\tau+T_2]$, $I(T_1+\tau+T_2)=I^l$ and $\frac{dI(T_1+\tau+T_2)}{dt} \leq 0$. Thus from the second equation of (2), (4) and (7), we easily see that

$$\frac{dI(T_1+\tau+T_2)}{dt} \geq (\lambda+b-pb-\zeta\beta)(\hbar\varpi-1)I^l > 0.$$

This contradiction. Hence, we get that $I(t) \geq I^l > 0$, for all $t \geq T_1$.

From (7), we have

$$\frac{dR(t)}{dt} > (\lambda+b-pb-\zeta\beta)(\hbar\varpi-1)I^l > 0,$$

This implies $R(t) \to +\infty$ as $t \to +\infty$. This is a contradiction to $R(t) \leq 1+\beta\tau$ for t large enough. Therefore, for any positive constant t_0, the inequality $I(t) < m_2^*$ can not hold for all $t \geq t_0$.

On the other hand, if $I(t) \geq m_2^*$ holds true for all t large enough, then our aim is obtained. On the other hand, $I(t)$ is oscillatory about m_2^*.

where,

$$m_2 = \min\{\frac{m_2^*}{2},\ m_2^* e^{-(\lambda+b-pb)\tau}\}.$$

Critical values of some parameters of systems (2) ($R_2 > 1$ must be satisfied). The condition for the permanence of epidemicdisease

$$\delta < \delta^*,\ \ \delta^* = 1 - \frac{(b+\theta)e^{(b+\theta)T}}{((1-\alpha)b+\theta)\hbar(e^{(b+\theta)T}-1)+(b+\theta)}$$

$$T > T^*,\ T^* = \frac{1}{b+\theta} In\left(1 + \frac{\delta(b+\theta)}{\hbar((1-\alpha)b+\theta)(1-\delta)-(b+\theta)}\right)$$

In the following, we shall show that $I(t) \geq m_2$. There exists two positive constant $\bar{t}, \psi$ such that $I(\bar{t}) = I(\bar{t}+\psi) = m_2^*$, and $I(t) < m_2^*$ for $\bar{t} < t < \bar{t}+\psi$. When $\bar{t}$ is large enough, the inequality $S(t) > \varpi$ holds true for $\bar{t} < t < \bar{t}+\psi$, since $I(t)$ is continuous and ultimately bounded and is not effected by impulses. We conclude that $I(t)$ is uniformly continuous.

Hence there exists a constant T_3 (with $0 < T_3 < \theta$ and T_3 is independent of the choice of $\bar{t}$) such that $I(t) > \frac{m_2^*}{2} I(t) > \frac{m_2^*}{2}$ for all $\bar{t} < t < \bar{t}+T_3$. If $\psi \leq T_3$, our aim is obtained.

If $T_3 < \psi \leq \theta$, from the second equation of system (2) we have that

$$\frac{dI(t)}{dt} \geq -(\lambda+b-pb)I(t) \ \text{ for } \ \bar{t} < t < \bar{t}+\psi.$$

Then we have

$$I(t) \geq m_2^* e^{-(\lambda+b-pb)\theta}, \text{ for } \bar{t} < t < \bar{t}+\psi \leq \bar{t}+\theta.$$

Since $I(t) = m_2^*$, it is obvious that $I(t) > m_2^*$ for $\bar{t} < t < \bar{t}+\psi$. If $\psi \geq \theta$, then we have that $I(t) \geq m_2$ for $\bar{t} < t \leq \bar{t}+\theta$. The same argument can be continued, we can obtain $I(t) \geq m_2$, for $\bar{t}+\theta \leq t \leq \bar{t}+\psi$. Since the interval $[\bar{t}, \bar{t}+\psi]$ is

arbitrarily chosen, we get that $I(t) \geq m_2$ for t large enough. In view of our arguments above, the choice of m_2 is independent of the positive solution of system (2) which satisfies that $I(t) \geq m_2$ for sufficiently large t. This completes the proof. □

Theorem 2.2 If $R_2 > 1$, then system (2) is uniformly permanent.

Proof: Suppose that $X(t) = (S(t), I(t))$ is any positive solution of system (2) with initial conditions (3). From the first and the fourth equations of (2), we have that

$$\begin{cases} \dfrac{dS}{dt} \geq b(1-\alpha)(1-p) - \beta - (b+\theta)S, & t \neq nT,\ n \in N. \\ S(t^+) = (1-\delta)S(t), & t = nT,\ n \in N. \end{cases} \tag{8}$$

we can get such t large enough and $\varepsilon > 0$ small enough that

$$S(t) \geq \frac{\left((1-\alpha)(b-pb)-\beta\right)}{b+\theta} \frac{(1-\delta)(1-e^{-(b+\theta)T})}{1-(1-\delta)e^{-(b+\theta)T}} - \varepsilon = m_1, \quad \text{for } t > T_4.$$

Set $D = \{(S, I) \in R^2 : \ m_1 \leq S(t) \leq 1, \ m_2 \leq I(t) \leq 1\}$. Then D is a bounded compact region in which has positive distance from coordinate hyperplanes. One obtains that every solution of system (2) eventually enters and remains in the region D. The proof is completed. □

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