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Rebalancing and Diversification Return – Evidence from the German Stock Market

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Abstract

The aim of this study is on the one hand to contribute to more clarification about the so-called diversification return especially related to portfolio rebalancing. On the other hand, because of the inconclusive theoretical results, this paper wants to ascertain through empirical tests whether rebalancing a portfolio is likely to be beneficial or not for an equally weighted German stock portfolio. It is shown that diversification returns tend to rise with an increasing rebalancing frequency in all considered periods whereas the variance reduction benefit hardly changes. Not rebalancing has the highest impact on the buy and hold (B&H) portfolio in all periods. However, the rebalancing return defined as the difference between the average geometric return of a rebalanced portfolio and the B&H portfolio sometimes turns out to be positive and sometimes negative. This suggests that rebalancing in the periods considered in this analysis would not always have been reasonable. Removing those stocks from the portfolio that follow a long-term trend and therefore have relatively high or low final weights in the B&H portfolio, leads to a revised portfolio where the assets' returns are more mean-reverting and which generates more positive rebalancing returns. However, mean-reverting returns are often associated with negative autocorrelations of returns, but autocorrelations over the whole period turn out not to be consistent for different time lags. Finally, the study shows no evidence that rebalancing generally leads to better risk adjusted performance or better portfolio diversification.

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1 Introduction

Typically, passive investment strategies include the weighting of investments in a portfolio according to a certain rule. If the originally determined weightings are to be maintained over time, a rebalancing of the portfolio is necessary with regard to the desired portfolio weights. On the one hand, rebalancing serves to adjust the current asset allocation to the one desired according to the investment policy. On the other hand, rebalancing can also be a trading strategy to yield higher portfolio returns.

An important factor is the frequency of rebalancing. The extreme case is the onetime portfolio allocation at the beginning of the investment period with no further adjustment up to the end of the period ("buy and hold", B&H). Such a strategy means that portfolio weights will vary as a result of price changes (rising stocks automatically get a higher weight compared to falling stocks). In case of keeping portfolio weights constant, stocks with a rising portfolio weight due to price changes must be sold, and stocks whose portfolio weight has been reduced have to be purchased ("buy low and sell high"). In this way, a positive effect can be achieved for the portfolio return (Hayley et al., 2015, pp. 1, 16, 22).

Such a rebalancing affects the so-called diversification return which is a term that Booth and Fama (1992) used in the context of a rebalanced portfolio with constant portfolio weights for each asset. Further research papers discussed the sources of these diversification returns.

According to Erb and Harvey (2006, p. 84), the diversification return of an equally weighted portfolio can be attributed, on the one hand, to the variance reduction benefit (due to the use of several individual assets in the portfolio) and, on the other hand, to the impact of not rebalancing in the case of an initially equally weighted but unrebalanced portfolio.

Gorton and Rouwenhorst (2006) refer to the diversification return as "a comparison between the geometric average returns of individual assets and the geometric average return of a portfolio" (p. 5). This definition is generalized to a portfolio in which the weights are not held constant. They show that in the special case of a portfolio that rebalances to equal weights an approximation of the diversification return will be "the difference between the average asset variances and their covariances" and add "It does not imply that rebalancing is the source of the diversification return (it is called the "diversification" return for a reason), but it allows an indirect calculation of its magnitude" (p. 5).

Willenbrock (2011, p. 42) points out that diversification return is an incremental return earned by a rebalanced portfolio and that the underlying source of the diversification return is the rebalancing. He concludes that the diversification return might be described as the "free dessert" (because "it is an incremental return earned while maintaining a constant risk profile") while "diversification is often described as the only free lunch in finance because it allows for the reduction of risk for a given expected return" (p. 48). Furthermore, he notes that "diversification return can be a significant source of return for any rebalanced portfolio of volatile assets" (p. 42).

Bouchy et al. (2012) explore the extra growth that can be generated from the systematic diversification and rebalancing of a portfolio and call this "volatility harvesting". In their paper they focus on equal weighting. Their advice is simply to diversify and rebalance as it enhances returns in the long term. They don't distinguish between mean-reverting and assets that follow a trend as they conclude that their advice applies to any set of volatile and uncorrelated assets that are sufficiently liquid.

Qian (2012) explores the diversification returns of leveraged portfolios. He notes that rebalancing and diversification cannot be separated and that rebalancing is essential for earning diversification returns. He found out that for long-only unleveraged portfolios rebalancing amounts to a mean-reverting strategy whereas for short and leveraged portfolios "rebalancing at the top-down level amounts to a trend-following strategy that detracts from diversification return" (p. 23).

Chambers and Zdanowicz (2014) find, while diversification return often is seen as a valuable source of added return, absent mean-reversion, "no justification for believing that diversification return provides increased expected value" (p. 65). They conclude that "portfolio rebalancing tends to increase the expected value of a portfolio when asset prices are mean-reverting. This enhanced growth emanates from applying a mean-reverting strategy (i.e. rebalancing) to prices that are mean-reverting. The added expected portfolio value is not attributable to either reduced volatility or increased diversification" (p. 74).

Dichtl, Drobetz and Wambach (2014) show with history-based simulations that different classes of rebalancing (periodic, threshold, and range balancing) outperform a B&H strategy where they use the Sharpe ratio, Sortino ratio, and Omega measure to measure the risk-adjusted performance. The results also suggest that the economic relevance of the choice of a specific rebalancing strategy is minor.

Hallerbach (2014) explores the difference between the growth rate of a rebalanced portfolio and the B&H portfolio. He decomposes this full return from rebalancing into the volatility return and the dispersion discount and provides approximations that allow for an intuitive interpretation of the sources of these two components. Since, depending on the circumstances, the rebalancing return can be positive or negative, Hallerbach concludes that rebalancing cannot serve as a general "volatility harvesting" strategy. But in case of a rebalanced portfolio that consists of assets with comparable growth rates, the volatility return is likely to dominate the dispersion discount (pp. 313-314).

In a more recent paper, Hayley et al. (2015) show that there is a misattribution between rebalancing returns and diversification returns. While the latter can be earned by both rebalanced and unrebalanced strategies, the rebalancing return is specific to the act of rebalancing. They conclude that "investors would be better advised to seek to minimize volatility drag by diversifying effectively and to rebalance no more than is necessary to keep their portfolio compositions adequately close to their target allocations" (p. 34).

The above mentioned authors in some cases use different names for the diversification return so that the terminology used in the research literature and the conclusions are a little confusing. Therefore, on the one hand, this paper tries to contribute to more clarifications with regard to the different approaches. On the other hand, because of the inconclusive theoretical results of the rebalancing return, empirical tests of different portfolios and holding periods are essential to find out whether rebalancing is likely to be beneficial or not.

This paper is structured as follows: Section 2 provides an overview of definitions and explanations with respect to the diversification return and the rebalancing impact. Section 3 discusses the relationship between the autocorrelation of returns and the rebalancing return and provides an example for a better understanding of the context and calculations. The empirical results for a German stock portfolio are presented and discussed for different holding periods and different rebalancing frequencies in section 4. Section 5 summarizes the main results of the study.

2 Diversification Return and Rebalancing Return

To derive the formula for the diversification return of a rebalanced portfolio, it is useful to look at the relationship between the arithmetic average return and the geometric average return. The arithmetic average return $\bar{r}$ of a series of returns $r_1,\ldots, r_n$ is defined as follows:

$$\bar{r} = \frac{1}{n} \times \sum_{t=1}^{n} r_t \tag{1}$$

A problem of the arithmetic average return is the fact that is not compatible with the initial and final value of an asset in a certain period. This problem is solved by using the geometric average return $\bar{r}^g$ (Mindlin, 2011, p. 4):

$$\bar{r}^g = -1 + \left(\prod_{t=1}^{n} (1 + r_t) \right)^{\frac{1}{n}} \tag{2}$$

Based on Booth and Fama (1992), Willenbrock (2011, pp. 42, 49) derives the following approximate relation:

$$\bar{r}^g \approx \bar{r} - \frac{1}{2} \times \sigma^2 \qquad \Leftrightarrow \qquad \bar{r} \approx \bar{r}^g + \frac{1}{2} \times \sigma^2 \tag{3}$$

where σ^2 is the variance of the returns. The return of a portfolio (r_{PF}) can be calculated from the returns of the single assets (r_i) as follows:

$$r_{PF} = \sum_{i=1}^{n} w_i \times r_i \quad ; \quad \sum_{i=1}^{n} w_i = 1 \tag{4}$$

where w_i is the weight or proportion of each asset in the portfolio. In case of constant weights the arithmetic average return of the portfolio ($\bar{r}_{PF}$) can be expressed in the following way:

$$\bar{r}_{PF} = \sum_{i=1}^{n} w_i \times \bar{r}_i \qquad \text{for } w_i = \text{constant} \tag{5}$$

where $\bar{r}_i$ is the arithmetic average return of asset i. Willenbrock (2011, p. 42) points out that this equation applies only to a rebalanced portfolio where the portfolio is rebalanced to the constant proportions at the end of each holding period. Using the above relations, equation 5 can be written as

$$\bar{r}_{PF} = \sum_{i=1}^{n} w_i \times \bar{r}_i \approx \sum_{i=1}^{n} w_i \times \left(\bar{r}_i^g + \frac{1}{2} \times \sigma_i^2 \right) \approx \bar{r}_{PF}^g + \frac{1}{2} \times \sigma_{PF}^2 \tag{6}$$

Furthermore, if the weights are held constant, the variance of a portfolio (σ_{PF}^2) can be expressed in the following way (Bruns and Meyer-Bullerdiek, 2013, p. 83):

$$\sum_{i=1}^{n} w_i \times \mathrm{Cov}(r_i, r_{PF}) = \sigma_{PF}^2 \tag{7}$$

where $\mathrm{Cov}(r_i, r_{PF})$ is the covariance of the single asset return with the portfolio return. If this expression is used in equation 6, the following equation will be obtained:

$$\bar{r}_{PF}^{g} + \frac{1}{2} \times \sum_{i=1}^{n} w_i \times \mathrm{Cov}(r_i, r_{PF}) \approx \sum_{i=1}^{n} w_i \times \left(\bar{r}_i^{g} + \frac{1}{2} \times \sigma_i^2 \right) \tag{8a}$$

$$\Leftrightarrow \quad \bar{r}_{PF}^{g} \approx \sum_{i=1}^{n} w_i \times \left(\bar{r}_i^{g} + \frac{1}{2} \times \sigma_i^2 - \frac{1}{2} \times \mathrm{Cov}(r_i, r_{PF}) \right) \tag{8b}$$

$$\Leftrightarrow \quad \bar{r}_{PF}^{g} \approx \sum_{i=1}^{n} w_i \times \bar{r}_i^{g} + \frac{1}{2} \times \sum_{i=1}^{n} w_i \times \left(\sigma_i^2 - \mathrm{Cov}(r_i, r_{PF}) \right) \tag{8c}$$

The diversification return can be obtained by taking the difference between the geometric return of the portfolio and the weighted average geometric returns of the individual assets (Erb and Harvey, 2006, pp. 85-86; Gorton and Rouwenhorst, 2006, p. 5; Willenbrock, 2011, p. 43). Thus, for a rebalanced portfolio, the diversification return according to Willenbrock (DR_W) is given in equation 9:

$$DR_W = \bar{r}_{PF}^{g} - \sum_{i=1}^{n} w_i \times \bar{r}_i^{g} \approx \frac{1}{2} \times \sum_{i=1}^{n} w_i \times \left(\sigma_i^2 - \mathrm{Cov}(r_i, r_{PF}) \right) \tag{9}$$

Willenbrock (2011, p. 43) emphasizes that “maintaining (nearly) constant weights is essential to obtain a diversification return”, so that this equation only applies to a rebalanced portfolio. Using equation 7, the diversification return of equation 9 can be written as follows:

$$DR_W \approx \frac{1}{2} \times \sum_{i=1}^{n} w_i \times \left(\sigma_i^2 - \mathrm{Cov}(r_i, r_{PF}) \right) \approx \frac{1}{2} \times \sum_{i=1}^{n} \left(w_i \times \sigma_i^2 \right) - \frac{1}{2} \times \sigma_{PF}^2 \tag{10}$$

This again is the diversification return of a rebalanced portfolio. According to Erb and Harvey (2006, p. 84) equation 10 expresses the "variance reduction benefit" of the equally weighted portfolio. In case of an initially equally weighted but unrebalanced portfolio they add another component which they call "impact of not rebalancing" and use average weights of asset i over all single holding periods ($\overline{w}_i$) in their calculations. Thus, in this case, the diversification return has these two components.

The impact of not rebalancing can be described as the covariance between the asset's return and the asset's weight in a portfolio and is called the "covariance drag" (Erb and Harvey, 2006, p. 84):

$$\text{covariance drag} \quad = \quad \sum_{i=1}^{n} \operatorname{Cov}(r_i, w_i) \tag{11}$$

The covariance drag considers the case that asset weights can vary. Besides, the covariance between r_i and w_i can be expressed as follows (Poddig, Dichtl and Petersmeier, 2003, p. 54.):

$$\operatorname{Cov}(r_i, w_i) = E((r_i - E(r_i)) \times (w_i - E(w_i))) = E(r_i \times w_i) - E(r_i) \times E(w_i) \tag{12}$$

where E(…) is the expected value of the return or weight, respectively. Hence, the following formula applies to the sum over all securities i:

$$\sum_{i=1}^{n} \operatorname{Cov}(r_i, w_i) = \sum_{i=1}^{n} E(r_i \times w_i) - \sum_{i=1}^{n} E(r_i) \times E(w_i) = \bar{r}_{PF} - \sum_{i=1}^{n} \overline{w}_i \times \bar{r}_i \tag{13}$$

Thus, the covariance drag equals the difference between the arithmetic average return of the portfolio and the weighted average of arithmetic average asset returns. Consequently, the approximate diversification return according to Erb and Harvey (2006) can be expressed as the sum of two terms:

$$DR_{E\&H} \approx \underbrace{\frac{1}{2} \times \sum_{i=1}^{n} \left(\overline{w}_i \times \sigma_i^2\right) - \frac{1}{2} \times \sigma_{PF}^2}_{\text{variance reduction benefit}} + \underbrace{\bar{r}_{PF} - \sum_{i=1}^{n} \overline{w}_i \times \bar{r}_i}_{\text{impact of not rebalancing}} \tag{14}$$

In case of a rebalanced portfolio, the following applies:

$$\text{Rebalanced portfolio:} \quad \bar{r}_{PF} = \sum_{i=1}^{n} \overline{w}_i \times \bar{r}_i \qquad , \qquad \overline{w}_i = w_i \tag{15}$$

Therefore, in this special case DR_W equals $DR_{E\&H}$:

$$DR_{E\&H} \approx \frac{1}{2} \times \sum_{i=1}^{n} \left(\overline{w}_i \times \sigma_i^2 \right) - \frac{1}{2} \times \sigma_{PF}^2 = \frac{1}{2} \times \sum_{i=1}^{n} \left(w_i \times \sigma_i^2 \right) - \frac{1}{2} \times \sigma_{PF}^2 \approx DR_W \quad (16)$$

Willenbrock (2011, p. 44) argues that the true source of the diversification return is not the variance reduction but the rebalancing whereas the variance reduction is just a consequence of diversification. He concludes that the diversification return is driven by the volatility of the assets in the portfolio and that the underlying source of the diversification return is the above mentioned "buy low and sell high"-strategy. He recommends the name "volatility return" instead of "diversification return" (p. 44). Furthermore, Bouchey et al. (2012, p. 30) use the term "rebalancing premium" for the diversification return. Hallerbach (2014, p. 304) points out that the name "diversification return" as used by Booth and Fama (1992) is misleading because even under perfect correlations there would be a positive diversification return. He follows Willenbrock (2011, p. 44) and uses the term "volatility return" for the difference between the growth rate of a rebalanced portfolio and the assets' average growth rate. However, he focuses on the rebalancing return as the difference between the growth rates of a rebalanced portfolio and a B&H portfolio and divides it into the volatility return and the so-called "dispersion discount". The dispersion discount equals the difference between the growth rate of a B&H portfolio and the weighted average of the assets' growth rates. Hallerbach (2014, p. 306) shows that this difference is positive when there is variation in the assets' growth rates. Thus, the rebalancing return according to Hallerbach (RR_H) can be expressed as follows:

$$RR_H = \bar{r}_{PF}^g - \bar{r}_{B\&H}^g = \underbrace{\left(\bar{r}_{PF}^g - \sum_{i=1}^{n} w_{i0} \times \bar{r}_i^g \right)}_{\text{Volatility return}} - \underbrace{\left(\bar{r}_{B\&H}^g - \sum_{i=1}^{n} w_{i0} \times \bar{r}_i^g \right)}_{\text{Dispersion discount}} \qquad (17)$$

where $\bar{r}_{PF}^g$ is the geometric average return of the portfolio (which is rebalanced), $\bar{r}_{B\&H}^g$ is the geometric average return of the B&H portfolio, and w_{i0} are the initial fixed weights of the assets. Hallerbach (2014, p. 307) notes that it is not possible to tell beforehand whether the rebalancing return is positive or negative. It can moreover vary in every period. Just focusing on the volatility return (or diversification return as named above) ignores the impact of the dispersion discount on the rebalancing return.

In order to refer the formula to those portfolios in which the time intervals underlying the return calculation deviate from the rebalancing time intervals (for example, portfolio adjustments on a four week basis versus portfolio return calculation on a weekly basis), the use of average assets' weights is proposed. This generalization leads to the following formula:

$$RR_H = \bar{r}^g_{PF} - \bar{r}^g_{B\&H} = \underbrace{\left(\bar{r}^g_{PF} - \sum_{i=1}^{n} \overline{w}_i \times \bar{r}^g_i\right)}_{\text{Volatility return}} - \underbrace{\left(\bar{r}^g_{B\&H} - \sum_{i=1}^{n} \overline{w}_i \times \bar{r}^g_i\right)}_{\text{Dispersion discount}} \tag{18}$$

In case of a portfolio that is rebalanced in every period, the initial assets' weights equal the average assets' weights (i.e. $w_{i0} = \overline{w}_i$).

3 Autocorrelation of Returns and Rebalancing Return

Following Chambers and Zdanowicz (2014, pp. 71 and 74), negative rebalancing returns imply trending asset prices whereas rebalancing returns should generally be positive when asset prices are mean-reverting. A mean-reverting strategy involves buying assets that have earned inferior returns and selling assets that have earned superior returns. In case of mean-reverting returns, rebalancing leads to asset sales before achieving relatively poor returns and to asset purchases before achieving relatively high returns.

Hayley et al. (2015, p. 14) point out that an increase in expected terminal wealth only occurs if there is rebalancing and negative autocorrelation in relative asset returns. Besides, if returns are mean-reverting they exhibit negative autocorrelation (also known as serial correlation, Chambers and Zdanowicz, 2014, p. 71). Autocorrelation of returns describes the correlation of an asset return with itself over specific time periods ("time lag"). According to Poddig, Dichtl and Petersmeier (2003, p. 99), the empirical autocorrelation c_k at lag k can be expressed as follows:

$$c_k = \frac{\frac{1}{n-k} \times \sum_{t=k+1}^{n} \left(r_t - \bar{r}\right) \times \left(r_{t-k} - \bar{r}\right)}{\frac{1}{n-1} \times \sum_{t=1}^{n} \left(r_t - \bar{r}\right)^2} \tag{19}$$

where k is the time lag, n is the number of observations (and also the current point of time or today, respectively), r_t is the return at time t, and $\bar{r}$ is the arithmetic average return.

In the following, a simple example will show that a negative autocorrelation of all assets in a portfolio does not necessarily lead to a positive rebalancing return:

Given is a portfolio (current value = € 1,000,000) of three assets that are equally weighted. Table 1 shows the values of a portfolio that is rebalanced in every time period to its initial weight of 1/3. Given are the prices P_i of the stocks and the corresponding returns r_i. A more detailed example of a similar calculation is provided by Meyer-Bullerdiek (2016, pp. 41-42).

Table 1: Example 1 – Rebalanced portfolio

t	P_A	r_A	Number of stocks A	w_A	P_B	r_B	Number of stocks B	w_B	P_C	r_C	Number of stocks C	w_C	Value of PF
0	50		6.666,7	33.33%	15		22,222.2	33.33%	20		16,666.7	33.33%	1,000,000
1	40	-20.00%	7.453,7	33.33%	14	-6.67%	21,296.3	33.33%	19	-5.00%	15,692.0	33.33%	894,444
2	41	2.50%	7.286,9	33.33%	13	-7.14%	22,981.9	33.33%	20	5.26%	14,938.2	33.33%	896,294
3	42	2.44%	7.107,4	33.33%	12	-7.69%	24,876.0	33.33%	21	5.00%	14,214.9	33.33%	895,537
4	29	-30.95%	9.680,8	33.33%	13	8.33%	21,595.7	33.33%	22	4.76%	12,761.1	33.33%	842,231
5	30	3.45%	9.563,9	33.33%	14	7.69%	20,494.0	33.33%	21	-4.55%	13,662.6	33.33%	860,747
6	31	3.33%	9.284,7	33.33%	13	-7.14%	22,140.5	33.33%	22	4.76%	13,083.0	33.33%	863,479
7	32	3.23%	9.688,8	33.33%	15	15.38%	20,669.5	33.33%	23	4.55%	13,480.1	33.33%	930,128
8	33	3.13%	9.084,6	33.33%	15	0.00%	19,986.1	33.33%	20	-13.04%	14,989.6	33.33%	899,377
9	34	3.03%	8.857,5	33.33%	14	-6.67%	21,511.1	33.33%	21	5.00%	14,340.7	33.33%	903,465
10	20	-41.18%	13.827,5	33.33%	17	21.43%	16,267.7	33.33%	20	-4.76%	13,827.5	33.33%	829,652

The empirical autocorrelations at lag 1 are -0.1643 (stock A), -0.2041 (stock B), and -0.4287 (stock C). Table 2 shows the data for a B&H portfolio. The results for example 1 are presented in Table 3.

Table 2: Example 1 – B&H portfolio

t	P_A	r_A	Number of stocks A	w_A	P_B	r_B	Number of stocks B	w_B	P_C	r_C	Number of stocks C	w_C	Value of PF
0	50		6,666.7	33.33%	15		22,222.2	33.33%	20		16,666.7	33.33%	1,000,000
1	40	-20.00%	6,666.7	29.81%	14	-6.67%	22,222.2	34.78%	19	-5.00%	16,666.7	35.40%	894,444
2	41	2.50%	6,666.7	30.52%	13	-7.14%	22,222.2	32.26%	20	5.26%	16,666.7	37.22%	895,556
3	42	2.44%	6,666.7	31.23%	12	-7.69%	22,222.2	29.74%	21	5.00%	16,666.7	39.03%	896,667
4	29	-30.95%	6,666.7	22.77%	13	8.33%	22,222.2	34.03%	22	4.76%	16,666.7	43.19%	848,889
5	30	3.45%	6,666.7	23.23%	14	7.69%	22,222.2	36.13%	21	-4.55%	16,666.7	40.65%	861,111
6	31	3.33%	6,666.7	23.97%	13	-7.14%	22,222.2	33.51%	22	4.76%	16,666.7	42.53%	862,222
7	32	3.23%	6,666.7	22.94%	15	15.38%	22,222.2	35.84%	23	4.55%	16,666.7	41.22%	930,000
8	33	3.13%	6,666.7	24.81%	15	0.00%	22,222.2	37.59%	20	-13.04%	16,666.7	37.59%	886,667
9	34	3.03%	6,666.7	25.53%	14	-6.67%	22,222.2	35.04%	21	5.00%	16,666.7	39.42%	887,778
10	20	-41.18%	6,666.7	15.79%	17	21.43%	22,222.2	44.74%	20	-4.76%	16,666.7	39.47%	844,444

Table 3: Example 1 – Results

	Rebalanced Portfolio	B&H Portfolio
Arithmetic average return of the portfolio: $\bar{r}_{PF}$	-1.7173%	-1.5621%
Geometric average return of the portfolio: $\bar{r}^g_{PF}$	-1.8502%	-1.6766%
Variance of portfolio returns: σ^2_{PF}	0.2598%	0.2249%
Weighted average of arithmetic average asset returns: $\sum_{i=1}^{n} \overline{w}_i \times \bar{r}_i$	-1.7173%	-1.2275%
Weighted average of geometric average asset returns: $\sum_{i=1}^{n} \overline{w}_i \times \bar{r}^g_i$	-2.4987%	-1.9167%
Impact of not rebalancing: $\bar{r}_{PF} - \sum_{i=1}^{n} \overline{w}_i \times \bar{r}_i$	0.0000%	-0.3346%
Variance reduction benefit: $\frac{1}{2} \times \sum_{i=1}^{n} \left(\overline{w}_i \times \sigma^2_i\right) - \frac{1}{2} \times \sigma^2_{PF}$	0.5433%	0.4910%
Approx. diversification return according to Erb and Harvey: $DR_{E\&H}$	0.5433%	0.1563%
Volatility return: $\bar{r}^g_{PF} - \sum_{i=1}^{n} \overline{w}_i \times \bar{r}^g_i$	0.6486%	0.2402%
Dispersion discount: $\bar{r}^g_{B\&H} - \sum_{i=1}^{n} \overline{w}_i \times \bar{r}^g_i$	0.8222%	0.2402%
Rebalancing return according to Hallerbach: $RR_H = \bar{r}^g_{PF} - \bar{r}^g_{B\&H}$	-0.1736%	0.0000%

Example 1 shows that the approximate diversification return according to Erb and Harvey (2006) of the rebalanced portfolio is larger than the one of the B&H portfolio because of the negative impact of not rebalancing. The variance reduction benefit is similar for both portfolios. Furthermore, the rebalancing return of the rebalanced portfolio is negative, i.e. the B&H portfolio leads to a better geometric average return than the rebalanced portfolio. Thus, in this example of negative autocorrelation (at lag 1) in asset returns, it would have been better not to rebalance. The reason is the much lower average weight of the low performing stock A in the B&H portfolio ($\overline{w}_i = 0{,}2681$) compared to the rebalanced portfolio ($\overline{w}_i = 0{,}3333$). Obviously, the strong negative performance of stock A over the whole period leads to the low weight in the B&H portfolio. However, the autocorrelation of the returns at lag 1 is negative.

In the appendix, a second example is constructed with positive autocorrelation of returns at lag 1 for all assets in the portfolio (Tables 12-14). It is shown that the rebalanced portfolio performs better than the B&H portfolio. Thus, in this example, there is a positive rebalancing return despite positive autocorrelations in asset returns.

However, in order to find out if rebalancing is beneficial in practice, empirical tests of different markets and holding periods are essential.

4 Empirical Results

The empirical analysis concentrates on the German stock market that is represented by 15 stocks that have been continuously included in the German stock index DAX since its start date in 1988:

Allianz, BASF, Bayer, BMW, Commerzbank, Daimler, Deutsche Bank, E.ON, Henkel, Linde, Lufthansa, RWE, Siemens, ThyssenKrupp, Volkswagen.

Please note that the preferred shares of Volkswagen are used, which replaced the ordinary shares in the DAX in 2009. Furthermore a few companies were rebranded (e.g. VEBA or VIAG to E.ON).

The data is taken from ariva.de and comprises the weekly closing prices of all stocks based on a Friday. In case that there is no trade on this day, the data from the previous trading date is taken. All share prices are adjusted for dividends as well as for subscription rights and share splits.

The data cover the period from January 2006 to December 2015. The total period is divided into several subperiods:

6 January 2006 – 25 December 2015 (520 weeks)
6 January 2006 – 4 January 2008 (104 weeks)
4 January 2008 – 25 December 2015 (416 weeks)
4 January 2008 – 1 January 2010 (104 weeks)
1 January 2010 – 25 December 2015 (312 weeks)
1 January 2010 – 30 December 2011 (104 weeks)
30 December 2011 – 25 December 2015 (208 weeks)
30 December 2011 – 27 December 2013 (104 weeks)
27 December 2013 – 25 December 2015 (104 weeks)

On the initial starting date of each period, all stocks have the same portfolio weight (=1/15). The portfolios are reallocated to these weights on a regular basis. The tests consider different rebalancing frequencies: weekly, every 2, 4, 13, 26, 52, and 104 weeks. Additionally, the B&H portfolio is included where no rebalancing takes place over the whole considered time period. Transaction costs are not considered in the tests.
Table 4 shows the empirical results for the total period. Please note that the results are based on weekly returns. Therefore, only in case of a weekly rebalancing the

initial assets' weights equal the average assets' weights of the rebalanced portfolio ($w_{i0} = \overline{w}_i$) used in the tests. For all other rebalancing frequencies, $\overline{w}_i$ is used so that the volatility return and the dispersion discount are calculated according to equation 18.

Table 4: Results based on weekly returns, 6 January 2006 – 25 December 2015

	Rebalancing frequency							
	every week	every 2 weeks	every 4 weeks	every 13 weeks	every 26 weeks	every 52 weeks	every 104 weeks	B&H
$\bar{r}_{PF}$	0.1820%	0.1823%	0.1792%	0.1732%	0.1704%	0.1692%	0.1747%	0.2091%
$\bar{r}^g_{PF}$	0.1167%	0.1172%	0.1145%	0.1091%	0.1068%	0.1061%	0.1130%	0.1485%
σ^2_{PF}	0.1285%	0.1281%	0.1272%	0.1257%	0.1248%	0.1240%	0.1210%	0.1192%
$\sum_{i=1}^{n} \overline{w}_i \times \bar{r}_i$	0.1820%	0.1822%	0.1827%	0.1850%	0.1869%	0.1914%	0.1995%	0.2572%
$\sum_{i=1}^{n} \overline{w}_i \times \bar{r}^g_i$	0.0621%	0.0623%	0.0629%	0.0655%	0.0673%	0.0722%	0.0815%	0.1426%
Impact of not rebalanc.	0.0000%	0.0001%	-0.0035%	-0.0118%	-0.0166%	-0.0223%	-0.0248%	-0.0481%
Variance red. benefit	0.0548%	0.0549%	0.0553%	0.0557%	0.0562%	0.0563%	0.0564%	0.0530%
$DR_{E\&H}$	0.0548%	0.0550%	0.0517%	0.0439%	0.0397%	0.0340%	0.0316%	0.0049%
Volatility return	0.0546%	0.0548%	0.0515%	0.0437%	0.0394%	0.0338%	0.0315%	0.0059%
Dispersion discount	0.0864%	0.0862%	0.0856%	0.0830%	0.0812%	0.0763%	0.0670%	0.0059%
RR_H	-0.0318%	-0.0313%	-0.0340%	-0.0394%	-0.0417%	-0.0424%	-0.0355%	0.0000%

The results show for the whole period considered that the approximate diversification return according to Erb and Harvey (2006) diminishes with a lower rebalancing frequency. The reason for it is the negative impact of not rebalancing whereas the variance reduction benefit is almost independent from the rebalancing frequency. For the whole period considered a rebalancing strategy leads to negative rebalancing returns regardless of the rebalancing frequency because of the lower geometric average return compared to the B&H portfolio. Thus, the dispersion discount is greater than the volatility return so that, for this long term period, the B&H strategy is superior. With regard to the geometric average return of the single portfolios, it can be ascertained that a more frequent rebalancing has a slightly positive effect. However, the B&H portfolio still achieves a much better return.

Tables 5 and 6 present the results for the other time periods.

Table 5: Results based on weekly returns for the 2 year periods

	Rebalancing frequency						
	every week	every 2 weeks	every 4 weeks	every 13 weeks	every 26 weeks	every 52 weeks	every 104 weeks (B&H)
	6 January 2006 – 4 January 2008						
Impact of not rebalancing	0.0000%	-0.0005%	-0.0034%	-0.0038%	-0.0105%	-0.0126%	-0.0338%
Variance reduction benefit	0.0343%	0.0343%	0.0343%	0.0343%	0.0346%	0.0350%	0.0359%
$DR_{E\&H}$	0.0343%	0.0338%	0.0309%	0.0306%	0.0240%	0.0224%	0.0021%
Volatility return	0.0340%	0.0335%	0.0306%	0.0302%	0.0237%	0.0220%	0.0017%
Dispersion discount	0.0388%	0.0385%	0.0374%	0.0334%	0.0270%	0.0179%	0.0017%
RR_H	-0.0049%	-0.0051%	-0.0068%	-0.0032%	-0.0033%	0.0041%	0.0000%
	4 January 2008 – 1 January 2010						
Impact of not rebalancing	0.0000%	-0.0008%	-0.0263%	-0.0560%	-0.0757%	-0.0950%	-0.1138%
Variance reduction benefit	0.1191%	0.1195%	0.1206%	0.1201%	0.1223%	0.1241%	0.1157%
$DR_{E\&H}$	0.1191%	0.1187%	0.0942%	0.0642%	0.0466%	0.0291%	0.0019%
Volatility return	0.1180%	0.1176%	0.0929%	0.0627%	0.0454%	0.0285%	0.0014%
Dispersion discount	0.0414%	0.0406%	0.0379%	0.0319%	0.0300%	0.0244%	0.0014%
RR_H	0.0766%	0.0770%	0.0549%	0.0309%	0.0154%	0.0041%	0.0000%
	1 January 2010 – 30 December 2011						
Impact of not rebalancing	0.0000%	-0.0034%	0.0005%	-0.0144%	-0.0096%	-0.0106%	-0.0300%
Variance reduction benefit	0.0424%	0.0424%	0.0424%	0.0429%	0.0431%	0.0430%	0.0431%
$DR_{E\&H}$	0.0424%	0.0390%	0.0430%	0.0285%	0.0335%	0.0324%	0.0131%
Volatility return	0.0419%	0.0385%	0.0425%	0.0280%	0.0330%	0.0318%	0.0126%
Dispersion discount	0.1052%	0.1044%	0.1029%	0.0905%	0.0827%	0.0612%	0.0126%
RR_H	-0.0633%	-0.0659%	-0.0604%	-0.0625%	-0.0497%	-0.0294%	0.0000%
	30 December 2011 – 27 December 2013						
Impact of not rebalancing	0.0000%	0.0015%	0.0067%	-0.0072%	-0.0064%	-0.0322%	-0.0375%
Variance reduction benefit	0.0402%	0.0402%	0.0401%	0.0403%	0.0403%	0.0389%	0.0379%
$DR_{E\&H}$	0.0402%	0.0417%	0.0468%	0.0331%	0.0339%	0.0067%	0.0004%
Volatility return	0.0397%	0.0412%	0.0463%	0.0325%	0.0334%	0.0062%	-0.0002%
Dispersion discount	0.0330%	0.0327%	0.0321%	0.0286%	0.0271%	0.0164%	-0.0002%
RR_H	0.0067%	0.0084%	0.0141%	0.0039%	0.0063%	-0.0102%	0.0000%
	27 December 2013 – 25 December 2015						
Impact of not rebalancing	0.0000%	0.0028%	0.0011%	0.0043%	-0.0126%	-0.0193%	-0.0230%
Variance reduction benefit	0.0366%	0.0366%	0.0365%	0.0368%	0.0368%	0.0364%	0.0358%
$DR_{E\&H}$	0.0366%	0.0394%	0.0377%	0.0410%	0.0242%	0.0170%	0.0128%
Volatility return	0.0378%	0.0406%	0.0389%	0.0423%	0.0253%	0.0181%	0.0138%
Dispersion discount	0.0346%	0.0345%	0.0337%	0.0308%	0.0253%	0.0216%	0.0138%
RR_H	0.0031%	0.0061%	0.0052%	0.0115%	0.0000%	-0.0035%	0.0000%

Table 6: Results based on weekly returns for the 8, 6, and 4 year periods

	Rebalancing frequency							
	every week	every 2 weeks	every 4 weeks	every 13 weeks	every 26 weeks	every 52 weeks	every 104 weeks	B&H
	4 January 2008 – 25 December 2015							
Impact of not rebalanc.	0.0000%	0.0002%	-0.0038%	-0.0154%	-0.0216%	-0.0308%	-0.0330%	-0.0485%
Variance red. benefit	0.0598%	0.0599%	0.0604%	0.0609%	0.0615%	0.0613%	0.0609%	0.0507%
$DR_{E\&H}$	0.0598%	0.0601%	0.0565%	0.0455%	0.0399%	0.0305%	0.0279%	0.0022%
Volatility return	0.0597%	0.0600%	0.0563%	0.0452%	0.0397%	0.0304%	0.0278%	0.0027%
Dispersion discount	0.0985%	0.0982%	0.0975%	0.0940%	0.0914%	0.0848%	0.0725%	0.0027%
RR_H	-0.0389%	-0.0383%	-0.0412%	-0.0488%	-0.0517%	-0.0544%	-0.0447%	0.0000%
	1 January 2010 – 25 December 2015							
Impact of not rebalanc.	0.0000%	0.0004%	0.0033%	-0.0029%	-0.0046%	-0.0123%	-0.0118%	-0.0454%
Variance red. benefit	0.0399%	0.0399%	0.0399%	0.0403%	0.0404%	0.0399%	0.0396%	0.0361%
$DR_{E\&H}$	0.0399%	0.0404%	0.0432%	0.0374%	0.0358%	0.0276%	0.0278%	-0.0093%
Volatility return	0.0400%	0.0405%	0.0433%	0.0375%	0.0359%	0.0277%	0.0279%	-0.0090%
Dispersion discount	0.0938%	0.0935%	0.0929%	0.0891%	0.0863%	0.0779%	0.0637%	-0.0090%
RR_H	-0.0537%	-0.0530%	-0.0496%	-0.0516%	-0.0504%	-0.0502%	-0.0359%	0.0000%
	30 December 2011 – 25 December 2015							
Impact of not rebalanc.	0.0000%	0.0022%	0.0040%	-0.0008%	-0.0085%	-0.0230%	-0.0242%	-0.0295%
Variance red. benefit	0.0384%	0.0384%	0.0384%	0.0386%	0.0387%	0.0382%	0.0376%	0.0363%
$DR_{E\&H}$	0.0384%	0.0406%	0.0424%	0.0378%	0.0302%	0.0151%	0.0134%	0.0068%
Volatility return	0.0388%	0.0410%	0.0428%	0.0382%	0.0305%	0.0154%	0.0138%	0.0072%
Dispersion discount	0.0546%	0.0544%	0.0538%	0.0512%	0.0480%	0.0429%	0.0344%	0.0072%
RR_H	-0.0158%	-0.0134%	-0.0110%	-0.0130%	-0.0175%	-0.0275%	-0.0207%	0.0000%

It can be seen that in the longer periods the rebalancing returns are also negative, whereas in the two-year time periods these returns are positive as well as negative. For this reason, no clear statement regarding the rebalancing return is possible for the shorter periods.

With respect to the diversification return, only slight differences can be found between $DR_{E\&H}$ and the volatility return. In most cases, they increase with a higher rebalancing frequency. While the variance reduction benefit hardly changes, the absolute value of the impact of not rebalancing tends to decrease with an increasing rebalancing frequency. As expected, not rebalancing has the highest impact on the B&H portfolio. In this portfolio, the diversification return is by far the lowest and approaches zero.

In order to find the reason for the negative rebalancing returns especially in the longer time periods, the empirical autocorrelations of the weekly returns of the individual stocks are considered in Table 7 for the total period.

Table 7: Autocorrelations of weekly returns for the total period

Stock	**Empirical autocorrelation for the period 6 January 2006 – 25 December 2015**			
	at lag 1	**at lag 2**	**at lag 3**	**at lag 4**
Allianz	-0.0878	0.0233	0.0743	-0.1004
BASF	-0.0747	0.0712	-0.0530	0.0197
Bayer	-0.0693	0.0056	-0.0578	-0.0203
BMW	-0.0313	-0.0130	-0.1029	-0.0079
Commerzbank	-0.0164	0.0224	0.0508	0.0323
Daimler	-0.0644	0.0449	-0.0680	0.0175
Deutsche Bank	-0.0781	0.0725	-0.0337	0.0187
E.ON	-0.1115	0.0419	0.0153	-0.0240
Henkel	-0.0227	0.0531	-0.0288	-0.0114
Linde	-0.0468	0.0013	-0.0041	-0.0900
Lufthansa	-0.0741	0.0520	-0.0093	0.0064
RWE	-0.0458	0.0739	-0.0074	0.0288
Siemens	-0.0666	0.0444	-0.0660	-0.0156
ThyssenKrupp	-0.0120	0.0780	0.0305	-0.0081
Volkswagen	0.0026	0.1279	-0.0041	0.0191

For a time lag of 1 week and 3 weeks, they are consistently negative for the entire time period with a few exceptions. The autocorrelations at lag 2 are almost all positive whereas at lag 4, 7 autocorrelations are positive and 8 are negative. Thus, these results are not consistent.

In general, negative rebalancing returns imply trending asset prices. To find out which stocks followed a certain trend over the total period, the proportions of the stocks at the beginning and at the end of the total period are considered. While the final weight of the Volkswagen stock in the B&H portfolio has more than doubled, the weights of several stocks have fallen below 2% (Commerzbank, Deutsche Bank, E.ON, and RWE). Hence, the weight concentration of the B&H portfolio has increased in comparison to the rebalanced portfolio. This is also shown by the correlations of the stock returns with the weekly rebalanced portfolio and the B&H portfolio. For example, the correlation of the Volkswagen stock with the B&H portfolio is much higher (0.698) than with the weekly rebalanced portfolio (0.606) while the correlation of the Commerzbank stock with the B&H portfolio (0.610) is markedly lower than with the weekly rebalanced portfolio (0.707).

For this reason, the above named five stocks are taken out of the portfolio. Thus, this step should improve the rebalancing return. However, the revised B&H

portfolio is still more weight concentrated than the rebalanced portfolio. But compared to the original B&H portfolio, the weight concentration is lower according to the normalized Herfindahl index H*(w). This index can be calculated as follows (Roncalli, 2014, pp. 126-127):

$$H^*(w) = \frac{n \times H(w) - 1}{n - 1} \tag{20}$$

where $H(w) = \sum_{i=1}^{n} w_i^2$ which is the Herfindahl index associated with w.

Tables 15 to 17 in the appendix show the results for this revised portfolio of the 10 stocks left. In all periods, the approximate diversification return according to Erb and Harvey (2006) tends to diminish with a lower rebalancing frequency because of the increasing absolute value of the impact of not rebalancing whereas again the variance reduction benefit is almost independent from the rebalancing frequency. Unlike the 15 stock-portfolio, the revised portfolio generates positive rebalancing returns for each rebalancing frequency with regard to the total time period and these returns rise with a higher rebalancing frequency. This meets the expectations because the selection of the stocks to take out of the portfolio has been based on the total period. In the other time periods (2, 4, 6, and 8 years), the rebalancing return is at least predominantly positive.

To determine whether rebalancing improves the risk adjusted performance, the return to risk ratio is calculated. This measure quantifies return per unit of risk where risk is defined by the standard deviation of the portfolio returns. Thus, this performance measure is based on total risk which is appropriate when the portfolio is sufficiently diversified so that it exhibits hardly any non-systematic risk (Culp and Mensink, 1999, p. 62).

$$\text{Return to risk ratio } = \frac{\bar{r}_{PF}}{\sigma_{PF}} \tag{21}$$

The results for the original and the revised portfolio are shown in Tables 8 and 9.

Table 8: Return to risk ratios for the original portfolio (15 stocks)

Time period	Rebalancing frequency							
	every week	every 2 weeks	every 4 weeks	every 13 weeks	every 26 weeks	every 52 weeks	every 104 weeks	B&H
6 Jan. 2006 – 25 Dec. 2015	5.08%	5.09%	5.02%	4.88%	4.82%	4.80%	5.02%	6.06%
6 Jan. 2006 – 4 Jan. 2008	19.63%	19.62%	19.53%	19.65%	19.60%	19.79%	19.54%	19.54%
4 Jan. 2008 – 1 Jan. 2010	-0.79%	-0.80%	-1.25%	-1.76%	-2.08%	-2.35%	-2.64%	-2.64%
1 Jan. 2010 – 30 Dec. 2011	0.32%	0.25%	0.39%	0.31%	0.65%	1.20%	1.99%	1.99%
30 Dec. 2011 – 27 Dec. 2013	18.47%	18.53%	18.73%	18.34%	18.43%	17.91%	18.31%	18.31%
27 Dec. 2013 – 25 Dec. 2015	0.75%	0.85%	0.82%	1.02%	0.61%	0.50%	0.62%	0.62%
4 Jan. 2008 – 25 Dec. 2015	2.96%	2.98%	2.90%	2.69%	2.61%	2.53%	2.78%	3.99%
1 Jan. 2010 – 25 Dec. 2015	5.33%	5.35%	5.46%	5.41%	5.46%	5.47%	5.94%	7.14%
30 Dec. 2011 – 25 Dec. 2015	8.83%	8.92%	9.00%	8.95%	8.81%	8.46%	8.71%	9.52%

Table 9: Return to risk ratios for the revised portfolio (10 stocks)

Time period	Rebalancing frequency							
	every week	every 2 weeks	every 4 weeks	every 13 weeks	every 26 weeks	every 52 weeks	every 104 weeks	B&H
6 Jan. 2006 – 25 Dec. 2015	7.62%	7.57%	7.49%	7.44%	7.39%	7.26%	7.17%	7.12%
6 Jan. 2006 – 4 Jan. 2008	17.99%	17.89%	17.78%	17.96%	17.89%	18.12%	17.09%	17.09%
4 Jan. 2008 – 1 Jan. 2010	0.36%	0.26%	0.01%	-0.33%	-0.55%	-0.80%	-0.92%	-0.92%
1 Jan. 2010 – 30 Dec. 2011	4.82%	4.77%	4.77%	4.73%	4.88%	4.85%	4.91%	4.91%
30 Dec. 2011 – 27 Dec. 2013	22.45%	22.46%	22.53%	22.38%	22.29%	22.28%	22.47%	22.47%
27 Dec. 2013 – 25 Dec. 2015	5.74%	5.74%	5.73%	5.81%	5.69%	5.23%	5.23%	5.23%
4 Jan. 2008 – 25 Dec. 2015	6.08%	6.03%	5.96%	5.85%	5.78%	5.58%	5.62%	5.79%
1 Jan. 2010 – 25 Dec. 2015	9.85%	9.83%	9.85%	9.84%	9.84%	9.63%	9.70%	9.90%
30 Dec. 2011 – 25 Dec. 2015	13.42%	13.42%	13.45%	13.45%	13.35%	13.02%	13.08%	13.45%

The results show that the return to risk ratios are very much depending on the time periods. But within a certain time period, there is not much evidence that rebalancing leads to better scores. In some cases it does, but in other cases it doesn’t. However, a rebalancing strategy seems to be more beneficial for the revised portfolio in terms of the return to risk ratio.

Finally the relationship between rebalancing and the diversification of a portfolio shall be explored. Choueifaty and Coignard (2008, p. 41) recommended the “diversification ratio” to measure the portfolio diversification which is defined as the ratio of the weighted average of assets’ volatilities divided by the portfolio volatility:

$$\text{Diversification ratio} = \frac{\sum_{i=1}^{n} w_i \times \sigma_i}{\sigma_{PF}} \tag{22}$$

Choueifaty, Froidure and Reynier (2013, p. 2) point out that “this measure embodies the very nature of diversification, whereby the volatility of a long-only portfolio of assets is less than or equal to the weighted sum of the assets’ volatilities.” In this paper, again, the average (weekly) weights are used in the numerator as weekly returns are used in this study, so that equation 22 changes to:

$$\text{Diversification ratio} = \frac{\sum_{i=1}^{n} \overline{w}_i \times \sigma_i}{\sigma_{PF}} \tag{23}$$

The results for the original and the revised portfolio are shown in Tables 10 and 11. These results are inconsistent. Thus, there is no evidence that a more frequent rebalancing increases the diversification ratio, i.e. the diversification of the portfolio. This result goes hand in hand with the considerations of Chambers and Zdanowicz (2014, p. 73) who point out that “rebalancing does not inherently keep a portfolio better diversified” although “diversification return advocates argue that rebalancing creates diversification return through maintaining better diversification than is obtained using a buy-and-hold strategy.” They conclude that “rebalancing can improve diversification in some cases and can increase idiosyncratic risk in other cases” (p. 74).

Table 10: Diversification ratios for the original portfolio (15 stocks)

Time period	Rebalancing frequency							
	every week	every 2 weeks	every 4 weeks	every 13 weeks	every 26 weeks	every 52 weeks	every 104 weeks	B&H
6 Jan. 2006 – 25 Dec. 2015	1.3303	1.3318	1.3361	1.3425	1.3478	1.3506	1.3602	1.3478
6 Jan. 2006 – 4 Jan. 2008	1.4832	1.4838	1.4827	1.4812	1.4815	1.4804	1.4862	1.4862
4 Jan. 2008 – 1 Jan. 2010	1.2903	1.2928	1.3005	1.3053	1.3143	1.3244	1.3218	1.3218
1 Jan. 2010 – 30 Dec. 2011	1.2445	1.2447	1.2453	1.2513	1.2538	1.2540	1.2562	1.2562
30 Dec. 2011 – 27 Dec. 2013	1.4578	1.4569	1.4544	1.4573	1.4567	1.4525	1.4427	1.4427
27 Dec. 2013 – 25 Dec. 2015	1.3289	1.3296	1.3288	1.3344	1.3392	1.3345	1.3296	1.3296
4 Jan. 2008 – 25 Dec. 2015	1.3124	1.3139	1.3186	1.3257	1.3313	1.3337	1.3410	1.2996
1 Jan. 2010 – 25 Dec. 2015	1.3258	1.3259	1.3257	1.3316	1.3349	1.3327	1.3309	1.3081
30 Dec. 2011 – 25 Dec. 2015	1.3896	1.3898	1.3883	1.3933	1.3967	1.3939	1.3884	1.3814

Table 11: Diversification ratios for the revised portfolio (10 stocks)

Time period	Rebalancing frequency							
	every week	every 2 weeks	every 4 weeks	every 13 weeks	every 26 weeks	every 52 weeks	every 104 weeks	B&H
6 Jan. 2006 – 25 Dec. 2015	1.2504	1.2499	1.2512	1.2568	1.2620	1.2610	1.2662	1.2536
6 Jan. 2006 – 4 Jan. 2008	1.4238	1.4247	1.4230	1.4196	1.4140	1.4117	1.4133	1.4133
4 Jan. 2008 – 1 Jan. 2010	1.2052	1.2040	1.2065	1.2143	1.2227	1.2257	1.2286	1.2286
1 Jan. 2010 – 30 Dec. 2011	1.1962	1.1959	1.1963	1.2006	1.2026	1.1982	1.1975	1.1975
30 Dec. 2011 – 27 Dec. 2013	1.3418	1.3416	1.3426	1.3411	1.3446	1.3446	1.3438	1.3438
27 Dec. 2013 – 25 Dec. 2015	1.2462	1.2465	1.2461	1.2510	1.2574	1.2459	1.2410	1.2410
4 Jan. 2008 – 25 Dec. 2015	1.2292	1.2286	1.2301	1.2367	1.2427	1.2418	1.2464	1.2304
1 Jan. 2010 – 25 Dec. 2015	1.2495	1.2494	1.2498	1.2535	1.2573	1.2519	1.2517	1.2431
30 Dec. 2011 – 25 Dec. 2015	1.2897	1.2898	1.2900	1.2923	1.2973	1.2915	1.2896	1.2871

It should be noted that the diversification ratios of the revised portfolio are lower than of the original portfolio. This can be attributed to the fact that the revised portfolio contains only 10 stocks compared to 15 stocks in the original portfolio. Because of the more “concentrated” weights, the revised portfolio is more poorly diversified and thus leads to lower diversification ratios (Choueifaty, Froidure and Reynier (2013, p. 3)).

5 Conclusion

In the literature doesn’t exist a consistent terminology regarding the diversification return. If a portfolio is rebalanced in every period, the diversification return according to Willenbrock (2011) equals the approximate diversification ratio according to Erb and Harvey (2006). However, in case of an equally weighted, unrebalanced portfolio, the diversification return consists of the two components “variance reduction benefit” and “impact of not rebalancing”. The latter can be described as the covariance drag. To find out if the rebalanced portfolio is superior to the B&H portfolio, the rebalancing return according to Hallerbach (2014) as the difference between the growth rates of the rebalanced portfolio and the B&H portfolio is more appropriate.

According to literature, negative rebalancing returns imply trending asset prices whereas rebalancing returns should generally be positive when asset prices are mean-reverting which is associated with negative autocorrelations of returns. However, it is shown with two simple examples that a negative autocorrelation of all asset returns in a portfolio does not necessarily lead to a positive rebalancing return and that conversely a positive autocorrelation not necessarily implies a negative rebalancing return.

Because of the inconclusive theoretical results of the rebalancing return, empirical tests of different portfolios and holding periods are essential to find out whether rebalancing is likely to be beneficial or not. The study for the German stock market based on an initially equally weighted portfolio of 15 selected stocks shows that especially for the total period considered, rebalancing returns turned out to be negative regardless of the rebalancing frequency. It should be noted that no transaction costs were taken into account in this study. So, a more frequent rebalancing would have even lowered the rebalancing return.

The empirical autocorrelations of the weekly returns of the individual stocks cannot provide an explanation for this result because for a time lag of 1 week and 3 weeks, they are consistently negative for the entire time period with a few exceptions. The autocorrelations at lag 2 are almost all positive whereas at lag 4, 7 autocorrelations are positive and 8 are negative. Thus, there are no consistent autocorrelation results. To find out which stocks followed a certain trend over the total period, the final proportions of the stocks in the B&H portfolio at the end of the total period are considered. As a consequence, five stocks are removed from the portfolio because their final weights (based on the total period) are either relatively high or relatively low. This step improves the rebalancing return significantly. The revised portfolio of the 10 stocks left is still quite concentrated but not as much as the original portfolio. It generates positive rebalancing returns for each rebalancing frequency with regard to the total time period and these returns rise with a higher rebalancing frequency.

With regard to all periods considered, the results of the original portfolio are not clear as the rebalancing returns in some cases turn out to be positive and in other cases to be negative. Hence, rebalancing obviously can be both profitable and adverse. However, in most of the time periods the rebalancing returns of the revised portfolio are mostly positive. Hence, for this portfolio, rebalancing seems to be a more reliable source of return than for the original portfolio.

With respect to the diversification return, only slight differences can be found between $DR_{E\&H}$ and the volatility return. In most cases, the diversification return increases with an increasing rebalancing frequency. While the variance reduction benefit hardly changes, the absolute value of the impact of not rebalancing tends to decrease with increasing rebalancing frequency. As expected, not rebalancing has the highest impact on the B&H portfolio. In this portfolio, the diversification return is by far the lowest and approaches zero. These results also apply to the revised portfolio which is the more mean-reverting portfolio.

Finally, the study shows no evidence that rebalancing generally leads to better risk adjusted performance although a rebalancing strategy seems to be more beneficial for the revised portfolio in terms of the return to risk ratio. Besides, it cannot be determined whether rebalancing generally causes better portfolio diversification measured by the diversification ratio. Thus, it cannot be said whether a portfolio is better or less diversified due to rebalancing.

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Appendix

Table 12: Example 2 – Rebalanced portfolio

t	P_A	r_A	number of stocks A	w_A	P_B	r_B	number of stocks B	w_B	P_C	r_C	number of stocks C	w_C	Value of PF
0	50		6,666.7	33.33%	15		22,222.2	33.33%	20		16,666.7	33.33%	1,000,000
1	49	-2.00%	6,266.1	33.33%	14	-6.67%	21,931.2	33.33%	17	-15.00%	18,061.0	33.33%	921,111
2	48	-2.04%	6,451.6	33.33%	13	-7.14%	23,821.4	33.33%	19	11.76%	16,298.9	33.33%	929,036
3	49	2.08%	6,636.8	33.33%	14	7.69%	23,228.8	33.33%	20	5.26%	16,260.1	33.33%	975,608
4	50	2.04%	6,920.0	33.33%	15	7.14%	23,066.5	33.33%	22	10.00%	15,727.2	33.33%	1,037,994
5	52	4.00%	6,695.5	33.33%	14	-6.67%	24,868.9	33.33%	23	4.55%	15,137.6	33.33%	1,044,494
6	53	1.92%	6,550.1	33.33%	13	-7.14%	26,704.1	33.33%	24	4.35%	14,464.7	33.33%	1,041,458
7	54	1.89%	6,139.5	33.33%	11	-15.38%	30,139.4	33.33%	24	0.00%	13,813.9	33.33%	994,600
8	51	-5.56%	6,396.7	33.33%	12	9.09%	27,185.9	33.33%	22	-8.33%	14,828.7	33.33%	978,693
9	50	-1.96%	6,465.5	33.33%	13	8.33%	24,867.3	33.33%	20	-9.09%	16,163.8	33.33%	969,825
10	47	-6.00%	7,622.4	33.33%	18	38.46%	19,903.1	33.33%	20	0.00%	17,912.8	33.33%	1,074,765

The empirical autocorrelations at lag 1 are 0.3420 (stock A), 0.1996 (stock B), and 0.1063 (stock C). Table 13 shows the data for a B&H portfolio. The results for example 2 are presented in Table 14.

Table 13: Example 2 – B&H portfolio

t	P_A	r_A	number of stocks A	w_A	P_B	r_B	number of stocks B	w_B	P_C	r_C	number of stocks C	w_C	Value of PF
0	50		6,666.7	33.33%	15		22,222.2	33.33%	20		16,666.7	33.33%	1,000,000
1	49	-2.00%	6,666.7	35.46%	14	-6.67%	22,222.2	33.78%	17	-15.00%	16,666.7	30.76%	921,111
2	48	-2.04%	6,666.7	34.57%	13	-7.14%	22,222.2	31.21%	19	11.76%	16,666.7	34.21%	925,556
3	49	2.08%	6,666.7	33.64%	14	7.69%	22,222.2	32.04%	20	5.26%	16,666.7	34.32%	971,111
4	50	2.04%	6,666.7	32.26%	15	7.14%	22,222.2	32.26%	22	10.00%	16,666.7	35.48%	1,033,333
5	52	4.00%	6,666.7	33.30%	14	-6.67%	22,222.2	29.88%	23	4.55%	16,666.7	36.82%	1,041,111
6	53	1.92%	6,666.7	33.90%	13	-7.14%	22,222.2	27.72%	24	4.35%	16,666.7	38.38%	1,042,222
7	54	1.89%	6,666.7	35.84%	11	-15.38%	22,222.2	24.34%	24	0.00%	16,666.7	39.82%	1,004,444
8	51	-5.56%	6,666.7	34.93%	12	9.09%	22,222.2	27.40%	22	-8.33%	16,666.7	37.67%	973,333
9	50	-1.96%	6,666.7	34.88%	13	8.33%	22,222.2	30.23%	20	-9.09%	16,666.7	34.88%	955,556
10	47	-6.00%	6,666.7	29.94%	18	38.46%	22,222.2	38.22%	20	0.00%	16,666.7	31.85%	1,046,667

Table 14: Example 2 – Results

	Rebalanced Portfolio	B&H Portfolio
Arithmetic average return of the portfolio: $\bar{r}_{PF}$	0.8530%	0.5769%
Geometric average return of the portfolio: $\bar{r}^g_{PF}$	0.7236%	0.4571%
Variance of portfolio returns: σ^2_{PF}	0.2629%	0.2420%
Weighted average of arithmetic average asset returns: $\sum_{i=1}^{n} \bar{w}_i \times \bar{r}_i$	0.8530%	0.7696%
Weighted average of geometric average asset returns: $\sum_{i=1}^{n} \bar{w}_i \times \bar{r}^g_i$	0.4077%	0.3450%
Impact of not rebalancing: $\bar{r}_{PF} - \sum_{i=1}^{n} \bar{w}_i \times \bar{r}_i$	0.0000%	-0.1927%
Variance reduction benefit: $\frac{1}{2} \times \sum_{i=1}^{n} \left(\bar{w}_i \times \sigma^2_i\right) - \frac{1}{2} \times \sigma^2_{PF}$	0.3488%	0.3347%
Approx. diversification return according to Erb and Harvey: $DR_{E\&H}$	0.3488%	0.1420%
Volatility return: $\bar{r}^g_{PF} - \sum_{i=1}^{n} \bar{w}_i \times \bar{r}^g_i$	0.3159%	0.1122%
Dispersion discount: $\bar{r}^g_{B\&H} - \sum_{i=1}^{n} \bar{w}_i \times \bar{r}^g_i$	0.0494%	0.1122%
Rebalancing return according to Hallerbach: $RR_H = \bar{r}^g_{PF} - \bar{r}^g_{B\&H}$	0.2665%	0.0000%

Table 15: Results for the revised portfolio (10 stocks) based on weekly returns, Jan. 2006 – Dec. 2015

	Rebalancing frequency							
	every week	every 2 weeks	every 4 weeks	every 13 weeks	every 26 weeks	every 52 weeks	every 104 weeks	B&H
$\bar{r}_{PF}$	0.2653%	0.2634%	0.2605%	0.2574%	0.2546%	0.2505%	0.2465%	0.2425%
$\bar{r}_{PF}^{g}$	0.2040%	0.2020%	0.1992%	0.1966%	0.1943%	0.1902%	0.1865%	0.1836%
σ_{PF}^{2}	0.1211%	0.1212%	0.1209%	0.1198%	0.1188%	0.1190%	0.1181%	0.1162%
$\sum_{i=1}^{n} \overline{w}_i \times \bar{r}_i$	0.2653%	0.2653%	0.2654%	0.2656%	0.2658%	0.2666%	0.2666%	0.2732%
$\sum_{i=1}^{n} \overline{w}_i \times \bar{r}_i^{g}$	0.1678%	0.1678%	0.1679%	0.1681%	0.1684%	0.1692%	0.1690%	0.1790%
Impact of not rebalanc.	0.0000%	-0.0019%	-0.0049%	-0.0082%	-0.0112%	-0.0161%	-0.0201%	-0.0306%
Variance red. benefit	0.0364%	0.0363%	0.0365%	0.0370%	0.0375%	0.0374%	0.0379%	0.0356%
$DR_{E\&H}$	0.0364%	0.0345%	0.0316%	0.0288%	0.0263%	0.0213%	0.0178%	0.0049%
Volatility return	0.0361%	0.0342%	0.0313%	0.0285%	0.0260%	0.0210%	0.0175%	0.0046%
Dispersion discount	0.0157%	0.0157%	0.0157%	0.0154%	0.0152%	0.0144%	0.0145%	0.0046%
RR_{H}	0.0204%	0.0184%	0.0156%	0.0130%	0.0108%	0.0066%	0.0029%	0.0000%

Table 16: Results for the revised portfolio (10 stocks) for the 2 year periods

	Rebalancing frequency						
	every week	every 2 weeks	every 4 weeks	every 13 weeks	every 26 weeks	every 52 weeks	every 104 weeks (B&H)
	6 January 2006 – 4 January 2008						
Impact of not rebalancing	0.0000%	-0.0025%	-0.0051%	-0.0015%	-0.0021%	-0.0015%	-0.0328%
Variance reduction benefit	0.0305%	0.0306%	0.0305%	0.0304%	0.0303%	0.0307%	0.0313%
$DR_{E\&H}$	0.0305%	0.0281%	0.0254%	0.0289%	0.0282%	0.0292%	-0.0015%
Volatility return	0.0302%	0.0278%	0.0251%	0.0286%	0.0279%	0.0289%	-0.0018%
Dispersion discount	0.0173%	0.0172%	0.0167%	0.0150%	0.0139%	0.0064%	-0.0018%
RR_H	0.0130%	0.0106%	0.0084%	0.0137%	0.0140%	0.0225%	0.0000%
	4 January 2008 – 1 January 2010						
Impact of not rebalancing	0.0000%	-0.0052%	-0.0193%	-0.0381%	-0.0492%	-0.0651%	-0.0727%
Variance reduction benefit	0.0708%	0.0705%	0.0711%	0.0727%	0.0745%	0.0751%	0.0746%
$DR_{E\&H}$	0.0708%	0.0653%	0.0517%	0.0346%	0.0253%	0.0100%	0.0018%
Volatility return	0.0701%	0.0646%	0.0510%	0.0336%	0.0242%	0.0090%	0.0008%
Dispersion discount	0.0114%	0.0114%	0.0109%	0.0093%	0.0093%	0.0063%	0.0008%
RR_H	0.0587%	0.0532%	0.0401%	0.0243%	0.0149%	0.0027%	0.0000%
	1 January 2010 – 30 December 2011						
Impact of not rebalancing	0.0000%	-0.0019%	-0.0025%	-0.0077%	-0.0070%	-0.0153%	-0.0278%
Variance reduction benefit	0.0299%	0.0299%	0.0299%	0.0303%	0.0305%	0.0300%	0.0299%
$DR_{E\&H}$	0.0299%	0.0280%	0.0274%	0.0227%	0.0235%	0.0147%	0.0021%
Volatility return	0.0295%	0.0276%	0.0270%	0.0223%	0.0231%	0.0142%	0.0017%
Dispersion discount	0.0326%	0.0325%	0.0321%	0.0290%	0.0247%	0.0167%	0.0017%
RR_H	-0.0030%	-0.0048%	-0.0051%	-0.0067%	-0.0016%	-0.0024%	0.0000%
	30 December 2011 – 27 December 2013						
Impact of not rebalancing	0.0000%	0.0001%	0.0012%	-0.0030%	-0.0077%	-0.0218%	-0.0272%
Variance reduction benefit	0.0264%	0.0264%	0.0265%	0.0264%	0.0266%	0.0261%	0.0258%
$DR_{E\&H}$	0.0264%	0.0265%	0.0277%	0.0234%	0.0189%	0.0043%	-0.0013%
Volatility return	0.0260%	0.0261%	0.0272%	0.0230%	0.0185%	0.0039%	-0.0018%
Dispersion discount	0.0218%	0.0217%	0.0213%	0.0196%	0.0187%	0.0081%	-0.0018%
RR_H	0.0042%	0.0044%	0.0059%	0.0034%	-0.0003%	-0.0041%	0.0000%
	27 December 2013 – 25 December 2015						
Impact of not rebalancing	0.0000%	-0.0003%	-0.0006%	0.0003%	-0.0044%	-0.0172%	-0.0203%
Variance reduction benefit	0.0236%	0.0236%	0.0236%	0.0239%	0.0243%	0.0236%	0.0233%
$DR_{E\&H}$	0.0236%	0.0233%	0.0230%	0.0243%	0.0199%	0.0064%	0.0029%
Volatility return	0.0238%	0.0235%	0.0232%	0.0244%	0.0201%	0.0065%	0.0031%
Dispersion discount	0.0095%	0.0095%	0.0093%	0.0086%	0.0081%	0.0068%	0.0031%
RR_H	0.0143%	0.0140%	0.0139%	0.0158%	0.0120%	-0.0003%	0.0000%

Table 17: Results for the revised portfolio (10 stocks) for the 8, 6, and 4 year periods

	Rebalancing frequency							
	every week	every 2 weeks	every 4 weeks	every 13 weeks	every 26 weeks	every 52 weeks	every 104 weeks	B&H
	4 January 2008 – 25 December 2015							
Impact of not rebalanc.	0.0000%	-0.0018%	-0.0050%	-0.0105%	-0.0148%	-0.0239%	-0.0249%	-0.0346%
Variance red. benefit	0.0378%	0.0377%	0.0379%	0.0386%	0.0392%	0.0391%	0.0395%	0.0360%
$DR_{E\&H}$	0.0378%	0.0360%	0.0329%	0.0281%	0.0244%	0.0151%	0.0146%	0.0014%
Volatility return	0.0375%	0.0357%	0.0326%	0.0277%	0.0241%	0.0148%	0.0142%	0.0011%
Dispersion discount	0.0243%	0.0243%	0.0243%	0.0239%	0.0231%	0.0212%	0.0196%	0.0011%
RR_H	0.0132%	0.0114%	0.0084%	0.0038%	0.0009%	-0.0064%	-0.0054%	0.0000%
	1 January 2010 – 25 December 2015							
Impact of not rebalanc.	0.0000%	-0.0006%	-0.0004%	-0.0022%	-0.0042%	-0.0125%	-0.0128%	-0.0269%
Variance red. benefit	0.0268%	0.0268%	0.0268%	0.0271%	0.0273%	0.0268%	0.0268%	0.0254%
$DR_{E\&H}$	0.0268%	0.0261%	0.0264%	0.0249%	0.0231%	0.0143%	0.0141%	-0.0014%
Volatility return	0.0266%	0.0259%	0.0262%	0.0247%	0.0229%	0.0141%	0.0138%	-0.0017%
Dispersion discount	0.0258%	0.0258%	0.0257%	0.0249%	0.0239%	0.0208%	0.0182%	-0.0017%
RR_H	0.0007%	0.0001%	0.0005%	-0.0002%	-0.0010%	-0.0067%	-0.0044%	0.0000%
	30 December 2011 – 25 December 2015							
Impact of not rebalanc.	0.0000%	-0.0001%	0.0004%	-0.0011%	-0.0061%	-0.0171%	-0.0188%	-0.0253%
Variance red. benefit	0.0250%	0.0251%	0.0251%	0.0252%	0.0255%	0.0250%	0.0249%	0.0244%
$DR_{E\&H}$	0.0250%	0.0250%	0.0254%	0.0241%	0.0194%	0.0079%	0.0061%	-0.0009%
Volatility return	0.0249%	0.0249%	0.0253%	0.0240%	0.0192%	0.0078%	0.0060%	-0.0010%
Dispersion discount	0.0243%	0.0242%	0.0240%	0.0230%	0.0220%	0.0186%	0.0146%	-0.0010%
RR_H	0.0007%	0.0006%	0.0013%	0.0010%	-0.0027%	-0.0108%	-0.0086%	0.0000%