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An investigation of MHD Flows of Nanofluid over an oscillating surface with Joule Heating

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Abstract

The present study aims at investigating unsteady hydromagnetic flow of viscous incompressible, conducting nanofluid over an infinite oscillating surface with Joule heating. The governing equations involved are transformed into dimensionless form using appropriate non-dimensional variables. The resulting dimensionless momentum equation is solved analytically by using the method of separation of variables. The Joule heating terms in the energy equation are expressed explicitly using this solution. With the help of the superposition principle, a complementary function is creatively synthesized and a particular integral is obtained by the method of undetermined coefficients. The complete solution is obtained by summing the two. The effects of magnetic field strength, oscillation frequency, nanoparticle concentration, Prandtl number and the Eckert number are illustrated graphically.

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Nomenclature

$\mathbf{V}$	velocity	Pr	Prandtl number
u	velocity component in the x-direction	Ec	Eckert number
MHD	magnetohydrodynamics	T	temperature
$\mathbf{B}$	magnetic field	θ	dimensionless temperature
ρ	density	υ	kinematic viscosity
$\mathbf{E}$	electrical field	ϕ	nanopaprticle concentration
c_p	specific heat capacity at constant pressure	k	thermal conductivity
$\mathbf{J}$	electrical current	$(\ )^*$	dimensionless variables
ω	oscillation frequency	∞	infinity
$(\ )_w$	surface/wall conditions	$(\ )_\infty$	free stream conditions

1 Introduction

The movement of electrically conducting hydromagnetic flow results to flow of electrical current, $\mathbf{J}$. This current results to a force $\mathbf{F} = \mathbf{J} \times \mathbf{B}$ called the Lorentz force and Joule heating expressed mathematically as $\mathbf{J} \cdot \mathbf{E}$. The heating effect is a manifestation of energy dissipation inform of heat due to opposition of current. This study aims at analyzing the amount of Joule heating which occurs as a result of nanofluid movement triggered by the oscillation of surface and the role played by nanofluid in removing the heat generated due to Joule heating. Studies on MHD flows have received a lot of attention by researchers due to their many applications in areas like astrophysics, geophysics, Plasma confinement, MHD pumps etc.. Such flows over oscillating surfaces have been done. However, the Joule heating which occurs due to the oscillation of the surface has not been given the seriousness it deserves. Ostrach [1] analyzed aerodynamic heating of an oscillating surface. It was demonstrated that aerodynamic heating increases with an increase on the amplitude of oscillation and that oscillation considerably increases mechanical dissipation. Soundalgelar [2] studied free convection effects on MHD flow past an infinite vertical oscillating plate with constant heat flux. It was found that velocity decreases with increasing frequency of oscillation but temperature increases with an increase in frequency of oscillation. Muthucumaraswamy and Meena [3] theoretically studied chemical reaction effects on vertical oscillating plate with variable temperature. It was found that chemical reaction has significant effect on heat transfer. An investigation on combined heat and mass transfer on MHD free convection flow past an oscillating plate embedded in porous medium was done by Chaudhanry and Jain [4]. Here, it was discovered that cooling the oscillating plate leads to a decrease in velocity and vice versa. Kishore et al. [5] used finite difference method to investigate the effect of thermal radiation and viscous dissipation on MHD heat on mass diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface conditions. This study showed that plate oscillation, variable mass diffusion, radiation, viscous dissipation and porous medium affect the flow pattern significantly. Ahmed and Kalita [6] very recently, analytically and numerically studied MHD flow over an infinite oscillating vertical surface bounded by porous medium in presence of chemical reaction. In this study, it was illustrated that results by Laplace technique compare very well with those by Crank-Nicolson numerical technique. It was also confirmed that an increase in chemical reaction parameter leads to a decrease in fluid velocity and concentration profiles. Very recently Abid et al. [7] investigated MHD flow of a casson fluid past an oscillating vertical plate with Newtonian heating. It was proved that flow separation can be controlled by increasing the value of Casson fluid parameter as well as increasing the Prandtl number. It was also shown that velocity decreases as Casson parameter increases and thermal boundary layer thickness increases with increasing Newtonian heating parameter. Another very recent study on MHD past a vertical oscillating plate with radiation and chemical reaction in a porous medium was done by Rudra and Nhaben [8]. It was discovered that temperature decreases with chemical parameter. This study also showed that concentration decreases as Schmidt and radiation parameters increases. It was found that velocity increases with decreasing magnetic parameter and chemical reaction. The science of nanofluid and technology was introduced by Nobel prizing winning physicist Richard in 1959. Since then much research work has been done to this new type of material because of its high rated properties and behavior associated with heat transfer as reported by Choi [9]. Xuan and Roetfel [10] investigated conceptions for heat transfer correlation of nanofluids. It was proved that thermal dispersions plays a key role in increasing heat transfer in the fluid and in the wall. Li and Xuan [11] examined the characteristics of Cu-water nanofluid. Here it was demonstrated that nanofluid thermal conductivity is superior to that of pure fluid. Raiskinmaki et al. [12] investigation showed that the important nanoparticles settles slowly than the larger particles and this prevents channel clogging and any other destructive effect. Wen and Ding [13] experimentally investigated convective heat transfer of nanofluids at the entrance region under laminar flow conditions. By measuring, it was found that heat transfer coefficient is greatest at the entry and that an increase in particle concentration enhances it. Thermally developing electroosmotic (with Joule heating) transport of nanofluids in a micro channels was investigated by Chakraborty and Roy [14]. It was demonstrated that pressure gradient as well as Joule heating affects velocity profile and temperature profile of the fluid significantly.Very recently, Ghasemi and Aminossadati [15] investigated mixed convection in a lid driven triangular enclosure filled with nanofluids. It was shown that addition of AL_2O_3 nano particles enhances heat transfer rate for every value of Richardson number and for each sliding wall motion. Chad [16] investigated the thermal instability of rotating nanofluid. The effects of the Taylor number, concentration Rayleigh number, Prandtl number and Lewis number were investigated for stability purposes. A Study on heat exchangers was done by Mehta [17]. This study gave a detailed description of heat exchangers, properties of nanofluids together with their preparation. A very important background of nanonfluids is also given. Anindya and Ashok [18], numerically analyzed heat transfer characteristics of combined electroosmotic (with Joule heating) and pressure-driven fully developed flow of power law nano-fluid in microchannels. Through this study it was observed that Nusset number decreases with decreasing viscosity ratio and increasing permittivity ratio but it increases with increasing resistivity ratio. Hady [19] studied the flow and heat transfer characteristics of a viscous nanofluid over a non linearly stretching sheet in the presence of thermal radiation. In this study it was found that an increase in solid volume and Eckert number yields an increment in the nanofluids temperature. In addition, an increase in the thermal radiation parameter and the non linear stretching sheet parameter yields a decrease in the nanofluids temperature. Very recently Mohammed [20] studied magnetohydrodynamic free convention of nanofluid over a vertical flat plate taking into account Newtonian heating boundary condition. It was found that dimensionless velocity and temperature distributions increase with the increase of Newtonian heating parameter. Ferdows and Khan [20] studied MHD boundary layer flow of a nanofluid over an exponentially stretching sheet. It was found that momentum, thermal and concentration boundary layer thickness increase as the viscous ratio parameter increases. On the other hand skin-friction coefficient, surface heat and mass transfer rate decreases. It was also observed that an increase in Eckert number results to increase in momentum and thermal boundary layer. Bakr and Raizah [21] investigated analytically MHD mixed convection flow of a viscous dissipating micropolar nanofluids in a bounding layer slip flow with Joule heating. The obtained results showed that physical parameters such as thermal Grashof number influence velocity, temperature and the concentration profiles. This literature review shows that, studies on MHD flow of nanofluid over oscillating surface have not been done yet. In this study we shall take the nanofluid as Copper water nanofluid. The properties this type of nanofluid are found in Anjali and Julie [22].

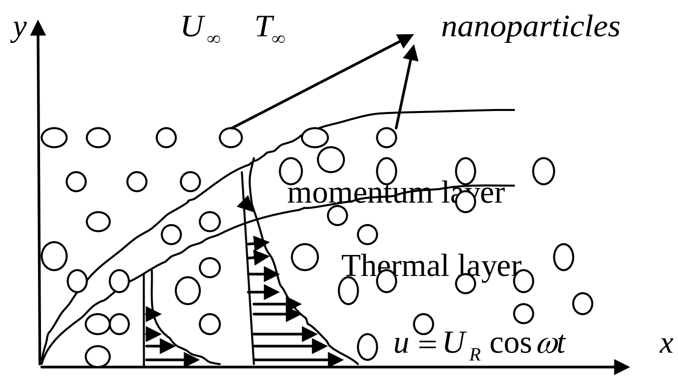

Figure 1: Physical configuration.

2 Mathematical formulation of the problem

We shall consider unsteady, forced convective MHD flow of incompressible and electrically conducting viscous nanofluid taking place over a doubly infinite surface oscillating in the x direction at a velocity $u = U_R \cos\omega t$ as shown in figure 1. At $t = 0$ the surface and the fluid are at the same temperature T_∞ everywhere. Also, a magnetic field of uniform strength is applied perpendicularly to the surface. A combination of the Navier-Stoke's (N-S) equations of fluid dynamics and Maxwell's equations of electromagnetism gives the general governing equations of this flow. Since the surface is doubly infinite, the velocity and the temperature of the fluid can be visualized in the tyu and tyT $3D$ geometries respectively. Since there is no motion in the y direction $v = 0$.

Now
$$\mathbf{J} = \sigma\mathbf{E}$$
$$= \sigma(\mathbf{V} \times \mathbf{B})$$

$$= \sigma \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u & 0 & 0 \\ 0 & B & 0 \end{vmatrix}$$

$$= \sigma Bu\mathbf{k} .$$

$$\text{The Lorentz force} = \mathbf{J} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \sigma Bu \\ 0 & B & 0 \end{vmatrix} = -\sigma uB^2\mathbf{i} .$$

And the Joule heating term is expressed as;

$$\mathbf{J} \cdot \mathbf{E} = (\sigma uB,0,0) \cdot (uB,0,0) = \sigma u^2 B^2 .$$

Since, the surface is double infinity, all gradients with respect to x and z of velocity components must varnish.

In view of the above discussion the governing equations which describe the flow become

Momentum

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_z{}^2 u}{\rho} \quad . \tag{1}$$

Energy

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_z{}^2 u^2}{\rho c_p} . \tag{2}$$

Subject to:

$$\begin{aligned} &u = 0, \quad v = 0, \quad T = T_\infty, \quad \text{for all} \quad y, \quad t \le 0 \\ &u = U_R \cos \omega t, \quad T = T_w \quad \text{at} \quad y = 0, \quad t > 0 \\ &u = 0, \quad T \to T_{\infty}, \quad \text{as} \quad y \to \infty, \quad t > 0 \end{aligned} \tag{3}$$

Here ,

$\frac{\sigma B^2 u}{\rho}$ is the MHD term and $\frac{\sigma B^2 u^2}{\rho c_p}$ represents Joule heating.

3 Solution of the problem

To seek the solution the following dimensionless variables are introduced:

$$t^* = \frac{t}{t_R}, \quad y^* = \frac{y}{L_R}, \quad u^* = \frac{u}{U_R},$$

$$\Pr = \frac{\mu c_p}{k}, \quad M = \frac{\sigma B^2 \upsilon}{\rho U_R^2}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \Delta T = T_w - T_\infty,$$

$$\omega^* = \omega t_R, \quad U_R = (\upsilon \Delta T)^{1/2}, \quad L_R = \upsilon^{1/2} \Delta T^{-1/2}, \tag{4}$$

$$t_R = (\Delta T)^{-1}, \quad Ec = \frac{U_R^2}{c_p \Delta T}.$$

In view of this variables equation (1) becomes

$$\frac{U_R}{t_R}\frac{\partial u^*}{\partial t^*} = \upsilon \frac{U_R}{L_R^{\,2}}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B^2 u^* U_R}{\rho}. \tag{5}$$

Multiply both sides of equation (5) by $\dfrac{\upsilon}{U_R^{\,3}}$ to get

$$\frac{\upsilon}{U_R^{\,2} t_R}\frac{\partial u^*}{\partial t^*} = \frac{\upsilon^2}{U_R^{\,2} L_R^{\,2}}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B^2 \upsilon u^*}{\rho U_R^{\,2}} \tag{6}$$

By making use of the dimesionless variables as in (4), (6) becomes

$$\frac{\partial u^*}{\partial t^*} = \frac{\partial^2 u^*}{\partial y^{*2}} - M u^* \tag{7}$$

Equation (7) is the momentum equation in dimensionless form.

In view of equation(4) equation (2) becomes

$$\left(\frac{T_w - T_\infty}{t_R}\right)\frac{\partial \theta}{\partial t^*} = \frac{k}{\rho c_p}\frac{T_w - T_\infty}{L_R^2}\frac{\partial^2 \theta}{\partial y^{*2}} + \frac{\sigma B^2 U_R^{\,2} u^{*2}}{\rho c_p} \tag{8}$$

Dividing both sides by $\dfrac{T_w - T_\infty}{t_R}$ yields

$$\frac{\partial\theta}{\partial t^*}=\frac{k}{\rho c_p}\frac{t_R}{L_R^2}\frac{\partial^2\theta}{\partial y^{*2}}+\frac{\sigma B^2 U_R{}^2 u^{*2} t_R}{\rho c_p\left(T_w-T_\infty\right)} \tag{9}$$

Putting the relevant dimensionless variables in (9) gives

$$\frac{\partial\theta}{\partial t^*}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^{*2}}+EcMu^{*2}. \tag{10}$$

EcM accounts for Joule heating.

Equation (10) is the energy equation in dimensionless form.

The ICS together with the BCS in dimensionless form are:

$$\begin{aligned}&u^*=0,\quad \theta=0,\quad \text{for all}\quad y^*\ ,\qquad t^*\le 0\\ &u^*=\cos\omega^* t^*,\quad \theta=1\quad \text{at}\quad y^*=0\\ &u^*=0,\quad \theta=0,\qquad \text{as}\quad y^*\to\infty,\quad t^*>0\end{aligned} \tag{11}$$

Let the solution of the momentum equation be

$u^*\left(y^*,t^*\right)=Y\left(y^*\right)T\left(t^*\right)$ or simply $u^*=YT$.

$$\frac{dT}{Tdt^*}+M^*=\frac{1}{Y}\frac{d^2Y}{dy^{*2}}=constant. \tag{12}$$

Since the LHS is a function of independent variable t^* and RHS is a function of independent variable y^*, and the two cannot be equal to each other unless both reduce to a constant value. Let the constant be k^2 so that

$\frac{dT}{Tdt^*}+M^*=k^2$ and $\frac{1}{Y}\frac{d^2Y}{dy^{*2}}=k^2$. The solutions are

$T=A_1\exp\left(k^2-M^*\right)t^*$ and $Y=A_2\exp\left(ky^*\right)+A_3\exp\left(-ky^*\right)$. It follows that

$$u^*\left(y^*,t^*\right)=\left(A\exp\left(ky^*\right)+B\exp\left(-ky^*\right)\right)\exp\left(k^2-M^*\right)t^*. \tag{13}$$

Now since $u^*\left(\infty,t^*\right)=0$, we must have $A=0$. The solution reduces to

$$u^*\left(y^*,t^*\right)=B\exp(-ky^*)\exp\left(k^2-M^*\right)t^*. \tag{14}$$

Applying the boundary condition $u(0,t^*) = \cos\omega^* t^*$ we have

$\mathrm{Re}\left(\exp\left(i\omega^* t^*\right)\right) = B\exp(-ky^*)\exp\left(k^2 - M^*\right)t^*$ meaning that

$B = 1$ and $k^2 - M^* = i\omega^*$ or $k = \sqrt{M^* + i\omega^*}$.

The unique solution becomes,

$u\left(y^*, t^*\right) = \exp(-ky^*)\exp\left(k^2 - M^*\right)t^*$ or

$$u^*\left(y^*, t^*\right) = \mathrm{Re}\left[\exp\left(-y^*\sqrt{M^* + i\omega^*} + i\omega^* t^*\right)\right] \tag{15}$$

or
$$u^*\left(y^*, t^*\right) = \mathrm{Re}\left[\exp\left(-y^*\left(M^{*^2} + \omega^{*2}\right)^{\frac{1}{4}}\cos\frac{1}{2}\left(\tan^{-1}\left(\frac{\omega^*}{M^*}\right)\right) + i\left(\omega^* t^* - y^*\left(M^{*^2} + \omega^{*2}\right)^{\frac{1}{4}}\sin\frac{1}{2}\left(\tan^{-1}\left(\frac{\omega^*}{M^*}\right)\right)\right)\right)\right]$$

or

$$u^*\left(y^*, t^*\right) = \mathrm{Re}\left[\exp\left(-y^*\left(M^{*^2} + \omega^{*2}\right)^{\frac{1}{4}}\cos\frac{1}{2}\left(\tan^{-1}\left(\frac{\omega^*}{M^*}\right)\right)\right)\times \times\exp i\left(\omega^* t^* - y^*\left(M^{*^2} + \omega^{*2}\right)^{\frac{1}{4}}\sin\frac{1}{2}\left(\tan^{-1}\left(\frac{\omega^*}{M^*}\right)\right)\right)\right]$$

or

$$u\left(y^*, t^*\right) = \exp\left(-y^*\left(M^{*^2} + \omega^{*2}\right)^{\frac{1}{4}}\cos\frac{1}{2}\left(\tan^{-1}\left(\frac{\omega^*}{M^*}\right)\right)\right)\times \cos\left(\omega^* t^* - y^*\left(M^{*^2} + \omega^{*2}\right)^{\frac{1}{4}}\sin\frac{1}{2}\left(\tan^{-1}\left(\frac{\omega^*}{M^*}\right)\right)\right) \tag{16}$$

Replacing u^{*2} and in equation (10) in view of (16) we have,

$$\frac{\partial \theta}{\partial t^*} - \frac{1}{(\Pr)_{nf}} \frac{\partial^2 \theta'}{\partial y^{*2}} = EcM^* \left(\exp\left(-2y^* \left(M^{*^2} + \omega^{*2} \right)^{\frac{1}{4}} \cos \frac{1}{2} \left(\tan^{-1} \left(\frac{\omega^*}{M^*} \right) \right) \right) \right) \times$$

$$\cos^2 \left(\omega t^* - y^* \left(M^{*^2} + \omega^{*2} \right)^{\frac{1}{4}} \sin \frac{1}{2} \left(\tan^{-1} \left(\frac{\omega^*}{M^*} \right) \right) \right) \quad (17)$$

equation (17) may be written in compact form as

$$\frac{\partial \theta}{\partial t} - \frac{1}{(\Pr)_{nf}} \frac{\partial^2 \theta}{\partial y^{*2}} = \frac{EcM^*}{2} \exp\left(-2ay^*\right) + \frac{Ec\, M^*}{2} \exp\left(-2ay^*\right) \cos\left(2\omega^* t^* - 2by^*\right) -$$

$$abE_c \exp\left(-2ay^*\right) \sin\left(2\omega^* t^* - 2by^*\right). \quad (18)$$

Where $a = \left(M^{*^2} + \omega^{*2}\right)^{\frac{1}{4}} \cos \frac{1}{2} \left(\tan^{-1} \left(\frac{\omega^*}{M^*} \right) \right)$,

$$b = \left(M^{*^2} + \omega^{*2}\right)^{\frac{1}{4}} \sin \frac{1}{2} \left(\tan^{-1} \left(\frac{\omega^*}{M^*} \right) \right),$$

Let

$$f(y^*, t^*) = \frac{Ec\, M^*}{2} \exp\left(-2ay^*\right) + \frac{Ec\, M^*}{2} \exp\left(-2ay^*\right) \cos\left(2\omega^* t - 2by^*\right)$$

$$- abEc \exp\left(-2ay^*\right) \sin\left(2\omega^* t - 2by^*\right)$$

So that

$$\frac{\partial \theta}{\partial t^*} - \frac{1}{(\Pr)_{nf}} \frac{\partial^2 \theta}{\partial y^{*2}} = f(y^*, t^*). \quad (19)$$

Now, $(\Pr)_{nf} = \dfrac{\upsilon_{nf}}{\alpha_{nf}}$

Where ν_{nf} and α_{nf} are the kinematic viscosity and thermal diffusivity of the nanofluid respectively. The kinematic viscosity is given by,

$$\upsilon_{nf} = \frac{\mu_{nf}}{\rho_{nf}} \tag{20}$$

And the thermal diffusivity is given by,

$$\alpha_{nf} = \frac{k_{nf}}{\rho_{nf}\left(c_p\right)_{nf}} \tag{21}$$

Thus the Prandtl number may be expressed as,

$$\left(\mathrm{Pr}\right)_{nf} = \frac{\mu_{nf}\left(c_p\right)_{nf}}{k_{nf}} \tag{22}$$

or

$$\frac{1}{\left(\mathrm{Pr}\right)_{nf}} = \frac{k_{nf}}{\mu_{nf}\left(c_p\right)_{nf}} = \frac{\mu_f}{k_{nf}}\frac{(1-\phi)\left(c_p\right)_f + \phi\left(c_p\right)_s}{\left(P_r\right)_{nf}^2 (1-\phi)^{2\cdot 5}} \tag{23}$$

Equation (23) may be written as:

$$\frac{\partial\theta}{\partial t^*} - \frac{\mu_f}{k_{nf}}\frac{(1-\phi)\left(c_p\right)_f + \phi\left(c_p\right)_s}{\left(\mathrm{Pr}\right)_{nf}^2 (1-\phi)^{2\cdot 5}}\frac{\partial^2\theta}{\partial {y^*}^2} = f(y^*, t^*)$$

or simply,

$$\frac{\partial\theta}{\partial t^*} - \lambda\frac{\partial^2\theta}{\partial {y^*}^2} = f(y^*, t^*). \tag{24}$$

Whereby, $\lambda = \frac{\mu_f}{k_{nf}}\frac{(1-\phi)\left(c_p\right)_f + \phi\left(c_p\right)_s}{\left(\mathrm{Pr}\right)_{nf}^2 (1-\phi)^{2\cdot 5}}$.

Assume a particular integral of the form

$$\theta_p\left(y^*, t^*\right) = A\exp\left(-2ay^*\right) + B\exp\left(-2ay^*\right)\cos\left(2\omega^* t^* - 2by^*\right) + C\exp\left(-2ay^*\right)\sin\left(2\omega^* t^* - 2by^*\right). \tag{25}$$

It follows that

$$\frac{\partial\theta_p}{\partial t^*} = -2B\omega^*\exp\left(-2ay^*\right)\sin\left(2\omega^* t^* - 2by^*\right) + 2C\omega^*\exp\left(-2ay^*\right)\cos\left(2\omega^* t^* - 2by^*\right). \tag{26}$$

$$\frac{\partial^2\theta_p}{\partial {y^*}^2} = 4a^2 A\exp(-2ay^*) + \left[\left(4a^2 - 4b^2\right)B + 8abC\right]\exp(-2ay^*)\cos\left(2\omega^* t^* - 2by^*\right)$$

$$+\left[-8abB + \left(4a^2 - 4b^2\right)C\right]\exp(-2ay^*)\sin\left(2\omega^* t^* - 2by^*\right) \quad . \qquad (27)$$

Substituting equation (26) and (27) into equation (24) and equating coefficients we get

$$A = \frac{-EcM^*}{8\lambda a^2} \quad ,$$

$$\begin{cases} \left(-2\omega^* + 8\lambda ab\right)B - \lambda\left(4a^2 - 4b^2\right)C = 0 \\ \left(2\omega^* - 8\lambda ab\right)C - \lambda\left(4a^2 - 4b^2\right)B = \dfrac{EcM}{2} \end{cases} \qquad (28)$$

After solving for B and C in equation (28) we get

$$B = \frac{-\dfrac{EcM}{2}\lambda\left(4a^2 - 4b^2\right)}{\left(2\omega^* - 8\lambda ab\right)^2 + \left(4\lambda a^2 - 4\lambda b^2\right)^2}$$

$$C = \frac{-\dfrac{EcM}{2}\lambda\left(8\lambda ab - 2\omega^*\right)}{\left(2\omega^* - 8\lambda ab\right)^2 + \left(4\lambda a^2 - 4\lambda b^2\right)^2}.$$

The particular integral becomes

$$\theta_p\left(y^*, t^*\right) = \exp\left(-2az^*\right)\left[\frac{-EcM}{8\lambda a^2} - \frac{\dfrac{EcM}{2}\lambda\left(4a^2 - 4b^2\right)}{\left(2\omega^* - 8\lambda ab\right)^2 + \left(4\lambda a^2 - 4\lambda b^2\right)^2}\cos\left(2\omega^* t^* - 2by^*\right)\right.$$

$$\left. - \frac{\dfrac{EcM}{2}\lambda\left(8\lambda ab - 2\omega^*\right)}{\left(2\omega^* - 8\lambda ab\right)^2 + \left(4\lambda a^2 - 4\lambda b^2\right)^2}\sin\left(2\omega^* t^* - 2by^*\right)\right] \qquad (29)$$

We shall creatively synthesize the complementary function of equation (24) from some solution of the homogeneous part of the equation so that the boundary conditions are satified. Some of these solutions include:

$\exp\left(-\sqrt{\frac{\omega^*}{\lambda}}y^*\right)\cos\left(2\omega^*t^*-\sqrt{\frac{\omega^*}{\lambda}}y^*\right)$, is a solution if the temperature of a stationary surface is specified to be harmonic of time.

$\exp\left(-\sqrt{\frac{\omega^*}{\lambda}}y^*\right)\sin\left(2\omega^*t^*-\sqrt{\frac{\omega^*}{\lambda}}y^*\right)$, is a solution if the temperature of a stationary surface is specified to be harmonic of time.

$erf\left(\frac{y^*}{2\sqrt{\lambda t^*}}\right)$ is a solution to or from a stationary surface due to a temperature difference or potential of $\frac{EcM}{8\lambda a^2}+T_w$.

By the principle of superposition the complementary function may be written as,

$$\theta_c\left(y^*,t^*\right)=$$

$$=\left[\frac{EcM}{8\lambda a^2}+\frac{\frac{EcM}{2}\lambda\left(4a^2-4b^2\right)}{\left(2\omega^*-8\lambda ab\right)^2+\left(4\lambda a^2-4\lambda b^2\right)^2}\exp\left(-\sqrt{\frac{\omega^*}{\lambda}}y^*\right)\cos\left(2\omega^*t^*-\sqrt{\frac{\omega^*}{\lambda}}y^*\right)+\right.$$

$$\left.+\frac{\frac{EcM}{2}\lambda\left(8\lambda ab-2\omega^*\right)}{\left(2\omega^*-8\lambda ab\right)^2+\left(4\lambda a^2-4\lambda b^2\right)^2}\sin\left(2\omega^*t^*-2by^*\right)\exp\left(-\sqrt{\frac{\omega^*}{\lambda}}y^*\right)\sin\left(2\omega^*t^*-\sqrt{\frac{\omega^*}{\lambda}}y^*\right)\right]$$

$$-\frac{EcM}{8\lambda a^2}erf\left(\frac{y^*}{2\sqrt{\lambda t^*}}\right)+\theta_w+\left(\theta_\infty-\theta_w\right)erf\left(\frac{y^*}{2\sqrt{\lambda t^*}}\right). \tag{30}$$

The sum of equation (29) and (30) gives us the complete solution

$$\theta\left(y^*,t^*\right)=\exp\left(-2ay^*\right)\left[\frac{-EcM}{8\lambda a^2}-\frac{\frac{EcM}{2}\lambda\left(4a^2-4b^2\right)}{\left(2\omega^*-8\lambda ab\right)^2+\left(4\lambda a^2-4\lambda b^2\right)^2}\cos\left(2\omega^*t^*-2by^*\right)\right.$$

$$\left.-\frac{\frac{EcM}{2}\lambda\left(8\lambda ab-2\omega^*\right)}{\left(2\omega^*-8\lambda ab\right)^2+\left(4\lambda a^2-4\lambda b^2\right)^2}\sin\left(2\omega^*t^*-2by^*\right)\right]$$

$$+\left[\frac{EcM}{8\lambda a^2}+\frac{\frac{EcM}{2}\lambda\left(4a^2-4b^2\right)}{\left(2\omega^*-8\lambda ab\right)^2+\left(4\lambda a^2-4\lambda b^2\right)^2}\exp\left(-\sqrt{\frac{\omega^*}{\lambda}}y^*\right)\cos\left(2\omega^*t^*-\sqrt{\frac{\omega^*}{\lambda}}y^*\right)+\right.$$

$$\left.+\frac{\frac{EcM}{2}\lambda\left(8\lambda ab-2\omega^*\right)}{\left(2\omega^*-8\lambda ab\right)^2+\left(4\lambda a^2-4\lambda b^2\right)^2}\sin\left(2\omega^*t^*-2by^*\right)\exp\left(-\sqrt{\frac{\omega^*}{\lambda}}z^*\right)\sin\left(2\omega^*t^*-\sqrt{\frac{\omega^*}{\lambda}}y^*\right)\right]-$$

$$-\frac{EcM}{8\lambda a^2}erf\left(\frac{y^*}{2\sqrt{\lambda t^*}}\right)+\theta_w+\left(\theta_\infty-\theta_w\right)erf\left(\frac{y^*}{2\sqrt{\lambda t^*}}\right). \tag{31}$$

Where the error function $erf(\ )$ is defined as $erf(\gamma)\equiv\frac{2}{\sqrt{\pi}}\int_0^\gamma \exp(-r^2)dr$ and has the properties $erf(0)=0$ and $erf(\infty)=1$.

4 Results and Discussions

Below are graphs of velocity and temperature profiles in connection to the parameters of interest.

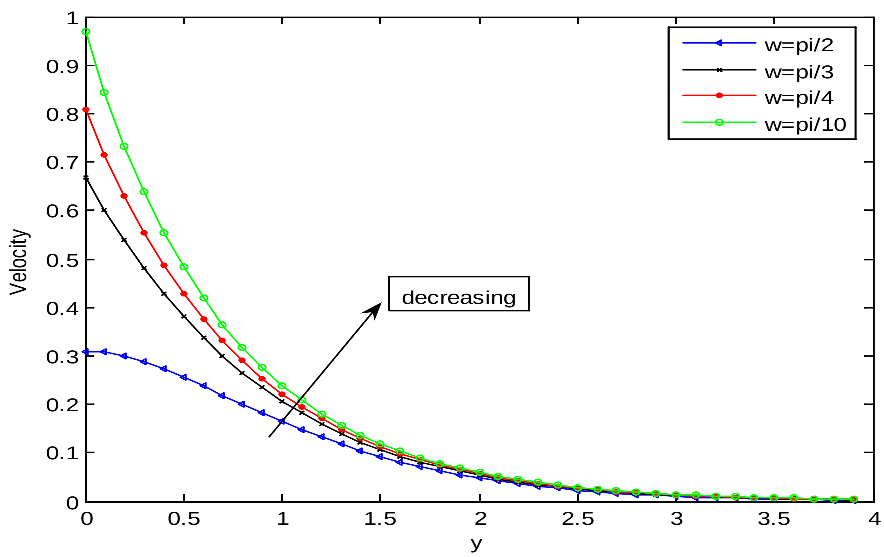

Figure 2: Velocity profiles for different values of ω when M=2 and t=0.8

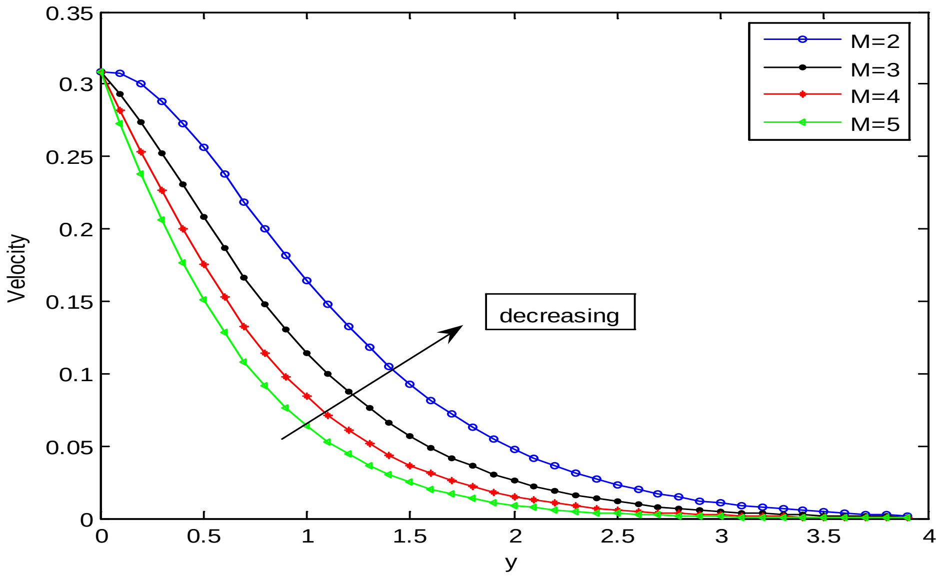

Figure 3: Velocity profiles for different values of M when $\omega = pi/2$ and t=0.8.

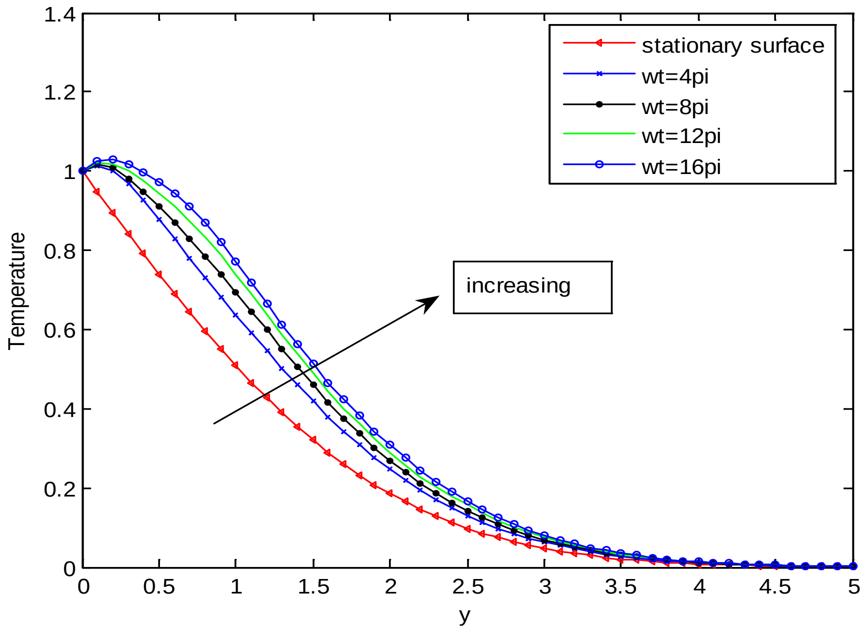

Figure 4: Temperature profiles for different values of ωt when M=2, t=10, E=0.3, Pr=7.02.

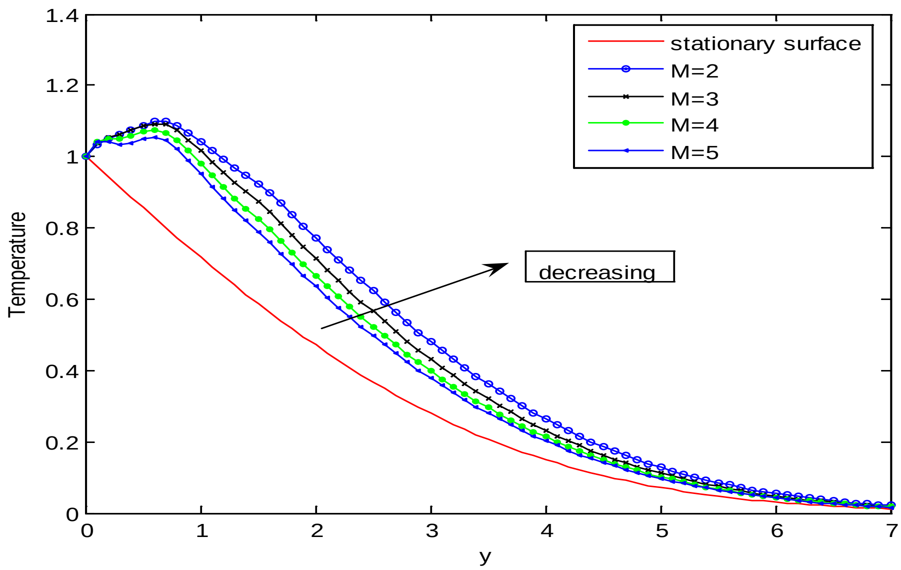

Figure 5: Temperature profiles for different values of M when $\omega = 4pi$, t=10, E=0.4, Pr=2.60.

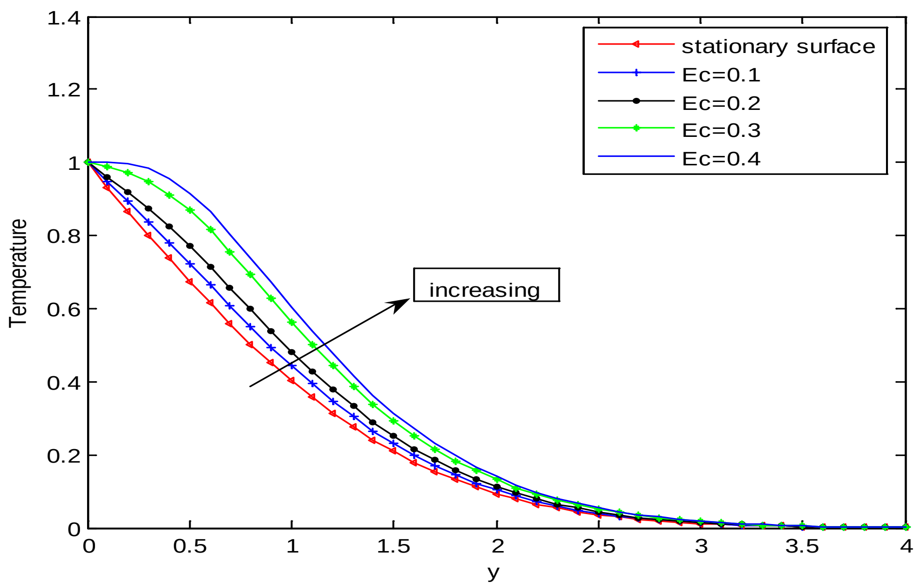

Figure 6: Temperature profiles for different values of Ec when $\omega = pi/2$, M=2, Pr=7.02 and t=5.

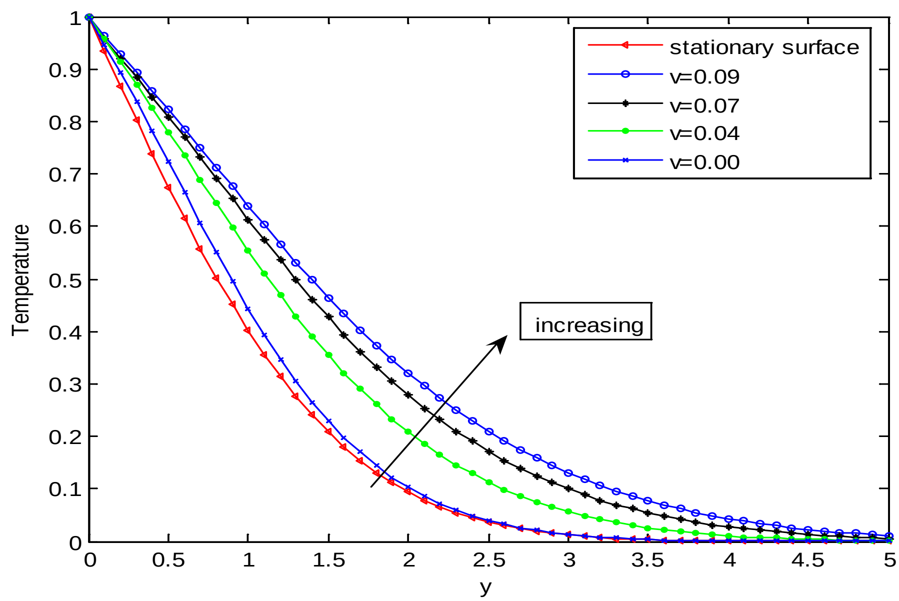

Figure 7: Temperature profiles for different values of v (nanoparticle concentration) when $\omega = pi/2$, M=2, Ec=0.1 and t=5.

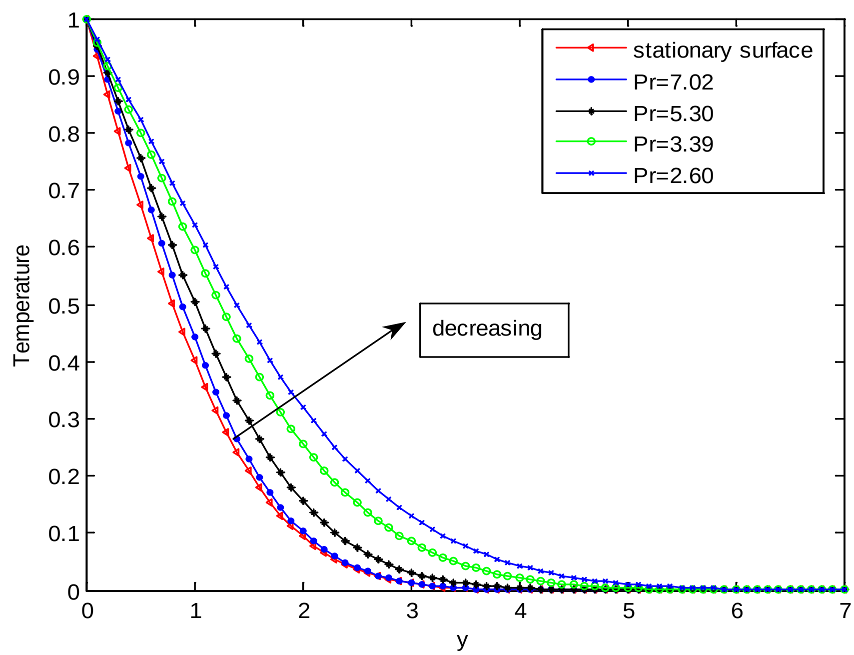

Figure 8: Temperature profiles for different values of Pr when $\omega = pi/2$, M=2, Ec=0.1 and t=5.

Figure 2 shows the variation of temperature with ω. Here, it can be seen that velocity decreases with an increase in ω. Figure 3 depicts the effect of M on velocity. Evidently, velocity decreases with an increase in M. increasing M leads to an increase of the Lorentz force which opposes the flow of the nanofluid. Figure 4 illustrates the relationship between temperature and oscillations frequency ω. It is observed that temperature increases with the increase of oscillation frequency. Figure 5 shows that temperature increases with a decrease in M which is attributed to the fact that smaller M corresponds to higher velocity which in turn leads to high electrical current hence the increase in Joule heating. Figure 6 indicates that temperature increases with an increase in Ec. Figure 7 demonstrates the effects of nanoparticle concentration on temperature. Clearly, the higher the concentration the higher the temperature. This is due to the fact that nanoparticles improves the thermal conductivity of the fluid. From figure 8 it is observed that temperature decreases with an increase in Pr. This is because Pr reduces the thermal conductivity of the nanofluid.

From Figures 4-8 it can be observed that the temperature of the nanofluid over a stationary surface is the lowest as compared to that of over oscillating surface. This means that oscillation of the surface leads movement of the nanofluid which in turn leads to flow of electrical current which results to Joule heating hence the increase in temperature.

5 Conclusion

Studies on MHD flow of nanofluid over an oscillating surface have been carried out. It was found out that velocity decreases with the frequency of oscillation as well as with magnetic parameter. It was also observed that temperature increases with an increase in Erkert number (Ec), nanoparticle concentration and the frequency of oscillation but decreases with an increase in magnetic parameter (M) and the Prandtl number (Pr). In summary, oscillation of the surface over which a nanofluid flows increases Joule heating depending on the applied magnetic field strength, oscillation frequency of the surface. Increasing nanoparticle concentration increases the thermal conductivity of the nanofluid and this improves the potential to remove heat added into the fluid as a result of Joule heating.

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