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The Spatial Equilibrium Monopoly Models of the Steamcoal Market

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Abstract

This paper replicates two spatial monopoly models by Takayama and Judge. The first one is a simple spatial monopoly model. The second model allows the activity of arbitrage between any pains of consumption regions if the price differential exceeds the corresponding unit transportation cost. With the additional constraints, the profit level must decrease unless the constraints are all redundant. The simulation of the Appalachian steamcoal market indicates that both models perform poorly either in terms of flow variables or in the case of consumption and production levels. This implies that the steamcoal market in our model is far from being either of the spatial monopoly models. The Appalachian steamcoal market, characterized by numerous coal mines as well as utility companies, simply cannot be modeled by the spatial monopoly models.

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1 Introduction

The applications of the spatial equilibrium model have spanned over nearly three decades from Fox's free-livestock model [1] to the international aluminum model of Newcomb *et al.* [2]. However, they were in general formulated based on the assumption of a competitive market: a market structure represents one extreme of a spectrum of various markets. On the other hand, the formulation of the spatial equilibrium monopoly model is due to Takayama and Judge ([3], Chapter 11) over twenty years ago; but the empirical applications of their model is at best scanty or none. The purpose of this paper is to implement the spatial equilibrium monopoly models to the Appalachian Steamcoal market using the parameters estimated by Yang [4], Labys and Yang [5], and Irwin and Yang [6]. The next section discusses the simple spatial equilibrium monopoly model; then a precautionary monopoly model with the arbitrage activities. Last we evaluate the performance of the steamcoal market based on the competitive and two spatial monopoly models.

2 A Simple Spatial Equilibrium Monopoly Model

Following the work by Takayama and Judge [3], we make four key assumptions: (1) the product produced by the monopolist is homogeneous; (2) the production and market power is controlled by a sole monopolist or central authority; (3) the input markets (e.g., labor, raw material) are competitive; and (4) the transportation industry plays a passive role in the model, i.e., unit transportation costs are constant. Within this framework, the objective function of the spatial monopolist is to maximize total profit from sales over n regions.

$$\text{Maximize} \quad \pi = \sum_j a_j y_j - \sum_j b_j y_j^2 - \sum_j c_i x_i - \sum_i d_i x_i^2 - \sum_i \sum_j t_{ij} x_{ij} \tag{1}$$

$$\text{Subject to} \quad \sum_i x_{ij} - y_j \geq 0, \ \forall \ i \in M ; \tag{2}$$

$$\sum_j x_{ij} - x_i \leq 0, \ \forall \ j \in N ; \tag{3}$$

$$y_j \geq 0, \ x_i \geq 0, \ x_{ij} \geq 0, \ \forall \ (ij) \in M \times N . \tag{4}$$

where $P_j = a_j - b_j y_j$ is the demand price of region j,

$TC_i = C_i x_i + d_i x_i^2$ is the total cost function for region i,

M , N = set of positive integers,

$M \times N$ = the Cartesian product of M and N ,

t_{ij} = unit transportation cost from region i to j ,

x_{ij} = commodity shipment from region i to j ,

x_i = production level of region i ,

y_j = consumption level of region j .

The formulation is almost identical to that of the competitive equilibrium except that the coefficient of $1/2$ is missing in both quadratic terms indicating the spatial monopolist can choose the best point on each of the regional demand function given his cost structures in each supply region in order to maximize total profit. The first-order condition of the lagrangian equations can be expressed as:

$$L = \sum a_j y_j - \sum b_j y_j^2 - \sum c_i x_i - \sum d_i x_i^2$$
$$- \sum\sum t_{ij} x_{ij} + \sum \lambda_j (\sum x_{ij} - y_i) + \sum \gamma_i (x_i - \sum x_{ij})$$

$$(\partial L / \partial y_j) = a_j y_j - 2b_j y_j - \lambda_j \leq 0 \text{ , } \forall \; j \in N \text{ ;} \quad (6)$$

$$(\partial L / \partial x_i) = -C_i - 2d_i x_i + \gamma_i \leq 0 \text{ , } \forall \; i \in M \text{ ;} \quad (7)$$

$$(\partial L / \partial x_{ij}) = -t_{ij} + \lambda_j - \gamma_i \leq 0 \text{ , } \forall \; (ij) \in M \times N \text{ ;} \quad (8)$$

$$(\partial L / \partial \lambda_j) = \sum x_{ij} - y_j \geq 0 \text{ , } \forall \; i \in M \text{ ;} \quad (9)$$

$$(\partial L / \partial \gamma_i) = x_i - \sum x_{ij} \geq 0 \text{ , } \forall \; j \in N \text{ .} \quad (10)$$

where λ_j = imputed marginal revenue of region j ,

γ_i = imputed marginal cost of region i .

Equation (6) implies that optimum marginal revenue is less than or equal to the imputed marginal revenue of region j ; equation (7) suggests that optimum marginal cost is greater than imputed marginal cost of region i ; and equation (8) states that the difference between imputed marginal price and cost is no less than the corresponding transportation cost. Note that all the decision variables are evaluated at optimum values and a strict equality sign holds for all positive values of decision variables.

3 A Precautionary Spatial Equilibrium Monopoly Model

The simple spatial equilibrium monopoly model from the previous section ignores the possibility of resale between two regions, i.e., the role of an arbitrager is not considered. This assumption is perhaps unrealistically restrictive. To take into consideration the possibility of arbitrage, we add the following constraints :

$$\overline{P_j} - \overline{P_k} = a_j - b_j \overline{y_j} - a_k + b_k \overline{y_k} \leq t_{jk} \text{ , } \forall \; j \neq k \text{ .} \quad (11)$$

Equation (11) states that the optimum demand prices between demand region j and demand region k cannot exceed its corresponding unit transportation costs, i.e., the arbitrage activity between region j and k are considered in the feasibility region. Adding equation (11) to the simple spatial equilibrium monopoly model of equations (1) through (4), we have the following Lagrange function:

$$\phi = L + \sum_j \sum_k \beta_j^k (t_{jk} - a_j + b_j y_j + a_k - b_k y_k) \, , \qquad (12)$$

where β_j^k is the lagrange multiplier of the arbitrage constraint between demand region j and k. Note that if equation (11) does not hold from the optimum solution set of the simple spatial equilibrium monopoly model, such a constraint of the arbitrage activity is not needed. We simply add k numbers of the nC_2 constraints to equations (1) through (4). The first-order conditions are the same as that of the simple spatial equilibrium monopoly model except for the following:

$$(\partial\phi / \partial y_j) = a_j - 2b_j y_j - \lambda_j + \sum_j \sum_k \beta_j^k b_j \le 0 \, , \ \forall \ j \ne k \, ; \qquad (13)$$

$$(\partial\phi / \partial \beta_j^k) = t_{jk} - a_j + b_j y_j + a_k - b_k y_k \ge 0, \ \forall \ j \ne k \, ; \qquad (14)$$

where variables are evaluated at optimum values; and the sign of -β_j^k is reversed for $\overline{P_j} < \overline{P_k}$. While equation (14) is the restatement of the arbitrage constraint, the interpretation of equation (13) is not the same as equation (6). It states that at optimality, the marginal revenue of the j th demand market is equal to or less than the imputed marginal revenue plus the change in the discrimination rent (see [3], p. 221). The equality sign holds for a positive consumption level of region j.

4 An Evaluation of the Spatial Equilibrium Monopoly Models

Based on the estimated parameters by Labys and Yang [5], we assume that, perhaps to the contrary of the true market structure of the Appalachian steamcoal, the production process is controlled by one single supplier. The marginal cost function in each region reflects the smoothed step functions much like that of the world copper industry [7]. The marginal cost functions of seven supply regions by Yang [4] are linearized step functions. Under this specification, we simulated the simple spatial monopoly model with the results reported in Table 1. Similarly, the simulation results of the precautionary monopolist model is again shown in Table 1. The latter results are derived from the simple spatial monopoly assumption with 21 additional resaling possibilities (see Table 2) which considers all the activities of arbitrage in the Appalachian market. The unit transportation costs between a pair of demand regions are approximated by the difference of the observed demand prices.

Of the 22 observed coal shipments, the simple spatial monopoly model predicts 12 flows while the precautionary monopoly model predicts only 10 flows.

Table 1: Optimum solutions under the competitive and monopoly market structures

	1 PA-MD	2 OH	3 Northern WV	4 Southern WV	5 VA	6 East. KY- TN	7 AL	Total Demand	Plant Price
New England					(0.042) [0.023] {0.03}			(0.042) [0.023] {0.03}	(50.574) [59.157] {55.825}
Mid Atlantic	(1.025) [0.597] {0.617}		(0.195) [0.043] {0.37}					(1.220) [0.639] {0.987}	(46.074) [65.294] {53.8}
IN-MI		(0.304) [0.012] {0.172}	(0.336) [0.354] {0.033}					(0.640) [0.367] {0.205}	(44.374) [49.976] {53.297}
North Central						(0.053) [0.05]		(0.053) [0.05] {0}	(47.574) [47.696] {49.7}
Ohio Valley		(0.708) [0.547] {0.407}		(0.035)		(0.628) [0.214] {0.352}		(1.37) [0.761] {0.759}	(43.874) [51.627] {51.653}
South Central					(0.081) [0.057]		(0.138) [0.107] {0.107}	(0.219) [0.164] {0.107}	(43.474) [44.482] {45.496}
South Atlantic					(0.138) [0.103] {0.156}	(0.653) [0.336] {0.274}		(0.791) [0.439] {0.43}	(44.074) [51.885] {52.084}
Total Supply	(1.025) [0.597] {0.617}	(1.012) [0.559] {0.579}	(0.531) [0.397] {0.403}	(0.035) [0] {0}	(0.261) [0.183] {0.186}	(1.334) [0.6] {0.621}	(0.138) [0.107] {0.107}		
Mine Price $cents/10^6$ Btu or Marginal Cost	(31.574) [29.663] {29.752}	(30.574) [28.659] {28.744}	(32.574) [30.661] {30.746}	(31.274) [30.4] {30.4}	(29.374) [27.456] {27.529}	(30.474) [28.562] {28.629}	(36.474) [39.579] {34.579}		

Sources: Mineral Year Book (1974), Steam-Electric Plant Factors (1974), Energy

Economics Vol.2 (April 1980), Competitive Transportation Costs of Supplying Low-Sulfur Fuels to Mid-western and Eastern Domestic Energy Market (1972).
Figures in ( ) are optimal values of the competitive solution; figures in ( ) and { } are optimum values under the simple monopoly and precautionary monopoly respectively.

In order to evaluate the predictive power in terms of the mean percentage of absolute error, we calculate the following index:

$$I_1 = \sum_j (\left| y_j - \overline{y_j} \right| / y_j), \quad (15)$$

$$I_2 = \sum_i (\left| x_i - \overline{x_i} \right| / x_i), \quad (16)$$

$$I_3 = \sum_i \sum_j (\left| x_{ij} - \overline{x_{ij}} \right| / x_{ij}), \quad (17)$$

where barred variables denote optimum values; unbarred variables represent actual values. A calculation reveals that the values of I_1 are 51.97% and 54.77% for the simple spatial monopoly and precautionary monopoly models respectively. It implies that the former explains 48.03% and the latter explains 45.23% of the consumption activities of the Appalachian steamcoal market. Similarly, the two spatial monopoly models explain only 43.88% (or $I_2 = 0.5612$) and 44.93% ($I_2 = 0.5507$) of the coal production activities.
Finally, of the 10 major coal shipments, i.e.,

$$x_{12}, x_{23}, x_{25}, x_{32}, x_{35}, x_{57}, x_{65}, x_{66}, x_{67} \text{ and } x_{76},$$

the simple spatial monopoly model explains 33.45% ($I_3 = 0.6655$) while the precautionary monopoly model explains 42.8% ($I_3 = 0.572$) of the major coal shipment activities.

A perusal of Table 1 indicates immediately all the delivered pieces under both spatial monopoly prices are higher than that under the competitive spatial equilibrium prices. This comes as no surprise since the objective function in both cases is to maximize the monopoly profit whereas the objective of the competitive spatial equilibrium model is consistent with zero profit condition. The marginal costs of the mouth of mines are all lower than that under the competitive spatial equilibrium level since the production levels at all regions are much lower than that of the competitive level. Hence, the spatial monopoly models would produce much higher demand prices and lower output levels.

Table 2: Anti-arbitrage constratnts

optimum demand price (higher price)	optimum demand price (lower price)	unit transportation cost
P_1	P_2	4.3
P_1	P_3	4.8
P_1	P_4	8.4
P_1	P_5	13.5
P_1	P_6	12.6
P_1	P_7	3.7
P_2	P_3	0.5
P_2	P_4	4.1
P_2	P_5	9.2
P_2	P_6	8.3
P_3	P_4	3.6
P_3	P_5	8.7
P_3	P_6	7.8
P_4	P_5	5.1
P_4	P_6	4.2
P_6	P_5	0.9
P_7	P_5	9.8
P_7	P_6	8.9
P_7	P_2	0.6
P_7	P_3	1.1
P_7	P_4	4.7

5 Concluding Remarks

In this paper, we replicate two spatial monopoly models by Takayama and Judge. The first one is a simple spatial monopoly model, which is no longer consistent with the zero profit condition. The second model allows the activity of arbitrage between any pains of consumption regions if the price differential exceeds the corresponding unit transportation cost. With the additional constraints, the profit level must decrease unless the constraints are all redundant. The simulation of the Appalachian steamcoal market indicates that both models perform poorly either in terms of flow variables or in the case of consumption and production levels. This implies that the steamcoal market in our model is far from being either of the spatial monopoly models. The Appalachian steamcoal market, characterized by numerous coal mines as well as utility companies, simply cannot be modeled by the spatial monopoly models.

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