

Forecasting a Composite Indicator of Economic Activity in Ghana: A Comparison of Data Science Methods

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Abstract

Of recent, data science methods have been used to study and forecast financial and economic problems. This paper uses historical data to build a more parsimonious predictive model for making short term forecasts of the future values for the Composite Indicator of Economic Activity (CIEA) in Ghana. Based on our studies of a variety of shrinkage methods and a dimension reduction technique, we show empirically that the estimated model based on the Adaptive Elastic Net (Adaptive ENET) algorithm offers the greatest forecasting potential for the CIEA. A major finding in this paper was that, the Adaptive ENET model outperformed the benchmark model: Principal Component Regression (PCR) according to the cross validation root mean square error difference Statistic.

Mathematics Subject Classification: G12; C15; G22

Keywords: Composite Index of Economic Activity; Least Absolute Shrinkage

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and Selection Operator; Elastic Net; Principal Component; Artificial Neural Network

1 Introduction

The aim of this paper is to use historical data to build a more parsimonious predictive model that can be used to make short term forecasts of the future values for the Composite Indicator of Economic Activity (CIEA) in Ghana. The CIEA symbolizes a point estimate that tracks the current state of an economy by providing useful picture of where the economy is headed. Policy makers in developing countries rely heavily on it as a way of judging how best an economy is performing [1]. CIEA is a measure that correlates with the current level of economic activity such as the real Gross Domestic Product (GDP) [2]. One of the biggest challenges in applied economic and financial research is finding a dependable model for forecasting macroeconomic and financial variables due to the simultaneous and comovements in most of these variables. Also, the use of only one indicator for short term forecasting is not reliable for the simple reason that most leading indicators produce erroneous signals [3]. To overcome some of these challenges is the emergence of a composite index that reflects a broader spectrum of the entire economy (real, monetary, fiscal, and external sector data) and [4] reviews the benefit of composite indexes. In 2003, economists at the Bank of Ghana (BoG) produced a working paper aimed at constructing a CIEA for Ghana and explored the likely use of the index to explain short run economic fluctuations. Using the conference board methodology synonymous to the Moore-Shiskin methodology, their findings indicate that economic activity for Ghana is still on the rising trend and that economic activity stagnated between February and April 2003. They concluded that, the composite indicator can be used to predict the overall direction of economic activity in Ghana[1].

In macroeconomic studies and monetary policy analysis, forecasting variables plays a crucial role. Authors of [5, 6] have established the importance of accurate prediction in having a better understanding of economic movements and implementation of effective monetary policies respectively. In the recent past, econometricians and statisticians have used Dynamic Factor Model

(DFM) and its modifications and extensions to obtain valuable insights from a large panel of time series [7, 8, 9]. In applying DPMs, a compromise between loss of information and curse of dimensionality has to be made. The use of a limited number of principal components to summarize most of the information in time series results in a loss of information because the remaining principal components also contain some amount of information. Also, in an attempt to use a large number of principal components enlarges the dimensionality of the model causing the degrees of freedom problem. For these reasons we put forward a different method of forecasting rooted in data science. Data science is a scientific discipline that combines statistics and computer science with foundations in mathematics [10]. The most common applications of data science techniques by companies are for tracking business processes and building a wide variety of fancy predictive models. For example, A credit card company uses predictive models to monitor its underwriting, pricing, and marketing activities. Healthcare insurers adjust payments and quality measures based on risk factors which are estimated from predictive models [11].

The proposed forecasting data science methods use the concept of shrinkage as opposed to dimension reduction to estimate model parameters. To explain in simple terms, the shrinkage methods operate by imposing a penalty on the least squares estimation method. Various assumptions have been made in the literature where the l_1 - norm and l_2 - norm or both which represent the penalty term were used to influence the parameter estimates in order to minimize the impact of collinearity. This paper focuses on two of the shrinkage methods - the least absolute shrinkage and selection operator (LASSO) and the elastic net (ENET). To improve upon the LASSO and the ENET results, we included the Adaptive LASSO and the Adaptive ENET. It is important to add that, with the advent of Big Data, low dimensional models suffer massive computational and statistical challenges such as scalability and storage problems, noise accumulation, spurious correlation, incidental endogeneity, and measurement error in capturing complex, dynamic patterns underlying large panels of time series [12]. In the context of an approximate factor model with dynamics, it has been shown that forecasts of a single time series based on principal components of a large number of predictors are first-order asymptotically efficient [7]. For this reason, we will use the principal component regression (PCR) as a benchmark to compare the proposed data science models. This paper is not

intended to repeat what has already been well covered in many standard data science textbooks, for detailed review on PCR, we direct readers to [13, 7]. The rest of the paper is partitioned into six sections. Section 2 provides a short conceptual overview of the shrinkage estimation of the forecasting models. Section 3 describes the time series data used. Section 4 presents the statistical procedures for evaluating out-of-sample predictive accuracies of the forecasting models. Section 5 shows detailed results and section 6 concludes.

2 Data Science Methods

This section reviews the data science methods used in constructing the forecasting models. These methods can be classified as either shrinkage or dimension reduction methods. For this paper, the emphasis is on shrinkage methods.

2.1 Shrinkage Methods

Consider a stationary autoregressive model with endogenous and exogenous regressors as follows:

$$y_t = \alpha + \sum_{l=1}^p \phi_l y_{t-l} + \sum_{i=1}^k \sum_{j=1}^{q_i} \beta_{ij} x_{it-j} + \epsilon_t \quad (1)$$

where y_{t-1}, \dots, y_{t-p} and x_{1t}, \dots, x_{kt} are the endogenous and exogenous regressors respectively. ϵ_t is a random sequence of independent Gaussian with $E\epsilon = 0$ and $E\epsilon^2 = \sigma^2$. Let $\theta = (\phi, \beta)$, then the ordinary least squares (OLS) estimate $\hat{\theta}^{OLS} = (\hat{\phi}, \hat{\beta})$ minimizes

$$RSS = \frac{1}{2} \sum_{t=1}^T \left(y_t - \alpha - \sum_{l=1}^p \phi_l y_{t-l} - \sum_{i=1}^k \sum_{j=1}^{q_i} \beta_{ij} x_{it-j} \right)^2 \quad (2)$$

The OLS estimates usually perform poorly in prediction and interpretation particularly when the size of the sample T is small relative to the number of predictors (both exogenous and endogenous variables). Shrinking or regularizing the coefficient estimates towards zero improves fit and significantly reduce

their variances. In this paper we consider important shrinkage techniques, the LASSO and the ENET and their variants.

In LASSO regression [14], the penalty term has the form of the sum of absolute values. The solution can be obtained solving (3):

$$\hat{\theta}^{LASSO}(\lambda) = \underset{\theta}{\operatorname{argmin}} RSS + \lambda \left(\sum_{l=1}^p |\phi_l| + \sum_{i=1}^k \sum_{j=1}^{q_i} |\beta_{ij}| \right) \quad (3)$$

where $\lambda \geq 0$. The LASSO simultaneous does continuous shrinkage and automatic variable selection with the imposition of an l_1 -penalty.

ENET regression [15] is basically a combination of RIDGE and LASSO procedures. The estimates from ENET method are defined by

$$\begin{aligned} \hat{\theta}^{ENET}(\lambda) = \underset{\theta}{\operatorname{argmin}} RSS + \lambda_2 \left(\sum_{l=1}^p \phi_l^2 + \sum_{i=1}^k \sum_{j=1}^{q_i} \beta_{ij}^2 \right) + \\ + \lambda_1 \left(\sum_{l=1}^p |\phi_l| + \sum_{i=1}^k \sum_{j=1}^{q_i} |\beta_{ij}| \right) \end{aligned} \quad (4)$$

The Adaptive LASSO seeks to minimize (5) [16]:

$$\hat{\theta}^{AdpLASSO}(\lambda) = \underset{\theta}{\operatorname{argmin}} RSS + \lambda \left(\sum_{l=1}^p \hat{w}_l |\phi_l| + \sum_{i=1}^k \sum_{j=1}^{q_i} \hat{w}_{ij} |\beta_{ij}| \right) \quad (5)$$

where \hat{w}_l is the adaptive data-riven weight and can be computed by $\hat{w}_l = \left(\left| \hat{\phi}_l(\text{ridge}) \right| \right)^{-\gamma}$. γ is a positive constant and $\hat{\phi}^{ini}$ is a consistent estimate of ϕ . Also, \hat{w}_{ij} is computed by $\hat{w}_{ij} = \left(\left| \hat{\beta}_{ij}(\text{ridge}) \right| \right)^{-\gamma}$ where $\hat{\beta}^{ini}$ is a consistent estimate of β .

The adaptive ENET can be viewed as a combination of the ENET and the adaptive LASSO. Thus, the Adaptive ENET solves the problem [15]

$$\begin{aligned} \hat{\theta}^{AdpENET}(\lambda) = \underset{\theta}{\operatorname{argmin}} RSS + \lambda_2 \left(\sum_{l=1}^p \phi_l^2 + \sum_{i=1}^k \sum_{j=1}^{q_i} \beta_{ij}^2 \right) + \\ + \lambda_1 \left(\sum_{l=1}^p \hat{w}_l |\phi_l| + \sum_{i=1}^k \sum_{j=1}^{q_i} \hat{w}_{ij} |\beta_{ij}| \right) \end{aligned} \quad (6)$$

where the adaptive weights \hat{w}_l and \hat{w}_{ij} are obtained by $\hat{w}_l = \left(\left| \hat{\phi}_l(\text{enet}) \right| \right)^{-\gamma}$ and $\hat{w}_{ij} = \left(\left| \hat{\beta}_{ij}(\text{enet}) \right| \right)^{-\gamma}$ respectively. Note that \hat{w}_l and \hat{w}_{ij} in (6) are estimators associated with the ENET algorithm.

3 Data

This section describes the data series and their associated transformations. The data set can be downloaded from the BoG website: <https://www.bog.gov.gh/>. The data set consists of 33 variables (Table 1 in Appendix A) spanning the period February 2000 – March 2016.

Table 2 displays the descriptive statistics of the original variables. These variables were all positive in terms of skewness and kurtosis, however, the JB test for normality indicates that all the original variables were not normally distributed ($p - values < 0.01$).

In accordance with standard practice in macroeconomic literature, we used two widely known methods: the Augmented Dickey Fuller (ADF) [17] and Phillip Perron (PP) [18] tests to check for the existence or otherwise of unit root in the original variables. The results of these tests are shown in Table 3. The ADF test indicates that apart from CPI-O ($p - value = 0.010$) where the null hypothesis of a unit root can be rejected, the null hypothesis of a unit root for the rest of the variables cannot be rejected at the 0.05 significance level. Of all the variables tested, the PP test shows that the null hypothesis of a unit root can be rejected at significance level of 0.05 for CIEARReal ($p - value = 0.010$), CPI-O ($p - value = 0.010$), and INF-F ($p - value = 0.022$). The time plot of the individual variables are shown in Figures 1 to 12 in Appendix B. It is evident that majority of the variables have unit root non-stationarity property.

Table 2: Descriptive statistics for the Original Variables

| Variable | N | Mean | SD | Skewness | Kurtosis | JB Normality Test | |
|---------------|-----|----------|------------|----------|----------|-------------------|---------|
| | | | | | | Statistic | p-value |
| CIEANom | 194 | 511.32 | 400.1743 | 0.7972 | 2.419 | 23.278 | 0.0000 |
| CIEAReal | 194 | 224.74 | 126.5762 | 2.1609 | 15.704 | 1455.6 | 0.0000 |
| GSE-ASI | 194 | 3693.10 | 2776.8922 | 0.8077 | 2.6749 | 21.948 | 0.0000 |
| INF-YOY | 194 | 16.93 | 7.8581 | 1.4372 | 4.5592 | 86.435 | 0.0000 |
| TBR-91 | 194 | 21.49 | 9.5100 | 0.7663 | 3.1644 | 19.207 | 0.0001 |
| BCROIL | 194 | 66.33 | 32.9065 | 0.2483 | 1.7412 | 14.803 | 0.0006 |
| BNCG | 194 | 2064.65 | 2636.3761 | 1.4457 | 3.7718 | 72.391 | 0.0000 |
| CIC | 194 | 2383.39 | 2406.0205 | 1.1117 | 3.0696 | 40.001 | 0.0000 |
| CITOB | 194 | 224.91 | 242.6918 | 1.0462 | 3.0679 | 35.426 | 0.0000 |
| COB | 194 | 2154.13 | 2164.5157 | 1.1317 | 3.1126 | 41.515 | 0.0000 |
| CocoaP | 194 | 2152.17 | 710.5970 | 0.0633 | 1.6579 | 14.689 | 0.0006 |
| CPI- F | 194 | 186.35 | 83.9133 | 0.3446 | 1.7483 | 16.505 | 0.0003 |
| CPI- NF | 194 | 225.45 | 127.9649 | 0.7529 | 2.45 | 20.773 | 0.0000 |
| CPI- O | 194 | 214.12 | 136.8050 | 3.6004 | 30.2735 | 6431.9 | 0.0000 |
| DD | 194 | 3101.02 | 3483.5252 | 1.2646 | 3.4464 | 53.318 | 0.0000 |
| FCD | 194 | 2857.99 | 3335.3403 | 1.4398 | 4.0926 | 21.948 | 0.0000 |
| GIR | 194 | 2443.80 | 1553.5424 | 0.2214 | 1.7737 | 13.741 | 0.0010 |
| IBKEXRENDMUSD | 194 | 1.45 | 0.8959 | 1.566 | 4.5682 | 99.171 | 0.0000 |
| IBKEXRMAVEUSD | 194 | 1.44 | 0.8813 | 1.558 | 4.5348 | 97.521 | 0.0000 |
| IBKXEMAVEGBP | 194 | 2.34 | 1.3286 | 1.4085 | 4.1928 | 75.644 | 0.0000 |
| IBKXRAVEEURO | 194 | 1.80 | 1.0817 | 0.9732 | 3.0501 | 30.645 | 0.0000 |
| IBKXRENDMEURO | 194 | 1.82 | 1.0921 | 1.0001 | 3.0808 | 32.392 | 0.0000 |
| IBKXRENDMGBP | 194 | 2.35 | 1.3346 | 1.408 | 4.1659 | 75.089 | 0.0000 |
| INF-F | 194 | 12.66 | 7.6997 | 1.0718 | 3.5357 | 39.465 | 0.0000 |
| INF-NF | 194 | 20.26 | 9.9586 | 1.699 | 5.7405 | 154.04 | 0.0000 |
| IntBkWAve | 194 | 19.62 | 8.5443 | 1.184 | 3.9632 | 52.825 | 0.0000 |
| M1 | 194 | 5261.35 | 5658.9232 | 1.2071 | 3.2764 | 47.729 | 0.0000 |
| M2 | 194 | 8647.06 | 9178.0168 | 1.1727 | 3.2887 | 45.137 | 0.0000 |
| M2+ | 194 | 11496.16 | 12458.4691 | 1.2332 | 3.444 | 50.769 | 0.0000 |
| MPR | 194 | 18.88 | 5.0663 | 0.4807 | 1.7904 | 19.3 | 0.0001 |
| RM | 194 | 3509.35 | 3903.0914 | 1.2613 | 3.4063 | 52.771 | 0.0000 |
| TBR-91 day | 194 | 21.49 | 9.5093 | 0.7687 | 3.1676 | 19.332 | 0.0001 |
| TotDep | 194 | 11730.07 | 9087.4002 | 1.0912 | 3.3666 | 39.587 | 0.0000 |

To guarantee stationarity of all variables before modeling, we transformed all variables by taking natural log and first difference. Descriptive statistics and the unit root test results on the transformed variables are shown in Tables 4 and 5 respectively. The transformed variables exhibited positive kurtosis and the Jarque-Bera (JB) test for normality shows that none of the transformed

Table 3: ADF and PP Tests for the Original Variables.

| Variable | ADF Test | | PP Test | |
|---------------|----------------|-----------------|----------------|-----------------|
| | Test Statistic | <i>p</i> -value | Test Statistic | <i>p</i> -value |
| CIEANom | 1.985 | 0.990 | -1.113 | 0.918 |
| CIEAReal | -2.765 | 0.069 | -12.394 | 0.010 |
| GSE-ASI | -1.727 | 0.415 | -1.737 | 0.686 |
| INF-YOY | -2.106 | 0.274 | -2.685 | 0.290 |
| TBR-91 | -2.053 | 0.294 | -1.824 | 0.650 |
| BCROIL | -2.122 | 0.268 | -1.488 | 0.791 |
| BNCG | -0.036 | 0.952 | -1.869 | 0.631 |
| CITOB | 1.762 | 0.990 | -0.746 | 0.965 |
| CNBP | 0.091 | 0.962 | -3.065 | 0.130 |
| COB | 1.820 | 0.990 | -0.570 | 0.978 |
| CocoaP | -1.625 | 0.452 | -2.831 | 0.228 |
| CPI- F | -1.804 | 0.386 | -1.369 | 0.840 |
| CPI- NF | -2.270 | 0.213 | -2.729 | 0.271 |
| CPI- O | -3.620 | 0.010 | -6.410 | 0.010 |
| DD | 4.099 | 0.990 | 1.018 | 0.990 |
| FCD | 3.252 | 0.990 | 0.086 | 0.990 |
| GIR | -1.305 | 0.572 | -2.839 | 0.225 |
| IBKEXRENDMUSD | 2.982 | 0.990 | -0.012 | 0.990 |
| IBKEXRMAVEUSD | 2.967 | 0.990 | 0.312 | 0.990 |
| IBKXEMAVEGBP | 1.186 | 0.990 | -0.874 | 0.953 |
| IBKXRAVEEURO | 0.593 | 0.989 | -1.842 | 0.642 |
| IBKXRENDMEURO | 1.304 | 0.990 | -1.441 | 0.810 |
| IBKXRENDMGBP | 1.008 | 0.990 | -1.154 | 0.912 |
| INF-F | -2.143 | 0.260 | -3.767 | 0.022 |
| INF-NF | -1.974 | 0.323 | -2.414 | 0.403 |
| IntBkWAve | -1.789 | 0.391 | -1.510 | 0.781 |
| M1 | 3.309 | 0.990 | 0.272 | 0.990 |
| M2 | 3.056 | 0.990 | 0.911 | 0.990 |
| M2+ | 5.356 | 0.990 | 1.496 | 0.990 |
| MPR | -1.047 | 0.667 | 0.294 | 0.990 |
| RM | 2.400 | 0.990 | 0.095 | 0.990 |
| TBR-91 day | -2.046 | 0.296 | -1.823 | 0.650 |
| TotDep | -0.272 | 0.920 | -1.286 | 0.875 |

variable was normal since p – values were all less than 0.01. While some of the transformed variables were skewed right, majority were left skewed. The null hypothesis of a unit root for the all transformed variables can be rejected at the 0.05 level of significance for both the ADF and the PP tests (Table 5). It can therefore be concluded that all the transformed variables have achieved stationarity.

Table 4: Descriptive statistics for the Transformed Variables

| Variable | N | Mean | SD | Skewness | Kurtosis | JB Normality Test | |
|---------------|-----|----------|---------|-----------|-----------|-------------------|---------|
| | | | | | | Statistic | p-value |
| CIEANom | 193 | 0.01454 | 0.09119 | -1.18734 | 69.36817 | 35467.00 | 0.0000 |
| CIEAReal | 193 | 0.00936 | 0.32143 | -0.35764 | 72.81053 | 39195.00 | 0.0000 |
| GSE-ASI | 193 | 0.00492 | 0.15246 | -10.87958 | 139.37037 | 153360.00 | 0.0000 |
| INF-YOY | 193 | 0.00083 | 0.09296 | 1.03531 | 16.02756 | 1399.30 | 0.0000 |
| BCROIL | 193 | 0.00189 | 0.09211 | -0.84348 | 4.14395 | 33.41 | 0.0000 |
| BNCG | 193 | 0.02105 | 0.22824 | -0.95372 | 7.70036 | 206.93 | 0.0000 |
| CIC | 193 | 0.02163 | 0.07714 | 1.38692 | 9.35858 | 387.01 | 0.0000 |
| CITOB | 193 | 0.02707 | 0.21117 | 1.85345 | 16.32924 | 1539.30 | 0.0000 |
| COB | 193 | 0.02091 | 0.06213 | 0.79845 | 4.14414 | 31.03 | 0.0000 |
| CocoaP | 193 | 0.00595 | 0.06895 | -0.37683 | 4.44314 | 21.32 | 0.0000 |
| CPI- F | 193 | 0.00414 | 0.08761 | -11.85668 | 157.40598 | 196250.00 | 0.0000 |
| CPI- O | 193 | 0.00597 | 0.16327 | -3.24883 | 63.82491 | 30091.00 | 0.0000 |
| DD | 193 | 0.02442 | 0.06747 | 0.10831 | 4.46293 | 17.59 | 0.0002 |
| FCD | 193 | 0.02406 | 0.07703 | -0.76251 | 16.48692 | 1481.50 | 0.0000 |
| GIR | 193 | 0.01267 | 0.09933 | 0.87737 | 4.55741 | 44.27 | 0.0000 |
| IBKEXRENDMUSD | 193 | 0.01183 | 0.03023 | -1.14607 | 25.54137 | 4128.30 | 0.0000 |
| IBKEXRMAVEUSD | 193 | 0.01208 | 0.06471 | 2.37768 | 61.14852 | 27373.00 | 0.0000 |
| IBKXEMAVEGBP | 193 | 0.01148 | 0.04033 | 0.20849 | 11.49324 | 581.48 | 0.0000 |
| IBKXRAVEEURO | 193 | 0.01324 | 0.18846 | -0.26362 | 86.58825 | 56189.00 | 0.0000 |
| IBKXRENDMEURO | 193 | 0.01270 | 0.04699 | -0.55862 | 16.97745 | 1581.10 | 0.0000 |
| IBKXRENDMGBP | 193 | 0.01140 | 0.04624 | 0.51966 | 13.24765 | 853.18 | 0.0000 |
| INF-F | 193 | 0.00009 | 0.16575 | 0.66202 | 11.62322 | 612.08 | 0.0000 |
| INF-NF | 193 | -0.00022 | 0.13351 | -0.25997 | 23.29625 | 3314.80 | 0.0000 |
| IntBkWAve | 193 | -0.00086 | 0.09291 | -0.01772 | 9.67280 | 358.08 | 0.0000 |
| M1 | 193 | 0.02253 | 0.05870 | -0.02386 | 15.44010 | 1244.50 | 0.0000 |
| M2 | 193 | 0.02300 | 0.03323 | 0.47490 | 3.44751 | 8.87 | 0.0119 |
| M2+ | 193 | 0.02350 | 0.03187 | 0.51824 | 7.68764 | 185.35 | 0.0000 |
| RM | 193 | 0.02233 | 0.09058 | -0.04048 | 12.26741 | 690.71 | 0.0000 |
| TotDep | 193 | 0.01217 | 0.28729 | -2.30957 | 63.13548 | 29252.00 | 0.0000 |

Table 5: ADF and PP Tests for the Transformed Variables.

| Variable | ADF Test | | PP Test | |
|---------------|----------------|-----------------|----------------|-----------------|
| | Test Statistic | <i>p</i> -value | Test Statistic | <i>p</i> -value |
| CIEANom | -16.479 | 0.01 | -30.271 | 0.01 |
| CIEAReal | -16.610 | 0.01 | -33.300 | 0.01 |
| GSE-ASI | -9.239 | 0.01 | -12.957 | 0.01 |
| INF-YOY | -8.386 | 0.01 | -12.387 | 0.01 |
| BCROIL | -8.560 | 0.01 | -10.507 | 0.01 |
| BNCG | -11.956 | 0.01 | -19.527 | 0.01 |
| CIC | -10.133 | 0.01 | -15.610 | 0.01 |
| CITOB | -14.436 | 0.01 | -24.426 | 0.01 |
| COB | -9.560 | 0.01 | -11.261 | 0.01 |
| CocoaP | -11.287 | 0.01 | -14.200 | 0.01 |
| CPI- F | -9.341 | 0.01 | -13.818 | 0.01 |
| CPI- O | -12.317 | 0.01 | -19.851 | 0.01 |
| DD | -10.693 | 0.01 | -17.413 | 0.01 |
| FCD | -13.266 | 0.01 | -20.002 | 0.01 |
| GIR | -9.199 | 0.01 | -16.435 | 0.01 |
| IBKEXRENDMUSD | -6.910 | 0.01 | -14.602 | 0.01 |
| IBKEXRMAVEUSD | -12.345 | 0.01 | -21.077 | 0.01 |
| IBKXEMAVEGBP | -9.073 | 0.01 | -14.695 | 0.01 |
| IBKXRAVEEURO | -16.027 | 0.01 | -29.680 | 0.01 |
| IBKXRENDMEURO | -9.674 | 0.01 | -16.754 | 0.01 |
| IBKXRENDMGBP | -9.535 | 0.01 | -16.538 | 0.01 |
| INF-F | -10.878 | 0.01 | -14.665 | 0.01 |
| INF-NF | -10.102 | 0.01 | -14.835 | 0.01 |
| IntBkWAve | -9.629 | 0.01 | -12.337 | 0.01 |
| M1 | -10.436 | 0.01 | -15.263 | 0.01 |
| M2 | -9.324 | 0.01 | -10.848 | 0.01 |
| M2+ | -9.566 | 0.01 | -15.598 | 0.01 |
| RM | -12.038 | 0.01 | -19.839 | 0.01 |
| TotDep | -9.694 | 0.01 | -13.984 | 0.01 |

4 Forecasting Accuracy Metrics

In this section, we describe the statistical metrics for comparing the out-of-sample predictive accuracies of alternative models. In particular, we focus on two quantitative methods. One, the Root Mean Square Error difference ($\Delta RMSE$) statistic similar to the one used by [19, 20] for assessing the predictive abilities of asset pricing models. In this paper, we prefer to call the metric Cross Validation Root Mean Square Error difference ($\Delta CV - RMSE$) statistic because we hold-out a sample of observations before fitting begins and once fitting is completed, the held-out sample is used to evaluate the predictive performance of the fitted models. Two, the Diebold-Mariano test [21] to assess the forecast accuracy of two predictive models.

4.1 The Cross Validation Root Mean Square Error Difference Statistic

The $\Delta CV - RMSE$ statistic is given by:

$$\Delta CV - RMSE = \sqrt{\frac{\sum_{t=T+1}^N (y_t - \bar{y}_t)^2}{N - T}} - \sqrt{\frac{\sum_{t=T+1}^N (y_t - \hat{y}_t)^2}{N - T}} \quad (7)$$

where $N - T$ represents the number of held-out sample forecasts, y_t is the observed value of variable being predicted at time t , $t = T + 1, \dots, N$, \bar{y}_t and \hat{y}_t are the forecasts from a benchmark model and a proposed model respectively. A positive $\Delta CV - RMSE$ is an indication that the proposed model has a better predictive power compared to the benchmark model.

4.2 The Diebold-Mariano test

The Diebold-Mariano test statistic is given by:

$$DM = \left[\frac{1}{N - T} \left(\hat{\gamma}_0 + 2 \sum_{j=1}^{r-1} \hat{\gamma}_j \right) \right]^{-\frac{1}{2}} \frac{1}{N - t} \sum_{t=T+1}^N d_t \quad (8)$$

where d_t is the difference of squared forecast errors which is defined as:

$$d_t = e_{j,t}^2 - e_{k,t}^2 \quad (9)$$

$\hat{\gamma}_r$ is the estimated j^{th} autocovariance of the time series d_t , and r denotes the forecast horizon. The test statistic has an asymptotic standard normal distribution under the null hypothesis of no difference between the expected forecast performance of models j and k .

5 Results

The data set used for empirical analysis comprises of the variables that were transformed to ensure stationarity. Specifically, 29 stationary time series variables where CIEARreal was the predictor variable were used. The original time series variables started from February 2000 to March 2016 and they were transformed to include up to lags of 4 in both the response (natural log transformation) variable and the predictor (first difference of natural log) variables. In total there were 128 predictor variables and the modeling period commenced from July 2000. We define a training set or an in-sample period starting from July 2000 to January 2013 (151 months in total) and a testing set or an out-of-sample period from February 2013 to March 2016 (38 months in total). The training set was used to estimate alternative predictive regression models including lags up to a maximum of 4. The estimated regression models were validated on the testing set to ascertain their predictive capabilities. The proposed predictive regression models were estimated using shrinkage methods discussed under Section 2. However, for purposes of benchmarking, the PCR was estimated.

Table 6 shows the estimation results using the LASSO, ENET, Adaptive LASSO, and Adaptive ENET algorithms and PCR on the training set. The MSEs shown on the table were computed based on the testing set. The LASSO and ENET algorithms at their optimal values of λ estimated 75 ($MSE = 0.16647$) and 13 ($MSE = 0.06255$) non-zero coefficients out of 128 respectively. When $\gamma = 1$, the Adaptive LASSO and Adaptive ENET algorithms at their optimal λ values gave respectively 8 ($MSE = 0.14628$) and 4 ($MSE = 0.04434$) non-zero coefficients out of 128. For the PCR, the minimum mean square error ($MSE = 0.04481$) was achieved with the first principal component in the regression. From a comparison of the MSEs of the various predictive regression models, it is obvious that Adaptive ENET model has the least MSE, followed

by PCR, ENET, Adaptive LASSO, and LASSO in that order. Clearly, the predictive power of Adaptive ENET model supersedes the rest of the models. Figures 12 to 16 (see Appendix C) display the graphs of out-of-sample comparison between the predicted values from each of the LASSO, ENET, Adaptive LASSO, Adaptive ENET, and PCR models to the $\ln(CIEAR_{real})$. A close observation of these figures reveals the superiority of the Adaptive ENET model relative to the other proposed models in terms of predictive accuracy.

A rigorous comparison between the proposed models (LASSO, ENET, Adaptive LASSO, and Adaptive ENET) and the benchmark model (PCR) using formal tests are displayed in Table 7. The $\Delta CV - MSE$ statistic for each comparison of LASSO, ENET, or Adaptive LASSO model to the PCR was negative. However, the sign of the $\Delta CV - MSE$ statistic for the comparison between Adaptive ENET and PCR was positive. These test results show that, the Adaptive ENET model has a better predictive power relative to the benchmark model. The LASSO, ENET, and Adaptive LASSO models underperformed relative to the benchmark model in terms of the ability to forecast.

Table 6: Estimated Non-Zero Coefficients

| Model | α | γ | $\lambda_{optimal}$ | Coefficients | MSE |
|----------------|----------|----------|---------------------|---------------------|------------|
| LASSO | 1.0 | - | 0.21227 | 75 | 0.16647 |
| ENET | 0.2 | - | 0.00038 | 13 | 0.06255 |
| Adaptive LASSO | 1.0 | 1.0 | 0.17027 | 8 | 0.14628 |
| Adaptive ENET | 0.7 | 1.0 | 0.16993 | 4 | 0.04434 |
| PCR | - | - | - | - | 0.04481 |

The DM tests show that there were statistically significant differences ($p - value < 0.01$) between the expected forecast performances of each of the LASSO, ENET, and Adaptive LASSO model to the benchmark model since the $p - values$ associated with these tests were less than 0.01 and all the DM statistics were negative. However, no statistically significant difference was observed between the expected forecast performance of the Adaptive ENET model and that of the PCR because the $p - value$ of the test was greater than 0.1 and the DM statistic was positive.

Table 7: Formal Tests for Model Comparisons

| Benchmark - Proposed Model | ΔCV -RMSE Statistic | DM-Test | |
|----------------------------|-----------------------------|---------|-----------------|
| | ΔCV -RMSE | DM | <i>p</i> -value |
| PCR - LASSO | -0.29555 | -8.1867 | 0.00000 |
| PCR - ENET | -0.05355 | -4.7108 | 0.00003 |
| PCR - Adaptive LASSO | -0.17079 | -2.4785 | 0.01802 |
| PCR - Adaptive ENET | 0.00059 | 0.0901 | 0.92870 |

6 Conclusions

Recent literature has amplified the significance of large information for forecasting and has recommended approaches based on factor models to handle problems with huge dimensionality. However, the fact remains that in statistics and econometrics, forecasting time series with a massive dimensional predictor space is an essential and stimulating problem.

This paper has studied a variety of shrinkage methods (LASSO, ENET, Adaptive LASSO, and Adaptive ENET algorithms) with a high dimensional predictor space to propose a more parsimonious predictive model for short term forecast of the future values for the CIEA in Ghana. In the modeling process, we have estimated the LASSO, ENET, Adaptive LASSO, Adaptive ENET, and the PCR models, and evaluated their out-of-sample predictive accuracies using the MSE metric. Results based on the MSE seem to suggest that the Adaptive ENET model has the greatest potential in accurately forecasting CIEA in Ghana, followed by PCR, ENET, Adaptive LASSO, and LASSO in that order.

A major finding in this paper was that, among the four proposed models, the Adaptive ENET model was the only one that outperformed the benchmark model (PCR) according to the $\Delta CV - MSE$ statistic. Using data science approaches for fashioning out forecasting models offer great tools for practicing statisticians and economists when dealing with intricate and high-dimensional economic and financial time series variables.

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Appendix A

Table 1: Definition of Variables

| NAME | DEFINTATION |
|---------------|--|
| CIEANom | Composite Index of Economic Activities (nominal) |
| CIEAReal | Composite Index of Economic Activities (real) |
| GSE-ASI | Ghana Stock Exchange All Share Index |
| INF-YOY | Overall Inflation |
| TBR-91 | 91 Day Treasury Bill Rate (%) |
| BCROIL | Brent Crude Oil |
| BNCG | Bank of Ghana Net Claims on Govt (GHC'm) |
| CIC | Currency in Circulation (GHC'm) |
| CITOB | Cash in the Tills of the Commercial Banks (GHC'm) |
| COB | Currency Outside Banks |
| CocoaP | Indicative Cocoa Prices: dollars/tonne |
| CPI- F | CPI-Food Index |
| CPI- NF | CPI-Non Food Index |
| CPI- O | CPI-Overall Index |
| DD | Demand Deposit (GHC'm) |
| FCD | Foreign Currency Deposit |
| GIR | Gross International Reserves (\$ million) |
| IBKEXRENDMUSD | Inter-Bank Exchange Rate End Month (GHC/US\$) |
| IBKEXRMAVEUSD | Inter-Bank Exchange Rates Monthly Average GHC/US\$ |
| IBKXEMAVEGBP | Inter-Bank Exchange Rates Monthly Average GHC/GBP |
| IBKXRAVEEURO | Inter-Bank Exchange Rates Monthly Aerage GHC/EURO |
| IBKXRENDMEURO | Inter-Bank Exchange Rates End Month GHC/EURO |
| IBKXRENDMGBP | Inter-Bank Exchange Rates End Month GHC/GBP |
| INF-F | Food Inflation |
| INF-NF | Non-Food Inflation |
| IntBkWAve | Inter-Bank Weighted Average |
| M1 | Narrow Money (GHC'm) |
| M2 | Broad Money (M2) (GHC'm) |
| M2+ | Total Liquidity (M2+)(GHC'm) |
| MPR | Monetary Policy Rate (%) |
| RM | Reserve Money (GHC'm) |
| TBR-91 day | 91 Day Treasury Bill Rate (%) |
| TotDep | Total Deposits |

Appendix B

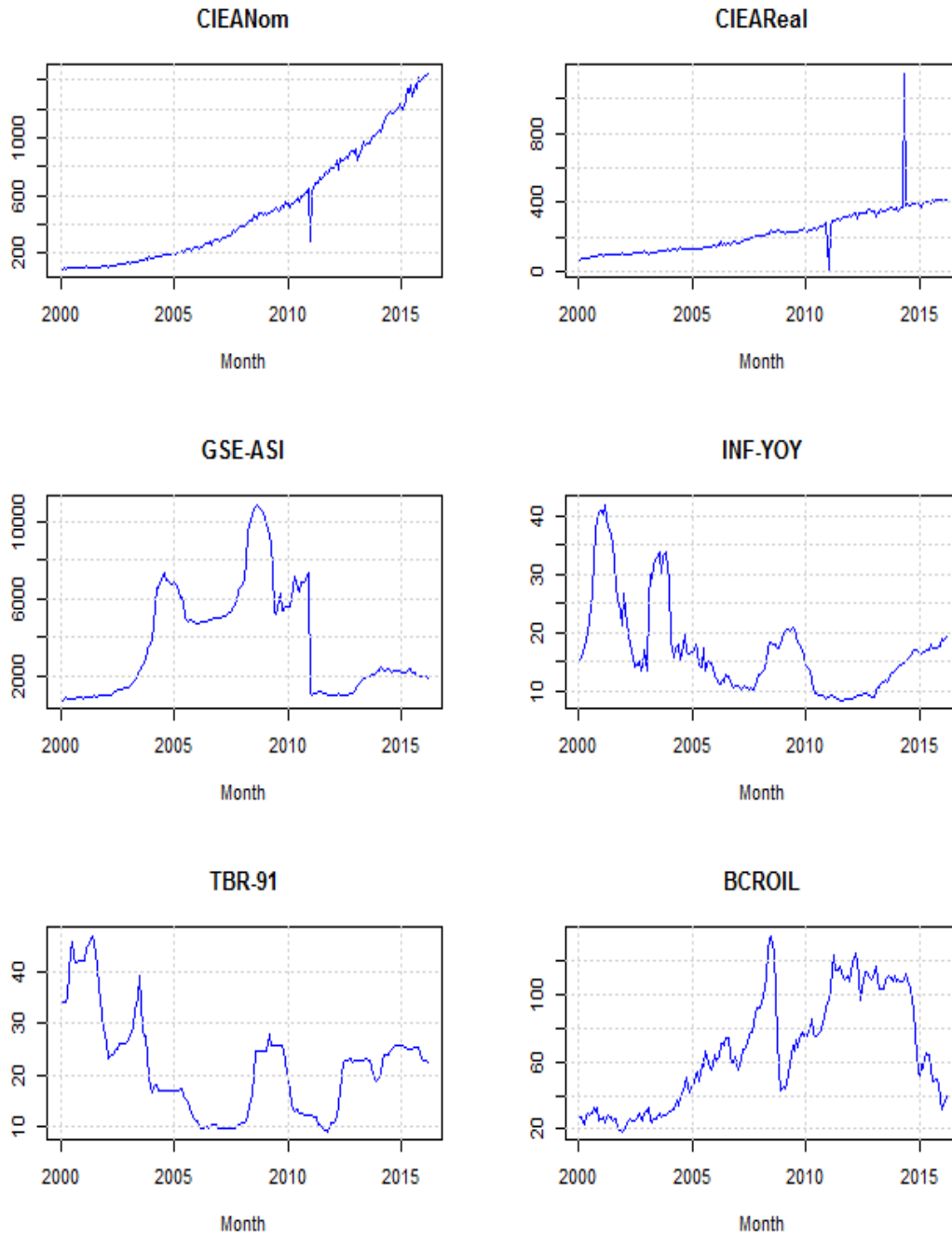


Figure 1: Time plot of the Original Variables

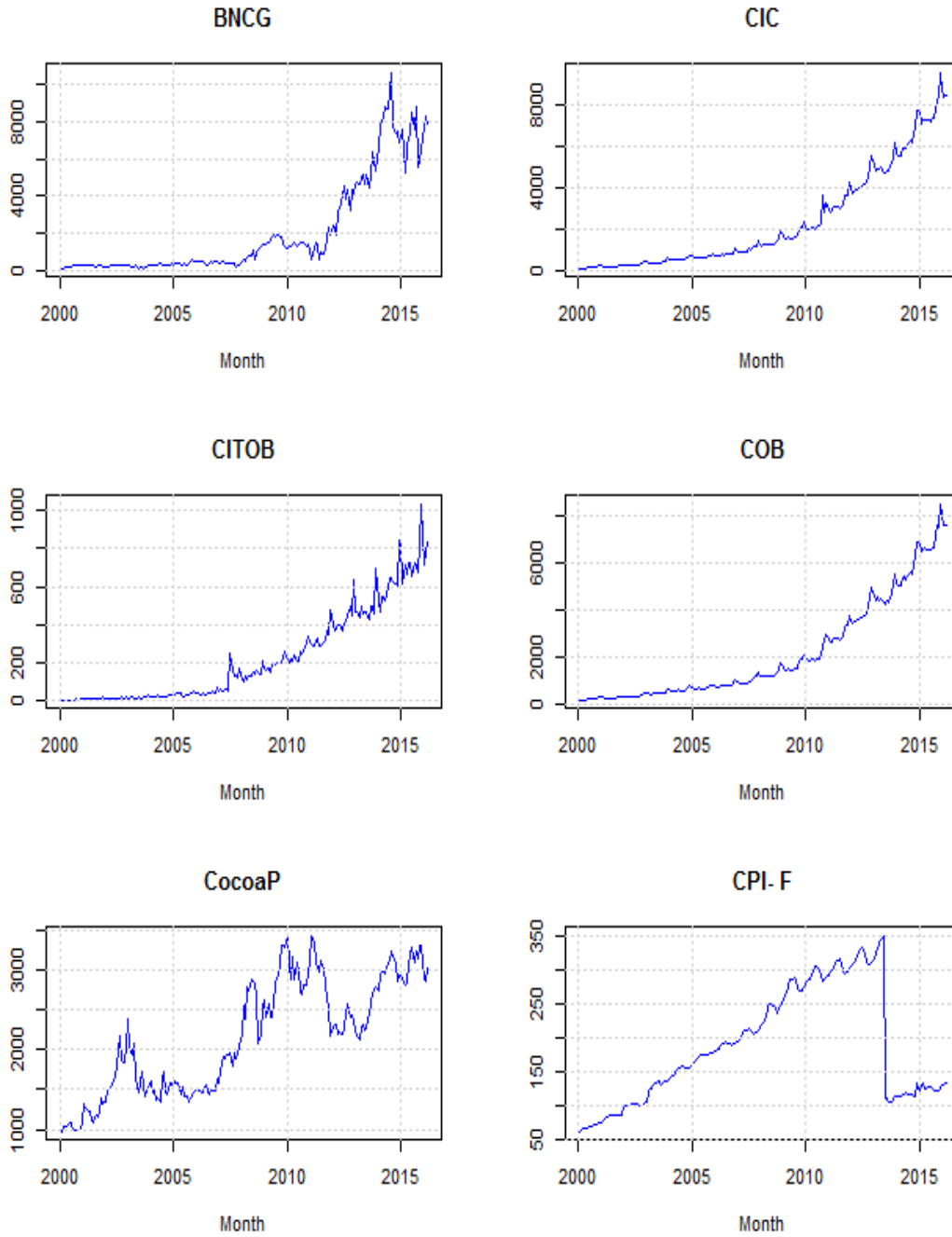


Figure 2: Time plot of the Original Variables

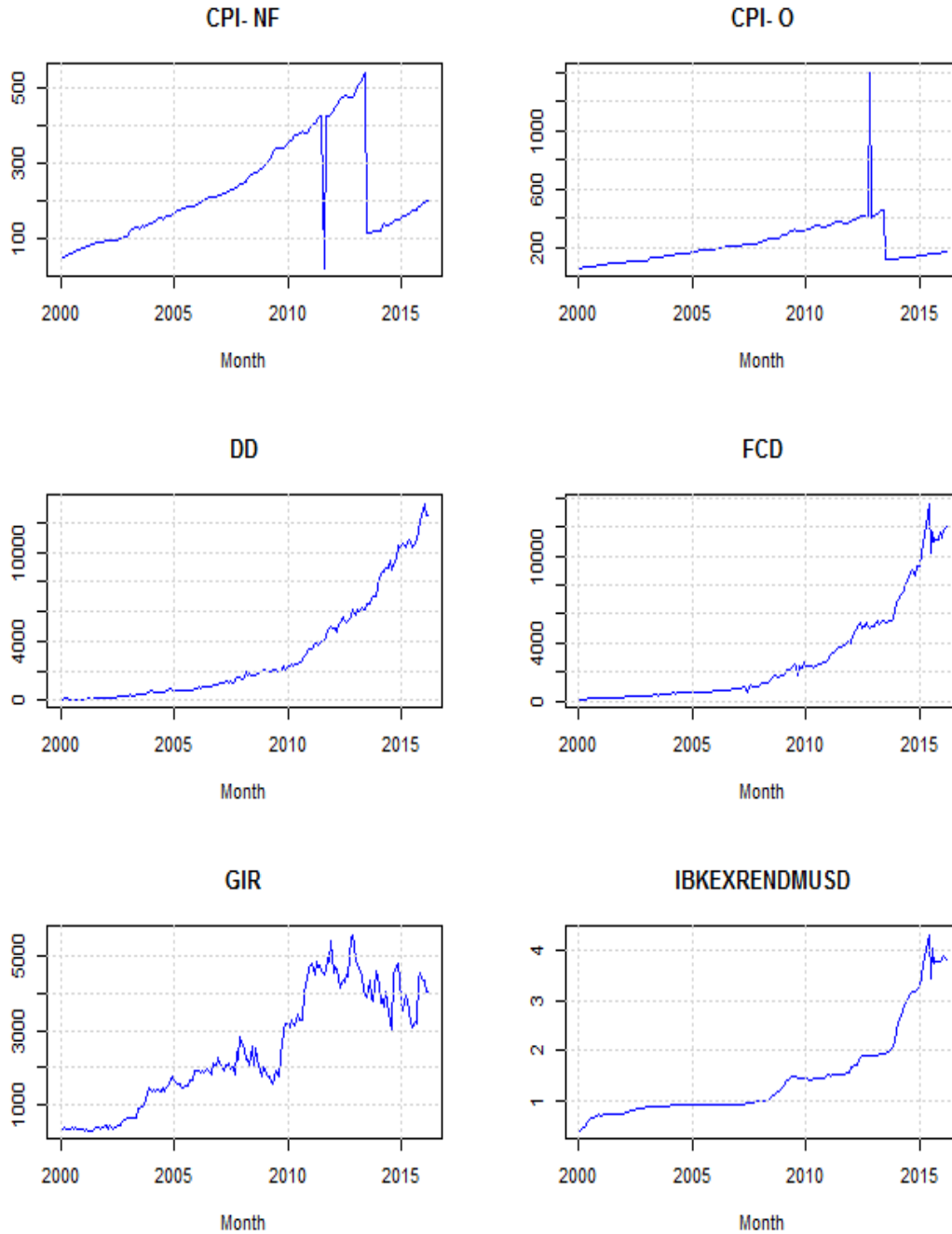


Figure 3: Time plot of the Original Variables

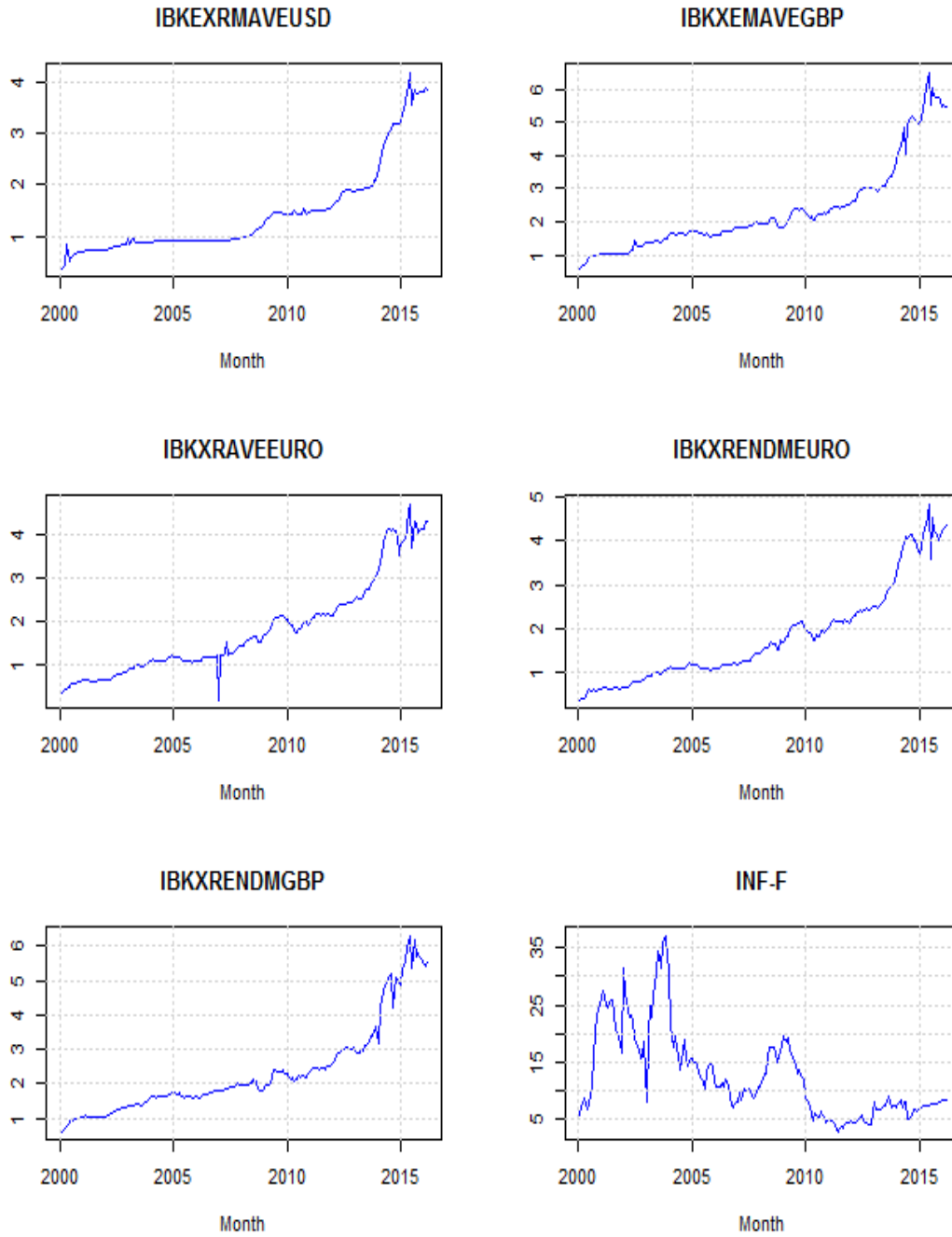


Figure 4: Time plot of the Original Variables

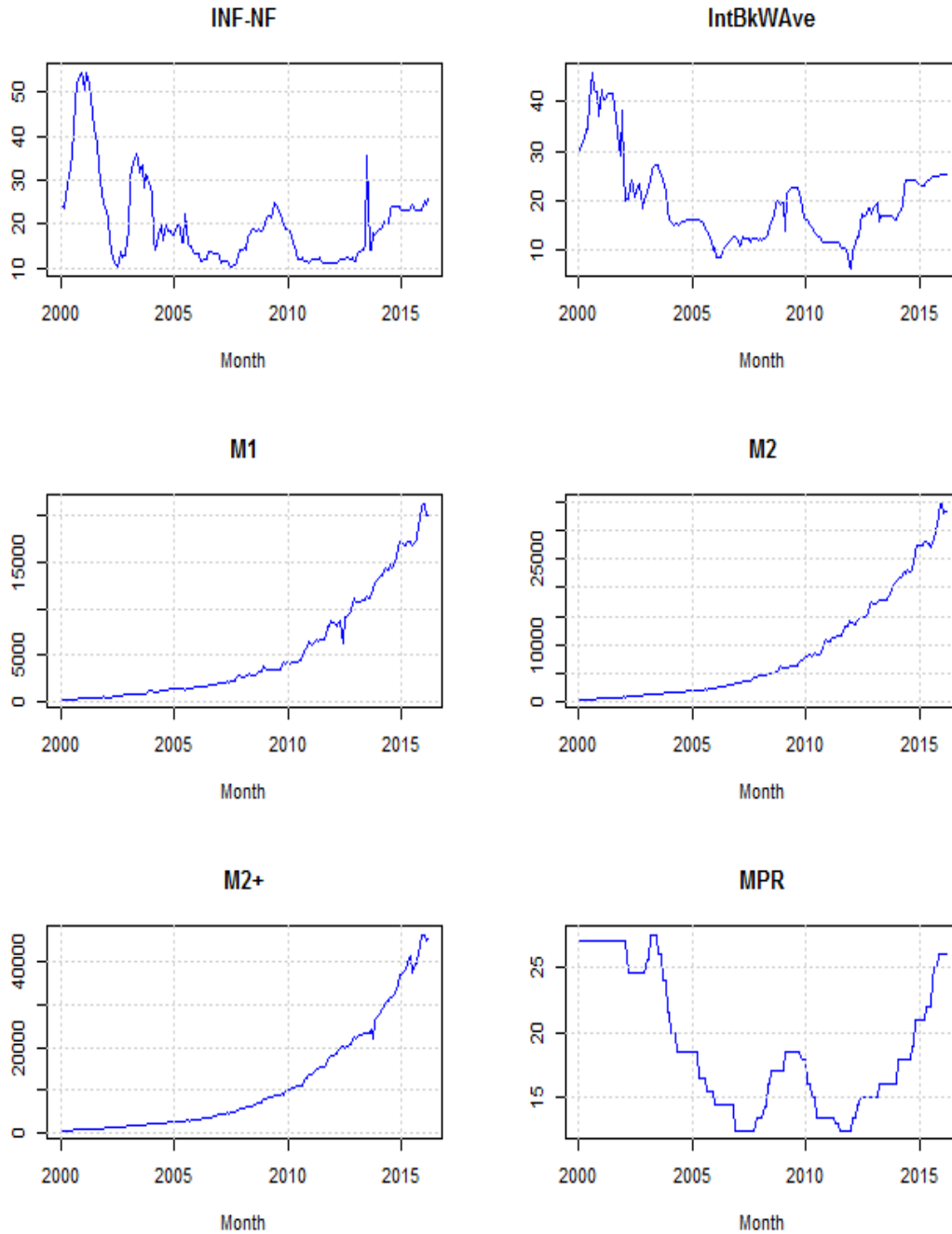


Figure 5: Time plot of the Original Variables

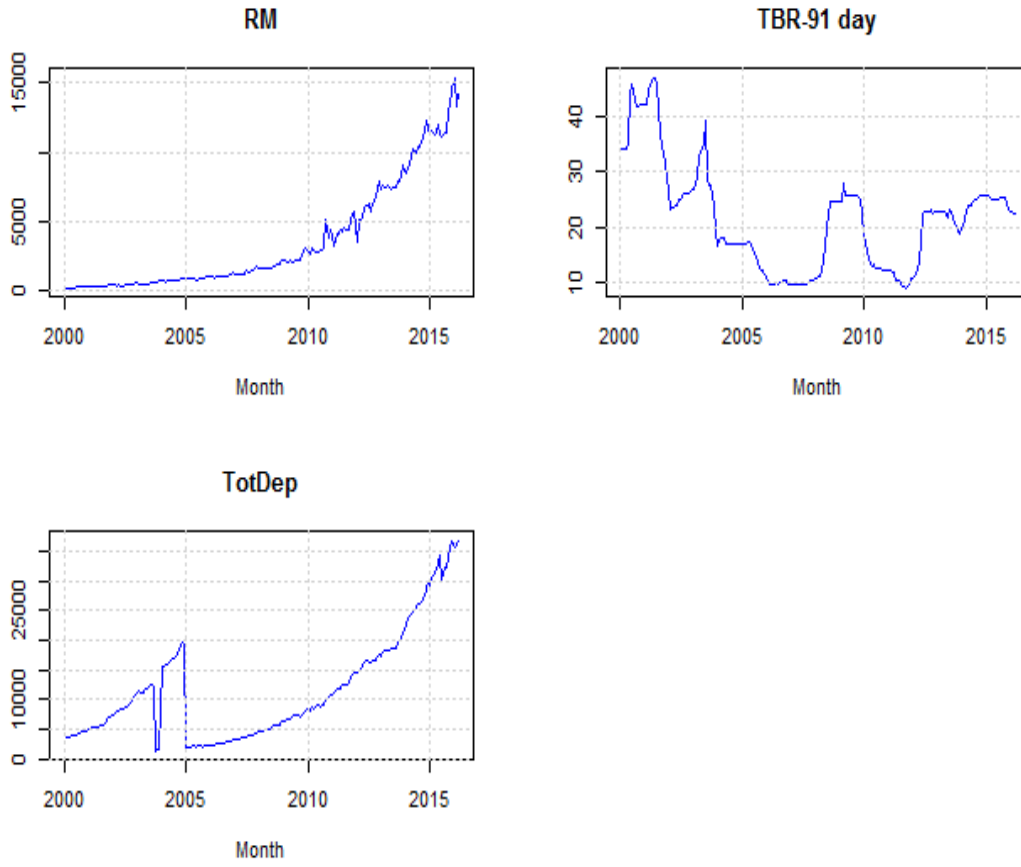


Figure 6: Time plot of the Original Variables

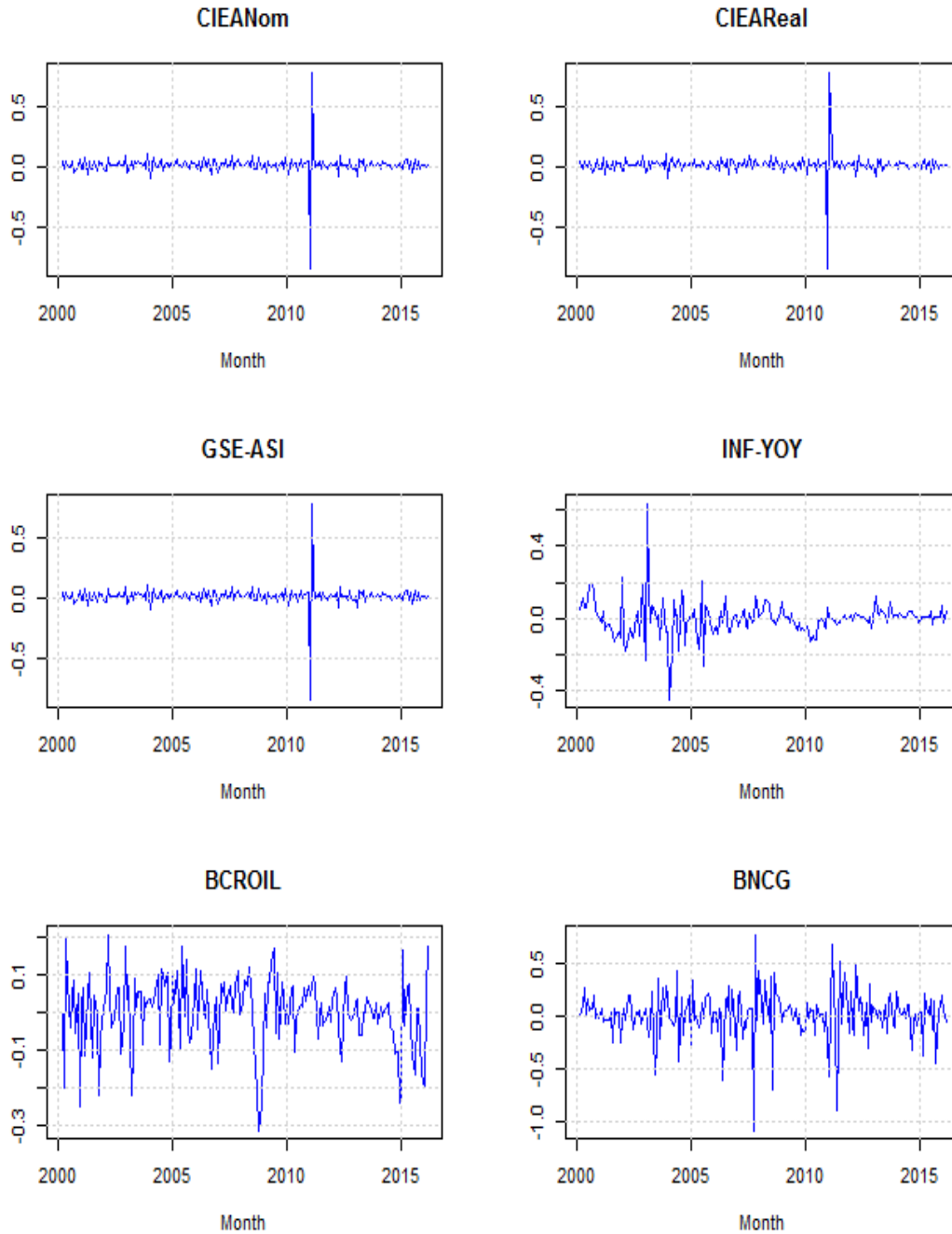


Figure 7: Time plot of the Transformed Variables

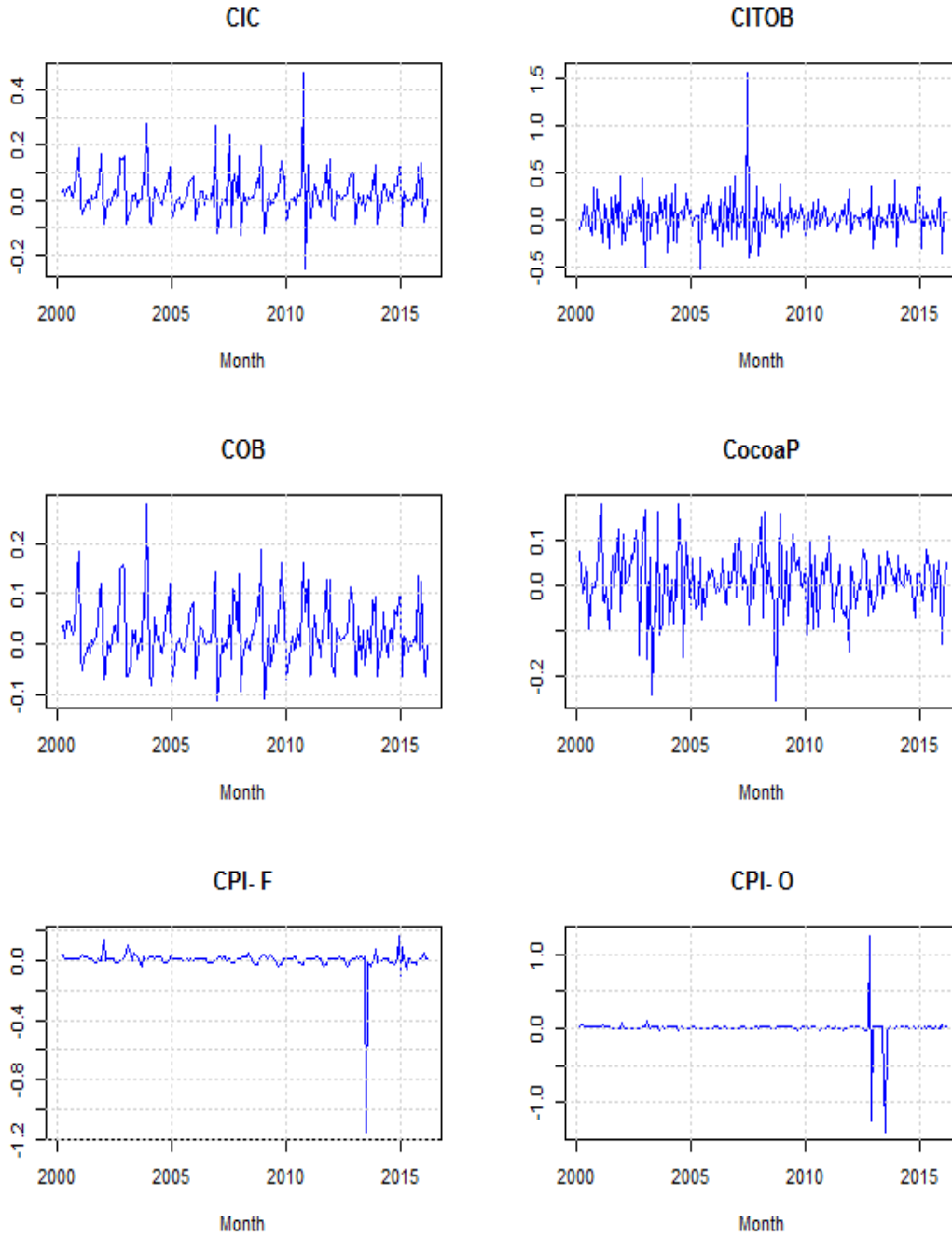


Figure 8: Time plot of the Transformed Variables

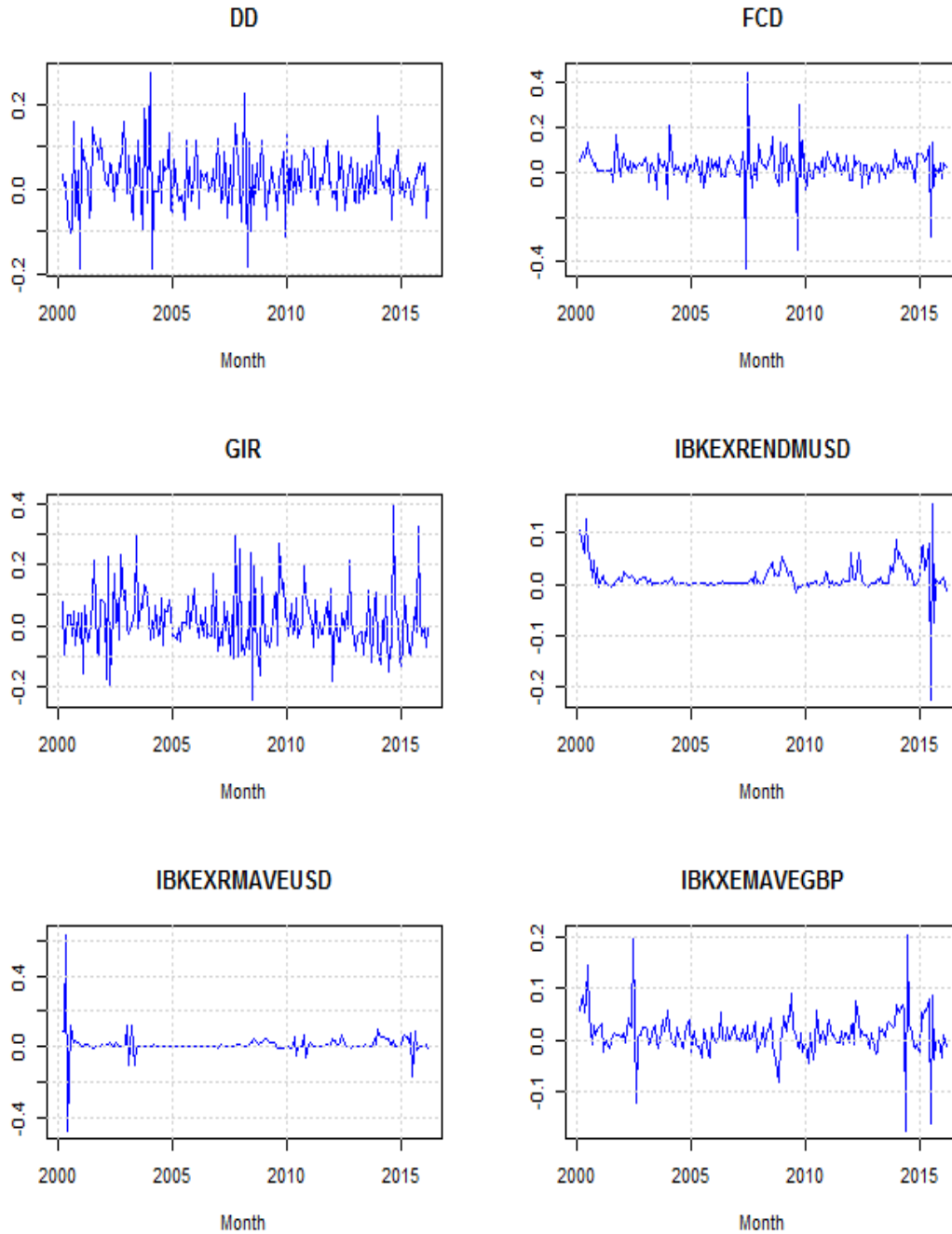


Figure 9: Time plot of the Transformed Variables

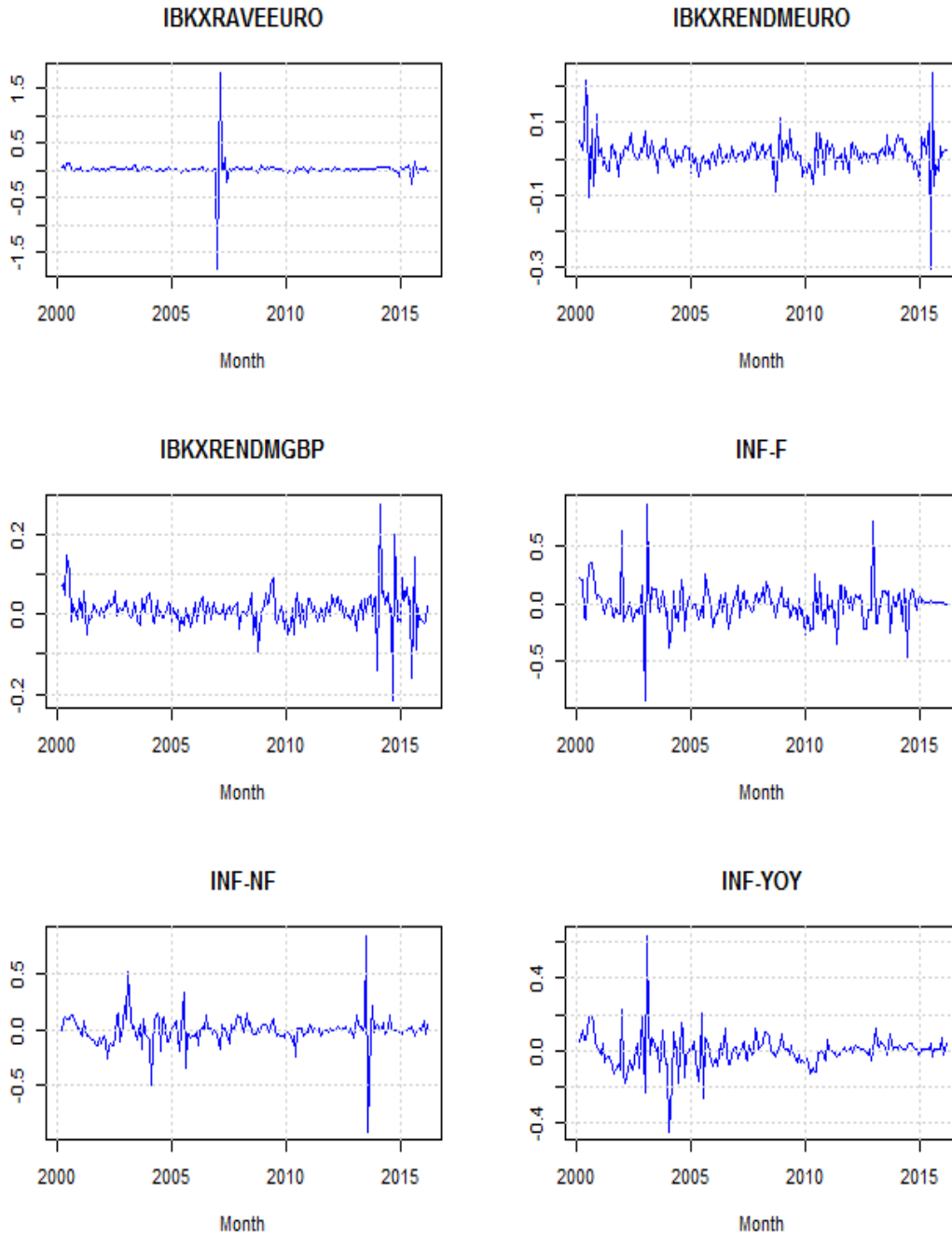


Figure 10: Time plot of the Transformed Variables

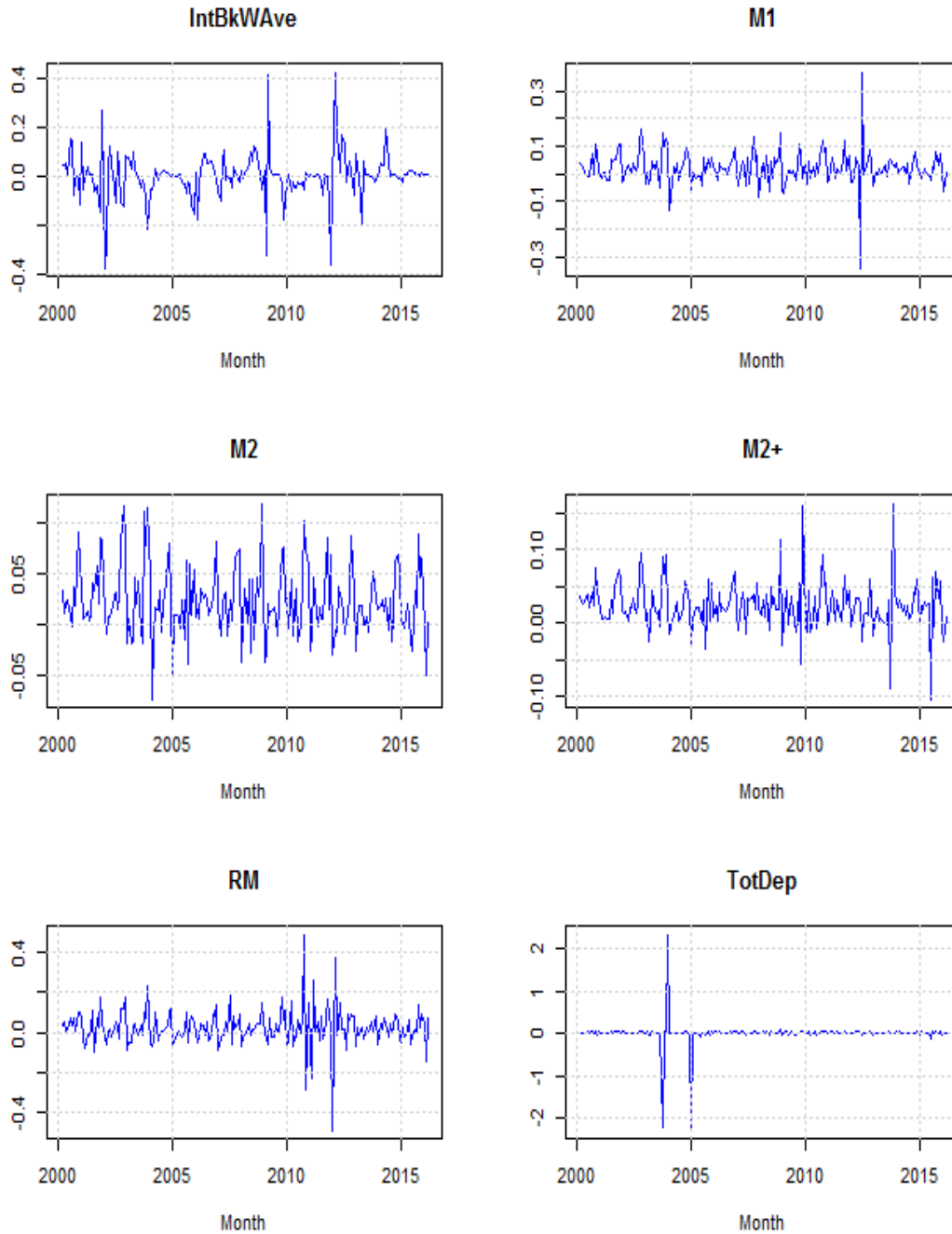


Figure 11: Time plot of the Transformed Variables

Appendix C

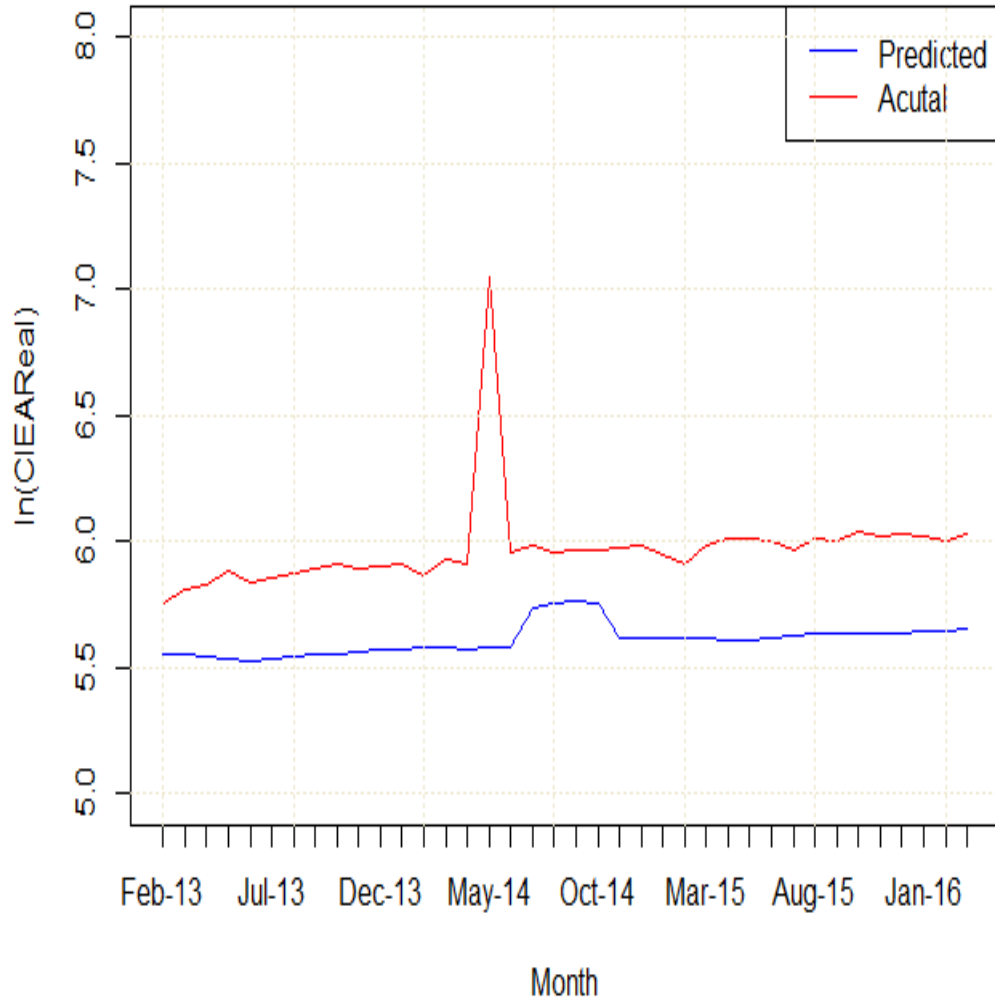


Figure 12: Out-of-sample comparison of predicted values from LASSO and $\ln(CIEAReal)$

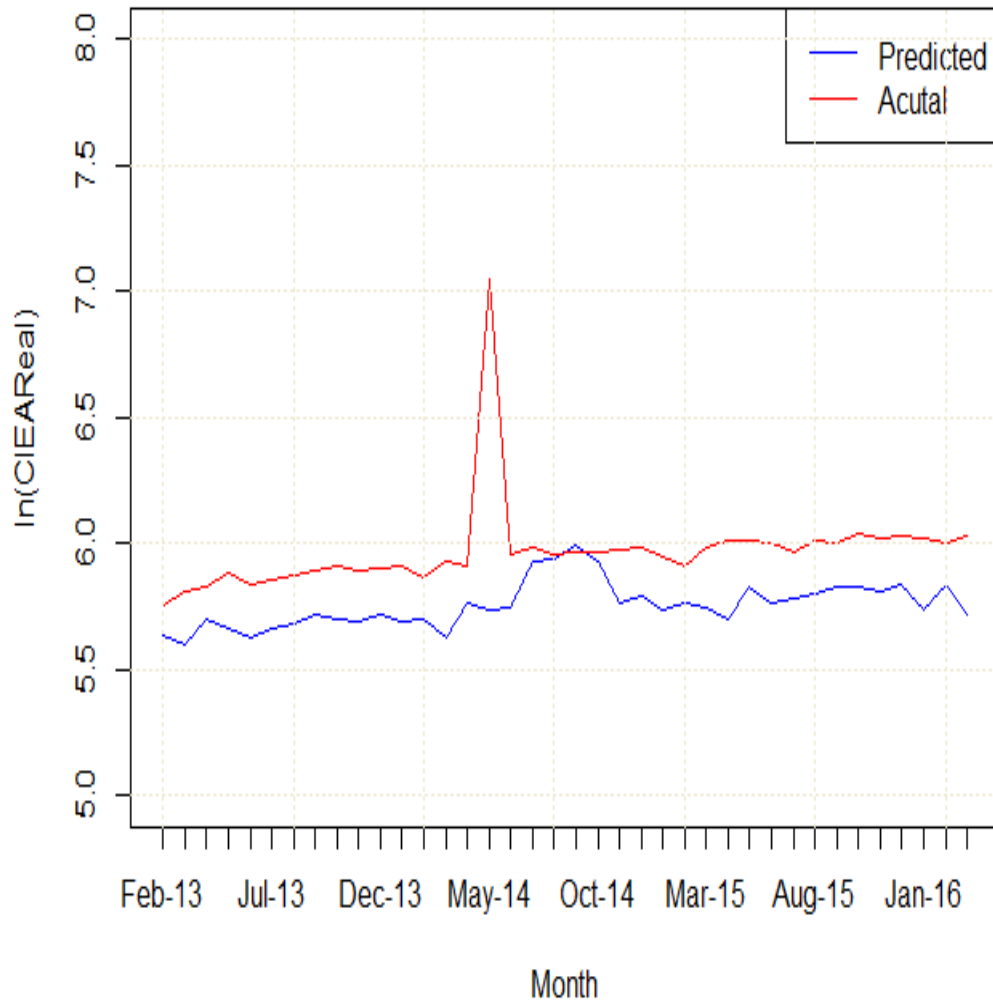


Figure 13: Out-of-sample comparison of predicted values from ENET and $\ln(CIEAR_{Real})$

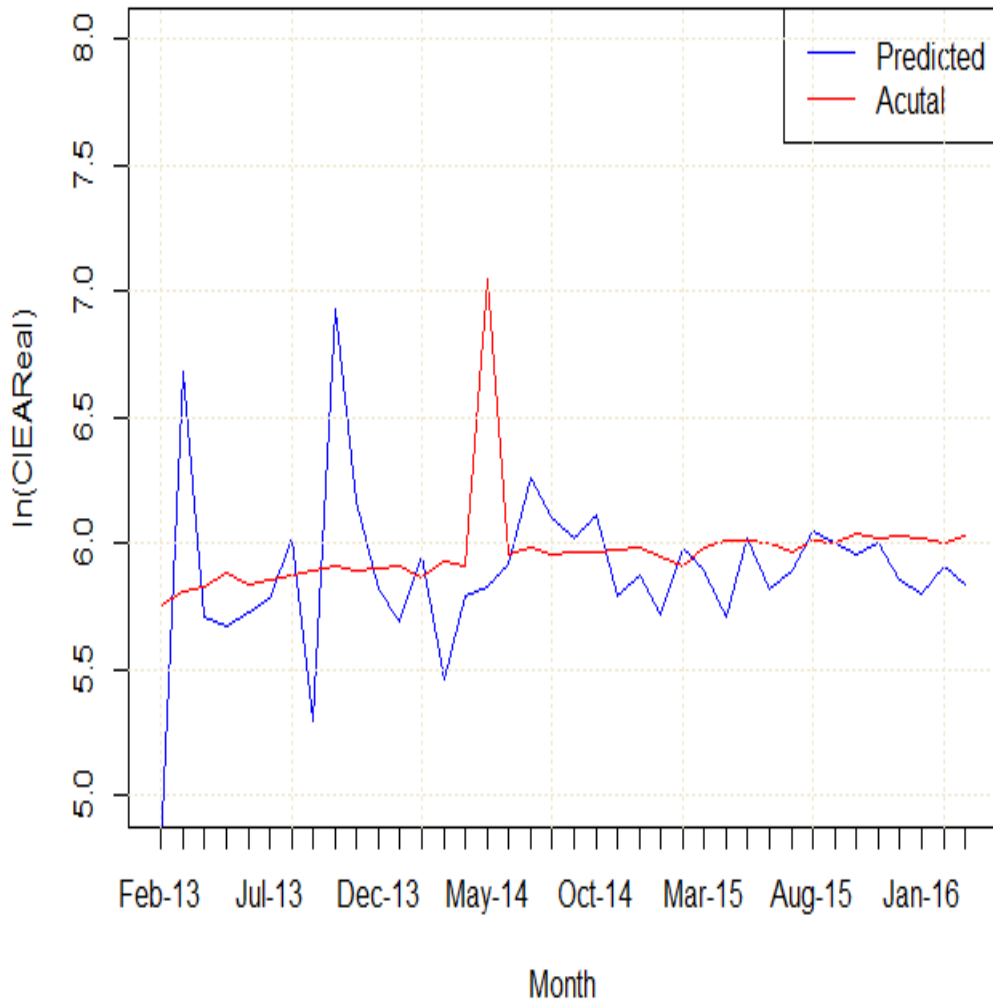


Figure 14: Out-of-sample comparison of predicted values from Adaptive LASSO and $\ln(CIEAR_{Real})$

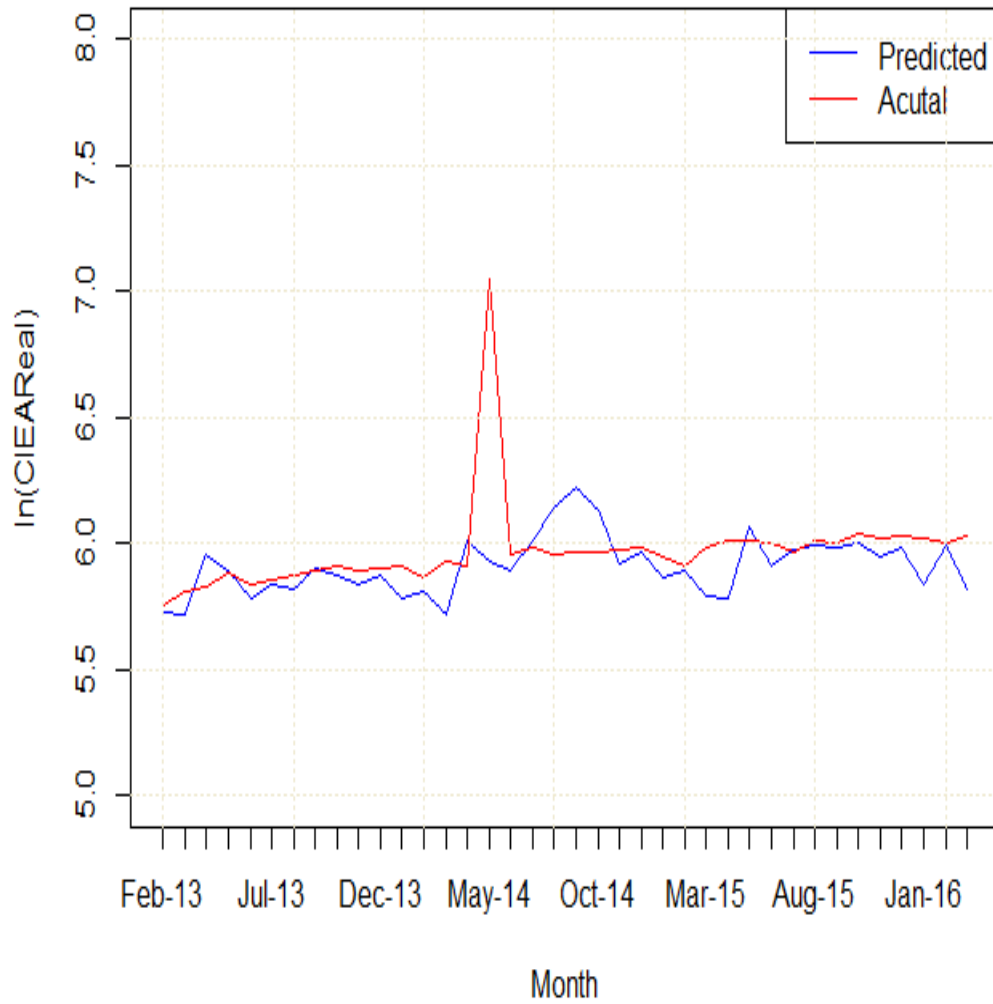


Figure 15: Out-of-sample comparison of predicted values from Adaptive ENET and $\ln(CIEAR_{Real})$

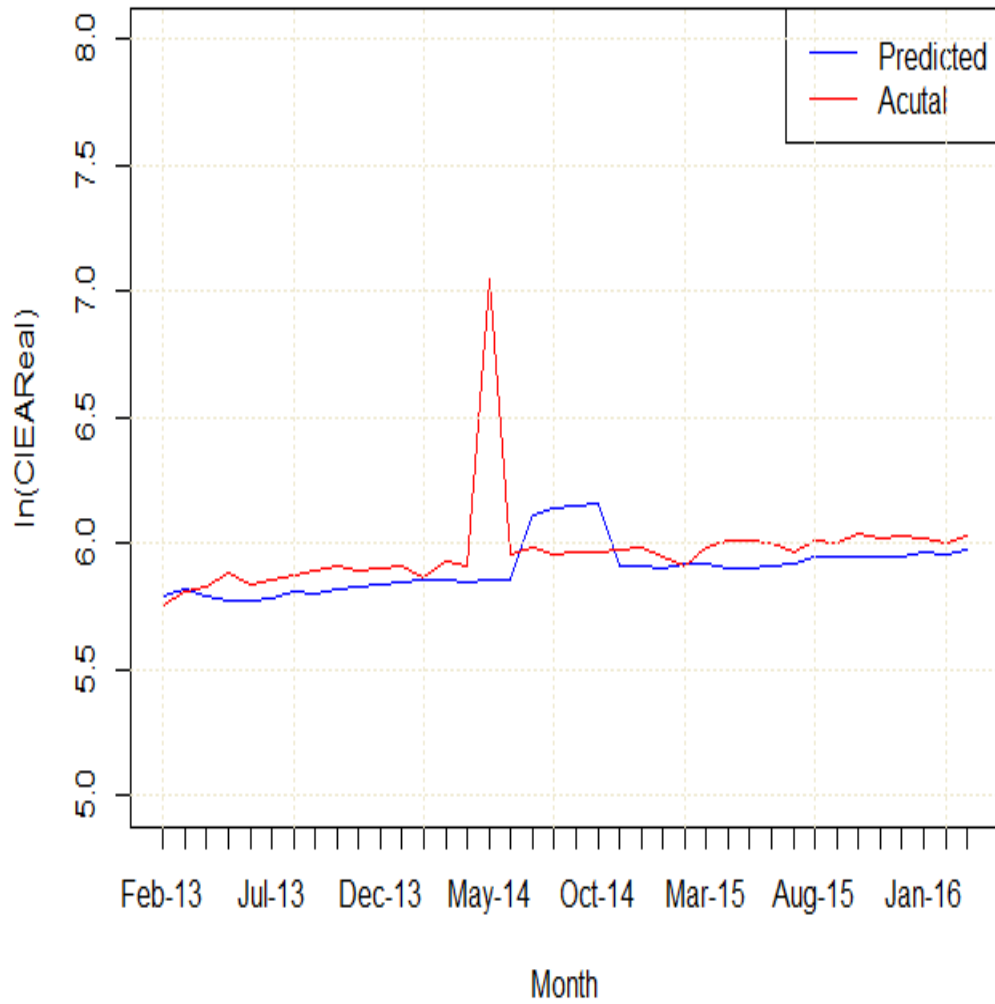


Figure 16: Out-of-sample comparison of predicted values from PCR and $\ln(CIEAR_{Real})$