

A Recommended Financial Model for the Selection of Safest portfolio by using Simulation and Optimization Techniques

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Abstract

Investment of portfolio known that there is an important level of uncertainty about the future worth of a portfolio. The concept of value at risk (VAR) has been used to help describe a portfolio's uncertainty. The current trend of investment in India is to invest in stock market which categorized as a high-risk level of investment. There are various methods to calculate the variance. Monte Carlo simulation method is one of the methods to calculate the VAR of the portfolio. Monte-Carlo simulation method is considered to be the optimization technique in which objective is to minimize/maximize the risk/profit before making any type of investment with portfolio. The Monte Carlo simulation method calculation for VAR of a portfolio can briefly be summarized in two steps. In the first step, a stochastic process is specified for financial variables. In the second step, financial variable of interest are simulated to get fictitious price path.

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The aim of the research is to develop the financial model for the safest portfolio selection based on VAR and Markowitz classical models. In the financial model, at first we measure the value at risk of Indian equity markets over short horizon of time (less than one year) by creating multiple scenarios by using Monte Carlo simulation. With the help of financial model, we rank measured values at risk by using statistical tools. Finally, financial model will suggest an optimal portfolio over the same horizon of time using a developed optimization model. A real case study was selected and introduced to find the safest allocation of a portfolio; eight of the most active share volume was selected to perform an analysis. The results obtained by financial model indicate that the reliability description of the portfolio's uncertainty and then gave highly reliable recommendation on portfolio optimization.

JEL classification numbers: C61, G17

Keywords: Monte Carlo Simulation, Value at risk, equity markets and Markowitz classical Mathematical.

1 Introduction

The current trend of investment in India is to invest in stock market which is considered as a high risk level of investment. There is an important level of uncertainty about the future worth of the portfolio. Recently, the concept of value at risk (VAR) is help to describe the; portfolio uncertainty. The VAR model allows to moderate rate risk of a given portfolio. It is very important to evaluate acceptable risk level when investment models are analyzed. For risk valuation different mathematical-statistical models are used too. But most models estimate risk separately from trade models and usually with already known trade results. Winston (2001) estimated value at risk of a portfolio at a future point in time is usually considered to be the fifth percentile of the loss in the portfolio's value at

that point.

There are various methods that are used to calculate the VAR. The Monte Carlo simulation (MCS) approach and the historical simulation approach are examples of full valuation methods. The delta-normal method is the best method to compute VAR for portfolio with linear position in which their distributions are close to the normal distribution. Kondapaneni (2005) uses Monte Carlo Simulation method to calculate the VAR of the portfolio.

The MCS is the stochastic process which is based on the random numbers and the probability density function. The MCS method has been used in different field of management like economics finance etc. The MCS method calculations of VAR of portfolio are divided into two parts. The first part, stochastic process is specified for a financial variable. In the second part, fictitious price path are simulated for all financial variables of interest Kondapaneni (2005).

The aim of the research is to develop the financial model for the safest portfolio selection based on VAR and Markowitz classical models. In the financial model, at first we measure the value at risk of Indian equity markets over short horizon of time (less than one year) by creating multiple scenarios by using Monte Carlo simulation. With the help of financial model, we rank measured values at risk by using statistical tools. Finally, financial model will suggest an optimal portfolio over the same horizon of time using a developed optimization model. A real case study was selected and introduced to find the safest allocation of a portfolio; eight of the most active share volume was selected to perform an analysis. The results obtained by financial model indicate that the reliability description of the portfolio's uncertainty and then gave highly reliable recommendation on portfolio optimization.

Value at risk methods are most advanced modern methods which allow to measure foreign currency rate risk. These methods encompass sensitivity and volatility measuring tools together with negative uncertainty influence measurement possibilities. Value at risk methods were started to be applied in financial

institutions and the big companies. In 1994 concept of value at risk was applied in J.P. Morgan created Credit Metrics methodology.

2 Literature Review

The process of selecting a portfolio may be divided into two stages. The first stage starts with observation of intended security or securities that one is interested in investing in them, so that security or securities can be understood which make one experience on them and ends with beliefs about the future performances of available securities. The second stage starts with relevant beliefs about the future performances and ends with the choice of portfolio Markowitz (1952).

According to Duffie and Pan (1997) VAR is becoming somewhat of a revolution. The new technology was adopted by the different organization. The VAR will give you the idea about the future performance of the stock. Many mathematical have been developed to select the optimal portfolio. Braun and Shioji (2001) developed a framework for identifying the sources of the big aggregate shocks that have buffed the Japanese households in the past decade. Because of that shocks the economic growth has been slowed, unemployment risk has been raised, and assets prices have been fallen to levels not seen since the early 1980's. In order to assess the sources of these shocks, they considered the perspective of a forwarded-looking risk-averse household and derive of the expected returns and time varying risk premia for each risk.

Goorbergh and Vlaar (1999) applied various VAR techniques to the Dutch stock Market index and to the Down Jones industrial average. They have concluded that emphasis-changing volatility over time is the most important characteristics of stock returns and for high confidence levels.

Sharpe (2004) has provided a proof of the assumption of a Lognormal Model for stock prices, where the stock prices at different time are strictly positive and

independent identically distributed. Markowitz (1952) was first to determine the minimum variance that yields a desired return in the 1950's for his work. Mathematically this methodology was defined by Duffie and Pan (1997).

3 Classification of VAR Financial model

Though existing VAR methods uses different methodologies, all of them have the same core structure which is defined by three features: 1) portfolio should be identified on the market; 2) it is necessary to estimate distribution of portfolio profitability; 3) calculation of VAR for portfolio.

The main difference between VAR methods is related with the second point, how they are solving possible portfolio value fluctuation measurement problems. One of the most commonly used VAR valuation methodologies is historical simulation. This method simplifies risk value calculation procedure because it does not require any distribution presumptions about portfolio profitability. Historical simulation is based on the selection of the random numbers. Then, profit of the portfolio is ranked in increasing manner and necessary quintile is presented according contiguous observation results. Calculation of next day VAR is moved by one observation and the procedure is repeated.

VAR model is a powerful tool for market risk valuation, but at the same time it is also a great challenge. All liquid assets have unspecified market values, what can be described by probability distribution functions. All risk sources come into these functions. Because VAR can be applied for all liquid assets and theoretically encompassing all the risk sources. In order to evaluate market risk of portfolio using VAR market value locations probability distribution has to be defined.

4 Research Methodologies

The present study is divided into three stages. First stage is to measuring the Value at Risk (VAR) for any given company by using Monte Carlo simulation (MCS). In the second part is to rank and select the certain equalities by designing a statistical analysis tool to measuring the significant difference between the mean of value at risk. In the third part we use to identify the optimal allocation of the portfolio by developing optimization model.

In the research, that may affect the expected price of stock are current stock price, expected return, stock volatility, and the stock holding time. Thus, the expected future price can be represented as follows which is adopted from Winston (2001):

$$S_t = S_0 e^{(\mu - 0.5\sigma^2)t + \sigma\sqrt{t}\varepsilon}$$

However, measuring VAR equalities on this study are based on several assumptions are as follows:

- (a) The study is on the equity market prices, specifications on closing prices. The securities prices reflect all information and expectations that is proved by Efficient Market Theory.
- (b) The measure of VAR is merely on the left tail since the investors is apprehensive of only the unexpected losses. In addition, the confidence level is fixed at 95% which makes level of confidence to be 5%.
- (c) The varying in stock prices is following Lognormal Distribution due to huge change in small period Winston (2001).
- (d) Montgomery (1999), is expected outcome scenarios should follow the Normal Distribution.
- (e) The portfolio's holding period is assumed to be short because of Lognormal Distribution assumption of stack volatility, less than one year, specification 3 months.

In the second stage, hypothesis is stated to measure the mean significant difference of the resulted VAR's. The hypothesis will be tested by using Analysis

of Variance (ANOVA). However, the stated hypotheses are as follows:

$$H_0 : v_1 = v_2 = \dots = v_n \quad \text{and} \quad H_1 : v_1 \neq v_2 \neq \dots \neq v_n$$

In the third step, the classical mathematical model that developed by Markowitz (1952) will be used to allocate the safest portfolio. The model objective is to minimize the risk subjected to accepted return, where the risk can be measured by the standard deviation and the return can be measured by calculating the average mean of the return.

The first step in formulating and optimization model is to chosen the appropriate indices for the different dimension of the problem. In these researches there are two parallel dimensions which are as follows:

i = Fund Number I ($i = 1, 2, \dots, 25$) and j = Fund Number j ($j = 1, 2, \dots, 25$)

The decision variables can be defined into two parallel categories based on how much of cash need to be deposited in every fund as follows:

$$x_i = \text{Portfolio allocation of fund } i \quad \text{and} \quad x_j = \text{Portfolio allocation of fund } j$$

Moreover, all input parameters that rae treated as constant have been assigned the following notations:

B : the total budget that is invested in portfolio

α : The percent of minimum return demanded by the particular investor.

U : the upper bound that the portfolio allocation in fund I cannot be exceeded.

μ_1 : Average return/profit in fund over the entire period T.

T : Entire period = three years

t : evaluation period

x_{it} : Return invested in fund I over period t and j over period.

σ_{ij} : Covariance of these funds

We formulate the model by using the above decision variables. The objective function and constraints of the optimal allocation of the portfolio as follows:

$$\text{Min} \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j$$

s.t

(i) The minimum returns demand:

$$\sum_{i=1}^n \mu_i x_i \geq \alpha B$$

(ii) The budget constrain:

$$\sum_{i=1}^n x_i = B$$

(iii) The upper limit constraint

$$x_i \leq u_i B$$

(iv) Nonnegative constrain:

$$x_i \geq 0$$

5 Conclusions

In this section, a case study is established on actual data. Assume that an investor wants to construct a safe portfolio. Then we consider nine of the most active share volume BSE Index. These ten shares are Dabur, L & T InfoTech, RIL, Patni, Dell Hex aware, ICICI, HDFC, 3i InfoTech and Accel Tree software Ltd. Base on the historical data over the last three years VAR's are measured for the selected shares for holding period of one year. Table 1. Shows the VAR's calculation based on Monte Carlo simulation methodology.

Based on the above results obtained for VAR, the experiments are designed. The designed experiments are having two factors. The first factor is the estimated VAR of the shares and it has nine levels that are Dabur, L & T InfoTech, RIL, Patni, Dell Hex aware, ICICI, HDFC, 3i InfoTech and Accel Tree software Ltd. The second factor is blocked and it is the replication and it has five levels that are the number of observations. The means that 50 experiments were performed.

Table1. VAR's for selected shares

Shares name	Calculated VAR's			
Dabur	-0.056	-0.064	-0.070	-0.065
L & T Info Tech	-0.123	-0.112	-0.127	-0.116
RIL	-0.046	-0.055	-0.045	-0.042
Patni	-0.176	-0.175	0.044	-0.186
Dell	0.048	0.045	-0.280	0.047
ICICI	-0.280	-0.376	0.042	-0.279
HDFC	-0.325	0.045	-0.453	0.043
3i InfoTech	-0.342	-0.344	-0.245	-0.346
Accel Tree Software Ltd	0.548	-0.239	0.568	-0.239

However, using SPSS software the hypothesis was tested and the ANOVA table was constructed. Table 2. Shows the ANOVA table the significant difference of VAR's values. The variance-Covariance matrix are calculated from the historical data for the last three years and given in Table 3.

Suppose that the invested amount is \$100,000.00 demands a daily return at least 22% (or \$22,000.00) and wish that no share will receive than 50% of his budget that is at most 50,000.00. The problem is to minimize risk, by neglecting the risk-free interest rate, and not allowing short selling. We start formulating the model:

The objective function is to minimize the above variance-covariance matrix using the following objective function:

$$\begin{aligned}
& 0.000699x_1^2 + 0.000143x_1x_2 + 0.000152x_1x_3 + 0.000145x_1x_4 + 0.000112x_1x_5 \\
& + 0.000243x_2^2 + 0.000561x_2x_3 + 0.000124x_2x_4 + 0.00789x_2x_5 + 0.000768x_3^2 \\
& + 0.000895x_3x_4 + 0.000164x_3x_5 + 0.0000455x_4^2 + 0.000446x_4x_5 + 0.000789x_5^2
\end{aligned}$$

Subject to the budget constraint as follows:

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 100,000$$

Subject to the return demand constraint as follows:

$$0.07365x_1 + 0.026645x_2 + 0.1256x_3 + 0.1879x_4 + 0.1789x_5 \geq 22,000$$

Subject to the following lower and upper limits of the shares:

$$x_1 \leq 50,000,$$

$$x_2 \leq 50,000,$$

$$x_3 \leq 50,000,$$

$$x_4 \leq 50,000,$$

$$x_5 \leq 50,000,$$

$$x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0, \quad x_5 \geq 0$$

By software QM+, provides the following solution

$$x_1 = 22289.15; x_2 = 2852.844; x_3 = 17897.45; x_4 = 1079.36; x_5 = 1255.782$$

Table 2. ANOVA

Source of Variation	Sum of squares	Degree of freedom	Mean square	F
Corrected model	3.607	8	0.390	37.915
Intercept	0.481	1	0.458	49.126
VAR factor	2.706	9		
Replication Factor (Blocked)	0.0366	4	0.0065	
Error	0.306	40	0.0764	
Total	3.285	50		
Corrected Total	2.156	49		

Table 3. Variance-Covariance matrix

Selected Shares	Dabur	L & T Info Tech	RIL	Patni	Dell
Dabur	0.000699	0.000143	0.000152	0.000145	0.000112
L & T Info Tech	0.000243	0.000561	0.000124	0.000789	0.000455
RIL	0.000127	0.000456	0.000789	0.000895	0.000164
Patni	0.000176	0.000666	0.000488	0.000455	0.000446
Dell	0.000121	0.000745	0.000145	0.000566	0.000789

6 Conclusions

The research paper tried to prepare the combination of VAR and financial model of portfolio selection. The portfolio theory stated that the process of selecting a portfolio may be divided into two stages. The first stage starts with the observation based on the intended securities that one is interested in investing in the securities and which observe the further performance of the choice of portfolio.

A real case study was introduced to find the safest allocation of the portfolio. Nine of the most active shares volumes at BSE index were selected to perform the case study. A historical data of three years were collected and VAR's were measured by implementing MCS technique. Then hypothesis stated and significant test was performed utilizing ANOVA table. Next, the mathematical model is constructed to allocate the invested amount. The recommendation allocation was to invest 22289.15 in Dabur, 2852.844 in L & T InfoTech, 17897.45 in RIL, 1079.36 in Patni and 1555.782 in Dell and keep the rest of as liquid cash.

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