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Reliability modeling of multistate systems: advances and applications

Ioannis S. Triantafyllou[1]

Abstract

In the present article we study multistate **k**-out-of-n structures composed of n independent mixed components ($k = (k_1, k_2, \ldots, k_\rho)$). We assume that, at time $t = 0$, n_i components are in state i, where $i = 1, 2, \ldots, \rho$. Such a situation occurs, whenever a certain number of components is supplied from a used system. The design parameters $k_1, k_2, \ldots, k_\rho$ are strongly connected to the systemPersonName's lifetime, since the state that the system is considered to be, depends on them. Closed formulae for the reliability function and the mean lifetime of the aforementioned structures are derived under the assumption that the components' lifetimes distributions belong to the family of Lehmann alternatives. Several numerical results are displayed and some interesting remarks are also concluded.

Mathematics Subject Classification: 62E15; 60K10; 62E15; 62P30
Keywords: Three-state systems; k-out-of-n systems; survival function; Lehmann alternatives; mean system lifetime

[1] Department of Statistics and Insurance Science, University of Piraeus,
80 Karaoli and Dimitriou str., 18534 Piraeus, Greece.
E-mail: itrantal@unipi.gr

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1 Introduction

In a multistate system modeling, both the system and its components are allowed to stand on more than two possible states, such as perfect functioning, partially working or complete failure state. Multistate reliability models have attracted a great research interest, because of their great variety of applications in many areas. For example, multistate models are useful in describing several engineering structures, such as oil supply systems, lightning grids or manufacturing apparatus, while many military equipments, such as missiles of high technology, are formulated in terms of multistate structures.

The well-known k-out-of-n: G system assumes that both the system and its components can stand on only two possible states, while in the multi-state modeling more than two states are available. Many generalizations of the aforementioned binary reliability structure have been proposed in the literature. Huang *et al.* [4] and Tian *et al.* [6] introduced generalized multi-state k-out-of-n: G systems considering a different number of components at each possible state at time $t = 0$. In addition, Levitin [5] brought in a new general model, named the multi-state vector k-out-of-n system and suggested an algorithm for evaluating its state probabilities. Moreover, Eryilmaz *et al.* [1] deal with three-state k-out-of-n systems composed of mixed components.

Consider a system consisting of n non-repairable components such that at time $t = 0$ n_1 components are in a partially working condition (state "1") and n_2 components are perfectly functioning (state "2") where $n = n_1 + n_2$ (hereafter, state "0" will be used to describe the non-functional (failed) state of the components). It is assumed that a direct transition from state "2" to state "0" is not feasible, while the transition from state "1" to state "2" is not attainable. Note that the components of the same type are identical. That is, the components that are in a perfectly functioning state and the components that are in a partially working state at the beginning of the mission are identical.

Let T_{1i} and T_{2i} denote respectively the time that the $i-th$ component enters into state "0" (lifetime of the $i-th$ component) and the time that the $i-th$ component enters into state "1" (lifetime of the $i-th$ component in state "2"), $i = 1, 2, ..., n$. If X_i expresses the time spent by the $i-th$ component in a partially working state, $i = 1, 2, ..., n.$, we may write

$$T_{1i} = \begin{cases} X_i, & i = 1, 2, ..., n_1 \\ T_{2i} + X_i, & i = n_1 + 1, n_1 + 2, ..., n. \end{cases}$$

In the present paper, we consider a three-state **k**-out-of-n system under the aforementioned modeling, with independent components. According to the definition given by Tian *et al* [7] , a three-state **k**-out-of-n system is in state "1" or above if at least k_1 components are in state "1" or above. Moreover, the system is said to be in state "2" if at least k_1 components are in state "1" or above and at least k_2 components are in state "2". It should be noted that the system parameter **k** involves both k_1 and k_2. Reliability evaluation of such systems has been considered by Eryilmaz *et al.* [1] , Zuo and Tian [9], Tian *et al* [6], Zhao and Cui [8] and Eryilmaz [8].

The lifetime or equivalently the time that is spent by the three-state **k**-out-of-n system in state "1" or above, can be represented as

$$T_1 = T_{1,n-k_1+1:n}, \tag{1}$$

where $T_{1,r:n}$ is the $r-th$ smallest among $T_{11}, T_{12}, ..., T_{1n}$. The time spent by the system in the perfect functioning state is given as follows

$$T_2 = \min(T_{1,n-k_1+1:n}, T_{2,n_2-k_2+1:n_2}), \tag{2}$$

where $T_{2,r:n_2}$ is the $r-th$ smallest among $T_{21}, T_{22}, ..., T_{2n_2}$.

2 General results

Let us denote by

$$F_1(t;1) = P(T_{1i} \leq t), \ \ i = n_1 + 1, n_1 + 2, ..., n_1 + n_2$$

$$F_2(t;1) = P(T_{1i} \leq t), \ \ i = 1, 2, ..., n_1$$

and

$$F_1(t;2) = P(T_{2i} \leq t), \ \ i = n_1 + 1, n_1 + 2, ..., n$$

the cumulative density functions of the random variables T_{1i} and T_{2i}. Let us next introduce the following notation

$$\begin{pmatrix} x \\ x_1, x_2, x_3 \end{pmatrix} = \frac{x!}{x_1!x_2!x_3!}, \tag{3}$$

where x, x_1, x_2, x_3 are non-negative integers provided that $x = x_1 + x_2 + x_3$ (otherwise $\begin{pmatrix} x \\ x_1, x_2, x_3 \end{pmatrix} = 0$).

The following proposition offers some expressions for the survival function of the lifetime of the three-state **k**-out-of-n system, that will be proved useful in the sequel.

Proposition 2.1 *For the three-state* $\boldsymbol{k}$*-out-of-n system consisting of* n_1 *components in a partially working state "1" and* n_2 *components in a perfect functioning state* ($n = n_1 + n_2$) *at time* $t = 0$*, the survival function of the lifetime of the system can be expressed as follows*

(i)

$$P(T_1 > t) = \sum_{i=k_1}^{n} \sum_{m=\max(0,i-n_1)}^{\min(n_2,i)} \sum_{r=0}^{m} \sum_{s=0}^{i-m} (-1)^{r+s} \begin{pmatrix} n_2 \\ r, m-r, n_2-m \end{pmatrix} \begin{pmatrix} n_1 \\ s, i-m-s, n_1-i+m \end{pmatrix} \times (F_1(t;1))^{n_2-m+r} (F_2(t;1))^{n_1-i+m+s} \quad (4)$$

(ii)

$$P(T_1 > t) = \sum_{i=k_1}^{n} \sum_{m=\max(0,i-n_1)}^{\min(n_2,i)} \sum_{q=0}^{n_2-m} \sum_{p=0}^{n_1-i+m} (-1)^{p+q} \begin{pmatrix} n_2 \\ q, m, n_2-q-m \end{pmatrix} \begin{pmatrix} n_1 \\ p, i-m, n_1-p-i+m \end{pmatrix} \times (\bar{F}_1(t;1))^{m+q} (\bar{F}_2(t;1))^{i-m+p} \quad (5)$$

where $\bar{F}_i(t;1) = 1 - F_i(t;1)$, $i = 1, 2$.

Proof.(i) Eryilmaz *et al.* [1] proved that the survival function of the lifetime of the three-state **k**-out-of-n system consisting of n_1 components in state "1" and n_2 components in state "2" can be expressed as follows

$$P(T_1 > t) = \sum_{i=k_1}^{n} \sum_{m=\max(0,i-n_1)}^{\min(n_2,i)} \begin{pmatrix} n_2 \\ m \end{pmatrix} \begin{pmatrix} n_1 \\ i-m \end{pmatrix} (\bar{F}_1(t;1))^{m} \times (F_1(t;1))^{n_2-m} (\bar{F}_2(t;1))^{i-m} (F_2(t;1))^{n_1-i+m}. \quad (6)$$

Since the following binomial expansions hold true

$$(\bar{F}_1(t;1))^m = \sum_{r=0}^{m} \binom{m}{r} (-1)^r (F_1(t;1))^r, (\bar{F}_2(t;1))^{i-m} = \sum_{s=0}^{i-m} \binom{i-m}{s} (-1)^s (F_2(t;1))^s \quad (7)$$

the desired result is readily deduced by substituting the above expressions in formula (6).
(ii) Making use of the following binomial expressions

$$(F_1(t;1))^{n_2-m} = \sum_{q=0}^{n_2-m} \binom{n_2-m}{q} (-1)^q (\bar{F}_1(t;1))^q, (F_2(t;1))^{n_1-i+m} = \sum_{p=0}^{n_1-i+m} \binom{n_1-i+m}{p} (-1)^p (\bar{F}_2(t;1))^p \quad (8)$$

and substituting next these formulae in equation (6), the proof is easily completed by carrying out some straightforward algebraic manipulations. □

It is worth mentioning that under the Lehmann-type alternatives (see, e.g. Gibbons and Chakraborti [3]) one may easily deduce some interesting formulae for the computation of the survival function of the three-state **k**-out-of-n system consisting of mixed components. More precisely, based on the results proved in Proposition 2.1, we conclude that the following hold true

1. Under the Lehmann alternatives $F_i(t;1) = (F(t))^{a_i}$, $i = 1,2$, where $a_i > 0$, the survival function of the lifetime of the three-state **k**-out-of-n system takes on the following form

$$P(T_1 > t) = \sum_{i=k_1}^{n} \sum_{m=\max(0,i-n_1)}^{\min(n_2,i)} \sum_{r=0}^{m} \sum_{s=0}^{i-m} \binom{n_2}{r, m-r, n_2-m} \binom{n_1}{s, i-m-s, n_1-i+m} \times (-1)^{r+s} (F(t))^{a_1(n_2-m+r)+a_2(n_1-i+m+s)}. \quad (9)$$

2. Under the Lehmann alternatives $F_i(t;1) = 1-(1-F(t))^{a_i}$, $i = 1,2$, where $a_i > 0$, the survival function of the lifetime of the three-state

k-out-of-n system takes on the following form

$$P(T_1 > t) = \sum_{i=k_1}^{n} \sum_{m=\max(0,i-n_1)}^{\min(n_2,i)} \sum_{q=0}^{n_2-m} \sum_{p=0}^{n_1-i+m} \binom{n_2}{q,\, m,\, n_2-q-m} \binom{n_1}{p,\, i-m,\, n_1-p-i+m} \times (-1)^{p+q} (\bar{F}(t))^{a_1(m+q)+a_2(i-m+p)}. \quad (10)$$

The following corollary offers some expressions for the survival function of the time spent by the three-state **k**-out-of-n system in the perfect functioning state.

Proposition 2.2 *For the three-state* ***k****-out-of-n system consisting of* n_1 *components in a partially working state and* n_2 *components in a perfect functioning state* ($n = n_1 + n_2$) *at time* $t = 0$*, the survival function of the time spent by the system in the perfect functioning state can be expressed as*
(i)

$$P(T_2 > t) = \sum_{m=0}^{\min(k_1-1,n_1)} \sum_{i_1=\max(k_2,k_1-m)}^{n_2} \sum_{i_2=k_2}^{i_1} \sum_{r=0}^{i_2} \sum_{s=0}^{i_1-i_2} \sum_{q=0}^{m} \binom{n_1}{q,\, m-q,\, n_1-m} \binom{n_2}{r,\, i_2-r,\, n_2-i_2}$$
$$\times \binom{n_2-i_2}{s,\, i_1-i_2-s,\, n_2-i_1} (-1)^{r+s+q} (F_1(t;1))^{n_2-i_1+s} (F_2(t;1))^{n_1-m+q} (F_1(t;2))^{i_1-i_2-s+r}$$
$$+ \sum_{m=k_1}^{n_1} \sum_{i_2=k_2}^{n_2} \sum_{r=0}^{i_2} \sum_{q=0}^{m} \binom{n_2}{i_2-r,\, r,\, n_2-i_2} \binom{n_1}{q,\, m-q,\, n_1-m}$$
$$\times (-1)^{r+q} (F_1(t;2))^{n_2-i_2+r} (F_2(t;1))^{n_1-m+q}. \quad (11)$$

(ii)

$$P(T_2 > t) = \sum_{m=0}^{\min(k_1-1,n_1)} \sum_{i_1=\max(k_2,k_1-m)}^{n_2} \sum_{i_2=k_2}^{i_1} \sum_{s=0}^{i_1-i_2} \sum_{r=0}^{n_2-i_1} \sum_{q=0}^{n_1-m} \binom{n_1}{m,\, q,\, n_1-m-q} \binom{i_1}{s,\, i_2,\, i_1-s-i_2}$$
$$\times \binom{n_2}{i_1,\, r,\, n_2-i_1-r} (-1)^{s+r+q} (\bar{F}_1(t;1))^{i_1-i_2-s+r} (\bar{F}_2(t;1))^{m+q} (\bar{F}_1(t;2))^{i_2+s}$$
$$+ \sum_{m=k_1}^{n_1} \sum_{i_2=k_2}^{n_2} \sum_{p=0}^{n_2-i_2} \sum_{q=0}^{n_1-m} \binom{n_2}{i_2,\, p,\, n_2-i_2-p} \binom{n_1}{m,\, q,\, n_1-m-q}$$
$$\times (-1)^{p+q} (\bar{F}_1(t;2))^{i_2+p} (\bar{F}_2(t;1))^{m+q}. \quad (12)$$

Proof. (i) Eryilmaz *et al.* [1] proved that the survival function of the time spent by the three-state **k**-out-of-n system consisting of n_1 components in state "1" and n_2 components in state "2" in the perfect functioning state, can be expressed as follows

$$P(T_2 > t) = \sum_{m=0}^{\min(k_1-1,n_1)} \sum_{i_1=\max(k_2,k_1-m)}^{n_2} \sum_{i_2=k_2}^{i_1} \binom{n_2}{i_2, i_1-i_2} \binom{n_1}{m} \times (\bar{F}_1(t;2))^{i_2} (\bar{F}_1(t;1) - \bar{F}_1(t;2))^{i_1-i_2} (F_1(t;1))^{n_2-i_1} (\bar{F}_2(t;1))^m (F_2(t;1))^{n_1-m} + \sum_{m=k_1}^{n_1} \sum_{i_2=k_2}^{n_2} \binom{n_2}{i_2} \binom{n_1}{m} (\bar{F}_1(t;2))^{i_2} (F_1(t;2))^{n_2-i_2} (\bar{F}_2(t;1))^m (F_2(t;1))^{n_1-m}. \tag{13}$$

The desired result is readily deduced by substituting the expressions

$$(\bar{F}_1(t;2))^{i_2} = \sum_{r=0}^{i_2} \binom{i_2}{r} (-1)^r (F_1(t;2))^r,$$

$$(\bar{F}_2(t;1))^m = \sum_{q=0}^{m} \binom{m}{q} (-1)^q (F_2(t;1))^q,$$

$$(\bar{F}_1(t;1) - \bar{F}_1(t;2))^{i_1-i_2} = \sum_{s=0}^{i_1-i_2} \binom{i_1-i_2}{s} (-1)^s (F_1(t;2))^{i_1-i_2-s} (F_1(t;1))^s$$

in equation (13).
(ii) On substituting the following binomial expressions

$$(F_1(t;1))^{n_2-i_1} = \sum_{r=0}^{n_2-i_1} \binom{n_2-i_1}{r} (-1)^r (\bar{F}_1(t;1))^r,$$

$$(F_2(t;1))^{n_1-m} = \sum_{q=0}^{n_1-m} \binom{n_1-m}{q} (-1)^q (\bar{F}_2(t;1))^q,$$

$$(F_1(t;2))^{n_2-i_2} = \sum_{p=0}^{n_2-i_2} \binom{n_2-i_2}{p} (-1)^p (\bar{F}_1(t;2))^p,$$

$$(F_1(t;2) - F_1(t;1))^{i_1-i_2} = \sum_{s=0}^{i_1-i_2} \binom{i_1-i_2}{s} (-1)^s (\bar{F}_1(t;1))^{i_1-i_2-s} (\bar{F}_1(t;2))^s$$

in equation (13), the proof is easily completed after some straightforward algebraic manipulations. □

It is worth mentioning that under the Lehmann-type alternatives, one may easily arrive out some interesting formulae for the computation of the survival function of the time spent by the system in the perfect functioning state. More precisely, based on the results proved in Proposition 2.2, we may effortlessly derive the following:

1. Under the Lehmann alternatives $F_i(t;1) = (F(t))^{a_i},\ i = 1,2,\ F_1(t;2) = (F(t))^{a_3}$ $(a_1, a_2, a_3 > 0)$, the survival function of the time spent by the three-state $\mathbf{k}$-out-of-n system in the perfect functioning state takes on the form

$$P(T_2 > t) = \sum_{m=0}^{\min(k_1-1,n_1)} \sum_{i_1=\max(k_2,k_1-m)}^{n_2} \sum_{i_2=k_2}^{i_1} \sum_{r=0}^{i_2} \sum_{s=0}^{i_1-i_2} \sum_{q=0}^{m} \binom{n_1}{q,\, m-q,\, n_1-m} \binom{n_2}{r,\, i_2-r,\, n_2-i_2} \times \binom{n_2-i_2}{s,\, i_1-i_2-s,\, n_2-i_1} (-1)^{r+q+s} (F(t))^{a_1(n_2-i_1+s)+a_2(n_1-m+q)+a_3(r+i_1-i_2-s)} + \sum_{m=k_1}^{n_1} \sum_{i_2=k_2}^{n_2} \sum_{r=0}^{i_2} \sum_{q=0}^{m} \binom{n_2}{i_2-r,\, r,\, n_2-i_2} \binom{n_1}{q,\, m-q,\, n_1-m} \times (-1)^{r+q} (F(t))^{a_2(n_1-m+q)+a_3(n_2-i_2+r)}. \tag{14}$$

2. Under the Lehmann alternatives $F_i(t;1) = 1-(1-F(t))^{a_i},\ i = 1,2$ $(a_1, a_2, a_3 > 0)$, the survival function of the time spent by the three-state $\mathbf{k}$-out-of-n system in the perfect functioning state takes on the following form

$$P(T_2 > t) = \sum_{m=0}^{\min(k_1-1,n_1)} \sum_{i_1=\max(k_2,k_1-m)}^{n_2} \sum_{i_2=k_2}^{i_1} \sum_{s=0}^{i_1-i_2} \sum_{r=0}^{n_2-i_1} \sum_{q=0}^{n_1-m} \binom{n_1}{m,\, q,\, n_1-m-q} \binom{i_1}{s,\, i_2,\, i_1-s-i_2} \times \binom{n_2}{i_1,\, r,\, n_2-i_1-r} (-1)^{r+q+s} (\bar{F}(t))^{a_1(i_1-i_2-s+r)+a_2(m+q)+a_3(i_2+s)} + \sum_{m=k_1}^{n_1} \sum_{i_2=k_2}^{n_2} \sum_{p=0}^{n_2-i_2} \sum_{q=0}^{n_1-m} \binom{n_2}{i_2,\, p,\, n_2-i_2-p} \binom{n_1}{m,\, q,\, n_1-m-q} \times (-1)^{r+q} (\bar{F}(t))^{a_2(m+q)+a_3(i_2+p)}. \tag{15}$$

In closing this section, we shall derive a closed formula for the mean of the random variables T_1, T_2 under the Lehmann distribution models $F_i(t;1) =$

$1-(1-F(t))^{a_i}$, $i=1,2$ and $F_1(t;2)=1-(1-F(t))^{a_3}$. Let us first denote by $\upsilon_F(k)$ the mean value of the greatest order statistic for a random sample $Z_1, Z_2, ..., Z_k$ from a distribution with cdf F, i.e.

$$\upsilon_F(k) = E(\max(Z_1, Z_2, ..., Z_k)),$$

with $Z_i\tilde{}\ F$, $i=1,2,...,k$.

Corollary 2.3 *Under the Lehmann alternatives $F_i(t;1)=1-(1-F(t))^{a_i}$, $i=1,2$ $(a_1,a_2>0)$, the mean lifetime of the three-state **k**-out-of-n system takes on the following form*

$$E(T_1) = \sum_{i=k_1}^{n} \sum_{m=\max(0,i-n_1)}^{\min(n_2,i)} \sum_{q=0}^{n_2-m} \sum_{p=0}^{n_1-i+m} \binom{n_2}{q,\, m,\, n_2-q-m} \binom{n_1}{p,\, i-m,\, n_1-p-i+m} \times (-1)^{p+q} \upsilon_F(a_1(m+q)+a_2(i-m+p))$$

Proof. Follows immediately from expression (6) by taking into account that $(\bar{F}(t))^{a_1(m+q)+a_2(i-m+p)}$ is the tail probability of the maximum of $a_1(m+q)+a_2(i-m+p)$ i.i.d. random variables following a distribution with cdf $F(t)$ and making use of the well known formula

$$E(T_1) = \int_0^\infty P(T_1>t)dt.$$

□

Corollary 2.4 *Under the Lehmann alternatives $F_i(t;1)=1-(1-F(t))^{a_i}$, $i=1,2$ and $F_1(t;2)=1-(1-F(t))^{a_3}$ where $a_1,a_2,a_3>0$, the mean time spent by the three-state **k**-out-of-n system in the perfect functioning state takes on the following form*

$$E(T_2) = \sum_{m=0}^{\min(k_1-1,n_1)} \sum_{i_1=\max(k_2,k_1-m)}^{n_2} \sum_{i_2=k_2}^{i_1} \sum_{s=0}^{i_1-i_2} \sum_{r=0}^{n_2-i_1} \sum_{q=0}^{n_1-m} \binom{n_1}{m,\, q,\, n_1-m-q} \binom{i_1}{s,\, i_2,\, i_1-s-i_2}$$
$$\times \binom{n_2}{i_1,\, r,\, n_2-i_1-r} (-1)^{r+q+s} \upsilon_F(a_1(i_1-i_2-s+r)+a_2(m+q)+a_3(i_2+s))$$
$$+ \sum_{m=k_1}^{n_1} \sum_{i_2=k_2}^{n_2} \sum_{p=0}^{n_2-i_2} \sum_{q=0}^{n_1-m} (-1)^{p+q} \binom{n_2}{i_2,\, p,\, n_2-i_2-p} \binom{n_1}{m,\, q,\, n_1-m-q}$$
$$\times \upsilon_F(a_2(m+q)+a_3(i_2+p)).$$

Proof. Follows immediately from expression (13) by using once more the argument stated in the proof of Corollary 2.3. □

3 Numerical results

Let us consider a three-state **k**-out-of-n system consisting of n components such that at time $t = 0$ n_1 components are in a partially working condition (state "1") and n_2 components are perfectly functioning (state "2") where $n = n_1 + n_2$. We next assume that the time that the $i - th$ component spends in states "1" and "2", e.g. the random variables T_{1i} and T_{2i}, follow a Lehmann-type distribution model, with the baseline $F(t)$ being Weibull distribution, i.e.

$$F_1(t;1) = P(T_{1i} \leq t) = (F(t))^{a_1}, i = n_1 + 1, n_1 + 2, ..., n_1 + n_2, a_1 > 0,$$
$$F_2(t;1) = P(T_{1i} \leq t) = (F(t))^{a_2}, i = 1, 2, ..., n_1, a_2 > 0,$$
$$F_1(t;2) = P(T_{2i} \leq t) = (F(t))^{a_3}, i = n_1 + 1, n_1 + 2, ..., n_1 + n_2, a_3 > 0,$$

where

$$F(t) = 1 - e^{-(\lambda t)^\beta}, t \geq 0.$$

In Tables 1, 2 and 3, we compute the mean lifetime $E(T_1)$ and the mean time $E(T_2)$ spent by the system in a perfect functioning state, when $\lambda = 1$ for selected values of the design parameters k_1, k_2, n_1, n_2, several values of the parameters a_1, a_2, a_3 and three different choices of the shape parameter β.

Each $E(T_1)$ cell contains the mean lifetime for two different Lehmann-type alternatives: the upper value corresponds to $(a_1 = 1, a_2 = 2)$ and the lower value corresponds to $(a_1 = 3, a_2 = 2)$. Moreover, each $E(T_2)$ cell contains the mean lifetime for three different cases: the upper value corresponds to $(a_1 = 1, a_2 = 2, a_3 = 3)$, the middle value corresponds to $(a_1 = 3, a_2 = 1, a_3 = 2)$ and the lower value corresponds to $(a_1 = 2, a_2 = 3, a_3 = 1)$.

Based on the numerical results displayed in the above tables, one may easily observe that, as intuitively suspected, the mean lifetimes $E(T_1)$ and $E(T_2)$ of the three-state **k**-out-of-n system are increasing with respect to n_1, n_2 and decreasing with respect to k_1, k_2.

Table 1: Mean lifetime of a three-state **k**-out-of-n system for design parameters $k_1 = 3, k_2 = 2$.

n_1	n_2	$E(T_1)$			$E(T_2)$		
		$\beta = 2$	$\beta = 0.5$	$\beta = 1$	$\beta = 2$	$\beta = 0.5$	$\beta = 1$
5	**3**	1.25092	2.91892	1.61253	1.22667	2.79215	1.56098
		1.38445	4.17959	1.95896	1.08390	1.85934	1.23939
					0.85380	1.01086	0.82456
	4	1.28369	3.19440	1.69412	1.31540	3.50303	1.77741
		1.43931	4.80849	2.11168	1.20716	2.63596	1.51346
					0.99146	1.52164	1.06905
	5	1.31293	3.45779	1.76878	1.36525	3.97927	1.90729
		1.48543	5.39383	2.24478	1.29401	3.33613	1.72554
					1.08852	2.00248	1.26357
	6	1.33934	3.71051	1.83769	1.39457	4.29150	1.98679
		1.52505	5.94096	2.36259	1.36080	3.97466	1.89923
					1.16279	2.45309	1.42500
	7	1.36340	3.95368	1.90174	1.41179	4.48525	2.03441
		1.55968	6.45461	2.46820	1.41486	4.56258	2.04660
					1.22255	2.87600	1.56294
	8	1.38551	4.18823	1.96161	1.42145	4.59165	2.06092
		1.59039	6.93881	2.56386	1.46015	5.10818	2.17469
					1.27232	3.27420	1.68337

In addition, Figures 1.1 and 1.2 provide a graphical representation of the probabilities $P(T_1 > t), P(T_2 > t)$ for several choices of the design parameters of the three-state **k**-out-of-n system and the Lehmann-type alternatives models.

Table 2: Mean lifetime of a three-state **k**-out-of-n system for design parameters $k_1 = 5, k_2 = 2$.

n_1	n_2	$E(T_1)$			$E(T_2)$		
		$\beta = 2$	$\beta = 0.5$	$\beta = 1$	$\beta = 2$	$\beta = 0.5$	$\beta = 1$
5	**3**	0.95207	1.03139	0.94325	1.01707	1.29090	1.06863
		1.10835	1.75553	1.26078	0.89849	0.85594	0.84744
					0.83193	0.81815	0.76906
	4	1.00301	1.23191	1.04093	1.03660	1.36217	1.10555
		1.18308	2.21767	1.42956	1.02367	1.33014	1.08346
					0.96065	1.22030	0.98790
	5	1.04586	1.42453	1.12724	1.05064	1.41236	1.13217
		1.24376	2.66052	1.57497	1.11671	1.80394	1.27925
					1.05052	1.59706	1.16029
	6	1.08293	1.61036	1.20494	1.07056	1.50425	1.17299
		1.29455	3.08295	1.70246	1.18942	2.26210	1.44455
					1.11897	1.95004	1.30276
	7	1.11564	1.79017	1.27582	1.09696	1.64992	1.23035
		1.33806	3.48554	1.81585	1.24860	2.70010	1.58707
					1.17399	2.28205	1.42441
	8	1.14493	1.96455	1.34114	1.12768	1.84094	1.29987
		1.37604	3.86949	1.91788	1.29825	3.11738	1.71214
					1.21987	2.59278	1.53074

In the diagrams, the more briskly the discontinuous line is, the smaller is the value of the parameter n_2. It is easy to observe that we are able to increase the value of the survival function of a three-state **k**-out-of-n system at time t by increasing the number of components (n_2) that are perfectly functioning (state "2") at time $t = 0$.

Table 3: Mean lifetime of a three-state **k**-out-of-n system for design parameters $k_1 = 2, k_2 = 3$.

n_1	n_2	$E(T_1)$			$E(T_2)$		
		$\beta = 2$	$\beta = 0.5$	$\beta = 1$	$\beta = 2$	$\beta = 0.5$	$\beta = 1$
5	**3**	1.44393	5.18412	2.14808	0.95490	1.22783	0.97897
		1.56503	6.87183	2.50577	0.79214	0.70222	0.70000
					0.51166	0.22222	0.33333
	4	1.47054	5.52182	2.22409	1.15060	2.20538	1.37789
		1.61220	7.63915	2.65315	0.99652	1.37473	1.05357
					0.71731	0.51389	0.58333
	5	1.49473	5.84423	2.29442	1.26613	3.05140	1.65090
		1.65226	8.34426	2.78190	1.11889	1.99808	1.30595
					0.84793	0.82722	0.78333
	6	1.51688	6.15302	2.35990	1.34825	3.80412	1.86175
		1.68697	8.99711	2.89614	1.20620	2.57453	1.50473
					0.94313	1.14389	0.95000
	7	1.53731	6.44953	2.42117	1.41179	4.48525	2.03441
		1.71754	9.60552	2.99877	1.27384	3.10962	1.66939
					1.01762	1.45613	1.09286
	8	1.55626	6.73492	2.47877	1.46348	5.10911	2.18093
		1.74479	10.1757	3.09192	1.32886	3.60882	1.81022
					1.07852	1.76060	1.21786

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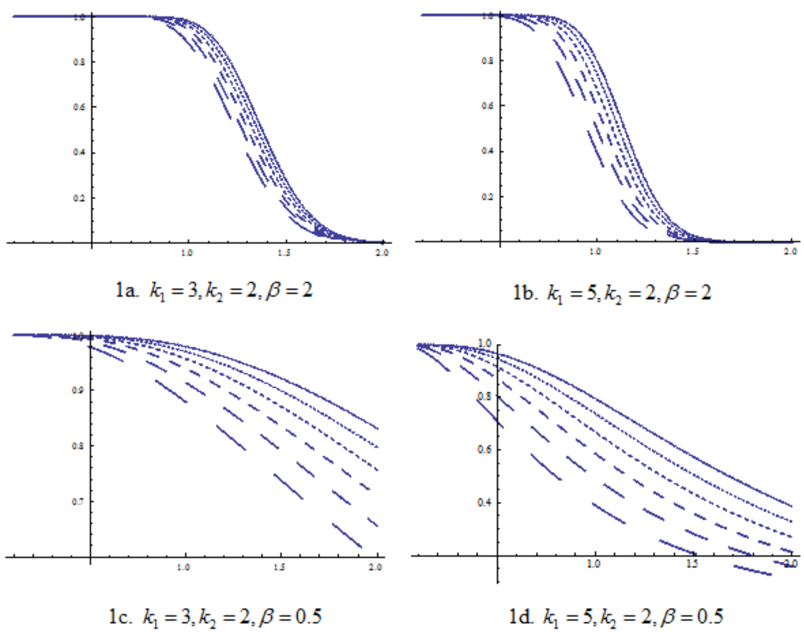

Figure 1: The survival function $P(T_1 > t)$ for $a_1 = 1, a_2 = 2, n_1 = 5$ and $n_2 = 3, 4, 5, 6, 7, 8$.

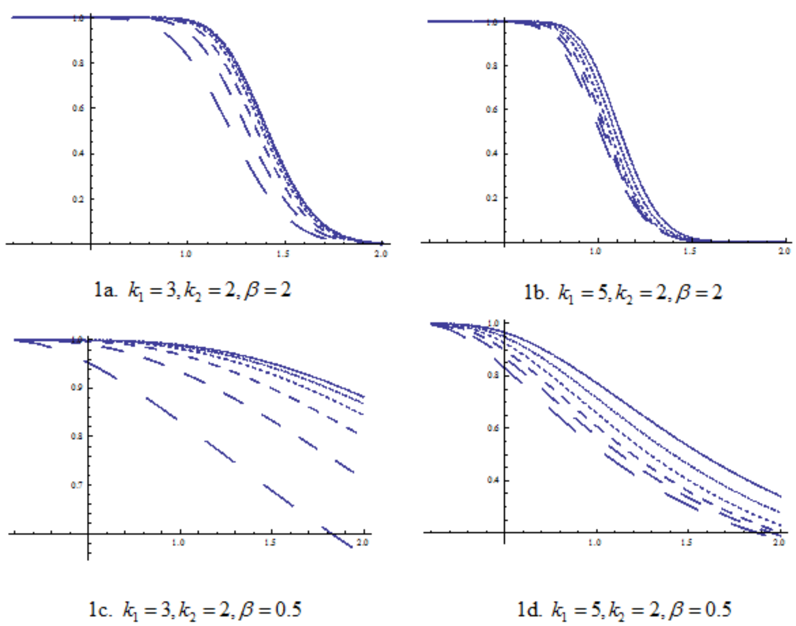

Figure 2: The survival function $P(T_2 > t)$ for $a_1 = 1, a_2 = 2, a_3 = 3, n_1 = 5$ and $n_2 = 3, 4, 5, 6, 7, 8$.

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