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Improvement of a five factor Altman model to assess the creditworthiness of an enterprise using the theory of fussy sets

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Abstract

In this paper we propose a method that uses the apparatus of the theory of fuzzy sets, together with the five-factor model of Altman to assess the creditworthiness of an enterprise. Altman's model is enhanced in two ways: applies integral approximation of the root mean square for the exact calculation of quantitative credit assessment (probability of bankruptcy), and applies the device of fuzzy sets for ordered sets according to the degree of confidence in the resulting probability. This paper shows the simulation procedures for the credit assessment and the capabilities of the model. Some real examples of the methodology of application are shown. The article is theoretical in nature, the findings made in the

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mathematical model have not been tested on a sufficiently large number of enterprises.

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1 Introduction

The problem of timely return of loans is relevant to any of the credit organization (Bank). Reliable solution to the problem largely depends on the "quality" of a reliable assessment of the creditworthiness of potential borrowers, carried out by experts on the basis of financial statements. It gives sufficient information about the financial condition of the company and allows us to develop objective and reliable methods for making decisions to grant the company a loan with minimal risk (blacksmiths, Carriers, 2008, pp. 84-106).In the modern practice of financial and economic activities of foreign firms to estimate the probability of bankruptcy the widest application was received by the models developed by E. Altman and Beaver (early December, 2014; Altman, 1968, pp. 589-609; Beaver, 1967, p. 71-111). The first Russian experience of using the approach of Altman is a relatively newly developed model Davydova-Belikov (Davydov, 1999, p. 13-20; Baranovskaya, Kovalenko, Urtenov, Karmazin, 2009, p. 250).

Currently, the theory of fuzzy sets is a well-developed scientific discipline, which is of great practical importance. The theory is widely used in the solution of technical problems (Hiyama, Sameshima, 1991, pp. 181-94). Expanding the use of fuzzy set theory in Economics and management of enterprises (diligenskii, Dimova, Sevastianov, 2004, Chapter 7; Kofman, Aloha, 1992, 223c.). Also one of the most promising areas of research in the field of analysis, forecasting and modeling of economic phenomena and processes is fuzzy logic (fuzzy logic) (Deluca, Termini, 1972, P. 301-312). But the application of measures of fuzziness of the sets is still insufficiently used in the analysis and the assessment of the creditworthiness of the company.

Lately a lot of popularity among the mathematical approaches to reproduce the studied processes or phenomena acquires simulation (Kharin, Malyugin, Cirilica, Lubach, Khatskevich, 1997, Chapter 3), which helps to not only adequately assess the creditworthiness of enterprises, but also to give a justification of the most rational solutions to decision makers.

Thus, the aim of this work consists in using the apparatus of fuzzy set theory and simulation modeling, using the model of Altman to improve effective methods of credit rating (bankruptcy) of the enterprises to develop a way of ordering fuzzy sets by calculating the measure of preference. To provide some real world examples on application of new methods of assessing the creditworthiness of enterprises and to conduct the simulation procedure for calculating the probability of bankruptcy.

2 Statement of the problem

Model Altman introduces the function p(z), which is equal to the probability of bankruptcy. The probability of bankruptcy is calculated according to the empirically established dependencies

$$p(z) = \begin{cases} [0.80, 1.0], & 0 \le z \le 1.8 \\ [0.35, 0.5], & 1.81 \le z \le 2.77 \\ [0.15, 0.2], & 2.8 \le z \le 2.99 \\ [0, \varepsilon], & 3 \le z < \infty \end{cases}, \tag{2}$$

When $z \geq 3$the probability of bankruptcy is $p = \varepsilon$ sufficiently small ($\varepsilon \to 0$ when $z \to \infty$) and is approximately equal to zero. Upon further presentation of the problem will take $\varepsilon = 0{,}05$. In Fig.1 shows the graph of the function *p(z)* of the model of Altman (1). We define two functions $f_1(z) = \min_{\forall z} p(z)$, $f_2(z) = \max_{\forall z} p(z)$. After that, solve the problem the mean integrated squared approximation sets Altman by a polynomial of sufficiently high 6-th degree

$$L_6(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 + a_5 z^5 + a_6 z^6$$

On a segment $z \in [0, z_4]$, $z_4 = 3.5$. A higher degree, as the calculations do not lead to different results that differ from the carried out. Smaller degree polynomials give rise to a lack of smoothness. Justification of the choice of the polynomial degree was considered as the subject of a separate work in economic-mathematical research (Bamadio, kuzjakina, Lebedev, 2014, №10(104)). The coefficients were of a minimization problem in seminar space R7 of the coefficients of the polynomial

$$a = \arg\left\{ \min_{a \in R^7} F(a) \right\} \tag{3}$$

Where $F(a) = \sum_{i=1}^{2} \int_0^{z4} \left(L_6(z) - f_i(z)\right)^2 dz$, $a = \{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}^T$, under natural additional restrictions

$$\left.\frac{dL_6(z)}{dz}\right|_{z=0} = 0, \qquad a_0 = 0,$$

$$L_6(z)\big|_{z=z4} = 0, \; a_0 + a_1 z_4 + a_2 z_4{}^2 + a_3 z_4{}^3 + a_4 z_4{}^4 + a_5 z_4{}^5 + a_6 z_4{}^6 = 0,$$

$$\left.\frac{dL_6(z)}{dz}\right|_{z=z4} = 0, \qquad a_1 + 2a_2 z_4 + 3a_3 z_4{}^2 + 4a_4 z_4{}^3 + 5a_5 z_4{}^4 + 6a_6 z_4{}^6.$$

From the segment, which is the approximation right end point of the selected z_4*=3.5*. The choice of this point to some extent arbitrary, but lines l_1, l_2 of the bounding region that contains the rectangles that intersect on the z-axis in one point with coordinate *z* = *3.5*. (Bamadio, Lebedev, 2014). A minimization problem was solved using a mathematical package MathCAD.

$$a = \{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}^T = \{0.936; 0; 0.22; -0.19; 0.011, 0.0025; 0.0003\}^T$$

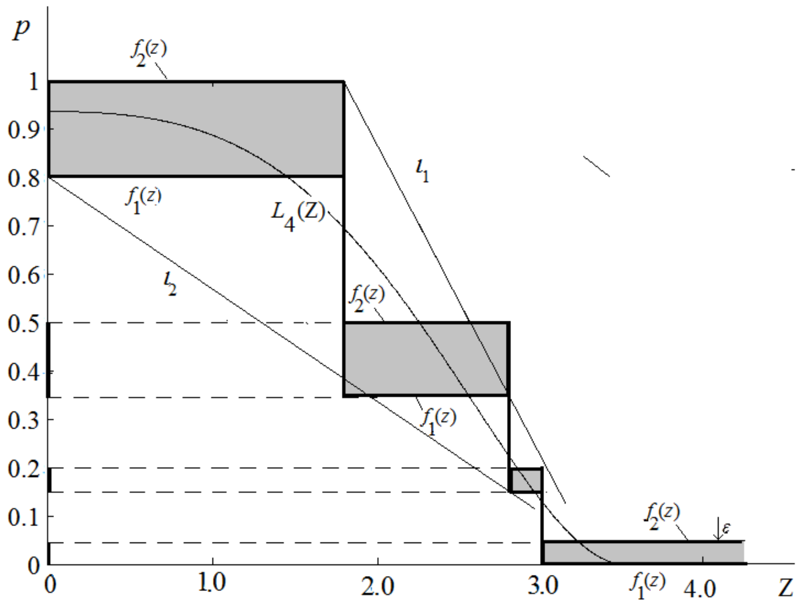

Figure 1. The graph of the function variable p(z) of the model of Altman. Using functions $f_1(z) = \min\limits_{\forall z} p(z)$, $f_2(z) = \max\limits_{\forall z} p(z)$, integral method root-mean-square approximation of the constructed polynomial of the sixth degree.

In model (1) the parameters ki and computed for them the parameter z cannot be measured accurately. Therefore, model (1) gives rise to fuzzy sets, which belong to the values of *p* and the values of membership functions of these sets coincide with the probability of bankruptcy. The Altman model, allows a first approximation to divide enterprises into four classes with the probability of bankruptcy A_i , $i = 1, \cdots, 4$. $A_1 = [0.8, 1.0]$. - "The probability of bankruptcy is high,”- $A_2 = [0.35, 0.50]$"average probability of bankruptcy", $A_3 = [0.15, 0.20]$ -

"the probability of bankruptcy is not large," $A_4 = [0, \varepsilon]$ - enterprise, “the probability of bankruptcy is small.” In the present example $p \in [0, 1]$.

For fuzzy set $\tilde{X}_i$ membership function $\mu_{\tilde{X}_i}(p): U \to \mu \in M = [0, 1] \in R$, (discussed in paragraph 4 below).

If the value of probability p was found in the models of Altman (1) with the use of $L_6(z)$ is one of a set A_i, the value of the membership function will be equal $\mu = 1$. This situation is shown in figure 2. In this case, the probability of bankruptcy is attributed to the resulting value $p = L_6(z) \in A_i$. If $p = L_6(z) \notin A_i$, then $\mu = 0$.

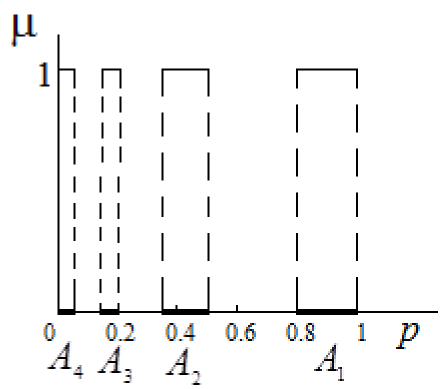

Figure 2. The values of the membership function $p \in X_i$.

The comfort A_i given by their distribution μ functions clearly.

The construction of function $L_6(z)$ is possible to obtain the value of p in the areas that lie outside the set of Altman, however, in such cases it is necessary to include the received value to one of the nearby sets of Altman, which it is proposed to use the theory of fuzzy sets, building the most simple piecewise linear continuous membership function (© BSTU. V. G. Shukhov, 2014, Chapter 2). When the value of probability p was found in the models of Altman (1) using $L_6(z)$does not fall within one of the sets $p = L_6(z) \notin A_i$, then the value of the membership function will be using are presented below (in paragraph 4) methods of using the apparatus of fuzzy sets. Currently, the fuzzy sets are actively used in practice when analyzing the risk of bankruptcy (Konyshev, Nazarov, 2011, pp. 170-179).

3 Membership function

The membership function $\mu_A(u)$ – is a function, the scope of which is the carrier U, ($u \in U$), and the range of values μ_A is the unit interval $[0;1]$ (Konyshev, Nazarov, 2011, p. 11; Ibragimov, 2010, p. 9). The higher the value $\mu_A(u)$, the higher the estimated degree of belonging of an element of the carrier U of fuzzy set A. In our case, as the carrier will select $U = \{X, 0 \le X \le 1\}$that has many A_i where $u = p$ – the probability of bankruptcy of the enterprise corresponding to the value z found using equation (1). On this we define the membership function: for values of $p_1 - \mu_{\tilde{X}_1}(p)$, $p_2 - \mu_{\tilde{X}_2}(p)$, $p_3 - \mu_{\tilde{X}_3}(p)$, $p_2 - \mu_{\tilde{X}_4}(p)$, and the first of them corresponds to a fuzzy subset $\tilde{X}_1$, the second – $\tilde{X}_2$, third – $\tilde{X}_3$, and fourth $\tilde{X}_4$, where – $\tilde{X}_1$"the possibility of bankruptcy is high" $\tilde{X}_2$– "the possibility of bankruptcy average", $\tilde{X}_3$– "possible bankruptcy", $\tilde{X}_4$ – "the possibility of bankruptcy is small."

Compute a value *z* according to the model of Altman (1), and calculate *p* by the formula $L_6(z)$ does not always gives the chance to carry the computed value of *p* to one of the sets A_i, that is one of the cases $p \in A_1 = [0.8, 1.0]$, $p \in A_2 = [0.35, 0.50]$, $p \in A_3 = [0.15, 0.20]$, $p \in A_4 = [0, 0.05]$. For example, if $p \in 0.7$, then *p* can be attributed to the set $A_1$and lot A_2.

In this regard, the input fuzzy sets $\tilde{X}_i$ which are defined by preference functions $\mu_{\tilde{X}_i}(u): U \to \mu \in M = [0, 1] \in R$ to determine the measure of fuzziness of the set $\tilde{X}_i$, in this case, the measure of fuzziness computed $p = L_6(z) \in \tilde{X}_i$.

The membership functions of the subsets $\tilde{X}_1$, $\tilde{X}_2$, $\tilde{X}_3$, $\tilde{X}_4$ have the following form:

$$\mu_{\tilde{X}_1} = \begin{cases} \dfrac{10p-5}{3}, & если \quad 0.5 \le p < 0.8, \\ 1, & если \quad 0.8 \le p \le 1; \end{cases}$$

$$\mu_{\tilde{X}_2} = \begin{cases} \dfrac{100p-20}{15}, & если \quad 0.2 \le p < 0.35, \\ 1, & если \quad 0.35 \le p < 0.5, \\ \dfrac{8-10p}{3}, & если \quad 0.5 \le p \le 0.8; \end{cases} \quad (6, 7)$$

$$\mu_{\tilde{X}_3} = \begin{cases} \dfrac{100p-5}{10}, & если \quad 0.05 \le p \le 0.15, \\ 1, & если \quad 0{,}15 < p \le 0{,}2, \\ \dfrac{35-100p}{15}, & если \quad 0.2 < p \le 0.35, \end{cases} \qquad \mu_{\tilde{X}_4} = \begin{cases} 1, & если \quad 0 \le p < 0.05, \\ \dfrac{15-100p}{10}, & если \quad 0.05 \le p \le 0.15. \end{cases}$$

(8, 9)

Then, the multiple write using traditional set theory notation (using the sign of the integral) (Konyshev, Nazarov, 2011, p. 13; Ibragimov, 2010, p. 116; Nedosekin, 204, p. 64):

$$\tilde{X}_1 = \int_{0.5 \le p \le 1} \mu_{\tilde{X}_1}(p)/p = \int_{0.5 \le p < 0.8} \left(\frac{10p-5}{3}\right) \Big/ p + \int_{0.8 \le p \le 1} 1/p; \quad (10)$$

$$\tilde{X}_2 = \int_{0.2 \le p \le 0.8} \mu_{\tilde{X}_2}(p)/p = \int_{0.2 \le p < 0.35} \left(\frac{100p-20}{15}\right) \Big/ p + \int_{0.35 \le p \le 0{,}5} 1/p + \int_{0.5 < p \le 0.8} \left(\frac{8-10p}{3}\right) \Big/ p;$$

(11)

$$\tilde{X}_3 = \int_{0.05 \le p \le 0.35} \mu_{\tilde{X}_3}(p)/p = \int_{0.05 \le p \le 0.15} \left(\frac{100p-5}{10}\right) \Big/ p + \int_{0.15 < p \le 0.2} 1/p + \int_{0.2 < p \le 0.35} \left(\frac{35-100p}{15}\right) \Big/ p; \quad (12)$$

$$\tilde{X}_4 = \int_{0 \le p \le 0.15} \mu_{\tilde{X}_4}(p)/p = \int_{0 \le p < 0.05} 1/p + \int_{0.05 \le p < 0.15} \left(\frac{15-100p}{10}\right) \Big/ p. \quad (13)$$

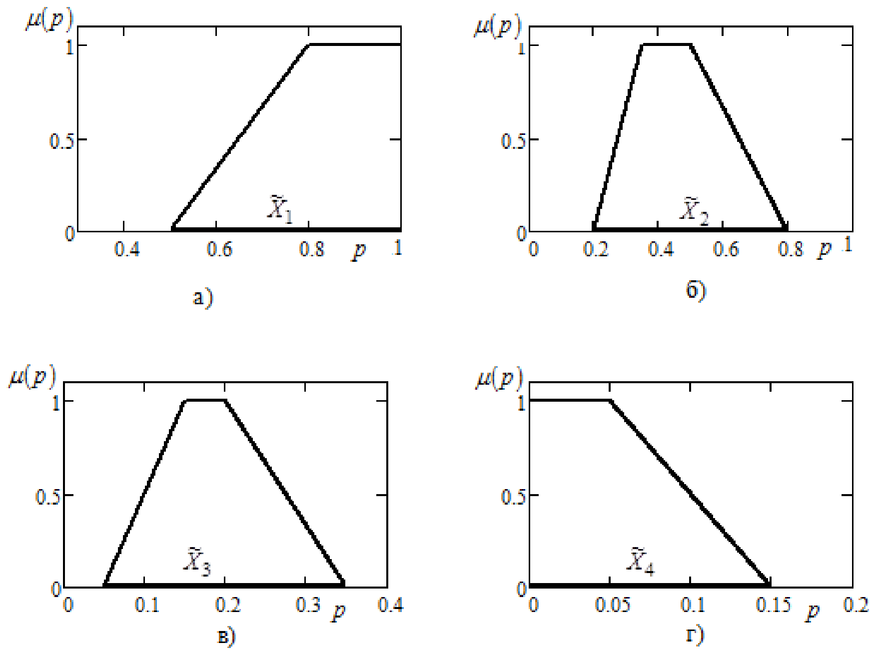

Figure 3. Graphs of membership functions of fuzzy subsets of a)– $\tilde{X}_1$, b) – $\tilde{X}_2$, C) – $\tilde{X}_3$, d) – $\tilde{X}_4$.meeting the sets Altman (figure 2).

If all graphs a) - d) to paint on in the same coordinate system, the abscissa of the intersection points of functions $\mu_{\tilde{X}_i}(p)$and $\mu_{\tilde{X}_{i+1}}(p)$, are equal $p_1 = 0.1$, $p_2 = 0.275$, $p_3 = 0.65$, and they meet the definition (14) of the nearest crisp sets X_{i0} (see section 5).

4 Measures of fuzzy sets

After calculating *z, p(z)* and calculate $\tilde{X}_i$ the measure rate is provided $\mu_{\tilde{X}_i}(p)$ for many terms of degree of fuzziness $\tilde{X}_i$, that is, we introduce the total ordering of sets by the degree of their fuzziness. To determine the degree of fuzziness of the set is used a measure of its fuzziness d , which is confined to the measurement of a measure of distinction between fuzzy set $\tilde{X}$ and crisp variety X_0 (Konyshev, Nazarov, 2011, p. 30; and Ibragimov, 2010, p. 149). The measure of fuzziness $\tilde{X}$ of the set is defined as the distance $d\left(\tilde{X}, X_0\right)$ from this set $\tilde{X}$ to the set nearest to him a clearly given set X_0: $d(\tilde{X}, X_0) = \rho(\mu_{\tilde{X}}, \mu_{X_0})$. Clear subset X_0 nearest to fuzzy $\tilde{X}$ with the membership function $\mu_{\tilde{X}}(u)$ $(\mu_i \in M[0,1] \subset R)$, called the subset $X_0 \in U$ characteristic function which has the form:

$$\mu_{X_0} = \begin{cases} 1, & если \quad \mu_{\tilde{X}} > 0.5, \\ 0, & если \quad \mu_{\tilde{X}} < 0.5, \\ 1 \quad или \quad 0, & если \quad \mu_{\tilde{X}} = 0.5. \end{cases} \tag{14}$$

Use the clear sets $X_{i0}: X_{10} = \{p : 0.65 < p \le 1.0\}$; $X_{20} = \{p : 0.275 < p \le 0.65\}$; $X_{30} = \{p : 0.1 < p \le 0.275\}$, $X_{40} = \{p : 0.0 < p \le 0.1\}$ is built, the decision-making function $I(p)$.

The decision-making function – the dependence of the index *I = {1,2,3,4}* indicating a clear near-subset X_{i0} of the set $\tilde{X}_i$, depending on the obtained probability with the use of the polynomial $L_6(z)$. It has the form shown in figure 4. Functions of the decision $I(p)$ clearly indicates one of the crisp subsets X_{i0} and thus generating it fuzzy set $\tilde{X}_i$. It also clearly defines many of Altman A_i as $A_i \subset X_{i0} \subset \tilde{X}_i$. Knowing *p* and function $\mu_{\tilde{X}_i}(p)$ $\tilde{X}_i$ the measure calculates a numerical value μ (which obviously will take values $0.5 \le \mu \le 1.0$) accessories (fuzzy) values of *p*, the corresponding fuzzy set $\tilde{X}_i$.

$$I(p)=\begin{cases} i=1,\text{если } p\in[p3;1,0] \\ i=2,\text{если } p\in[p2;p3] \\ i=3,\text{если } p\in[p1;p2] \\ i=4,\text{если } p\in[0,0;p1] \end{cases}$$

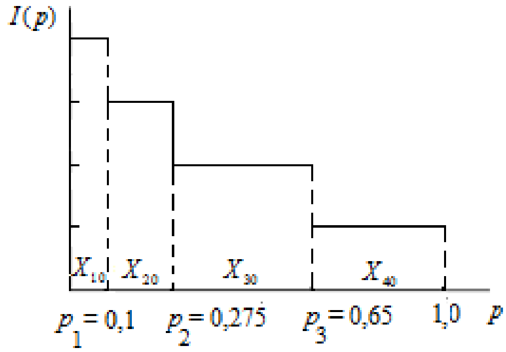

Figure 4. The decision-making function: a) analytical functions for the decision $I(p)$; b) the graph of the function decision-making on clear sets X_{i0} .

A clear subset of X_{10}, X_{20}, X_{30}, X_{40} respectively, to the next is not clearly specified $\tilde{X}_1$, $\tilde{X}_2$, $\tilde{X}_3$ и $\tilde{X}_4$,, and will be:

$$X_{10}=\int\limits_{0,5\le p\le 1}\mu_{\tilde{X}_1}(p)/p=\int\limits_{0,5\le p<0.8}0/p+\int\limits_{0.8\le p\le 1}1/p; \tag{15}$$

$$X_{20}=\int\limits_{0,2\le p\le 0,8}\mu_{\tilde{X}_2}(p)/p=\int\limits_{0,2\le p<p2}0/p+\int\limits_{p2\le p\le p3}1/p+\int\limits_{p3<p\le 0,8}0/p; \tag{16}$$

$$X_{30}=\int\limits_{0,05\le p\le 0,35}\mu_{\tilde{X}_3}(p)/p=\int\limits_{0,05\le p\le p1}0/p+\int\limits_{p1<p\le p2}1/p+\int\limits_{p2<p\le 0,35}0/p; \tag{17}$$

$$X_{40}=\int\limits_{0\le p\le 0,15}\mu_{X_4}(p)/p=\int\limits_{0\le p<0,05}1/p+\int\limits_{0,05\le p<p1}0/p. \tag{18}$$

The decision-making function $I(p)$ with a polynomial $L_6(z)$ solution will receive a clear economic meaning: the p value indicates the probability of bankruptcy and hence the financial condition of the company. The numerical value of the index *I(p)* allows to learn any of the formulas $\mu_{\tilde{X}_i}(p)$ (figure 3) to calculate the numeric value of the measure μ, which shows how the measure of confidence value p belongs to the corresponding set of A_i Altman (figure 2 and 3).

Crisp sets X_{i0} allow you to sort $\tilde{X}_i$ by degree of fuzziness to obtain additional criterion of confidence to get on the financial viability of the enterprise.

In the space of *Q[0,1]* of piecewise continuous functions having a finite number of discontinuities, we can determine the distance between sets $\tilde{X}$ and X_0, as the RMS distance between the membership functions (Kolmogorov A. N., S. V. Fomin, 1976, p. 48; Konysheva, Nazarov, 2011, p. 30; and Ibragimov, 2010, p. 149). This article discusses a class of piecewise linear continuous membership functions of fuzzy sets, i.e. a much more simple class, contained in *Q[0,1]*, and in the case of crisp sets, the membership function has at most two finite discontinuities at the ends of the set. Therefore, we can determine the distance between sets by the formula (19)

$$d(\tilde{X}, X_0) = \rho(\mu_{\tilde{X}}, \mu_{X_0}) = \sqrt{\int_0^1 \left(\mu_{\tilde{X}} - \mu_{X_0}\right)^2 dx} \;. \tag{19}$$

Find the measure of fuzziness previously defined subsets $\tilde{X}_1, \tilde{X}_2, \tilde{X}_3, \tilde{X}_4$, computing measures of fuzziness on the metric of Euclid:

$$d^E(\tilde{X}_1, X_{10}) = \rho(\mu_{\tilde{X}_1}, \mu_{X_0}) = \sqrt{\int_0^1 \left(\mu_{\tilde{X}_1} - \mu_{X_{10}}\right)^2 dx} \approx 0.158; \tag{20}$$

$$d^E(\tilde{X}_2,\ X_{20}) = \rho(\mu_{\tilde{X}_2}, \mu_{X_0}) = \sqrt{\int_0^1 \left(\mu_{\tilde{X}_2} - \mu_{X_{20}}\right)^2 dx} \approx 0.194; \tag{21}$$

$$d^E(\tilde{X}_3,\ X_{30}) = \rho(\mu_X, \mu_{X_0}) = \sqrt{\int_0^1 \left(\mu_{\tilde{X}_3} - \mu_{X_{30}}\right)^2 dx} \approx 0.144; \tag{22}$$

$$d^E(\tilde{X}_4,\ X_{40}) = \rho(\mu_{X_4}, \mu_{X_{40}}) = \sqrt{\int_0^1 \left(\mu_{\tilde{X}_4} - \mu_{X_{40}}\right)^2 dx} \approx 0.091\,. \tag{23}$$

From these calculations it follows that the subset $\tilde{X}_2$ is more fuzzy compared to the subsets $\tilde{X}_1$, $\tilde{X}_3$, and $\tilde{X}_4$. It is similar to: $\tilde{X}_1$ – more clearly defined in comparison with $\tilde{X}_3$ and $\tilde{X}_4$; many $\tilde{X}_3$ – more clearly defined compared with $\tilde{X}_4$.

Let means $X \succ Y$ that the fuzzier X set than Y . Then $\tilde{X}_1, \tilde{X}_2$, $\tilde{X}_3$ $\tilde{X}_4$, on the basis of vagueness, to order as follows: $\tilde{X}_2 \succ \tilde{X}_1 \succ \tilde{X}_3 \succ \tilde{X}_4$. Right-many, among $\tilde{X}_2 \succ \tilde{X}_1 \succ \tilde{X}_3 \succ \tilde{X}_4$ the more reliable judgment about the probability of bankruptcy, related to it. Therefore, the totality $\{\tilde{X}_1, \tilde{X}_2, \tilde{X}_3, \tilde{X}_4\}$ of the most clearly specified is $\tilde{X}_2$ – "the possibility of bankruptcy average", and most clearly defined is $\tilde{X}_4$ "the possibility of bankruptcy is small." This means that the credibility judgment about the possible bankruptcy of the enterprise increases from left to right in a row $\tilde{X}_2 \succ \tilde{X}_1 \succ \tilde{X}_3 \succ \tilde{X}_4$.

5 Examples of usage models

Consider a few concrete examples of the use of Altman model as a method of assessing the probability of bankruptcy.

Example 1. Using the accounting balance sheet of JSC "Concern Rosenergoatom" (O. A. On "Concern Rosenergoatom", 2013) over three years (2009 – 2011 and 2013) compute the values of the coefficients ki and size -Altman (1) (see tab. 1).

Table 1. Metrics and Altman probability of bankruptcy of **JSC** "Concern Rosenergoatom"

Indicators	k_1	k_2	k_3	k_4	k_5	**z**	probability of bankruptcy
2009 y.	0.10	0.05	0.05	5.83	0.31	4.18	$z \geq 3\ p \to 0$
2010 y.	0,11	0.12	0.04	10.59	0.28	7.06	$z \geq 3\ p \to 0$
2011 y.	0.06	0.11	0.00	5.28	0.21	3.62	$z \geq 3\ p \to 0$
-	-	-	-	-	-	-	-
2013 y	0.07	0.16	0.01	5.77	0.19	3.99	$z \geq 3\ p \to 0$

The table shows that of the three years (2009 – 2011), the study applies only to $\tilde{X}_4$ the enterprise (the possibility of bankruptcy is small). Among $\tilde{X}_2 \succ \tilde{X}_1 \succ \tilde{X}_3 \succ \tilde{X}_4$ the most clearly defined is $\tilde{X}_4$ – "the possibility of bankruptcy is small, so the smallness of the quantity p the probability of bankruptcy with the highest possible, in the framework of this model, reliability. This means that the company was going bankrupt and the forecast of its creditworthiness reliable with the highest possible degree of reliability.

Example 2: Calculate various ratios Altman when using statistical data of the balance sheet of the company JSC "TEPLOSET" (O. A. "heating system", 2014) [27] for three years (2009 – 2011). The results are presented in table 2.

Table 2. Metrics and Altman probability of bankruptcy of JSC "TEPLOSET"

Indicators	k_1	k_2	k_3	k_4	k_5	**z**	probability of bankruptcy
2009 y.	2.60	0.10	0.07	0.38	2.60	4.32	$z \geq 3$ $p \to 0$
2010 y.	1.60	0.11	0.10	0.35	1.60	3.19	$z \geq 3$ $p \to 0$
2011 y.	0.90	0.85	0.60	0.33	0.90	5.03	$z \geq 3$ $p \to 0$
-	-	-	-	-	-	-	-
2013 y.	0.66	0.14	0.16	0.77	2.24	4.22	$z \geq 3$ $p \to 0$

For the entire review period (i.e. 2009 – 2011 and 2013) to the value of Altman was $z \geq 3$. This means that it belongs to the set $\tilde{X}_4$ (the possibility of bankruptcy is small), therefore, a measure of fuzziness related to the same subset $\tilde{X}_4$ on the developed model, the least clearly defined in comparison with others and this judgment is most reliable ($\tilde{X}_2 \succ \tilde{X}_1 \succ \tilde{X}_3 \succ \tilde{X}_4$), as in the previous case.

Example 3. Using the accounting balance sheet of OAO Lomolomo" (O. A. "Lomolomo", 2013) [28] for three years (2009 – 2011), we compute the values of the coefficients ki and size - Altman (1) (see tab. 3).

Table 3. Metrics and Altman probability of bankruptcy of OAO Lomolomo"

Indicators	k_1	k_2	k_3	k_4	k_5	**z**	probability of bankruptcy
2009 y.	0.01	0.04	0.07	1.42	0.96	2.12	$1.81 \le z \le 2.77$ $p \in [0.35, 0.5]$
2010 y.	0.23	0.10	0.15	0.82	1.04	2.46	$1.81 \le z \le 2.77$ $p \in [0.35, 0.5]$
2011 y.	0.73	0.21	0.38	2.40	2.31	6.16	$z \ge 3$ $p \to 0$

The table shows that of the three years, investigated the company twice refers to $\tilde{X}_2$ (the possibility of bankruptcy average) and once to $\tilde{X}_4$ (the possibility of bankruptcy is small), and the first two conclusions in 2009 and 2010 deserve less confidence than the last the third case related to 2011., as are left in an orderly row $\tilde{X}_2 \succ \tilde{X}_1 \succ \tilde{X}_3 \succ \tilde{X}_4$, then how many is the most clear $\tilde{X}_4$. It can be concluded that done the calculations showed that the company is not facing bankruptcy, and in this case with a sufficient degree of certainty. Unfortunately information about the company for subsequent years is not available.

6 Simulation

In the model the source parameters k_i form the inputs to the system (input variables), allows obtaining the value of the parameter z-Altman. The system can move from one state to another under the influence of random input variables k_i . The value of z will be random, as it depends on random indices k_i. The values k_i are set randomly in MathCAD. The function generates random input variables, then using the model of Altman, the approximating function L_6, the function of decision making $I(p)$ and algorithm for computing preferences μ get the number set i, which belongs to the a number of sets ordered according to the measure of fuzziness $\tilde{X}_2 \succ \tilde{X}_1 \succ \tilde{X}_3 \succ \tilde{X}_4$

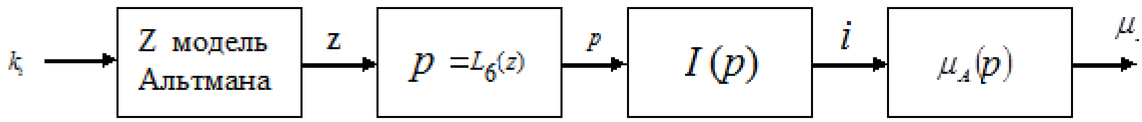

Figure 5. The scheme of calculation in the assessment of the creditworthiness of the company. The input source data are sequentially $k_i z$, p, i, μ for ordered sets $\tilde{X}_2 \succ \tilde{X}_1 \succ \tilde{X}_3 \succ \tilde{X}_4$.

The simulation modeling allows simulating the time-different situations for the same test, and given their number. The results of the tests will be determined by random simulation nature of the choice of input parameters. For the chosen simulation parameters, it is possible to obtain stable statistics. The model of Altman with the use of computer functions $p^j = L_6(z^j)$ and allows real values for the input parameters of the enterprises to replace the random values of the simulation model.

Played simulation of a random variable z that meets a certain set of random variables k_i. The parameter z was set randomly with a function randomly generating a uniformly distributed value in the interval [0, 3,5] the field of the function definition $p^j = L_6(z^j)$. Each input value of a random scalar z^j value was the probability $p^j = L_6(z^j)$ and using the membership functions of *I(p)* is the index $i = I(p^j)$ and hence the lot $\tilde{X}_i$ to which a company belongs, and the measure is evaluated facilities for identifying the enterprise to the many in the system are organized according $\tilde{X}_i$ to the degree of fuzziness (trust) $\tilde{X}_2 \succ \tilde{X}_1 \succ \tilde{X}_3 \succ \tilde{X}_4$.

Example 4: Let the input random variables are generated using random function. For example, a specific single realization of random uniformly distributed values on the interval [0, 3.5] would be equal $z = 2.710$. Using the model of Altman and approximating function $L_6(z)$ will get $p = L_6(z) = 0.266$. Based on the obtained values of this function, select index i with the decision-making function $i = I(p) = 2$. The decision-making function $I(p)$ allows you to choose in this case, the index $i = 2$ that meet the variety $\tilde{X}_2$ that is considering the case relates to $\tilde{X}_2$ (the possibility of bankruptcy). Measure the preference $\tilde{X}_2$ of many is to streamline the sets $\tilde{X}_2 \succ \tilde{X}_1 \succ \tilde{X}_3 \succ \tilde{X}_4$, first place to the right, and calculates the confidence measure $\mu_{\tilde{X}_2}(p) = \mu_{\tilde{X}_2}(0.266) = 0.56$ included identifying the enterprise to the many X_2 in the system are organized according to the degree of fuzziness.

Was conducted m = 1-1000 simulations of a random variable *z*, the results of the model are given in table 4 below. In the second column the expected value, the third is the RMS deviation of the quantities z, p, i, μ.

Table 4. Mathematical expectation and standard deviation of the variables zz, p, i, μ.

	M	$\sigma = \sqrt{D}$
z	1.741	1.025
p	0.599	0.33
i	1.815	1.071
μ	0.91	0.147

In Fig.5-8 presents the simulation results for different values:

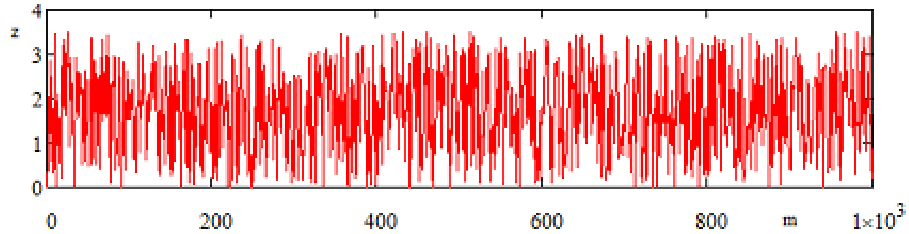

Figure 5. Simulation realization of a random process *Z(m)*.

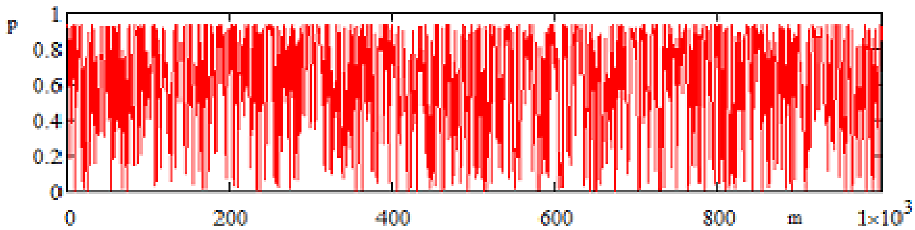

Figure 6. Simulation realization of a random process *p*(m).

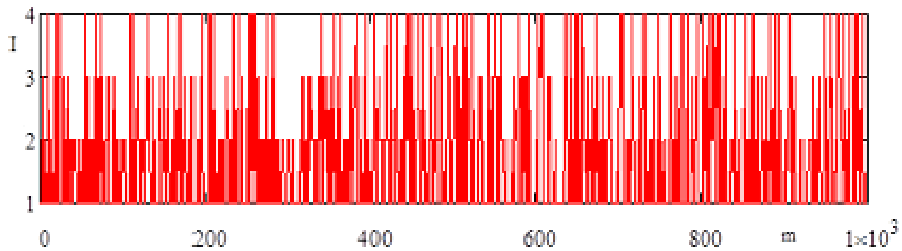

Figure 7. Simulation realization of the random process i(m) numbers of fuzzy sets.

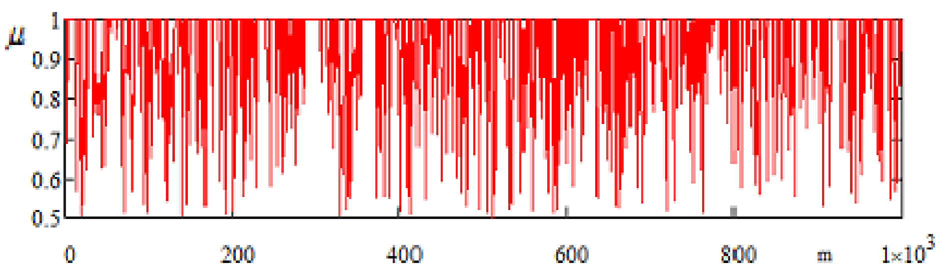

Figure 8. Simulation realization of random process μ (m).

In figure 5, it is seen that the random variable *z* lie in the interval $z \in [0, 3.5]$. Simulated random values z reliable and does not reach the *p=1* value Fig.6, because of the properties of the functions which $\max\limits_{z \in [0,3.5]} L_6(z) = L(0) = 0.936$. With regard to figure 7, the expected value is $M = 1.815$, because of the asymmetric properties of the selection function and with large enough root mean square deviation σ = 1.071. Figure 8 shows the level of preference μ to the interval of Altman and in simulated cases, the function of the decision μ> 0.5 Fig.3. The mathematical expectation of fuzziness is close to one $M(\mu)$ = 0.91 with a small RMS deviation $\sigma(\mu) = 0.147$, which indicates a high degree of confidence to the obtained probabilities of bankruptcy. Work based on the program (Bamadio, Lebedev, 2014). The results of the study show the possibility of application of the methodology to the cases of calculating the probability of bankruptcy.

7 Conclusion

The above-described mathematical model complements the model of Altman procedure continuous calculate the probability of bankruptcies of companies with higher-order polynomial obtained by mean integrated squared approximation, and also introduced a procedure of calculation of values of membership functions of fuzzy sets that allows you to specify which of the subsets is more clearly or unclearly specified. The function of decision-making $l(p)$ and is determined by its economic sense. Give the scheme of calculation in the assessment of the creditworthiness of company. Input source data $c_o$nsequentially are *z, p, i,* μ for ordered sets $\tilde{X}_2 \succ \tilde{X}_1 \succ \tilde{X}_3 \succ \tilde{X}_4$.

The simulation study conducted in this paper confirms the conclusions about the capabilities of the model and gave a set of stable statistics. Using the proposed model, the lenders can more than justify making decisions about the creditworthiness of the enterprise. Developed a method for evaluation of fuzziness can be applied to other models assesses the creditworthiness of the enterprise: models Davydov, Zaitseva, saifullina for and other relevant with the necessary modification.

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