

Long-run Cointegration and Market Equilibrium in Large Cap Stocks

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Abstract

Although stock returns are thought to be stationary and showing mean-reverting behaviors, stock price levels don't have to follow this manner. This paper finds that the general market condition has a commanding power on stock price level movements which are non-stationary individually but with statistically significant long-run cointegration relationships within sub-groups of large cap stocks in the U.S. market. Moreover, the vector error-correction models provide significant evidences that the short-run stock price level movements can be very volatile and show a reluctant behavior of returning to the long-run equilibrium. However, the estimated and the predicted cointegration parameters provide statistical evidences that the long-run equilibrium relationships are solid and stationary over time.

JEL classification numbers: G12; G14; G17

Keywords: Cointegration, Market Equilibrium, Stock Price Level

1 Introduction

Market risk is recognized as one major risk factor in the stock market. Put this another way, the general market condition should have impacts on all stocks that are being traded in the market. Therefore, the general market condition is thought to be acting as the law of gravity. Both the three-factor model (Fama and French, 1995) and the five-factor model (Fama and French, 2015) confirmed this empirically by showing that the market risk factor is significant in describing the expected average stock returns. However, most of finance literatures are focusing on the stock returns rather than on the stock price levels. Stock returns are exhibiting mean-reverting behaviors, because companies' future growths (Lakonishok, Shleifer and Vishny, 1994) and profitability (Fama and French, 2000) are highly mean-reverting. This phenomenon is partly due to the fact that continuously beating the market expectation is difficult. Hence, companies' future valuations are prone to grow at a slower rate which will drag down the return, and vice versa.

Although stock returns are subject to the mean-reverting behavior, stock price levels don't have to comply with this. Stock price levels should be free to continue

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going up if the general market condition is sound or to continue going down if the opposite market condition preserves. In other words, stock price levels should have the ability of continuously drifting away from its long-run mean, showing non-stationary behaviors. However, assuming that the market risk has the general command on the entire stock market, then, the stock price level movements should also give respect to it. As a consequence, cointegration relationships are expected to exist among stock price levels.

2 Literature Review

Lakonishok, Shleifer and Vishny (1994) argued that value stocks have no more risk than growth stocks. The reason that value stocks outperform growth stocks is due to the mean-reverting of future company growth rates and the agency problems between professional managers and investors. Campbell and Shiller (1998) claimed that, if the valuation ratios, such as dividend-to-price ratio (D/P) and price-to-earnings ratio (P/E), are at extremely levels, then the stock prices have to move accordingly to bring these ratios back to the historical levels.

Fama (1991) argued that short-term stock returns are predictable from past returns. However, long-term stock returns are not able to be predicted precisely by past returns. This implies that, in the long-term, stock price levels can move freely and show non-stationary behaviors. Fama and French (1995) proposed the three-factor model which includes the market risk factor, size factor and the book-to-market-ratio factor. While, Fama and French (2015) added profitability factor and investment factor as two new factors to their previous three-factor model to come up with the five-factor model. The market risk is significant in both the three-factor model and the five-factor model. While, the book-to-market-ratio factor becomes redundant after the two new factors joined in. This result indicates that the market risk factor might always remain significant in describing expected average stock returns, but other factors might be subject to specific samples and periods. This also gives more confidence to this research paper, since the long-run cointegration relationships among stock prices need a general market force to command.

Brenner and Kroner (1995), by using a no-arbitrage, cost-of-carry asset pricing model, demonstrated that cointegration relationships exist between the spot and the futures prices. They also found that conditions for cointegration relationships are more likely to be satisfied in the currency markets than in the commodity markets. Christopoulos and Tsionas (2004) investigated the long-run relationship between financial depth and economic growth. They identified a single long-run cointegration equilibrium relationship between the two variables. They concluded that the sole cointegration relationship indicates the causality from financial depth to growth. Hui and Fong (2015) found that there is cointegration relations between sovereign credit default swap (CDS) and currency option markets. Their findings suggest that credit risk generates impacts on the option market expectation in the long-run, while deviations are persistent in the short-run.

3 Data and Methodology

3.1 Data

Dow Jones Industrial Average is a widely recognized price-weighted market index for the U.S. stock market. It consists of thirty U.S. large cap companies. Therefore, its performances reflect the general U.S. stock market sentiment and healthiness.

In order to test the long-run cointegration relationships and the speed of the error corrections, companies are selected from the Dow Jones Industrial Average. For the fact that the companies consisting of the Dow index have been changing over time and the fact that the vector error-correction model, which is based on a G-variable VAR(P) model framework, requires time series variables to have long and balanced data panel, a total of 21 companies are selected from the index. The stock daily price level data and stock daily return data each covers for a total of 25 years from December 28th, 1992 to December 29th, 2017 (6,300 consecutive trading days for every company).

Companies that are included in the sample are: American Express, Boeing, Caterpillar, Cisco Systems, DuPont, Walt Disney, The Home Depot, IBM, Intel, Johnson & Johnson, Coca-Cola, McDonald's, 3M, Merck & Company, Microsoft, Nike, Pfizer, Procter & Gamble, UnitedHealth Group, United Technologies, Walmart.

3.2 Stationarity

Suppose a time series variable can be modeled by an AR(P) model as shown in equation (1). Then, the time series variable is said to be stationary if the underlying AR polynomials contain no unit root solution and the error term (ε_t) follows the white noise process. However, if the model contains one or more unit roots, then the variable is non-stationary.

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_p Y_{t-p} + \varepsilon_t \quad (1)$$

A white noise error should satisfy the following three conditions:

- (i) $E[\varepsilon_{t-j}] = 0$ for any time period "j"
- (ii) $E[(\varepsilon_{t-j})^2] = \sigma$ (a constant) for any time period "j"
- (iii) $E[\varepsilon_{t-j}, \varepsilon_{t-j-s}] = 0$ for any time periods "j" and "s"

3.3 Testing for Stationarity

The essence of stationarity test is to examine whether or not the time series variable contains a unit root solution. However, if a variable is correctly specified as a high order AR(P) model, then, it is extremely difficult to find solutions to its AR polynomials. As a consequence, testing will be technically impossible to conduct. Fortunately, a practical method is to express a time series variable as an AR(1) model only for testing purpose regardless of its true specifications. The reason is that, if the variable is non-stationary, the absolute value of the coefficient on Y_{t-1} of the AR(1) model will be equal to one. The unit root test results will unveil this information.

This research paper uses the Augmented Dickey-Fuller unit root test with one lag term for testing the stationarity condition. The testing equation is shown in equation (2). The corresponding hypotheses are $H_0: \alpha_1 = 0$ (non-stationary) versus $H_a: \alpha_1 < 0$ (stationary)

$$\Delta y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 t + \delta_1 \Delta y_{t-1} + \varepsilon_t \quad (2)$$

3.4 Cointegration and the Error-Correction Model

3.4.1 Cointegration in the Single-Equation Framework

A stationary time series variable should exhibit constant long-run mean, constant long-run variance and mean-reverting behavior. In comparison, a non-stationary time series variable will violate one or more of these conditions. Its behaviors are not predictable and will not show the pattern of mean-reverting. However, two non-stationary time series variables could potentially form a single stationary relationship, which is said to be “cointegrated”.

$$Y_t = \beta X_t + \varepsilon_t \quad (3)$$

Equation (3) shows a simple two-variable case, where both Y_t and X_t are non-stationary and $I(1)$ ². If $[Y_t - \beta X_t]$ is $I(0)$, then Y_t and X_t are cointegrated. Or, we could say that both Y_t and X_t are non-stationary, but the error term (ε_t) is stationary which contains the information about the long-run equilibrium relationship of Y_t and X_t . In this simple single-equation setting, Y_t and X_t forms a long run equilibrium relationship, although there might be short-run deviations from the equilibrium. The long-run equilibrium relationship is specified by the “cointegrating vector” as shown in vector (4)³. The testing for the existence of cointegration relationship for the single-equation framework can be conducted by implementing the Engle-Granger test.

$$\begin{bmatrix} 1 \\ -\beta \end{bmatrix} \quad (4)$$

3.4.2 Cointegration in the Multiple-Equation Framework

In the multiple-equation framework, there will be “G” time series variables: $Y1_t, Y2_t, Y3_t \dots YG_t$. One requirement for potential cointegration relationships to exist is that all variables must be $I(1)$. The maximum cointegration relationships can be up to “G-1” in a G-variable setting.

The first cointegration relationship can be written as:

$$Y1_t = \beta_1 Y2_t + \beta_2 Y3_t + \dots + \beta_{G-1} YG_t + \varepsilon_t.$$

The second cointegration relationship can be written as:

$$Y1_t = \alpha_1 Y2_t + \alpha_2 Y3_t + \dots + \alpha_{G-1} YG_t + \varepsilon_t.$$

The third cointegration relationship can be written as:

$$Y1_t = \delta_1 Y2_t + \delta_2 Y3_t + \dots + \delta_{G-1} YG_t + \varepsilon_t.$$

...

The (G-1)th cointegrating relationship can be written as:

$$Y1_t = \varphi_1 Y2_t + \varphi_2 Y3_t + \dots + \varphi_{G-1} YG_t + \varepsilon_t.$$

Likewise, the cointegration matrix can be expressed as matrix (5). Each column within the matrix represents one cointegration relationship. The maximum dimension that the cointegration matrix can take is $G \times (G-1)$.

² $I(1)$ means that the underlying time series variable is non-stationary and contains one unit root.

³ The cointegration parameter of Y_t is normalized to “1”.

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ -\beta_1 & -\alpha_1 & -\delta_1 & \dots & -\phi_1 \\ -\beta_2 & -\alpha_2 & -\delta_2 & \dots & -\phi_2 \\ \dots & \dots & \dots & \dots & \dots \\ -\beta(G-1) & -\alpha(G-1) & -\delta(G-1) & \dots & -\phi(G-1) \end{bmatrix} \quad (5)$$

3.4.3 The Johansen Rank Test and the Error-Correction Model

The major drawback of the Engle-Granger test is that it can only detect one cointegration relationship for a single-equation (two-variable) framework. For a typical G -variable framework, the Johansen Rank Test is more efficient and can detect up to $(G-1)$ cointegration relationships.

The G variables are firstly expressed as a VAR(P) model as shown in equation (6) with all variables being $I(1)$. \mathbf{Z}_t is the matrix that contains all the dependent variables. \mathbf{Z}_{t-1} through \mathbf{Z}_{t-p} each contains 1-period lagged independent variables through P -period lagged independent variables. $\mathbf{\Pi}_1$ through $\mathbf{\Pi}_p$ contain coefficients of \mathbf{Z}_{t-1} through \mathbf{Z}_{t-p} .

$$\mathbf{Z}_t = \mathbf{\Pi}_1 \mathbf{Z}_{t-1} + \mathbf{\Pi}_2 \mathbf{Z}_{t-2} + \mathbf{\Pi}_3 \mathbf{Z}_{t-3} + \dots + \mathbf{\Pi}_p \mathbf{Z}_{t-p} + \boldsymbol{\varepsilon}_t \quad (6)$$

Equation (6) is undergone a process called ‘‘cointegration transformation’’. The transformed equation is the error-correction model as shown in equation (7). The error-correction model requires that the original \mathbf{Z}_{t-1} through \mathbf{Z}_{t-p} to be $I(1)$, since cointegration is one type of long-run relationship between non-stationary variables with one unit root. Importantly, stationary time series variables are not able to form cointegration relationships.

$$\Delta \mathbf{Z}_t = \mathbf{\Gamma}_1 \Delta \mathbf{Z}_{t-1} + \mathbf{\Gamma}_2 \Delta \mathbf{Z}_{t-2} + \mathbf{\Gamma}_3 \Delta \mathbf{Z}_{t-3} + \dots + \mathbf{\Gamma}_{P-1} \Delta \mathbf{Z}_{t-(P-1)} + \mathbf{\Pi} \mathbf{Z}_{t-P} + \boldsymbol{\varepsilon}_t \quad (7)$$

In equation (7), $\Delta \mathbf{Z}_{t-1}$ through $\Delta \mathbf{Z}_{t-(p-1)}$ represent the differenced variables of \mathbf{Z}_{t-1} through $\mathbf{Z}_{t-(p-1)}$. Because \mathbf{Z}_{t-1} through $\mathbf{Z}_{t-(p-1)}$ are supposed to be $I(1)$, then $\Delta \mathbf{Z}_{t-1}$ through $\Delta \mathbf{Z}_{t-(p-1)}$ are transformed to be $I(0)$. However, \mathbf{Z}_{t-P} is not differenced, hence, still being $I(1)$. The matrices $\mathbf{\Gamma}_1$ through $\mathbf{\Gamma}_{P-1}$ are the coefficients of $\Delta \mathbf{Z}_{t-1}$ through $\Delta \mathbf{Z}_{t-(p-1)}$, therefore containing short-term information. More importantly, derived from the cointegration transformation, the $\mathbf{\Pi}$ matrix represents $(\mathbf{I} - \mathbf{\Pi}_1 - \mathbf{\Pi}_2 - \mathbf{\Pi}_3 - \dots - \mathbf{\Pi}_p)$ with a dimension of $G \times G$. It includes the coefficients of \mathbf{Z}_{t-P} and is the only source of long-run information. In summary, the error-correction model contains both the error-correction parameters (from $\mathbf{\Gamma}_1$ through $\mathbf{\Gamma}_{P-1}$) and the cointegration parameters (from $\mathbf{\Pi}$).

Because the cointegration relationship is one type of long-run information, we can test and detect this information by investigating the $\mathbf{\Pi}$ matrix. The essence of the Johansen Rank Test is to determine the rank of the $\mathbf{\Pi}$ matrix. The test follows a sequential testing process, starting from testing $\text{rank}(\mathbf{\Pi}) = 0$. If the testing results suggest that $\mathbf{\Pi}$ matrix has a rank of 0 (zero-rank) or G (full-rank), then no possible cointegration relationship could exist. In contrast, if the testing results support that the $\mathbf{\Pi}$ matrix has a rank that is between 0 and G , then the number of cointegration relationships is equal to the rank. For example, if the Johansen Rank Test suggests that the rank of $\mathbf{\Pi}$ matrix is 3 for a 5-variable framework, this will lead to the

conclusion that 3 cointegration relationships should exist among the 5-variable framework. Moreover, the Π matrix is able to be decomposed into two components: $\Pi = \alpha \times \beta'$ where

- (i) Π has a dimension of $G \times G$
- (ii) α has a dimension of $G \times r$ (r is the rank of Π matrix)
- (iii) β' has a dimension of $r \times G$
- (iv) β' is the cointegration relationship matrix
- (v) α is the error correction parameters

4 Empirical Results

This section contains four major subsections. The first subsection focuses on companies' historical stock return correlations and the testing results of their stationarity conditions. The second subsection presents the companies' historical stock price level correlations and the testing results of their stationarity conditions. The third subsection performs the Johansen Rank Test for the stock price level data and analyzes the cointegration conditions. The last subsection shows the vector error-correction model and the corresponding estimated long-run equilibrium parameters.

4.1 Stationarity of Stock Returns

Table 1 provides the correlation matrix that shows the correlation coefficients for each pair of companies' historical returns. A total of 21 large cap stocks that participating in consisting of the Dow index are included in the table. For each company, the stock daily return data covers for a time span of 25 years from December 28th, 1992 to December 29th, 2017 (each with 6,300 consecutive trading days). In order to avoid the "multiple comparisons" problem (false discovery problem), Bonferroni adjustments are incorporated when performing the correlation test. The results are very impressive since all pair of companies' historical returns are significantly correlated at 1% significance level. This confirmed that the market risk has the universal impact on stock returns. Past literatures, such as Fama and French (1995) and Fama and French (2015), found that the market risk is one of the major risk factors to describe the expected average stock returns. By including more recent data, table 1 offers empirical support to those related past literatures.

Since market risk is thought to have the command on stock returns, then, can it form long-run cointegration relationships among them? Before we can answer this question, we firstly have to investigate the time series behaviors of stock returns. Table 2 presents the Augmented Dickey-Fuller unit root test for companies' historical returns. The testing equation includes both drift and trend. In order to ensure uncorrelated testing equation errors, one augmentation term is also included. The results are again very astonishing. We can see that the testing statistic for each company is very significant. Therefore, we can easily reject the null hypothesis for every individual company, meaning that each company's historical returns are stationary. Like discussed in the section 3.2, a stationary time series variable shows constant long-run mean and variance, as well as a mean-reverting behavior. Alternatively, if the stock return drifts away from its long-run mean, there will be forces to pull it back. Table 2 provides empirical supports with more recent data to past literatures, such as Fama and French (1995) and Siegel and Thaler (1997).

Table 1: Correlation Matrix of Companies' Historical Returns

This table shows the correlation coefficients for each pair of companies' historical returns for a total of 21 large cap stocks that participating in consisting of the Dow index. For each company, the stock daily return data covers for a total of 25 years from December 28th, 1992 to December 29th, 2017 (each with 6,300 consecutive trading days). All companies are represented by their tickers. In order to counteract the problem of multiple comparisons, the correlation coefficients are adjusted by the Bonferroni Adjustment. The single asterisk "*" in this table denotes that the correlation relationship is significant at 1% significance level.

	AXP	BA	CAT	CSCO	DD	DIS	HD	IBM	INTC	JNJ	KO
AXP	1										
BA	0.4071*	1									
CAT	0.4637*	0.4060*	1								
CSCO	0.3914*	0.2963*	0.3257*	1							
DD	0.4764*	0.4179*	0.5326*	0.3111*	1						
DIS	0.4673*	0.3862*	0.3942*	0.3670*	0.4096*	1					
HD	0.4702*	0.3614*	0.3962*	0.3605*	0.3931*	0.4167*	1				
IBM	0.3811*	0.2972*	0.3387*	0.4680*	0.3377*	0.3629*	0.3515*	1			
INTC	0.3961*	0.3151*	0.3429*	0.5815*	0.3279*	0.3659*	0.3609*	0.4788*	1		
JNJ	0.3354*	0.2909*	0.2736*	0.2057*	0.3142*	0.3082*	0.2939*	0.2585*	0.2337*	1	
KO	0.3375*	0.2965*	0.2844*	0.2094*	0.3301*	0.3085*	0.3193*	0.2294*	0.2329*	0.3936*	1
MCD	0.3253*	0.2839*	0.2868*	0.2467*	0.3067*	0.3039*	0.3475*	0.2575*	0.2355*	0.2936*	0.3294*
MMM	0.4515*	0.4035*	0.4785*	0.2986*	0.5395*	0.3863*	0.3884*	0.3233*	0.3299*	0.3421*	0.3468*
MRK	0.3331*	0.2859*	0.2632*	0.2219*	0.3231*	0.3023*	0.2892*	0.2533*	0.2435*	0.5135*	0.3392*
MSFT	0.4033*	0.3204*	0.3431*	0.5310*	0.3346*	0.3829*	0.3757*	0.4442*	0.5788*	0.2905*	0.2745*
NKE	0.3481*	0.2998*	0.3032*	0.2677*	0.3401*	0.3226*	0.3597*	0.2702*	0.2793*	0.2631*	0.2553*
PFE	0.3785*	0.2987*	0.2933*	0.2584*	0.3270*	0.3251*	0.3176*	0.2787*	0.2596*	0.5237*	0.3462*
PG	0.3229*	0.2652*	0.2775*	0.2006*	0.3475*	0.2774*	0.2973*	0.2245*	0.2121*	0.4250*	0.4504*
UNH	0.2998*	0.2605*	0.2506*	0.1931*	0.2569*	0.2633*	0.2653*	0.1923*	0.1860*	0.2954*	0.2494*
UTX	0.4849*	0.5261*	0.5084*	0.3546*	0.5006*	0.4374*	0.4257*	0.3575*	0.3512*	0.3227*	0.3366*
WMT	0.3544*	0.2832*	0.2930*	0.2770*	0.3257*	0.3199*	0.5182*	0.2788*	0.2806*	0.3172*	0.3218*
	MCD	MMM	MRK	MSFT	NKE	PFE	PG	UNH	UTX	WMT	
MCD	1										
MMM	0.2981*	1									
MRK	0.2688*	0.3033*	1								
MSFT	0.2555*	0.3192*	0.2764*	1							
NKE	0.2625*	0.3245*	0.2289*	0.2819*	1						
PFE	0.2777*	0.3284*	0.5647*	0.3138*	0.2442*	1					
PG	0.3365*	0.3597*	0.3588*	0.2326*	0.2343*	0.3673*	1				
UNH	0.2053*	0.2784*	0.2750*	0.2231*	0.2234*	0.2844*	0.2487*	1			
UTX	0.3305*	0.5129*	0.3099*	0.3851*	0.3280*	0.3481*	0.3374*	0.2880*	1		
WMT	0.3147*	0.3319*	0.2911*	0.3130*	0.2913*	0.3191*	0.3239*	0.2108*	0.3417*	1	

Table 2: Augmented Dickey-Fuller Unit Root Test for Companies' Returns

This table presents the results for testing the stationarity condition for each individual company's historical returns for a total of 21 large cap stocks that participating in consisting of the Dow index. For each company, the stock daily return data covers for a total of 25 years from December 28th, 1992 to December 29th, 2017 (each with 6,300 consecutive trading days). The Augmented Dickey-Fuller Unit Root Test equation includes both drift and trend. One augmentation term is included to ensure uncorrelated testing equation errors. The null hypothesis: the time series contains unit root. The 1% critical value is -3.96. The notation "R***" means that the null hypothesis is rejected at 1% significance level.

Company	Test Statistic	Hypothesis	Stationarity
AXP	-60.274	R***	I(0)
BA	-57.264	R***	I(0)
CAT	-57.112	R***	I(0)
CSCO	-59.914	R***	I(0)
DD	-58.358	R***	I(0)
DIS	-59.348	R***	I(0)
HD	-58.531	R***	I(0)
IBM	-58.008	R***	I(0)
INTC	-58.407	R***	I(0)
JNJ	-60.894	R***	I(0)
KO	-58.534	R***	I(0)
MCD	-59.267	R***	I(0)
MMM	-60.038	R***	I(0)
MRK	-58.031	R***	I(0)
MSFT	-59.014	R***	I(0)
NKE	-57.898	R***	I(0)
PFE	-60.514	R***	I(0)
PG	-59.990	R***	I(0)
UNH	-56.628	R***	I(0)
UTX	-59.605	R***	I(0)
WMT	-59.836	R***	I(0)

However, one major implication from table 2 is that, since all sample stocks' historical returns are stationary [I(0)], it is impossible to form cointegration relationships among them. Because cointegration requires the underlying time series variables to be non-stationary and each contains only one unit root [I(1)]. Table 1 and table 2 together exhibit evidences that, although market risk has a general command on stock returns across different firms, it doesn't foster stock returns to form long-run cointegration relationships. In other words, long-run cointegration relationships among stock returns are not supported by the findings.

4.2 Stationarity of Stock Price Levels

Table 3 displays the correlation matrix that shows the correlation coefficients for each pair of companies' historical price level movements. Unlike the return correlation matrix (table 1), although most stock price level pairs are significantly correlated at 1% level, several pairs are not statistically significant such as UNH and CSCO, PFE and JNJ. This gives us a hint that the stock price level movements may enjoy a little bit more freedom than the stock returns do.

Table 4 is very important in describing the behaviors of stock price level movements. The testing equation includes both drift and trend, as well as one augmentation term for ensuring uncorrelated testing equation errors. However, the results are so surprising, because it is completely the opposite picture to table 2. While all stock returns are statistically stationary, most stock price levels are non-stationary (FTR the null hypothesis), except for two companies in the sample (NKE and WMT). The results strongly support the argument that stock price levels may behave completely different than stock returns do. Since we have 19 companies in our sample that are non-stationary on the price level, sequential follow-up tests are needed. The reason is that the Augmented Dickey-Fuller unit root test can only tell us whether or not the underlying time series data contains unit root, but without telling how many. Therefore, if the null hypothesis is failed to reject on the level data, then testing on the first-differenced data should be performed. The process needs to continue until the first time that we reject the null hypothesis. The number of differencing tells us the total number of unit roots that are contained in the original time series data, because each differencing removes 1 unit root. The sequential follow-up unit root tests on the first-differenced data of all 19 companies show that they are all stationary (the null hypothesis is rejected). Therefore, we have strong evidences to conclude that all 19 companies are non-stationary on the original level data with 1 unit root inside [I(1)]. In other words, the stock price levels by themselves are not stationary. But the first-differenced stock price levels are stationary. This finding is so exciting and crucial, because I(1) is the necessary condition to have potential cointegration relationships, and the further vector error-correction model depends on this condition.

Table 3: Correlation Matrix of Companies' Historical Price Levels

This table shows the correlation coefficients for each pair of companies' historical price level movements for a total of 21 large cap stocks that participating in consisting of the Dow index. For each company, the stock daily price level data covers for a total of 25 years from December 28th, 1992 to December 29th, 2017 (each with 6,300 consecutive trading days). All companies are represented by their tickers. In order to counteract the problem of multiple comparisons, the correlation coefficients are adjusted by the Bonferroni Adjustment. The single asterisk "*" in this table denotes that the correlation relationship is significant at 1% significant level.

	AXP	BA	CAT	CSCO	DD	DIS	HD	IBM	INTC	JNJ	KO
AXP	1										
BA	0.4061*	1									
CAT	0.2300*	0.6060*	1								
CSCO	0.4197*	-0.0788*	-0.1568*	1							
DD	0.5646*	0.5057*	0.4382*	0.5426*	1						
DIS	0.4685*	0.7292*	0.4721*	0.3708*	0.7428*	1					
HD	0.6289*	0.6750*	0.4037*	0.2874*	0.6803*	0.8132*	1				
IBM	0.5589*	0.5759*	0.3204*	-0.0549*	0.3226*	0.4575*	0.4674*	1			
INTC	0.2988*	-0.1328*	-0.0866*	0.8710*	0.6305*	0.3768*	0.3137*	-0.1237*	1		
JNJ	0.6714*	0.6532*	0.1584*	0.1579*	0.4257*	0.5638*	0.7314*	0.6502*	0.028	1	
KO	0.1548*	-0.1975*	-0.1862*	0.4022*	0.1890*	-0.0216	-0.2269*	0.1538*	0.3336*	0.0122	1
MCD	0.3645*	0.7199*	0.5324*	0.0713*	0.3402*	0.7122*	0.6221*	0.6648*	-0.0354	0.5583*	-0.0347
MMM	0.4225*	0.5241*	0.3507*	-0.0899*	0.3087*	0.4602*	0.7014*	0.3394*	-0.0955*	0.5651*	-0.3660*
MRK	0.6074*	0.0777*	-0.2060*	0.5893*	0.5615*	0.3620*	0.4189*	0.2820*	0.5691*	0.4371*	0.3712*
MSFT	0.3171*	-0.0414	-0.1349*	0.8063*	0.6531*	0.4226*	0.4190*	-0.0855*	0.8777*	0.1740*	0.2984*
NKE	-0.0076	0.3986*	0.4911*	-0.2587*	0.1496*	0.2100*	0.0901*	0.2153*	-0.2209*	-0.0138	-0.0823*
PFE	0.2116*	-0.1971*	-0.1150*	0.7307*	0.5990*	0.2980*	0.2685*	-0.2170*	0.8534*	-0.025	0.2632*
PG	0.5285*	0.2144*	0.0642*	0.3122*	0.5735*	0.3322*	0.4112*	0.3651*	0.3365*	0.3888*	0.2255*
UNH	0.4224*	0.4921*	0.3285*	0.0115	0.4309*	0.4744*	0.7427*	0.1729*	0.0559*	0.5886*	-0.3122*
UTX	0.5487*	0.6194*	0.5321*	0.1370*	0.6268*	0.6310*	0.6236*	0.6062*	0.1132*	0.5957*	0.0208
WMT	0.4654*	0.3270*	0.1353*	-0.3197*	-0.1556*	0.0994*	0.3881*	0.6090*	-0.4336*	0.5678*	-0.2310*
	MCD	MMM	MRK	MSFT	NKE	PFE	PG	UNH	UTX	WMT	
MCD	1										
MMM	0.4315*	1									
MRK	0.0282	0.2979*	1								
MSFT	0.0107	0.1188*	0.7006*	1							
NKE	0.2386*	-0.0830*	-0.3884*	-0.3272*	1						
PFE	-0.1828*	-0.0629*	0.6365*	0.8532*	-0.2565*	1					
PG	0.0770*	0.3720*	0.6998*	0.4504*	-0.1483*	0.3776*	1				
UNH	0.2530*	0.8080*	0.3162*	0.2373*	-0.0316	0.1523*	0.3360*	1			
UTX	0.5019*	0.4543*	0.4114*	0.2215*	0.2512*	0.2065*	0.4914*	0.5009*	1		
WMT	0.4420*	0.5844*	0.1215*	-0.3454*	-0.0088	-0.4420*	0.1634*	0.3681*	0.3442*	1	

Table 4: Augmented Dickey-Fuller Unit Root Test for Companies' Price Levels

This table presents the results for testing the stationarity condition for each individual company's historical price levels for a total of 21 large cap stocks that participating in consisting of the Dow index. For each company, the stock price level data covers for a total of 25 years from December 28th, 1992 to December 29th, 2017 (each with 6,300 consecutive trading days). The Augmented Dickey-Fuller Unit Root Test equation includes both drift and trend. One augmentation term is included to ensure uncorrelated testing equation errors. The null hypothesis: the time series contains unit root. The 1% critical value is -3.96. The notation "R***" means that the null hypothesis is rejected at 1% significance level and "FTR" means that the null hypothesis is "fail to reject" at 1% significance level. If the null hypothesis is "FTR" for the level data, then unit root test on first-differenced data is necessary and required. If the null hypothesis is "R***", then no further unit root test is needed.

Company	Testing on Level Data		Testing on first-differenced Data		Stationarity
	Test Statistic	Hypothesis	Test Statistic	Hypothesis	
AXP	-2.433	FTR	58.060	R***	I(1)
BA	-1.532	FTR	-57.181	R***	I(1)
CAT	-3.184	FTR	-56.687	R***	I(1)
CSCO	-2.537	FTR	-58.860	R***	I(1)
DD	-2.418	FTR	-58.320	R***	I(1)
DIS	-1.533	FTR	-59.062	R***	I(1)
HD	-1.647	FTR	-57.704	R***	I(1)
IBM	-3.285	FTR	-55.339	R***	I(1)
INTC	-2.153	FTR	-56.833	R***	I(1)
JNJ	-2.734	FTR	-58.685	R***	I(1)
KO	-3.868	FTR	-57.317	R***	I(1)
MCD	-1.899	FTR	-57.930	R***	I(1)
MMM	-2.341	FTR	-58.527	R***	I(1)
MRK	-2.077	FTR	-56.764	R***	I(1)
MSFT	-1.583	FTR	-58.444	R***	I(1)
NKE	-4.128	R***	N/A	N/A	I(0)
PFE	-2.249	FTR	-59.117	R***	I(1)
PG	-3.317	FTR	-57.084	R***	I(1)
UNH	-1.978	FTR	-56.457	R***	I(1)
UTX	-3.727	FTR	-56.548	R***	I(0)
WMT	-4.086	R***	N/A	N/A	I(1)

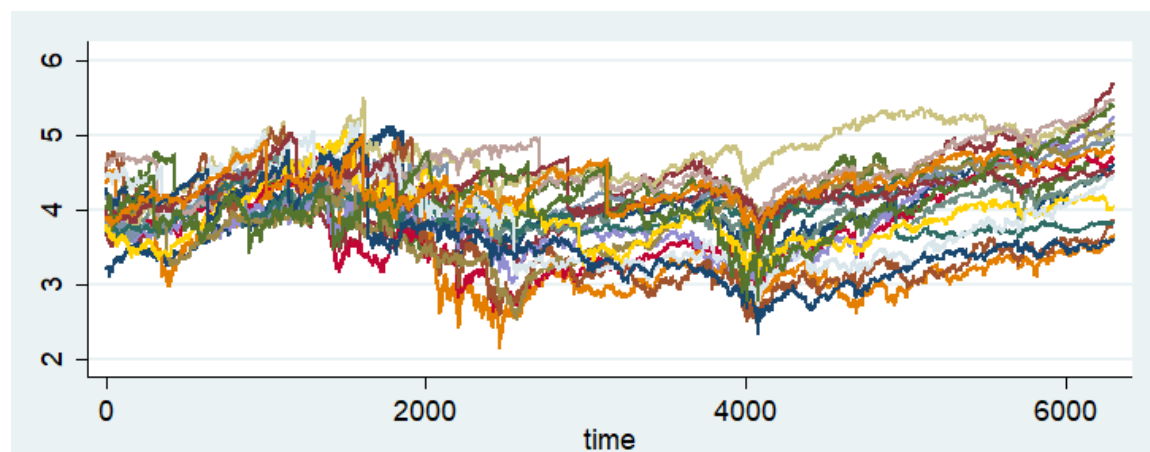
4.3 Cointegration of Stock Price Levels

The last section identified that 19 out of 21 companies are I(1). Therefore, this finding makes us to expect that there are possibilities that cointegration relationships are existing among those companies or among sub-groups of companies. Graph 1 shows the time series plots of all 19 companies' historical stock price level

movements⁴. The price levels are in the logged form [i.e. $\ln(\text{price})$], covering 6,300 consecutive trading days (25 years) from December 28th, 1992 to December 29th, 2017. The graph embraces a sense that stock price levels move coherently throughout time. Meanwhile, the spreads between price levels seem stable and trend together. Although Graph 1 is not a formal statistical test, nevertheless, it provides us with a visual clue that supports the potential existence of cointegration relationships among stock price levels.

Graph 1: Time Series Plots of Companies' Prices Level Movements

This graph displays time series plots of companies' historical price level movements for a total of 19 large cap stocks that participating in consisting of the Dow index. All 19 companies' stock price level movements are I(1). For each company, the stock price level data covers for a total of 25 years from December 28th, 1992 to December 29th, 2017 (each with 6,300 consecutive trading days). The vertical axis represents stock price level in the logged scale [i.e. $\ln(\text{price})$]. The horizontal axis represents time, with "0" representing the 1st trading day of the sample. Company legends are not displayed, since individual company's stock price level movements are not focused while the cohort stock price level movements are more interested in.



Although, no cointegration relationship is found by conducting the Johansen Rank Test for all 19 companies as a whole, cointegration relationships do exist among sub-groups of companies. The first company group (denote Group 1) includes companies: AXP, BA, CAT, CSCO, DD, DIS, HD, IBM, INTC, JNJ. The second company group (denote Group 2) includes companies: KO, MCD, MMM.MRK, MSFT, PFE, PG, UNH, UTX. Table 5 shows the formal Johansen Rank Test for the Group 1 companies. The essence of the Johansen Rank Test is to determine the rank of the Π matrix, which is discussed in detail in section 3.4.3. Since there are 10 companies within group 1, the minimum rank could be 0 [denote $\text{rank}(0)$] and the maximum rank could be 10 [denote $\text{rank}(10)$]. Both $\text{rank}(0)$ and $\text{rank}(10)$ implies that there is no cointegration relationship existing among selected companies. However, a $\text{rank}(r)$, where $0 < r < 10$, confirms the existence of the cointegration relationships and the number of cointegration relationships is equal to "r". The rank test follows a sequential testing process until the first time that the null hypothesis is not rejected. In table 5, we can see the trace statistic for testing rank of $\Pi = 0$ is 258.67, which is well

⁴ NKE and WMT are excluded from the graph and will not be included in the following vector error-correction models, because both companies' stock price level movements are stationary [I(0)]. However, cointegration requires time series variables to be I(1)

greater than the 5% critical value of 233.13. Therefore, it is statistically significant to reject the null hypothesis and conclude that the rank of Π is great than 0. The trace statistic for testing rank of $\Pi = 1$ is 173.31,

Table 5: Johansen Rank Test for Cointegration (Group 1)

The Johansen Rank Test for cointegration is based on Johansen's maximum likelihood framework. The essence of the Johansen rank test is to determine the rank of the Π matrix (detailed in section 3.4.3). Group 1 includes 10 companies: AXP, BA, CAT, CSCO, DD, DIS, HD, IBM, INTC, JNJ. For a 10-variable testing setup, both rank (0) and rank (10) imply no cointegration relationship. Rank (r), where $0 < r < 10$, implies r # of cointegration relationships. The Johansen rank test follows the sequential testing process until the null hypothesis is not rejected. All testing results are based on 5% significance level. The star symbol "★" indicates the correctly identified rank for this group of companies.

Rank	Parameters	LL	Eigenvalue	Trace Statistic	5% Critical Value
0	110	149843.69	.	258.67	233.13
1	129	149886.38	0.01364	173.31★	192.89
2	146	149911.78	0.00814	122.49	156.00
3	161	149932.09	0.00651	81.89	124.24
4	174	149943.69	0.00373	58.68	94.15
5	185	149952.13	0.00271	41.80	68.52
6	194	149959.38	0.00233	27.30	47.21
7	201	149965.96	0.00212	14.13	29.68
8	206	149970.54	0.00147	4.97	15.41
9	209	149972.63	0.00067	0.80	3.76
10	210	149973.03	0.00013		

Table 6: Johansen Rank Test for Cointegration (Group 2)

The Johansen Rank Test for cointegration is based on Johansen's maximum likelihood framework. The essence of the Johansen rank test is to determine the rank of the Π matrix (detailed in section 3.4.3). Group 2 includes 9 companies: KO, MCD, MMM.MRK, MSFT, PFE, PG, UNH, UTX. For a 9-variable testing setup, both rank (0) and rank (9) imply no cointegration relationship. Rank (r), where $0 < r < 9$, implies r # of cointegration relationships. The Johansen rank test follows the sequential testing process until the null hypothesis is not rejected. All testing results are based on 5% significance level. The star symbol "★" indicates the correctly identified rank for this group of companies.

Rank	Parameters	LL	Eigenvalue	Trace Statistic	5% Critical Value
0	90	140009.23	.	199.61	192.89
1	107	140036.52	0.00863	145.04★	156.00
2	122	140061.98	0.00805	94.13	124.24
3	135	140077.71	0.00498	62.66	94.15
4	146	140088.85	0.00353	40.38	68.52
5	155	140098.09	0.00293	21.89	47.21
6	162	140104.33	0.00198	9.42	29.68
7	167	140107.46	0.00099	3.15	15.41
8	170	140109.01	0.00049	0.07	3.76
9	171	140109.04	0.00001		

while the corresponding 5% critical value is 192.89. Therefore, we fail to reject the null hypothesis that the rank of $\Pi = 1$. Because this is the first time that the null hypothesis is not rejected, the sequential testing process stops. As a conclusion, the Johansen Rank Test provides statistical evidences that there is 1 cointegration relationship existing among group 1 companies. On the other hand, table 6 shows the formal Johansen Rank Test for group 2 companies, which follows the same logic in table 5. As we can see, the trace statistic for testing rank of $\Pi = 0$ is 199.61, which is greater than the 5% critical value of 192.89. Therefore, the null hypothesis is rejected for the first test. In the following second test, the trace statistic for testing rank of $\Pi = 1$ is 145.04 which is smaller than the 5% critical value of 156.00. Consequently, we fail to reject the null hypothesis that the rank of $\Pi = 1$. Hence, the Johansen Rank Test indicates that there is also 1 cointegration relationship existing among group 2 companies. As a short summary for table 5 and table 6, we can conclude that, for each group of companies, 1 cointegration relationship is identified. It gives us the evidences that, although stock price levels are non-stationary, they form long-run equilibrium relationships among sub-groups.

4.4 Long-run Equilibrium and Error-Corrections

The identification of cointegration relationships among sub-groups of companies hands us the necessary condition to investigate the long-run time series behaviors of stock price level movements. The vector error-correction model offers two valuable sets of estimated parameters. The first set are the short-run estimated error-correction parameters, which measure the speed of convergence to the long-run equilibrium if the underlying variable system experiences shocks. The second set are the long-run estimated cointegration parameters that identify the equilibrium relationship.

Table 7-1 presents the vector error-correction model for group 1 companies. One cointegration relationship is assumed in this model, since one cointegration relationship is identified by the Johansen Rank Test. "Alpha" refers to the estimated short-run error-correction parameters for each variable within the model. The model takes on stock price levels that are in the logged scale [i.e. $\ln(\text{price})$], with one lag being assumed by the model for its transformation process. The most important statistics in table 7-1 are the estimated error-correction parameters (Alpha). We can see that 6 out of 10 estimated error-correction parameters are individually significant. And they are also jointly significant. This finding gives us the evidences that stock price levels (group 1) are actively eliminating disequilibrium after the group experiences market shocks in order for them to restore their long-run equilibrium relationship. However, the average of the Alpha coefficients in absolute value is only 0.001 for group 1 companies. It means that, given the significant error-correction behaviors, the stock price level movements could still be very volatile in the short-run. On the other hand, table 7-2 shows the vector error-correction model for group 2 companies, which follows the same statistical logic as in table 7-1. Similar to group 1 companies, 5 out of 9 estimated error-correction parameters are individually significant within group 2 companies, while they are also jointly significant. One interesting and surprising finding is that the average of the Alpha coefficients in absolute value is 0.00154 for group 2 companies, which are very close to group 1 companies'. This provides us with the evidences that the short-run volatile price level movements might be a common behavior within different groups of companies that formed cointegration relationships.

Table 7-1: Vector Error-Correction Model for Group 1

This table presents the vector error-correction model for group 1 companies, which include AXP, BA, CAT, CSCO, DD, DIS, HD, IBM, INTC, JNJ. The stock price levels are in the logged scale [i.e. $\ln(\text{price})$]. “LD” means the variable is time lagged and differenced. One cointegration relationship is assumed in this model, since one cointegration relationship is identified by the Johansen Rank Test. “Alpha” refers to the estimated short-term error-correction parameters. “*” denotes significance at 10% level, “**” denotes significance at 5% level. “***” denotes significance at 1% level. AIC: -48.19. HQIC: -48.14. SBIC: -48.05. Log Likelihood: 149,886.4

	APX			BA			CAT			CSCO		
	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat
Alpha	-0.0015***	0.0005	-3.00	-0.0001	0.0004	-0.15	-0.0016***	0.0005	-3.26	0.002***	0.0006	2.79
LD.AXP	-0.058***	0.0147	-3.91	-0.0107	0.0118	-0.91	-0.0107	0.0144	-0.74	-0.011	0.0190	-0.59
LD.BA	0.037**	0.0179	2.05	-0.0143	0.0143	-1.00	-0.0043	0.0175	-0.25	-0.018	0.0231	-0.80
LD.CAT	-0.010	0.0147	-0.67	0.0097	0.0118	0.82	0.0053	0.0144	0.37	-0.013	0.0190	-0.71
LD.CSCO	0.011	0.0109	1.05	0.0023	0.0088	0.26	0.0057	0.0107	0.53	-0.021	0.0142	-1.48
LD.DD	-0.023	0.0195	-1.17	-0.0013	0.0157	-0.08	0.0288	0.0192	1.50	0.011	0.0253	0.45
LD.DIS	0.010	0.0159	0.60	0.0094	0.0127	0.74	-0.0029	0.0156	-0.19	-0.061**	0.0205	-2.99
LD.HD	0.022	0.0163	1.34	0.0149	0.0131	1.14	0.0006	0.0160	0.04	0.014	0.0212	0.67
LD.IBM	0.005	0.0181	0.29	-0.0211	0.0145	-1.46	-0.0086	0.0178	-0.48	0.011	0.0235	0.46
LD.INTC	0.011	0.0121	0.90	0.0057	0.0097	0.58	-0.0015	0.0119	-0.12	0.014	0.0157	0.90
Constant	0.000	0.0003	0.53	0.0003	0.0003	1.11	0.0001	0.0003	0.32	-0.0001	0.0004	-0.26
	DD			DIS			HD			IBM		
	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat
Alpha	0.00004	0.0004	0.11	-0.0004	0.0004	-0.82	0.0011**	0.0004	2.51	0.00063*	0.0004	1.64
LD.AXP	-0.0132	0.0112	-1.18	-0.022*	0.0132	-1.69	-0.0194	0.0129	-1.50	-0.0193*	0.0116	-1.66
LD.BA	-0.0069	0.0136	-0.51	-0.0032	0.0160	-0.20	-0.0172	0.0156	-1.10	-0.0245*	0.0141	-1.74
LD.CAT	0.0147	0.0112	1.32	-0.0109	0.0132	-0.82	-0.0090	0.0129	-0.70	0.0057	0.0116	0.49
LD.CSCO	-0.0002	0.0083	-0.02	0.0126	0.0098	1.29	-0.0006	0.0096	-0.07	-0.0011	0.0086	-0.13
LD.DD	-0.032**	0.0149	-2.18	0.055***	0.0175	3.08	-0.0077	0.0171	-0.45	0.0078	0.0154	0.51
LD.DIS	0.0049	0.0121	0.41	-0.0504***	0.0142	-3.55	-0.0016	0.0139	-0.12	-0.0097	0.0125	-0.78
LD.HD	0.0299**	0.0124	2.41	0.0214	0.0146	1.46	0.0307*	0.0143	2.15	-0.0071	0.0129	-0.55
LD.IBM	-0.0073	0.0138	-0.53	-0.0021	0.0162	-0.13	-0.0115	0.0159	-0.73	0.0517***	0.0143	3.62
LD.INTC	0.0019	0.0092	0.21	0.0093	0.0109	0.86	0.0204*	0.0106	1.92	-0.0059	0.0095	-0.62
Constant	0.0001	0.0002	0.37	0.0001	0.0003	0.46	0.0002	0.0003	0.53	0.0002	0.0003	0.71
	INTC			JNJ								
	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat						
Alpha	-0.0027***	0.0006	-4.67	-0.0003	0.0003	-0.91						
LD.AXP	-0.0238	0.0175	-1.36	-0.0031	0.0105	-0.29						
LD.BA	-0.0277	0.0212	-1.31	0.0021	0.0127	0.17						
LD.CAT	-0.0070	0.0175	-0.40	-0.0073	0.0105	-0.70						
LD.CSCO	0.0133	0.0130	1.02	-0.0085	0.0078	-1.10						
LD.DD	-0.0284	0.0232	-1.22	0.0050	0.0139	0.36						
LD.DIS	-0.0256	0.0188	-1.36	-0.0124	0.0113	-1.10						
LD.HD	-0.0102	0.0194	-0.52	-0.0124	0.0116	-1.07						
LD.IBM	-0.0133	0.0215	-0.62	-0.0131	0.0129	-1.02						
LD.INTC	0.0061	0.0144	0.42	0.0036	0.0086	0.41						
Constant	-0.0002	0.0004	-0.42	0.0002	0.0002	0.67						

Table 7-2: Vector Error-Correction Model for Group 2

This table presents the vector error-correction model for group 2 companies, which include KO, MCD, MMM, MRK, MSFT, PFE, PG, UNH, UTX. The stock price levels are in the logged scale [i.e. $\ln(\text{price})$]. "LD" means the variable is time lagged and differenced. One cointegration relationship is assumed in this model, since one cointegration relationship is identified by the Johansen Rank Test. "Alpha" refers to the estimated short-term error-correction parameters. "*" denotes significance at 10% level, "**" denotes significance at 5% level. "***" denotes significance at 1% level. AIC: -44.44. HQIC: -44.40. SBIC: -44.32. Log Likelihood: 140,036.5

	KO			MCD			MMM		
	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat
Alpha	-0.0016***	0.001	-2.68	0.0002	0.001	0.34	-0.003***	0.001	-4.77
LD.KO	-0.003	0.013	-0.21	0.011	0.014	0.78	0.025*	0.014	1.81
LD.MCD	0.006	0.012	0.45	0.006	0.013	0.46	-0.009	0.013	-0.7
LD.MMM	-0.010	0.013	-0.74	-0.020	0.014	-1.47	-0.023*	0.014	-1.69
LD.MRK	-0.021	0.014	-1.5	-0.003	0.015	-0.2	-0.018	0.015	-1.22
LD.MSFT	-0.007	0.009	-0.73	-0.001	0.009	-0.05	0.000	0.009	0.02
LD.PFE	-0.004	0.010	-0.4	0.006	0.011	0.54	-0.004	0.010	-0.34
LD.PG	-0.005	0.013	-0.4	-0.011	0.014	-0.76	-0.029**	0.014	-2.07
LD.UNH	-0.002	0.009	-0.28	-0.018*	0.009	-1.89	0.001	0.009	0.12
LD.UTX	-0.006	0.011	-0.53	0.003	0.012	0.26	0.013	0.011	1.09
Constant	0.000	0.000	-0.05	0.000	0.000	0.85	0.000	0.000	0.4
	MRK			MSFT			PFE		
	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat
Alpha	-0.001	0.001	-1.25	-0.0014*	0.001	-1.64	-0.001	0.001	-0.94
LD.KO	-0.009	0.014	-0.65	0.0001	0.020	0.01	0.0136	0.018	0.75
LD.MCD	-0.007	0.012	-0.59	0.018	0.018	0.98	-0.001	0.017	-0.04
LD.MMM	-0.025*	0.013	-1.93	-0.025	0.019	-1.31	-0.025	0.018	-1.42
LD.MRK	0.074***	0.014	5.26	-0.016	0.021	-0.75	-0.0011	0.019	-0.06
LD.MSFT	-0.006	0.009	-0.66	-0.004	0.013	-0.34	-0.006	0.012	-0.47
LD.PFE	0.002	0.010	0.15	0.003	0.015	0.18	-0.019	0.014	-1.42
LD.PG	-0.028**	0.013	-2.13	-0.033	0.020	-1.7	-0.0132	0.018	-0.74
LD.UNH	-0.012	0.009	-1.31	-0.015	0.013	-1.14	0.007	0.012	0.60
LD.UTX	-0.005	0.011	-0.47	-0.018	0.016	-1.12	0.003	0.015	0.21
Constant	0.00004	0.0002	0.15	-0.00001	0.00034	-0.04	-0.0001	0.0003	-0.40
	PG			UNH			UTX		
	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat	Coef	Std.Err	Z-Stat
Alpha	0.0008	0.001	1.25	0.0017**	0.001	1.94	-0.0032***	0.001	-4.34
LD.KO	0.009	0.014	0.63	0.0082	0.020	0.41	0.0146	0.017	0.87
LD.MCD	-0.017	0.013	-1.31	0.018	0.018	0.97	0.013	0.015	0.84
LD.MMM	-0.0125	0.013	-0.93	-0.0042	0.019	-0.22	-0.0040	0.016	-0.25
LD.MRK	-0.028**	0.014	-1.98	0.002	0.021	0.08	-0.032*	0.017	-1.86
LD.MSFT	-0.015*	0.009	-1.69	-0.0169	0.013	-1.29	-0.0169	0.011	-1.53
LD.PFE	0.0006	0.010	0.05	0.006	0.015	0.40	0.017	0.012	1.39
LD.PG	-0.002	0.014	-0.18	0.0174	0.020	0.89	-0.0027	0.016	-0.17
LD.UNH	0.004	0.009	0.48	0.011	0.013	0.81	0.012	0.011	1.08
LD.UTX	-0.0025	0.011	-0.22	0.0065	0.016	0.40	0.0031	0.014	0.23
Constant	0.0001	0.0002	0.43	0.0002	0.0003	0.69	0.0001	0.0003	0.38

Table 8-1: Long-run Cointegration Parameters with Johansen Normalization for Group 1

This table shows the estimated long-run cointegration parameters generated by the vector error-correction model for group 1 companies, which include AXP, BA, CAT, CSCO, DD, DIS, HD, IBM, INTC, JNJ. The estimated coefficient of AXP is normalized to “1”. The “Z-stat” is the test statistic for individual coefficient’s significance. The “Chi-sq” is the test statistic for coefficients’ joint significance. “*” denotes significance at 10% level, “**” denotes significance at 5% level. “***” denotes significance at 1% level.

	Coef	Std.Err	Z-Stat	P-Value	Chi-Sq	P>Chi-Sq
AXP	1	N/A	N/A	N/A		
BA	0.225	0.365	0.62	0.54		
CAT	1.310***	0.374	3.51	0.00		
CSCO	-2.005***	0.291	-6.89	0.00		
DD	-3.662***	0.619	-5.92	0.00	112.86	0.00
DIS	1.123***	0.324	3.47	0.00		
HD	-2.184***	0.347	-6.28	0.00		
IBM	-0.972***	0.292	-3.33	0.00		
INTC	2.639***	0.354	7.46	0.00		
JNJ	3.311***	0.556	5.96	0.00		

Table 8-2: Long-run Cointegration Parameters with Johansen Normalization for Group 2

This table shows the estimated long-run cointegration parameters generated by the vector error-correction model for group 2 companies, which include KO, MCD, MMM.MRK, MSFT, PFE, PG, UNH, UTX. The estimated coefficient of KO is normalized to “1”. The “Z-stat” is the test statistic for individual coefficient’s significance. The “Chi-sq” is the test statistic for coefficients’ joint significance. “*” denotes significance at 10% level, “**” denotes significance at 5% level. “***” denotes significance at 1% level.

	Coef	Std.Err	Z-Stat	P-Value	Chi-Sq	P>Chi-Sq
KO	1	N/A	N/A	N/A		
MCD	-0.607***	0.137	-4.42	0.00		
MMM	2.20***	0.329	6.69	0.00		
MRK	-0.655***	0.247	-2.65	0.01		
MSFT	0.267	0.208	1.28	0.20	62.84	0.00
PFE	0.121	0.223	0.54	0.59		
PG	-1.071***	0.353	-3.03	0.00		
UNH	-1.257***	0.223	-5.63	0.00		
UTX	1.470***	0.309	4.75	0.00		

The results regarding the estimated error-correction parameters from table 7-1 and table 7-2 are very consistent, although two models take on two groups of companies that are completely different. Both models confirmed the time series behavior that stock price levels are making efforts to restore their long-run equilibrium relationship. However, the short-run price level movements are thought to be volatile.

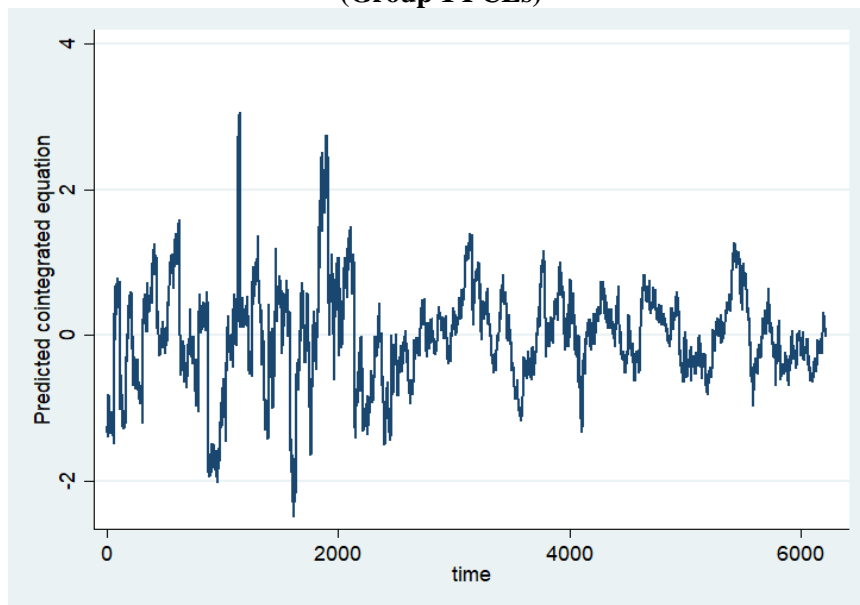
The estimated long-run cointegration parameters, as shown in table 8-1 and 8-2, provide coefficients that identify the equilibrium relationships within each group of companies. The Johansen normalization standardizes the first company’s coefficient to “1”. From table 8-1, we can see 8 out of 9 estimated cointegration parameters are individually significant with the first company’s coefficient normalized to “1”. The joint significance test yields a Chi-square statistic of 112.86 accompanied by a

corresponding P-value of 0.00. On the other hand, table 8-2 also yields very similar results. 6 out of 8 estimated cointegration parameters are individually significant with the first coefficient normalized to “1”. The joint significance test generates a Chi-square statistic of 62.84 with a corresponding P-value of 0.00. Both tables offer us the evidences that the underlying long-run cointegration relationship within each group is statistically significant. Therefore, the findings confirm that the long-run stock price levels within each identified group should move coherently throughout the time.

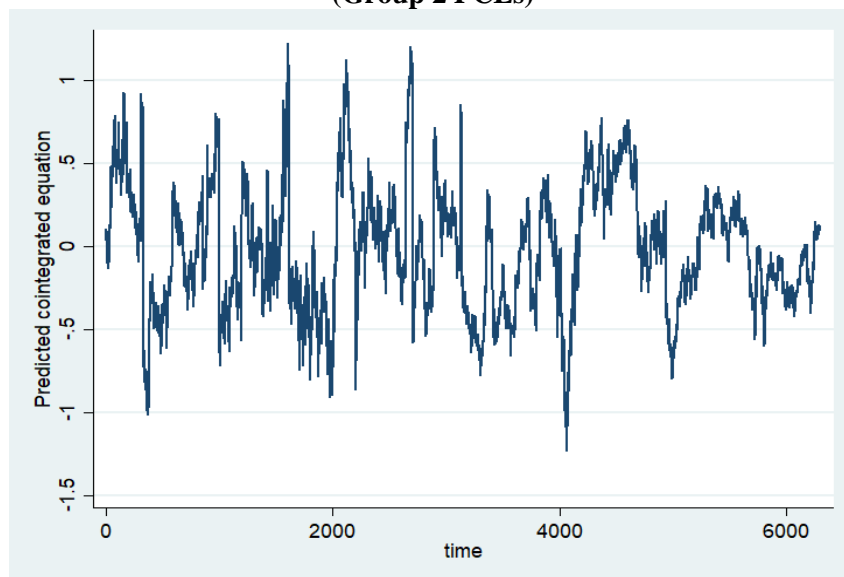
Graph 2: Predicted Cointegration Equations (PCEs)

The two graphs show the time series plots of the predicted cointegration equations for Group 1 companies and Group 2 companies respectively. The purpose of the two graphs is to assess the time series behaviors of PCEs, which provide additional information about the long-run cointegration relationships as identified in the above analysis.

(Group 1 PCEs)



(Group 2 PCEs)



Besides the identified long-run cointegration relationships within each group of companies, graph 2 shows the predicted cointegration equations (PCEs) for each group. The PCEs can illustrate how the cointegration equations are expected to behave in the future. From graph 2, we can see that both the PCEs for group 1 and PCEs for group 2 present a stationary behavior in the foreseeable future, as PCEs are fluctuating around the long-run mean with no time-dependent long-run trend. Consequently, the evidences incorporated in graph 2 support the argument that the stock price level cointegration relationships among groups of companies are expected to persist in the future, which are expected to hold strong and solid.

5 Conclusions

The market risk is thought to be the general risk factor in the stock market. The evidences presented in this research paper show that stock historical returns are highly correlated and statistically stationary [I(0)]. The findings indicate that stock returns are mean-reverting, which is consistent with many previous literatures. However, the behaviors of stock price level movements don't have to follow the same manner as stock returns do. Opposite to the evidences that stock returns are stationary, most individual stock price level movements are non-stationary and containing one unit root [I(1)]. This embraces the argument that stock price levels could continue drifting away from its long-run mean. Moreover, by using the Johansen Rank Tests, long-run cointegration relationships are identified within each sub-group of large cap stocks in the U.S. market, meaning that stock price levels are formed with long-run equilibrium relationships under the general market condition.

Furthermore, the vector error-correction models provide significant evidences that short-run stock price level movements can be very volatile and show a reluctant behavior of returning to the long-run equilibrium. Nevertheless, the estimated and the predicted long-run cointegration parameters offer statistical evidences that the equilibrium relationships are solid and stationary over time.

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