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Panel Bayesian VAR Modeling for Policy and Forecasting when dealing with confounding and latent effects

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Abstract

The paper develops empirical implementations of the standard time-varying Panel Bayesian VAR model to deal with confounding and latent effects. Bayesian computations and mixed hierarchical distributions are used to generate posteriors of conditional impulse responses and conditional forecasts. An empirical application to Eurozone countries illustrates the functioning of the model. A survey on policy recommendations and business cycles convergence are also conducted. The paper would enhance the more recent studies to evaluate idiosyncratic business cycles, policy-making, and structural spillovers forecasting. The analysis confirms the importance to separate common shocks from propagation of country- and variable-specific shocks.

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1 Introduction

The paper develops an approach to jointly model and quantify latent behaviors[2] and confounding effects[3]. The underlying logic is very useful in econometrics in order to avoid "omitted biases" for the case of serial correlations and not (directly) observed linkages among economic-financial variables, mainly if the lagged endogenous variables tend to show some kind of irregular or common behaviour. I estimate a structural time-varying Extended Panel Bayesian VAR (EPBVAR) model by specifying hierarchical priors through a mixture of distributions. In this study, Bayesian methods and Maximum Likelihood Estimates (MLE) are used to reduce the dimensionality of the model, put structure on the time variations, and simultaneously evaluate omitted variable biases and issues of endogeneity. In the case of fully hierarchical priors, a MCMC implementation is employed to calculate posterior distributions of Conditional Generalized Impulse Response Fuctions (CGIRFs) and Conditional Forecasts (CFs) experiments[4] to unexpected perturbations in the innovations of the factors of the system. A Normal Linear Regression (NLR) model is evaluated in order to work with smaller systems in which the regressors are observable, directly measured, and time-varying linear combinations of the right-hand side variables of the structural EPBVAR. The advantage of this approach is that it is easier to match endogenous variables with omitted factors and the framework is valid if and only if prior assumptions are satisfied and a hierarchical structure is provided. Thus, an analysis of joint and conditional densities and sequential factorizations are required.

An empirical application to a pool of the current members of the European Monetary Union (EMU) illustrates the functioning of the model, with par-

[2]Latent or hidden factors are variables that are not directly observed but are rather inferred from other variables that are observed and, hence, directly measured.

[3]Confounding factors are variables that affect both dependent and independent variables causing spurious association. This latter cannot be described and, hence, evaluated in terms of correlations or associations being a causal concept.

[4]See e.g., Chib [6], Chib [7], Pesaran and Shinb [29], and Chib and Jeliazkov [9].

ticular attention on the recent recession and post-crisis consolidation. To be more precise, the paper focuses on three interesting macroeconomic-financial issues left even now unanswered. First, how dimension and intensification of spillovers over time affect commonality, interdependence, and heterogeneity across countries and among variables. Second, how different transmission channels essentially affect the spreading of spillovers in real and financial dimension given an unexpected shock. Third, the importance of economic and institutional implications in driving the transmission of shocks. A survey on policy recommendations and business cycle convergence are also assessed. This implies that idiosyncratic business cycles among the G7 countries or international business cycles are not discussed, unless they contain interesting results from the viewpoint of the present paper. Likewise, the analysis does not review studies focusing on regional cycles in Europe. Overall, the analysis is consistent and robust with the more recent literature on business cycles, which recognizes the importance of accounting for both group-specifc and global factors in evaluating cross-country spillovers and to separate common shocks from propagation of country- and variable-specific shocks when studying economic-financial linkages.

Growth spillovers are evaluated through structural Bilateral Net Spillover Effects (BNSEs) and Systemic Contributions (SCs). The former incorporate feedback effects from the impulse variables and temporary or persistent long-run effects of a potential shock. The SC index represents the amplification contribution of the impulse variable to the response variable and is able to capture sequential feature associated with systemic events. Finally, the Generalized Theil (GT) index is estimated in order to investigate on business cycle convergence and, hence, divergence across member states.

The outline of this paper is as follows. Section 2 reviews the existing literature on business cycles and cross-country spillovers. Section 3 presents the econometric model and empirical specifications. Section 4 provides the empirical analysis. The final section contains some concluding remarks.

2 Discussion and Relationship with the literature

In recent years, several theoretical and applied models have given a new impulse to the literature on business cycles and policy making. Nevertheless, these studies often reach very different or mixed conclusions due to the selection of variables used, diverging methodologies to identify spillover effects and alternative ways to investigate co-movement, heterogeneity, and interdependence across countries.

For example, Blanchard and Perotti (see [2]) estimated dynamic effects of shocks in government spending and taxes on U.S. activity in the postwar period, by using a mixed structural VAR model. The results consistently show positive government spending shocks as having a positive effect on output, and positive tax shocks as having a negative effect. However, the model developed is not consistent with large sample and the computations are costly.

Pesaran *et al.* (see [28]) developed a multicountry Global Vector Autoregression (GVAR) approach in which estimate spillover effects of a domestic budget balance shock on the members of the EA by combining all country-specific VAR models in one multicountry model and treating all variables as endogenous. They focused on identification of shocks to the US economy, particularly the monetary policy shocks, and considered the time profiles of their effects on the EA. From a policy perspective, they showed that: (*i*) financial shocks (equity and bond prices) tend to be transmitted much faster than shocks to real output and/or inflation; (*ii*) the transmission of real shocks is rather slow, normally taking 2-3 years, or in some cases even more, before their full impacts are felt; (*iii*) the long run impacts of the real shocks are larger than what might be expected from a simple trade perspective; and (*iv*) the transmission of shocks does not take place only through trade, but also as importantly through the impact of real shocks on financial variables with subsequent spillover effects on real variables. However, they concluded that the GVAR presents a complicated spatio-temporal structure for the analysis of the world economy and, hence, it need to be modified and extended further.

Geweke (see [17]), Raftery and Lewis (see [30]), and Raftery *et al.* (see [31]) developed Bayesian econometrics analyses and forecasting via posterior simulations using MCMC integration. To be more precise, they described different methods to use posterior simulation output to estimate integrated likelihoods. They focused on the basic Laplace-Metropolis estimator for models without random effects. For models with random effects, they introduced a compound Laplace-Metropolis estimator. Then, they computed a Bayes factor as the ratio of two integrated likelihoods in order to compare the two models. In this way, they were able to obtain good approximations for integrated likelihoods in hierarchical models, favoring the model incorporating random effects. Nevertheless, these methods have been discussed in time since they refer to a broad class of conditionally independent hierarchical models and the integrals involved are of low dimension.

Canova and Ciccarelli (see [5]) estimated a Bayesian multicountry VAR model to assess economic-financial linkages, to test specification hypotheses and to conduct policy exercises, with cross-unit interdependencies, unit-specific dynamics, and time variations in the coefficients. They conducted an empirical analysis on G7 countries by addressing Bayesian computations. Impulse responses and conditional forecasts are obtained with the output of an MCMC routine. The transmission of certain shocks across countries are analyzed. However, the model is devoid of a structural framework in order to be able to add time variant factors.

Giannone *et al.* (see [18]) and Koop (see [23]) evaluated large Bayesian VARs through hierarchical priors. They examined both forecasting accuracy and structural analysis of the effect of a monetary policy shock adding additional macroeconomic variables and sectoral information on the results of De Mol and co-workers (2008). The shrinkage is obtained by using Minnesota priors that often is uninformative since it is based on an approximation which involves replacing the variance-covariance matrix with an estimate. Moreover, the framework is extended by using a set of dummy variables and, hence, difficult to model.

In a quite recent work, Ciccarelli *et al.* (see [11]) investigated heterogeneity and spillovers in macro-financial linkages across developed economies, focusing on the most recent recession. They developed a time-varying panel Bayesian VAR model including real and financial variables and identified a statistically significant common component. The main findings they found were the need to allow for cross-country and cross-variable interdependencies when studying real-financial linkages and the presence of a heterogeneous pattern across members with a common component. Nevertheless, the empirical model used is non-structural and constrained due to time-invariant or exogenous factors in the system and, hence, unable in identifying structural and institutional differences across countries, different reactions behind a common unexpected shock, and the causality among real and financial variables.

According to the above-mentioned literature, my empirical specifications differ from previous studies in four ways. First, I represents a structural time-varying approach as a sum of three terms: (i) a vector of lagged endogenous variables; (ii) a vector of serially correlated factors to deal with confounding effects; and (iii) a vector of latent factors to minimize further omitted biases. All terms are treated as endogenous in order to capture and discriminate statistical relationships from structural and causal connections. Second, dynamic relationships between these terms hold and are allowed to be country- and variable-specifics minimizing heterogeneity biases. Third, interdependence, heterogeneity and commonality can be evaluated without overlooking the contribution of different channels and/or structural constraints through which these effects might have occurred. Fourth, synchronization and convergence of business cycles can be achieved and shocks can be directly linked to policy recommendations, without imposing particular short-run and long-run restrictions.

3 Econometric Model and Empirical Specifications

3.1 Model Estimation

The econometric model has the form:

$$Y_{i,t} = A_{it,0} + \sum_{\lambda=1}^{l} \left[A_{it,j}(L) Y_{i,t-\lambda} + B_{it,j}(L) W_{i,t-\lambda} + C_{it,j}(L) Z_{i,t-\lambda} \right] + \varepsilon_{it} \quad (1)$$

where the subscripts $i, j = 1, 2, \ldots, N$ are country indeces, $t = 1, 2, \ldots, T$ denotes time, L stands for the lag operator, $A_{it,0}$ is an $N \cdot 1$ vector of intercepts, $A_{it,j}$ is an $NM \cdot NM$ matrix of coefficients for each j, $Y_{i,t-\lambda}$ is an $NM \cdot 1$ vector of variables lagged, $B_{it,j}$ is an $NQ \cdot NQ$ matrix of coefficients for each j, $W_{i,t-\lambda}$ is a $NQ \cdot 1$ vector including serially correlated factors, $C_{it,j}$ is an $N\Xi \cdot N\Xi$ matrix of coefficients for each j, $Z_{i,t-\lambda}$ is a $N\Xi \cdot 1$ vector of hidden factors, $\varepsilon_{it} \sim i.i.d.N(0, \Sigma)$ is a $N \cdot 1$ vector of disturbance terms. The subscripts $\lambda = 1, 2, \ldots, l$ are lags for each of the $m = 1, \ldots, M$ endogenous variables, $q = 1, \ldots, Q$ serially correlated factors, and $\xi = 1, \ldots, \Xi$ hidden factos.

Firstly, let $k = 1+N[m+q+\xi]l$ be the number of coefficient in each equation of the EPBVAR, a $Nk \cdot Nk$ matrix $X_t = (1, Y'_{t-1}, Y'_{t-2}, \ldots, Y'_{t-\lambda}, W'_t, W'_{t-1}, W'_{t-2}, \ldots, W'_{t-\lambda}, Z'_t, Z'_{t-1}, Z'_{t-2}, \ldots, Z'_{t-\lambda})'$ can be defined containing endogenous variables, serially correlated and hidden factors. Then, I define a $Nk \cdot 1$ vector $\gamma_t = vec(\Gamma^k_{it})$[5] which contains, stacked into a vector, the intercepts and all the M, Q and Ξ coefficients of the matrices $A_{it,j}$, $B_{it,j}$ and $C_{it,j}$, respectively, with $\Gamma^k_{it} = (A'_{it,0}, A^{1'}_{it}, A^{2'}_{it}, \ldots, A^{m'}_{it}, B^{1'}_{it}, B^{2'}_{it}, \ldots, B^{q'}_{it}, C^{1'}_{it}, C^{2'}_{it}, \ldots, C^{\xi'}_{it})'$. With all these definitions, I can write the EPBVAR in terms of multivariate normal distribution:

$$Y_t = (I_{Nk} \otimes X_t)\gamma_t + E_t \tag{2}$$

where $Y_t = (y'_{1t}, \ldots, y'_{Nt})'$ and $E_t = (\varepsilon'_{1t}, \ldots, \varepsilon'_{Nt})'$ are $N \cdot 1$ vectors containing the dependent variables and the random disturbances of the model, respectively. Here, $E_t \sim N(0, \Sigma \otimes I_t)$ and $Y_{i,t}$ is expressed in terms of X_t and contains, stacked, all T observations on the first dependent variable, then all T observations on the second dependent variable, and so on. Moreover, there is no subscript i since variables of all countries in the system are stacked in X_t.

Now, since the coefficient vectors in γ_t vary in different time periods for each country-variable pair and there are more coefficients than data points, following the framework in Koop (see [23]) and Ciccarelli *et al.* (see [11]), a flexible factorization for γ_t can be assumed to estimate them without restrictions and loss of efficiency:

$$\gamma_t = \sum_{k=1}^{K} D_k \cdot \beta_{kt} + u_t = D \cdot \beta_t + u_t \quad with \quad u_t \sim N(0, \Sigma \otimes V) \tag{3}$$

where $D_k = [D_1, D_2, \ldots, D_K]$ are conformable matrices of coefficients, with elements equal to zero and one, u_t captures unmodelled variations present in

[5]The vec operator transforms a matrix into a vector by stacking the columns of the matrix one underneath the other.

γ_t that are not captured from the system, $V = \sigma^2 \cdot I_k$ is a $k \cdot k$ matrix with σ^2 known by assumption, and $dim(\beta_t) \ll dim(\gamma_t)$ by construction. The idea is to shrink γ_t into β_{kt} obtaining a much smaller dimensional vector.

In (3), all factors are permitted to be time-varying and, hence, time variant structures can be obtained via implementations of MCMC algorithms. Moreover, time variations in the variance of shocks u_t to the factors β_{kt} are also allowed in order to capture in Y_t possible heterogeneity, interdependence, and commonality across countries and among variables. Running (2) and (3) for (1), the factorization is:

$$D_k \cdot \beta_{kt} = D_1 \cdot \beta_{1t} + D_2 \cdot \beta_{2t} + \ldots + D_K \cdot \beta_{Kt} \tag{4}$$

Finally, the factorization of γ_t becomes exact as long as σ^2 converges to zero.

With these specifications, the reduced-form EPBVAR model in (2) can be trasformed into a Normal Linear Regression (NLR) model with an error covariance matrix of an Inverse Wishart distribution[6]. By (2) and (3), the NLR model[7] can be written as:

$$Y_t = \Theta_t(D\beta_t + u_t) + E_t \equiv \chi_t\beta_t + \eta_t \quad with \quad \Theta_t = (I_{Nk} \otimes X_t) \tag{5}$$

where $\chi_t \equiv \Theta_t D$ is a $N \cdot k$ matrix which stacks all the coefficients of the system, β_t is a $k \cdot 1$ vector containing all the regression coefficients of the NLR model, $\eta_t \equiv \Theta_t u_t + E_t$ is a $N \cdot 1$ vector of random disturbances and distributed as a normal with zero-mean and covariance matrix equals to a $N \cdot N$ matrix Ω, with $\Omega = (\Sigma \otimes I_{kT})$ by assumption[8].

By construction, χ_{it} are linear combinations of right-hand side variables of the system and correlated among each other. The correlation decreases as k increases. The vectors of M endogenous variables, Q serially-correlated factors and Ξ hidden factors depend on a small number of observable indices, χ_{it}, and the factors β_{it} load on the indices. They are time-varying vectors to

[6]The Wishart distribution is a multivariate extension of χ^2 distribution and, in Bayesian statistics, corresponds to the conjugate prior of the inverse covariance-matrix of a multivariate normal random vector.

[7]To be more precise, it would correspond to the parsimonious Seemingly Unrelated Regression (SUR) model developed in literature. See for istance, Canova and Ciccarelli [5] and Ciccarelli *et al.* [11].

[8]See e.g., Kadiyala and Karlsson [22].

be estimated with a gain in efficiency. In fact, they are smooth (observed) linear functions of the lagged variables.

3.2 Prior Information

In hierarchical models, many problems involve multiple parameters which can be regarded as related in some way by the structure of the problem. A joint probability model for those parameters should reflect their mutual dependence. Typically, the dependence can be summarized by viewing these parameters as a sample from a common population distribution. Hence, the problem can be modelled hierarchically, with observable outcomes (Y_i) created conditionally on certain parameters (α_t), which themselves are assigned a distribution in terms of further (possibly common) parameters, hyperparameters (ϕ). This hierarchical thinking may help solve the trade-off between inaccurate fit and overfitting, and also plays an important role in developing computational strategies.

Given the NLR model described in (5), accounting for j and stacking all i country indices, it can be alternatively written as:

$$Y_j = \chi_j \beta + \eta_j \quad with \quad i = 1, 2, \ldots, N \quad and \quad j = 1, 2, \ldots, J \tag{6}$$

where $Y_j = (Y_{1j}, Y_{2j}, \ldots, Y_{Nj})^{'}$ is a $N \cdot 1$ vector, $\chi_j = diag(\chi^{'}_{1j}, \chi^{'}_{2j}, \ldots, \chi^{'}_{Nj})$ is a $N \cdot k$ matrix, $\beta = (\beta_1, \beta_2, \ldots, \beta_N)^{'}$ is a $k \cdot 1$ vector, and $\eta_j = (\eta_{1j}, \eta_{2j}, \ldots, \eta_{Nj})^{'}$ is a $N \cdot 1$ vector, with $\eta_j \sim N(0, \Sigma)$. Stacking further:

$$Y = \chi\beta + \eta \quad with \quad \eta \sim N(0, \Omega) \quad and \quad \Omega = (\Sigma \otimes I) \tag{7}$$

where $Y = (Y_1, Y_2, \ldots, Y_J)^{'}$, $\chi = (\chi_1, \chi_2, \ldots, \chi_J)^{'}$, $\eta = (\eta_1, \eta_2, \ldots, \eta_J)^{'}$, and $\Omega = diag(\Sigma^{'}, \Sigma^{'}, \ldots, \Sigma^{'})$.

The specifications in (6) and (7) allow heteroskedasticity and assume zero correlation across countries in order to obtain no perfect collinearity. The non-zero covariances imply that (7) is related and individual regressions are tied into a system of equations that can be analyzed together. However, variances can also differ across j, while η_j are independent across i. Possible heterogeneity, interdependence, and commonality across countries and among variables are absorbed in the coefficient vectors β_t, assuming the following state-space structure:

$$Y_t = \chi_t \beta_t + \eta_t \tag{8}$$

$$\beta_t = \beta_{t-1} + \upsilon_t \quad with \quad \upsilon_t \sim N(0, \mathcal{H}) \tag{9}$$

where $\upsilon_t \sim N(0, \Sigma \otimes V)$, $\mathcal{H} = diag(\bar{\mathcal{H}_1}, \ldots, \bar{\mathcal{H}_F})$ is a block diagonal matrix, and $\mathcal{H}_f = h_f \cdot I$, with $f = 1, \ldots, F$ and h_f controls the tightness of factor i in the coefficients. In the model (8), it is assumed a general AR structure and the block diagonality of $\mathcal{H}$ is needed to guarantee the orthogonality of the factors, which is preserved a-posterior and, hence, their identifiability. In this way, prior assumptions can be specified and, hence, Bayesian computations are feasibles.

3.3 Prior Assumptions

Let $\phi_0 = (\Omega^{-1}, h_f, \beta_0)$ to be the prior densities, two tentative beliefs (assumptions) can be defined accounting for the model described in (7). (i) Conditional Normality: $p(\eta|\phi_0) = N(0, \Omega)$. This is a hierarchical prior for η. (ii) Conditional Independence: $p(\eta|\phi_0) = p(\eta|\phi_0)p(\chi|\phi_0)$.

A hierarchical prior for η has been already specified. Thus, in order to complete the model, a prior moments on $(\Omega^{-1}, h_f, \beta_0)$ need to be defined. The likelihood function can be derived from the sampling density $p(Y|\phi_0)$ and it can be shown to be of a form that breaks into a mixture of distributions. In words: (i) A Normal distribution for factors β given Ω; (ii) a Wishart distribution for Ω^{-1}; (iii) an Inverse Gamma distribution for h_f, where $h_f = vec(\mathcal{H})$. That is:

$$\beta|\Omega^{-1}, Y \sim N(\hat{\beta}, \Omega^{-1} \otimes (\chi^{'}\chi)^{-1}) \tag{10}$$

$$\Omega^{-1}|Y \sim W(S^{-1}, T - k - 1) \tag{11}$$

$$h_f|Y \sim IG\left\{\frac{\bar{\omega}}{2}, \frac{vS}{2}\right\} \tag{12}$$

where $\hat{\beta} = (\sum_t \chi_t^{'}\chi_t)^{-1}(\sum_t \chi_t^{'}Y_t)$ is the OLS estimate of β, $S = (Y_t - \hat{\beta}\chi_t)^{'}(Y_t - \hat{\beta}\chi_t)$ is the sum of squared errors, and $\hat{\Sigma} = S/(T-k)$ is the OLS estimate of Σ.

Furthermore, such prior assumptions will generally be influenced, for example, by common or subjective beliefs about marginal effects of economic variables. Hence, Independent Normal Wishart Prior is used in this analysis, since it assumes that tentative beliefs on $\phi_0 = (\beta_0, \Omega^{-1}, h_f)$ derive from separate considerations.

According to (8) and (9), a MCMC methods and alternatives can be employed to conduct inference on the time-varying β_t (with $\mathcal{H}_{it} \neq 0$) and construct their exact posterior distributions. The result would be a Bayesian Model Averaging (BMA). Let data run from $(-\tau, T)$, where $(-\tau, 0)$ is a training sample used to estimate features of the prior, when such a sample is unavailable, it is sufficient to modify the expressions for the prior moments in (10), (11), and (12) as:

$$p(\Omega^{-1}, h_f, \beta_0) = p(\Omega^{-1}) \cdot \Pi_f p(h_f) \cdot p(\beta_0) \tag{13}$$

where, $p(\Omega^{-1}) = W(\beta_1, z_1)$, $p(h_f) = IG\big(\frac{\bar{\omega}_0}{2}, \frac{S_0}{2}\big)$, $p(\beta_0|\mathsf{F}_{-1}) = N(\bar{\beta}_0, \bar{R}_0)$, and F_{-1} denotes the information available at time *-1*. Here, *N()* stands for Normal, *W()* for Wishart, and *IG()* for Inverse Gamma distributions. The prior for β_0 and the law of motion for the factors imply that $p(\beta_t|\mathsf{F}_{-1}) = N(\bar{\beta}_{t-1|t-1}, \bar{R}_{t-1|t-1} + \mathcal{H}_t)$, where $\bar{\beta}_{t-1|t-1}$ and $\bar{R}_{t-1|t-1}$ are, respectively, the mean and the variance-covariance matrix of the conditional distribution of $\bar{\beta}_{t|t}$. The hyperparameters are all known. To be more precise, collecting them in a vector δ, where $\delta = (z_1, Q_1, \bar{\omega}_0, S_0, \bar{\beta}_0, \bar{R}_0)$, they are treated as fixed and are either obtained from the data to tune the prior to the specific applications (this is the case for $\bar{\beta}_0$ and β_1) or selected a-priori to produce relatively loose priors (this is the case for z_1, S_0, $\bar{R}_0$).

Whenever Ω is not replaced by an estimate[9], the only fully Bayesian approach which leads to analytical results requires the use of a natural conjugate prior. Here, the prior, likelihood and posterior come from the same family of distribution. According to (10), (11), and (12), and letting time variant factors, the natural conjugate prior has the form:

$$\beta_t|\Omega^{-1}, Y^T \sim N(\bar{\beta}_{t|t}, \Omega \propto \bar{R}_{t|t}) \quad or \quad p(\beta_t|\Omega^{-1}, Y^T) = N(\bar{\beta}_{t|t}, \Omega \propto \bar{R}_{t|t}) \tag{14}$$

$$\Omega^{-1}|Y^T \sim W(\beta_1, z_1) \qquad or \qquad p(\Omega^{-1}|Y^T, \beta) = W(\beta_1, z_1) \tag{15}$$

[9]For istance, the Minnesota priors are based on an approximation which involves replacing Ω with an estimate, $\hat{\Omega}$. See Doan *et al.* [14] and Litterman [25].

where, $\bar{\beta}_{t|t}$ and $\bar{R}_{t|t}$, correspond to hyperparameters collected in the vector δ, and GIRFs and CFs can be obtained with the same approach by Monte Carlo integration. That is, draws of $\mathcal{H}$ derive from (9), Ω^{-1} can be obtained from (8) and, conditional on these, draws of β can be taken from (8). Then, draws of impulse responses can be computed using these drawn values of Ω^{-1} and β. If $\mathcal{H} = 0$, allowing for time variant factors, draws of h_f can be taken from a Normal-Inverse Gamma distribution.

According to the natural conjugate prior, $\beta_t|\Omega$ and Ω^{-1} have Normal and Wishart distributions, respectively. The fact that the prior for β_t depends on Ω implies that β_t and Ω are not independent of one another. To be more precise, the estimation works with a prior which has VAR coefficients and error covariance being independent of one another. To allow for different equations in the VAR to have different explanatory variables, previous specification have to be modified. Given the NLR model in (8), a general prior which does not involve the restrictions inherent in the natural conjugate prior is the independent Normal-Wishart prior:

$$p(\beta_t, \Omega^{-1}|Y^T) = p(\beta_t|Y^T) \cdot p(\Omega^{-1}|Y^T) \tag{16}$$

where,

$$\beta_t|Y^T \sim N(\bar{\beta}_{t|t}, \bar{R}_{t|t}) \qquad or \qquad p(\beta_t|Y^T) = N(\bar{\beta}_{t|t}, \bar{R}_{t|t}) \tag{17}$$

$$\Omega|Y^T \sim iW(z_1, \beta_1) \qquad or \qquad p(\Omega|Y^T) = iW(z_1, \beta_1) \tag{18}$$

where $iW()$ stands for inverse Wishart distribution. Here, the prior allows for the prior covariance matrix, $\bar{R}_{t|t}$[10], to be anything the researcher chooses, rather the restrictive $(\Omega|Y^T \otimes \bar{R}_{t|t})$ form of the natural conjugate prior.

3.4 Posterior Distributions and MCMC Integration

The posterior distributions for $\phi = (\Omega^{-1}, h_f, \{\beta_t\}_{t=1}^T)$ are calculated combining the prior with the (conditional) likelihood on initial conditions of the data, which is proportional to:

$$L(Y^T|\phi) \propto (\Omega)^{-\frac{T}{2}} exp\Big\{ -\frac{1}{2}\Big[\Sigma_t(Y_t - (\Theta_t D)\beta_t)'\Omega^{-1}\Sigma_t(Y_t - (\Theta_t D)\beta_t)\Big]\Big\} \tag{19}$$

[10]These implementations do not allow to use the Minnesota prior since its covariance matrix is written in terms of blocks which varies across equations.

where, $Y^T = (Y_1, \ldots, Y_T)$ denotes the data, $\phi = (\Omega^{-1}, h_f, \{\beta_t\})$ refers to the unknowns whose joint distribution needs to be found, with ϕ_{-k} standing the vector ϕ excluding the parameter k. Despite the dramatic parameter reduction obtained with (8), analytical computation of posterior distributions $(\phi|Y^T)$ is unfeasible. Thus, through Monte Carlo techniques, a variant of Gibbs sampler approach can be used in this framework by making use of Kalman filter[11] so it only requires knowledge of the conditional posterior distribution of ϕ. The latter is extremely useful for investigating the issue of parameters constancy, because it is an updating method producing estimates for each time period based on the observations available up to the current period. To be more precise, the Kalman filter technique consists of two equations: the transition equation describing the evolution of the state variables and the measurement equation describing how the observed data are generated from the state variables. For the conditional posterior of $(\beta_1, \ldots, \beta_T|Y^T, \phi_{-\beta_t})$, it gives the following recursions:

$$\beta_{t|t} = \tilde{\beta}_{t-1|t-1} + \left[\tilde{R}_{t|t-1}(\Theta_t D) F^{-1}_{t|t-1}\right]\left[Y_t - (\Theta_t D)\theta_t\right] \tag{20}$$

$$R_{t|t} = \left[I - \tilde{R}_{t|t-1}(\Theta_t D)' F^{-1}_{t|t-1}(\Theta_t D)\right]\left(\tilde{R}_{t-1|t-1} + \mathcal{H}\right) \tag{21}$$

$$F_{t|t-1} = (\Theta_t D)' \tilde{R}_{t|t-1}(\Theta_t D) + \Omega_t \tag{22}$$

Hence, in order to obtain a sample $\{\beta_t\}$ from the joint posterior distribution $(\beta_1, \ldots, \beta_T|Y^T, \phi_{-\beta_t})$, the output of the Kalman filter is used to simulate β_T from $N(\beta_{T|T}, R_{T|T})$, β_{T-1} from $N(\beta_{T-1}, R_{T-1})$, and β_1 from $N(\beta_1, R_1)$. The recursion can be started choosing $R_{0|0}$ to be diagonal with elements equal to small values, whereas $\beta_{0|0}$ can be estimated in the training sample or initialized using a constant coefficient version of the model. Convergence only requires the algorithm to be able to visit all partitions of the parameter-space in a finite number of iterations. Thus, the marginal distributions of β_t can be computed averaging over draws in the nuisance dimensions and the posterior distributions for ϕ are:

$$\beta_t|Y^T, \phi_{-\beta_t} \sim N(\tilde{\beta}_{t|T}, \bar{R}_{t|T}) \qquad or \qquad p(\beta_t|Y^T, \phi_{-\beta_t}) = N(\tilde{\beta}_{t|T}, \bar{R}_{t|T}) \tag{23}$$

$$\Omega|Y^T, \phi_{-\Omega} \sim iW(\hat{z}_1, \hat{\beta}_1) \qquad or \qquad p(\Omega|Y^T, \phi_{-\Omega}) = iW(\hat{z}_1, \hat{\beta}_1) \tag{24}$$

[11] See e.g., Chib and Greenberg [8].

$$h_f|Y^T, \phi_{-h_f} \sim IG\left\{\frac{\bar{\omega}_f}{2}, \frac{\bar{S}}{2}\right\} \qquad or \qquad p(h_f|Y^T, \phi_{-h_f}) = IG\left\{\frac{\bar{\omega}_f}{2}, \frac{\bar{S}}{2}\right\} \tag{25}$$

where,

$$\tilde{\beta}_{t|T} = \tilde{R}_{t|T}\left[\bar{R}^{-1}_{t|T}\bar{\beta} + \sum_{t=1}^{T}(\Theta_t D)'\Omega^{-1}(\Theta_t D)\hat{\beta}\right] \tag{26}$$

$$\tilde{R}_{t|T} = \left[\bar{R}^{-1}_{t|T} + \sum_{t=1}^{T}(\Theta_t D)'\Omega^{-1}(\Theta_t D)\right]^{-1} \tag{27}$$

The $\hat{\beta}$ is the GLS estimator, with $\hat{\beta} = \left[(\Theta_t D)'\Omega^{-1}(\Theta_t D)\right]^{-1} \cdot (\Theta_t D)'\Omega^{-1}Y_t$. Rearraging terms, the equation (26) can be rewritten as:

$$\tilde{\beta}_{t|T} = \tilde{R}_{t|T}\left[\bar{R}^{-1}_{t|T}\bar{\beta} + \sum_{t=1}^{T}(\Theta_t D)'\Omega^{-1}Y_t\right] \tag{28}$$

where $\tilde{\beta}_{t|T}$ and $\tilde{R}_{t|T}$ denote the smoothed one-period-ahead forecasts of β_t and of the variance-covariance matrix of the forecast error, respectively, $\hat{z}_1 = z_1 + T$, $\hat{\beta}_1 = \beta_1 + \sum_t(Y_t - (\Theta_t D)\beta_t)$, $\bar{\omega}_f = K + \bar{\omega}_0$, $\bar{S} = S_0 + \Sigma_t(\beta^f_t - \beta^f_{t-1})'(\beta^f_t - \beta^f_{t-1})$, with β^f_t denoting the f^{th} subvector of β_t and f refers to the factors described in (3).

In this framework, it is common to burn some number of samples at the beginning and, hence, consider only any nth sample when averaging values to compute expectation. Moreover, the regressors of the NLR model in (8) are correlated, but the presence of correlation, even of extreme form, does not create problems in identifying the loadings as long as the priors are proper. In addition, the choice of making E_t and u_t correlated allows conjugation between the prior and the likelihood, avoids identification issues, and greatly simplifies the computation of the posterior. Here, the forecast error $\eta = Y_t - (X_t\Xi)\beta_t$ has the form $(\eta|\sigma^2) \sim N(0, \sigma_t\Omega)$ and, unconditionally, η_t has a multivariate t-distribution centered at *0*, with a scale matrix proportional to Ω and ν_n degrees of freedom. Hence, the innovations of the model described in (8) are endogenously allowed to have fat tails. Finally, since the fit improves when $\sigma^2 \to 0$, the model in (8) presents an exact factorization of γ_t.

4 Application

4.1 Theoretical Background

The Euro Area is an unique form of a monetary union without historical precedence. The member states of the euro have assigned the framing of monetary policy to a common monetary authority, the European Central Bank (ECB), set up as a highly independent central bank to insure that it will be able to carry out a policy of price stability. The adoption of a common monetary policy in Europe has eliminated the possibility to use monetary policy for the stabilization of country-specific shocks. This is generally considered as the main cost of forming a monetary union. How large this cost actually is depends on what alternative mecchanisms are available to ensure economic adjustment to idiosyncratic shocks. As monetary policy can no longer address country-specific shocks and factor mobility does not solve the problem being notoriously low within and across countries, the only remaining instrument in the hands of national authorities and capable to stabilize local macroeconomic-financial conditions is fiscal policy. In the context of the European Monetary Union (EMU), fiscal flexibility is hampered by large public debts and formal institutional constraints (such as the Maastricht rules and the Stability and Growth Pact). Thus, policy coordinations became an important issue in order to assess whether decisions about a given policy instrument should be taken at the central level or be decentralized. Even though the interaction with the ECB is a key aspect to determine whether coordination is desirable, the debate often remain focused on the magnitude and the signs of the cross-country spillovers that could justify or not a more cooperative approach in the member states in response to bad shocks. Nevertheless, the theoretical literature does not provide a clear-cut analytical answer. For istance, in classic analyses of policy coordination[12], ad-hoc fixed-price models generally assume direct and positive demand spillovers. By contrast, micro-founded models[13] of EMU tend to conclude in favour of negative spillovers.

In the last decade, the recent financial crisis that started in mid-2007 and affected the whole world by September 2008 was one of the most challenge

[12] See e.g., Mundell [27], Imbs [20], Imbs [21], and Tabellini [34].

[13] See e.g., Sorensen and Yosha [33], Blanchard and Perotti [2], and Beetsma and Jensen [1].

episodes for policy makers both at governments and central banks since the introduction of the euro. In a worldwide context, the effects of this disruption was not limited to the financial sector. Global real output and trade declined dramatically, and central banks took unprecedented coordinated action, in part, to alleviate the adverse impacts of the financial markets shocks on real activity. To be more precise, the theoretical literature[14] achieved three main findings. First, there are institutional and economic interdependencies across countries, specially between Eurozone countries having relinquished independent monetary and exchange rate policies. Second, there may still be a substantial degree of heterogeneity with some common behaviours in economic-financial linkages across countries and that those linkages may have changed over time due to different transmission channels. Third, the need to allow for cross-country and cross-variable interdependencies when studying real and financial linkages. Nevertheless, when dealing with multicountry data, these studies have not yet achieved an empirical consensus or have reached very different conclusions. Most of these differences can be related to diverging methodologies to assume structural relationships or lagged interdependencies across factors and alternative ways to assess spillover effects (such as, time invariant factors, exogenous variables, restrictions on selected priors).

For the above reasons, it makes sense to try to estimate the structural EPB-VAR model described in (1). To be more precise, I focused on a pool of the current members of the EMU addressing three important macroeconomic-financial issues, with particular attention on the recent financial crisis and post-crisis consolidation. First, how dimension and intensification of spillovers over time affect commonality, interdependence, and heterogeneity across countries and among variables. Second, how different transmission channels essentially affect the spreading of spillovers in real and financial dimension given an unexpected shock. Third, the importance of economic and institutional implications in driving the transmission of shocks. A survey on policy recommendations and business cycle convergence are also assessed.

[14]For istance, I refer to some main studies, such as: Lane and Milesi-Ferretti (see [24]), Mastrogiacomo *et al.* (see [26]), Facchini *et al.* (see [16]), Degiannakis *et al.* (see [13]), Crespo-Cuaresma and Fernandez-Amadorb (see [12]), Canova and Marrinan (see[4]), Reinhart and Rogoff (see [32]), and Ciccarelli and Rebucci (see [10]).

4.2 Empirical Analysis

My baseline model consists of 8 EA countries: Italy (IT), Spain (ES), France (FR), Austria (AT), Germany (DE), Ireland (IE), Greece (GR), and Portugal (PT). The dataset contains the following collection of variables. (i) Six endogenous variables are involved to describe real ($real_{it,j}$) and financial ($fin_{it,j}$) economy, so split: three real variables (general government spending, gross fixed capital formation, GDP growth rate) and three financial variables (general government debt, current account balance, interest rate). (ii) Bilateral flows of trade ($rweights_{it,j}$) and capital ($fweights_{it,j}$) are used to capture inter-linkages between country- and variable- specific factors in real and financial dimension. (iii) Five (directly) observed variables are used as *proxy* for accounting for latent macroeconomic-institutional implications ($structures_{it,j}$), so shared: one indicator monitoring external positions (net international investments); one indicator capturing competitiveness developments and catching-up effects (nominal labour cost); and three indicators reflecting internal imbalances (general government consumption, private sector consumption, and change of unemployment rate). (iv) The productivity is used to evaluate the effects of structural spillovers on total economy given an unexpected shock and corresponds to the logarithm of the GDP per capita for each country. The $weights_{it,j}$[15] and the $structures_{it,j}$ components are treated endogenously and used to investigate confounding and latent effects, respectively.

The series are expressed in standard deviation with respect to the same quarter of the previous year (q_t/q_{t-1}) and seasonally and calendar adjusted. All EA data points are originated from the Eurostat database. The estimation sample covers the period from March, 1999 to December, 2013. It amounts, without restrictions, to 3360 regression parameters. To be more precise, each equation of the structural EPBVAR has $k = [1 + 8(6 + 2 + 5) \cdot 1] = 105$ coefficients and there are 60 equations in the system. Since this span of data includes a sufficient number of quarters describing the recent financial crisis and fiscal consolidation, the model is able to capture not only possible time variation around business cycle phases, but also time variation caused by possible structural and economic changes. Finally, according to the Schwartz-

[15] The $weights_{it,j}$ component corresponds to the sum of $rweights_{it,j}$ and $fweights_{it,j}$.

Bayesian Information Criterion, the model is estimated with only one lag of all variables and factors in the system.

Here, the structural EPBVAR (defined in (1)) has the form:

$$Y_{it} = A_{it,0} + A_{it,j}(L)Y_{i,t-1} + B_{it,j}(L)W_{i,t-1} + C_{it,j}(L)Z_{i,t-1} + \varepsilon_{it} \tag{29}$$

where $i, j = 1, 2, \ldots, 8$ are the country indeces, $A_{it,j}$ is a $[(8 \cdot 6) \cdot (8 \cdot 6)]$ matrix of $real_{it,j}$ and $fin_{it,j}$ coefficients, $B_{it,j}$ is a $[(8 \cdot 2) \cdot (8 \cdot 2)]$ matrix of $weights_{it,j}$ coefficients, $C_{it,j}$ is a $[(8 \cdot 5) \cdot (8 \cdot 5)]$ matrix of $structures_{it,j}$ coefficients, $Y_{i,t-1}$ is a $[(8 \cdot 6) \cdot 1]$ vector of the endogenous variables lagged containing real and financial variables, $W_{i,t-1}$ is a $[(8 \cdot 2) \cdot 1]$ vector including bilateral trade and financial flows, and $Z_{i,t-1}$ is a $[(8 \cdot 5) \cdot 1]$ vector containing (directly) observed variables used as *proxy* for economic-institutional implications. The analysis assumes that the coefficient vector γ_t in (3) depends on eight factors. Each of them is the result of estimates of interaction terms. Thus,

$$D\beta_t = D_1\beta_{1t} + D_2\beta_{2t} + D_3\beta_{3t} + D_4\beta_{4t} + D_5\beta_{5t} + D_6\beta_{6t} + +D_7\beta_{7t} + +D_8\beta_{8t} \tag{30}$$

$D_{1,it} = \sum_{m_1=1}^{3} \cdot \sum_j y_{itm_1k_1l-j}$ and $D_{2,it} = \sum_{m_2=4}^{6} \cdot \sum_j y_{itm_2k_1l-j}$ are matrices of dimensions $NM_1k_1 \cdot N$ and $NM_2k_1 \cdot N$ respectively, with $k_1 = [1+NM]l$ and $M_1, M_2 \leq M$ denote real and financial variables, β_{1t} and β_{2t} are mutually orthogonal $NM_1k_1 \cdot 1$ and $NM_2k_1 \cdot 1$ vectors, respectively. They capture movements in the coefficient vector γ_t which are country-specifics. They account for the only $real_{it,j}$ and $fin_{it,j}$ components in order to evaluate restricted structural spillover effects and obtain a basis for matching additional effects.

$D_{3,it} = \sum_{m_1=1}^{3} \cdot \sum_j y_{itm_1k_2l-j}$ and $D_{4,it} = \sum_{m_2=4}^{6} \cdot \sum_j y_{itm_2k_2l-j}$ are matrices of dimensions $NM_1k_2 \cdot N$ and $NM_2k_2 \cdot N$ respectively, with $k_2 = [1+NM+Nq]l$, β_{3t} and β_{4t} are mutually orthogonal $NM_1k_2 \cdot 1$ and $NM_2k_2 \cdot 1$ vectors, respectively. They capture movements in the coefficient vector γ_t which are country-specifics, accounting for two components: $real_{it,j}$ with $rweights_{it,j}$ and $fin_{it,j}$ with $fweights_{it,j}$. These factors are able to assess the role of transmission channels in driving the spreading of a shock in real and financial dimension, capturing possible homogeneous and/or heterogeneous patterns across countries.

$D_{5,it} = \sum_{m_1=1}^{3} \cdot \sum_j y_{itm_1kl-j}$ and $D_{6,it} = \sum_{m_2=4}^{6} \cdot \sum_j y_{itm_2kl-j}$ are matrices of dimensions $NM_1k \cdot N$ and $NM_2k \cdot N$ respectively, with $k = [1 + NM + Nq + N\xi]l$, β_{5t} and β_{6t} are mutually orthogonal $NM_1k \cdot 1$ and $NM_2k \cdot$

1 vectors, respectively. They capture movements in the coefficient vector γ_t which are country-specifics, accounting for two components: $real_{it,j}$ with $rweights_{it,j}$ and $structures_{it,j}$ and $fin_{it,j}$ with $fweights_{it,j}$ and $structures_{it,j}$. These factors are able to investigate the effective importance and impact of economic-institutional implications in driving the propagation and transmission of country-specific shocks in real and financial dimension, capturing possible hidden and/or not directly observed interdependencies across countries.

$D_{7,it} = \sum_{i=1}^{8} \cdot \sum_j y_{itm_g l-j}$ is a matrix of dimension $NMk \cdot M_g$ and β_{7t} is mututally orthogonal $NM_g \cdot 1$ vector capturing movements in γ_t which are variable-specifics, where $M_g = 1, 2, 3, 4 \leq M$ denotes the number of variable groups. Thus, it corresponds to four groups of variables: (i) $real_{it,j}$ with $rweights_{it,j}$; (ii) $fin_{it,j}$ with $fweights_{it,j}$; (iii) $real_{it,j}$ with $rweights_{it,j}$ and $structures_{it,j}$; and (iv) $fin_{it,j}$ with $fweights_{it,j}$ and $structures_{it,j}$. The factor is able to capture possible commonality, heterogeneity and interdependence across variables accounting for both confounding and latent effects.

$D_{8,it} = \sum_{i=1}^{8} \sum_{m_c=1}^{2} \cdot \sum_j y_{itm_c l-j}$ is a matrix of dimension $NM_c k \cdot 1$ and β_{8t} is mutually orthogonal $N \cdot 1$ vector capturing movements in γ_t which are common across all countries and variables, with $M_c = 1, 2 \leq M$ denotes the number of common variable groups. Thus, it accounts for two groups: $real_{it,j}$ and $fin_{it,j}$ with $weights_{it,j}$ and $real_{it,j}$ and $fin_{it,j}$ with $structures_{it,j}$. It highlights possible reasons about different reactions or co-movements across countries and variables given a common unexpected shock.

Hence, $\beta_t = (\beta_{1t}^{'}, \beta_{2t}^{'}, \beta_{3t}^{'}, \beta_{4t}^{'}, \beta_{5t}^{'}, \beta_{6t}^{'}, \beta_{7t}^{'}, \beta_{8t}^{'})'$ is a $(54 \cdot 1)$ vector and the parsimonius NLR model having the form:

$$\begin{aligned} Y_t &= \sum_{i=1}^{8} X_t^i (D_f \beta_{ft} + u_t) + E_t \equiv \\ &\equiv \chi_{ft} \beta_{ft} + \eta_t \equiv \\ &\equiv \chi_{1t}\beta_{1t} + \chi_{2t}\beta_{2t} + \ldots + \chi_{8t}\beta_{8t} + \eta_t \quad with \quad f = 1, 2, \ldots, 8 \end{aligned} \tag{31}$$

where $\chi_t \equiv X_t \cdot D$, with $D = [D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8]$, $\beta_t = \beta_{t-1} + \upsilon_t$, and f stands for the factorization assumed for γ_t in order to construct hierarchical priors and obtain posterior mixed distributions. In (31), $\chi_{1t}\beta_{1t}$, $\chi_{2t}\beta_{2t}$, $\chi_{3t}\beta_{3t}$, $\chi_{4t}\beta_{4t}$, $\chi_{5t}\beta_{5t}$, and $\chi_{6t}\beta_{6t}$ are observable country-specific indicators for Y_t, $\chi_{7t}\beta_{7t} = \chi_{7t}^1\beta_{7t}^1, \chi_{7t}^2\beta_{7t}^2, \chi_{7t}^3\beta_{7t}^3, \chi_{7t}^4\beta_{7t}^4$ is observable cross-country variable-specific groups indicator for Y_t, and $\chi_{8t}\beta_{8t} = \chi_{8t}^1\beta_{8t}^1, \chi_{8t}^2\beta_{8t}^2$ is observable common indicator for Y_t. The hyperparameters are all known[16]. To be more precise, the values used are: $z_1 = N \cdot (M + 4 + 2)$, $Q_1 = \hat{Q}_1$, $\bar{\omega}_0 = 10^{4+2}$, $S_0 = 1.0$, $\bar{\theta}_0 = \hat{\theta}_0$, and $\bar{R}_0 = I_f$. Here, $\hat{Q}_1$ is a block diagonal matrix, with $\hat{\theta}_1 = diag(Q_{11}, \ldots, Q_{1N})$ and Q_{1i} is the estimated covariance matrix for each country VAR, and $\hat{\beta}_0$ is obtained with the OLS version of (31).

In order to show how dynamic analysis can be undertaken, accurate simulations are run. To be more precise, the total number of draws is $5000+1000 = 6000$, which corresponds to the sum of final number of draws to discard and draws to save, respectively. The study checked convergence recursively calculating the first two moments of the posterior of the parameters using 1000, 2000, 3000, 4000, 5000 draws and found that convergence is obtained about 1000 draws. A total of 1000 draws were used to conduct posterior inference at each t. The CGIRFs are computed as the difference between conditional[17] and unconditional[18] projection of output growth for each country in the period $1999q1 : 2016q1$. Here, outcomes absorb conditional forecasts computed on the time frame of 9 quarters (2.25 years). The natural conjugate prior refers to two subsamples, 2006q4-2009q4 and 2010q1-2013q4, in order to highlight the impact of the recent financial crisis and fiscal consolidation, respectively.

Finally, Table 1[19] shows some main diagnostic tests in order to verify the robustness and consistency of the model. The estimates would be asymptotically consistent given the absence of serial correlations between residuals. According to time variant factors and the Schwarz approximation, the (conditional) ML is tested. The latter confirms the exact γ_t's factorization as the p-value of the test equals $1.41e^{-4}$.

[16]Own computations.

[17]The conditional projection for output growth is the one the model would have obtained over the same period conditionally on the actual path of unexpected shock for that period.

[18]The unconditional projection is the one the model would obtain for output growth for that period based only on historical information and consistent with a model-based forecast path for the other variables.

[19]Here, LGB_m stands for Multivariate Ljung-Box Test on the series, with lags $m = 30$. P_m refers to Portmanteau (Asymptotic) Test on the residuals, with lags $m = 30$. MLE_f is the Marginal (Conditional) Likelihood Estimation Test obtained through the Schwartz approximation, with $f = 8$.

Table 1: Diagnostic Tests

Test	Test Statistics	degree of freedoms	p-value
LGB_m	5762	2430	0.00
P_m	1137.2	1215	0.9451
MLE_f	38.22	12	0.000141

4.3 Spillover Effects and Shock Transmission

Given the benchmark model in (31), structural spillover effects given an unexpected shock in real and financial dimension, accounting for both counfounding and latent effects, can be assessed. The output deriving from the model is able to absorbe each single draw obtained from the posterior of regression coefficients. Firstly, I construct a spillover matrix (see Table 2)[20] in order to define (individual) Bilateral Spillover Effects (BSEs). The latter describe the dynamics of impulse responses from a shock in real and financial variables within the EA as weighted average of responses of each variable. Since BSEs can either be negative or positive, two components can be defined: the average sum of the impulse responses to others defines Bilateral *OUT* Spillover Effects (32) and the average sum of the impulse responses from others defines Bilateral *IN* Spillover Effects (33). They incorporate feedback effects from the impulse variables and temporary or persistent long-run effects of a potential shock.

Table 2: Structural Spillover Matrix

Shock/Response	y_1	y_2	$\dots$	y_n	**To Others**	
y_1	$IR_{y_1\to y_1}$	$IR_{y_1\to y_2}$	$\dots$	$IR_{y_1\to y_n}$	$\sum_{j=1}^{N} IR_{y_1\to y_j}$	$j\neq 1$
y_2	$IR_{y_2\to y_1}$	$IR_{y_2\to y_2}$	$\dots$	$IR_{y_2\to y_n}$	$\sum_{j=1}^{N} IR_{y_2\to y_j}$	$j\neq 2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	
y_n	$IR_{y_n\to y_1}$	$IR_{y_n\to y_2}$	$\dots$	$IR_{y_n\to y_n}$	$\sum_{j=1}^{N} IR_{y_n\to y_j}$	$j\neq n$
From Others	$\sum_{j=2}^{N} IR_{y_j\to y_1}$	$\sum_{j=1}^{N} IR_{y_j\to y_2}$	$\dots$	$\sum_{j=1}^{N-1} IR_{y_j\to y_n}$	$\sum_{j=1}^{N}(IR_{y_i\to y_j}-IR_{y_j\to y_i})$	

[20]Here, row variables are the origin of the unexpected shock. Column variables are the respondents or spillover receivers.

$$BSE_{OUT,y_i \to *} = \sum_{j=1}^{N} IR_{y_i \to y_j} \quad with \quad i = j = 1, \ldots, 8 \tag{32}$$

$$BSE_{IN,* \to y_i} = \sum_{j=1}^{N} IR_{y_j \to y_i} \quad with \quad i = j = 1, \ldots, 8 \tag{33}$$

By the same token, Bilateral Net Spillover Effects (BNSEs) can be defined as the difference between the conditional impulse responses sent and received to/from another variable (34). When the BNSE is positive/negative, the variable (country) is a net sender/net receiver of the system, respectively.

$$BNSE_{y_{i,j}} = BSE_{OUT,y_i \to *} - BSE_{IN,* \to y_i} \quad with \quad i = j = 1, \ldots, 8 \tag{34}$$

where $\sum_{j=1}^{N}(IR_{y_i \to y_j} - IR_{y_j \to y_i}) = 0$.

Thus, I have suitable instruments to study dimension and intensification of spillover effects. To be more precise, I calculate the Systemic Contribution index, defined as the ratio between the BNSEs and the Total Net Positive Spillover (TNPS) of the system (35). It represents the amplification contribution of the impulse variable to the response variable and is able to capture sequential feature associated with systemic events.

$$SC_{y_{i,j}} = \frac{BNSE_{y_{i,j}}}{TNPS_{y_{i,j}}} \tag{35}$$

Systemic Contributions of the productivity given a 1% shock to real and financial dimension are drawn in standard deviation of the variables in the system. They account for $real_{it,j}$ (Figure 1) and $fin_{it,j}$ (Figure 2) components, corresponding to $\chi_{1t}\hat{\beta}_{1t}$ and $\chi_{2t}\hat{\beta}_{2t}$ cross-country indicators, respectively.

Overall, there is a heterogenous pattern across countries in financial dimension ($\hat{S}_{fin} = 0.310$) and even more in real dimension ($\hat{S}_{real} = 0.342$), with some common behaviors. These commonalities seem to be larger in financial dimension. There are not significant findings about the signs of the spillovers, confirming the need to consider further empirical evidence. In real dimension, most countries tend to be net receivers of the system and, hence, unexpected country-specific shocks directly affect own output growth in financial dimension and then in real economy because of consistent cross-country interdependencies.

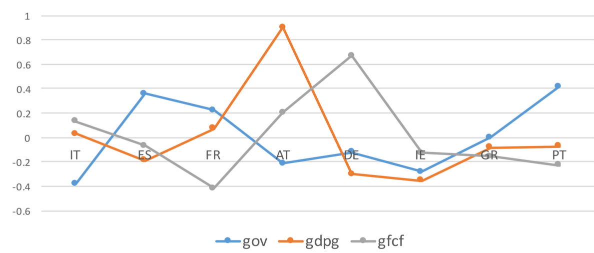

Figure 1: $\chi_{1t}\hat{\beta}_{1t}$ Factor. Overall Period

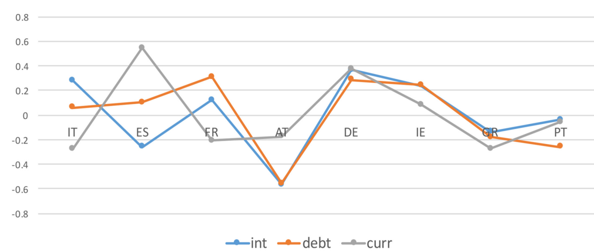

Figure 2: $\chi_{2t}\hat{\beta}_{2t}$ Factor. Overall Period

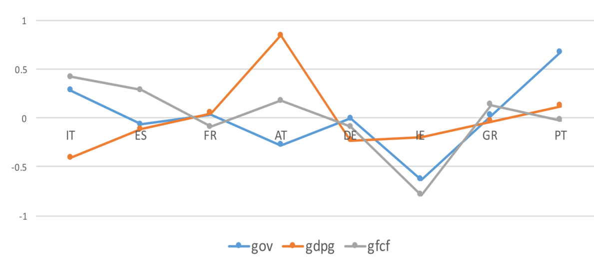

Figure 3: $\chi_{3t}\hat{\beta}_{3t}$ Factor. Overall Period

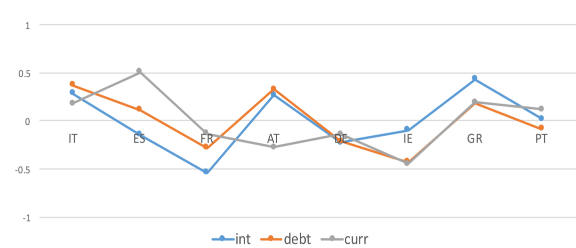

Figure 4: $\chi_{4t}\hat{\beta}_{4t}$ Factor. Overall Period

Accounting for the only $weights_{it,j}$ component (Figures 3 and 4) important modelling and policy perspectives are found. A significant common pattern emerges in both real and financial dimension. In the latter, commonalities tend to largely occurr. From a modelling perspective, it highlights that the transmission is more intense across countries in their financial dimension and, from a policy perspective, that consolidations over time occurred simultaneously behind more coordinated fiscal actions across member states. Focusing on the spreading of spillovers, there is a larger intensification, while their signs do not show significant considerations. A deeper homogeneity is observed in financial dimension. Moreover, a consistent degree of heterogeneity persists across countries, mainly in real dimension ($\hat{S}_{real} = 0.375$) than in financial dimension ($\hat{S}_{fin} = 0.306$).

Finally, considering both components (Figures 3 and 4), I prove that economic and institutional differences among national policy are implicitly involved when assessing growth spillovers and different transmission channels. To be more precise, most countries are net receivers in real economy and net senders in financial dimension, proving that shock transmissions are larger among capital flows than trade exposures. Cross-country homogeneity and co-movements in financial dimension are driven by large public debts and formal institutional constraints[21] (e.g., the Maastricht rules and the Stability and Growth Pact, which forbids public deficits exceeding 3% of GDP). Higher heterogeneity is observed across countries in their real dimension ($\hat{S}_{real} = 0.342$) and, hence, despite a common monetary policy, national policies of investments and structural reforms in labour and complementary markets remain heterogeneous across the EA. It might have contributed to the current emergence of different country-specific developments of competitiveness, consumption, investment, and production affecting national economy. From a policy perspective, despite its large size, Germany shows a limited role in generating growth spillovers. This result, in part, reflects Germany's own dependence on growth in the rest of the Eurozone. Furthermore, inward spillovers tend to be greater than outward spillovers, proving large trade exposures with other european countries. Finally, PIIGS countries[22] show larger conditional responses due to their high dimension in cross-border trade accounts.

Different results have been found for the variable-specific factor ($\chi_{7t}\hat{\beta}_{7t}$). To be more precise, accounting for the only $weights_{it,j}$ component (Figure

[21] See e.g., Eichengreen and Wyplosz [15] and Buti *et al.* [3].

[22] They correspond to Portugal, Ireland, Italy, and Greece (IMF, 2011).

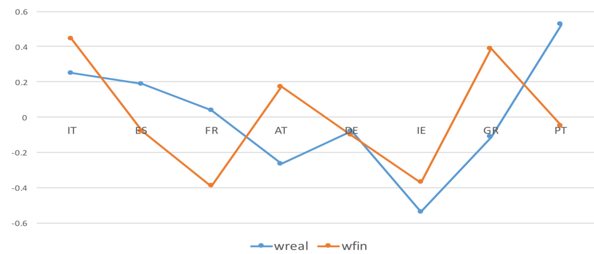

Figure 5: $\chi^1_{7t}\hat{\beta}^1_{7t}$ and $\chi^2_{7t}\hat{\beta}^2_{7t}$ Factors. Overall Period

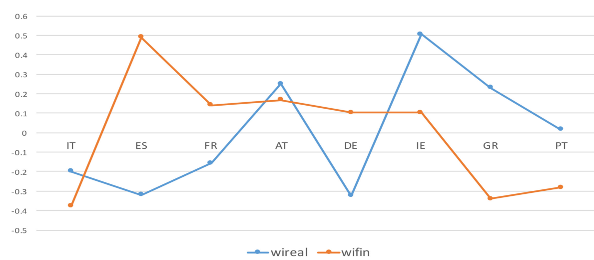

Figure 6: $\chi^3_{7t}\hat{\beta}^3_{7t}$ and $\chi^4_{7t}\hat{\beta}^4_{7t}$ Factors. Overall Period

5), on average, countries tend to generate larger outward growth spillovers in financial dimension ($\bar{\beta}^2_{7t} = 0.33$) than in real economy ($\bar{\beta}^1_{7t} = 0.25$). Thus, I confirm the consistent role of capital income flows in absorbing the effects of variable-specific shocks in contrast of standard theoretical models[23]. In addition, despite a substantial degree of heterogeneity, Figure 5 shows a certain homogeneity across countries and, hence, the analysis is in line with rapidly increasing cross-border trade and financial linkages.

Matching both $weights_{it,j}$ and $structures_{it,j}$ components (Figure 6), most countries tend to be net senders of the system in financial dimension, confirming that trade channels matter relatively less than financial channels. Moreover, outward and inward growth spillovers follow a homogeneous pattern across members, except Spain (possibly due to larger capital exposures). Thus, highly indebted countries were forced into equally taking wide-ranging austerity measures, having lost access to the financial markets. This has led to call for stronger cross-country differentiation and for temporary stimulus measures in countries not facing financial market pressure. Moreover, cross-border spillovers seem to have hampered the effects of consolidations, accounting for

[23]See e.g., Gordon and Bovenberg [19] and Sorensen and Yosha [33].

a substantial degree of heterogeneity in real dimension and a deeper interdependence in financial dynamic.

The last factor ($\chi_{8t}\hat{\beta}_{8t}$) shows that there is a significant common component across eurozone members in financial dimension and country-specific heterogeneities tend to matter more in real economy. In fact, in Figure 7, in which the common indicator is matched with the corresponding transmission channels for real and financial dimension, I demonstrate that growth shocks appear to be predominantly transmitted via financial linkages and there is a more important consistent common component in financial dimension because of stronger interdependencies among variables. The Figure 8, in which the common indicator is matched with the only variables capturing economic-institutional factors in real and financial dimension, shows larger heterogeneities among growth spillovers observed across countries. The analysis confirms that, in the EA, structural reforms without coordinated national fiscal actions negatively affect the adjustment capacity of the currency union as a whole because of high degree of divergence.

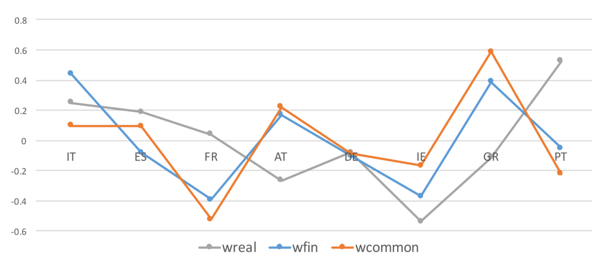

Figure 7: $\chi^1_{8t}\hat{\beta}^1_{8t}$ Factor. Overall Period

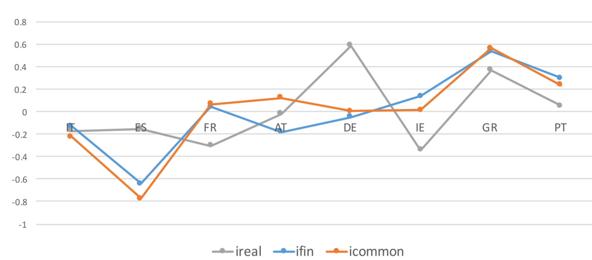

Figure 8: $\chi^2_{8t}\hat{\beta}^2_{8t}$ Factor. Overall Period

4.4 Crisis Period and Post-Crisis Consolidation

In recent years, the transmission of shocks is faster and deeper in financial dimension because of stronger economic and structural interdependencies. I found deeper co-movents and larger degree of homogeneity across countries in real dimension and even more in financial dimension during the recent recession (Figures 9 and 11) compared to successive post-crisis consolidations (Figures 10 and 12). These results cast three main findings. First, interdependencies because of strongly common economic-institutional linkages matter more during trigger events. Second, coordinated fiscal actions do not necessarily yield better outcomes. Third, the sign of growth spillovers are not significant in determining whether coordination should lead to a more expansionary or more restrictive fiscal stance in the member states.

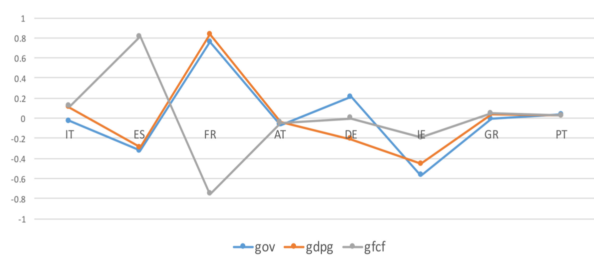

Figure 9: $\chi_{5t}\hat{\beta}_{5t}$ Factor. Crisis Period

Figure 10: $\chi_{5t}\hat{\beta}_{5t}$ Factor. Post Recession

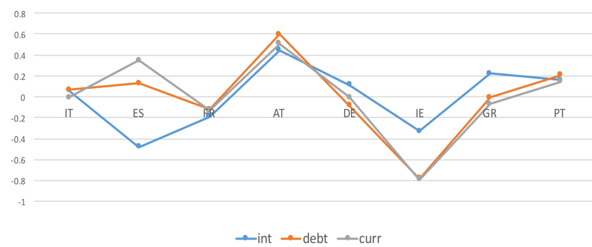

Figure 11: $\chi_{6t}\hat{\beta}_{6t}$ Factor. Crisis Period

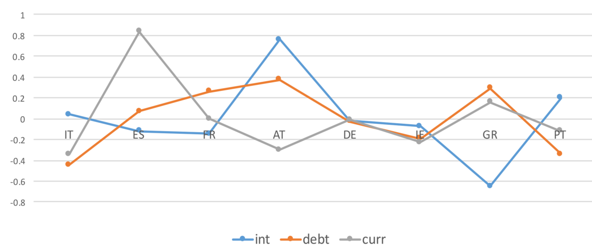

Figure 12: $\chi_{6t}\hat{\beta}_{6t}$ Factor. Post Recession

Thus, although recent theoretical studies suggest that the imbalances have been reduced and the macroeconomic policy mix with a discretionary fiscal expansion and a neutral monetary policy were likely to mitigate output growth during recession and successive consolidations, without the appropriate adjustment of the private and public sector, Eurozone imbalances and different degrees of productivity growth would tend to persist in the future (Figure 13).

Accounting for the last two groups of the variable-specific indicator ($\chi^3_{7t}\hat{\beta}^3_{7t}$ and $\chi^4_{7t}\hat{\beta}^4_{7t}$), I confirm that trade channels matter relatively less than capital linkages. In fact, most countries tend to be net senders in financial dimension and net receivers in real economy (Figures 14 and 15). During post-crisis consolidations, inward growth spillovers are a lot more frequent and large because of tight institutional and economic interdependencies. Thus, cross-border spillovers have exacerbated the negative effects of consolidations due to (individual) domestic policies designed to counteract the events of the recession and that, when successive consolidations occured, proved to be ineffective and counter-productive for the domestic economy.

The common indicator ($\chi_{8t}\hat{\beta}_{8t}$) shows that economic-institutional interdependencies matter more than different transmission channels in driving the

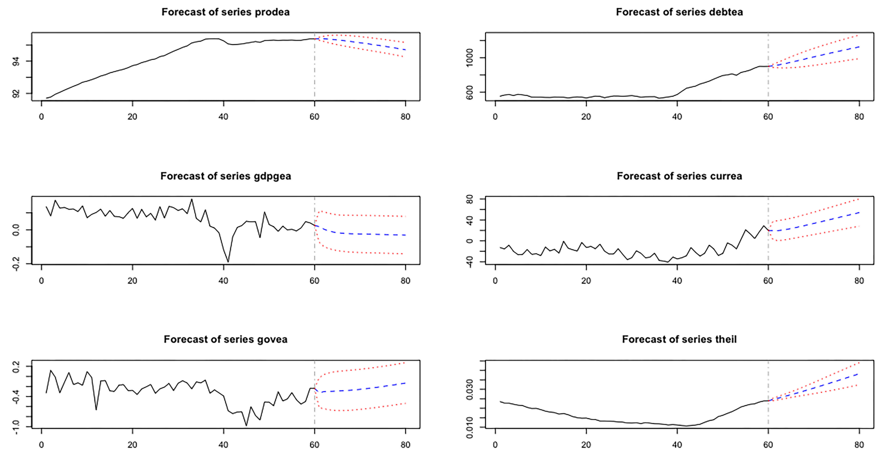

Figure 13: The Figure draws conditional forecasts of the productivity (*prodea*), general government debt (*debtea*), real GDP growth rate (*gdpgea*), current account balance (*currea*), general government spending (*govea*), and the generalized entropy index (*theil*) in the eurozone from 1999$q1$ to 2020$q2$. The latter corresponds to Theil's Entropy and is computed by weighing the GDP with the population in terms of proportions with respect to the total. It can be viewed as a measure of divergence and economic inequality. Here, forecasts from 2016$q1$ to 2020$q2$ correspond to conditional projections of each variable drawn in the EPBVAR (29).

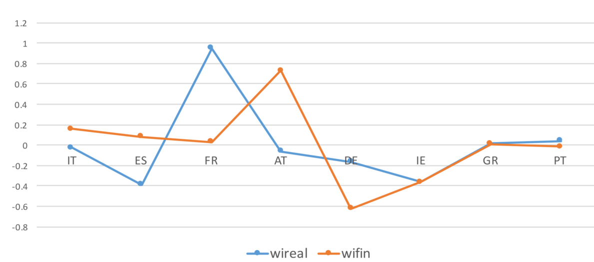

Figure 14: $\chi^3_{7t}\hat{\beta}^3_{7t}$ and $\chi^4_{7t}\hat{\beta}^4_{7t}$ Factors. Crisis Period

spreading of common unexpected shocks. To be more precise, a larger homogeneous pattern is observed during successive consolidations (Figures 16 and 17). It would seem to demonstrate that several countries actually started to put in practice a fiscal consolidation package and national fiscal actions have been adopted in some sort of coordinated way.

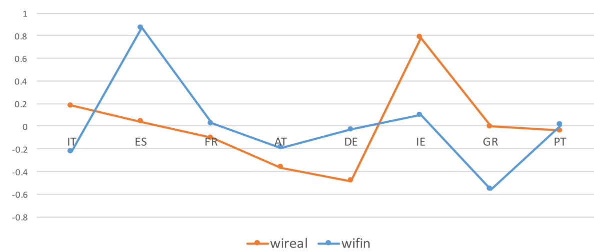

Figure 15: $\chi^3_{7t}/\hat{\beta}^3_{7t}$ and $\chi^4_{7t}/\hat{\beta}^4_{7t}$ Factors. Post Recession

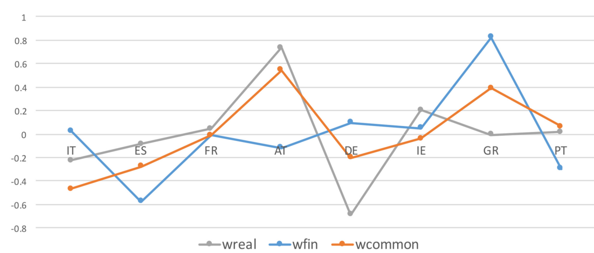

Figure 16: $\chi^1_{8t}/\hat{\beta}^1_{8t}$ Factor. Post Recession

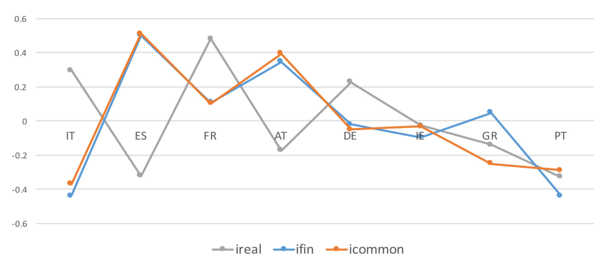

Figure 17: $\chi^2_{8t}/\hat{\beta}^2_{8t}$ Factor. Post Recession

5 Conclusion

The paper develops empirical implementations to conduct inference in time-varying coefficients by using a structural unconstrained Panel Bayesian Multi-country VAR model, in order to deal with confounding and latent effects. The aim of the project is to assess an analytical contribution to the recent literature on idiosyncratic business cycles and policy making, minimizing omitted biases that occurr when dealing with multicountry data. Bayesian computations are used to allow cross-unit interdependencies and unit-specific dynamics and restrict the coefficients to have a low-dimensional time-varying factor structure. The econometric model uses hierarchical priors for the vector of components in order to permit exchangeability, time variations, and endogeneity in the innovations in the factors. The identified setup only requires minimal prior input from the researcher. In this way, an overparametrized VAR can be transformed into a parsimonius Normal Linear model, where the regressors are all (directly) observed and the loadings are the time-varying coefficient factors. Generalized Conditional IRFs and Conditional Forecasts are obtained with the output of an MCMC routine and a variant of Gibbs sampling.

An empirical application to a pool of the current members of the European Monetary Union illustrates the functioning of the model, with particular attention on the recent recession and successive consolidations. Three interesting macroeconomic-financial issues left even now unanswered have been addressed. First, how length and intensification of spillovers over time affect commonality, interdependence, and heterogeneity across countries and among variables. Second, how different transmission channels essentially affect the spreading of spillovers in real and financial dimension given an unexpected shock. Third, the importance of economic and institutional implications in driving the transmission of shocks. The analysis is consistent and robust with the more recent literature on idiosyncratic business cycles, which recognizes the importance to separate common shocks from propagation of country- and variable-specific shocks when studying economic-financial linkages.

From a modelling perspective, I demonstrate that growth schocks spill over in a heterogeneous way across countries, although a significant common component held, mainly during the crisis period and even more during post-crisis consolidations. Overall period, commonalities are stronger in financial dimension where the shock transmission is more intense. The results prove the presence of higher interdependencies among variables and that consolidations occurred simultaneously behind more coordinated fiscal actions across member states. Accounting for variable-specific shocks, the spreading of spillovers show a homogeneous pattern across countries and deeper divergences whether economic-institutional linkages are involved.

From a policy perspective, despite a currency union, different country-specific developments of competitiveness, consumption, investment, and production affecting national economy should be designed in order to shrink growth divergencies across countries. With the advent of the financial crisis, fiscal expansion has been associated with smaller output growth loss and national fiscal actions have been used in some sort of coordinated way. After gradual economic recovery began to be observed, several countries started to put in practice a fiscal consolidation package. Nevertheless, even though the macroeconomic policy mix with a discretionary fiscal expansion and a neutral monetary policy were likely to mitigate output growth during recession and successive consolidation, without coordination efforts going beyond what already exists in the set of rules given in the Maastricht Treaty, eurozone imbalances and different degrees of productivity growth would tend to persist in the future.

I wish to express my sincere gratitude to Prof. Thierry Vissol, ex Special Advisory to the European Commission - Representation in Italy, and to Prof. Matteo Ciccarelli, Economist at the European Central Bank, rendering their help during the period of my work.

A Conditional Generalized Impulse Responses

Conditional impulse responses are generally computed as the difference between two realizations of $y_{t+\tau}$, with $\tau = 1,2, \ldots$, which are identical up to time t. Thus, between $t+1$ and $t+\tau$, one can assume two time impulses in the *jth* component of $e_{t+\tau}$. First, one that occurs only at time $t+1$. Second, the other that no shocks take place at all dates between $t+1$ and $t+\tau$. In a model with time-varying coefficients, the approach is inadequate since it overlooks that between $t+1$ and $t+\tau$, structural coefficients may also change. Therefore, impulse responses are obtained as the difference between two conditional expectations of $y_{t+\tau}$. In both cases, they are conditioned on the history of the data Y_t and of the factors β^t, the parameters of the law of motion of the coefficients, and all future shocks. However, impulse responses are conditioned on a random draw for the current shocks, whereas in the other the current shocks is set to its unconditional value. Hence, they are worked out on future shocks instead of integrating them out because, computationally, such a choice gives more stable responses, even though this makes standard error bands larger than in the case where future shocks are integrated out. There are two potential types of impulses. First, one to the variables of the system. Second, one to the factors.

Here, the reparametrized NLR is:

$$y_t = \chi_t \beta_t + (E_t + X_t u_t) \qquad with \qquad \beta_t = \beta_{t-1} + v_t \tag{36}$$

where $\beta_t = [\beta'_{1t}, \beta'_{2t}, \ldots, \beta'_{Ft}]'$, $\chi_t = [\chi_{1t}, \ldots, \chi_{Ft}]$, $\chi_{it} = DX_t$, $X_t = [Y_{t-1}, W_t, W_{t-1}, Z_t, Z_{t-1}]$. Let $\mho = [(E_t + X_t u_t)', v']'$ be the vector of reduced-form shocks and $P_t = [L_t^{-1}(E_t + X_t u_t)', L_t^{-1} v_t']'$ be the vector of structural shocks where $E_t = L_t v_t$, $L_t L_t' = \Omega$ so that $var(v_t) = \sigma^2 I_k$ and $L_t = J \cdot K_t$ where $K_t K_t' = I$ and J is a matrix that orthogonalizes the VAR shocks.

Here, a Choleski system is obtained setting $K_t = I, \forall t$, and choosing J to be lower triangular whereas more structural identification schemes are obtained letting J be an arbitrary square root matrix and K_t a matrix implementing certain theoretical restrictions. The identification matrix K_t is allowed to be time-varying since, when recursive estimations are used, estimates of Ω depends on t.

Let $\mathbb{P}_t = (\Omega, \sigma^2, H_t, \Phi)$, let $\bar{P}_{j,t}$ be a particular realization of $P_{j,t}$ and $P_{-j,t}$ indicate the structural shocks, excluding the one in the *jth* compo-

nent. Let $F_t^1 = \{Y^{t-1}, \beta^t, \mathbb{P}_t, L_t, P_{j,t}\}$, with $P_{j,t} = \{\bar{P}_{j,t}, P_{-j,t} \cdot \mho_{t+1}^{t+\tau}\}$, and $F_t^2 = \{Y^{t-1}, \beta^t, \mathbb{P}_t, L_t,$
$P_{j,t}\}$, with $P_{j,t} = \{EP_{j,t}, P_{-j,t} \cdot \mho_{t+1}^{t+\tau}\}$ be two conditioning sets. Thus, responses to a shock at t in the *jth* component of P_t are obtained as:

$$IR(t, t+\tau) = E(Y_{t+\tau}|F_t^1) - E(Y_{t+\tau}|F_t^2) \qquad t = 1, 2, \ldots \tag{37}$$

In order to see what definition in (37) involves, rewrite the original EPB-VAR model in a companion form, without intercept:

$$Y_{t+\tau} = A_{t+\tau}Y_{t+\tau-1} + B_{t+\tau}W_{t+\tau-1} + C_{t+\tau}Z_{t+\tau-1} + E_{t+\tau} \tag{38}$$

and let

$$\gamma_{t+\tau} = D[\beta_{t+\tau-1} + \upsilon_{t+\tau}] + u_{t+\tau} \tag{39}$$

where $\gamma_{t+\tau} = [vec(A_{1t+\tau}), vec(B_{t+\tau}), vec(C_{t+\tau})]$ and $A_{1t+\tau}$ is the first row of $A_{t+\tau}$. Taking $Y^{t-1} = (Y_{t-1}, Y_{t-2}, \ldots, W_{t-1}, W_{t-2}, \ldots, Z_{t-1}, Z_{t-2}, \ldots)$, $A^t = (A_t, A_{t-1}, \ldots)$, $B^t = (B_t, B_{t-1}, \ldots)$, $C^t = (C_t, C_{t-1}, \ldots)$, and $L_{t+\tau} = L_t$ for $\forall\tau$ as given. Solving backward, the equations (38) and (39) can be rewritten as:

$$\begin{aligned} Y_{t+\tau} \;=\; & \left(\prod_{k=0}^{\tau} A_{t+\tau-k}\right) Y_{t-1} + B_{t+\tau}W_{t+\tau-1} + C_{t+\tau}Z_{t+\tau-1} + \sum_{l=1}^{\tau}\left(\prod_{k=0}^{l-1} A_{t+\tau-k}\right) \cdot \\ & \cdot B_{t-\tau-l}W_{t+\tau-l-1} \cdot C_{t-\tau-l}Z_{t+\tau-l-1} + L_{t-\tau}\eta_{t+\tau} + \sum_{l=1}^{\tau}\left(\prod_{k=0}^{l-1} A_{t+\tau-k}\right) \cdot \\ & \cdot L_{t+\tau-l}\eta_{t+\tau-l} \end{aligned} \tag{40}$$

and as

$$\gamma_{t+\tau} = D\beta_{t-1} + D\sum_{k=0}^{\tau} \upsilon_{t+\tau-k} + u_{t+\tau} \tag{41}$$

Consider first the case of a *(m+1)*-period impulse in the *jth* component of υ. For example: $\upsilon_{j,t+k} = \bar{\upsilon}_{j,t+k}$; $\upsilon_{-j,t+k}$, $k = 0,\ 1,\ 2,\ \ldots,\ m$ and $\upsilon_{t+m'}$, with

$\forall m' > m$, are restricted. Then,

$$\begin{aligned} IR_{t,t+\tau} &= E_t\Big[Y_{t+\tau}|Y_{t-1}, A^t, B^t, C^t, \mathbb{P}_t, L_t, \{\bar{\eta}_{jt+m}\}_{k=o}^{m}, \{\eta_{-jt+k}\}_{k=0}^{m}, \{\eta_{t+k}\}_{k=m+1}^{\tau}\Big] - \\ &\quad - E_t\Big[Y_{t+\tau}|Y^{t-1}, A^t, B^t, C^t, \mathbb{P}_t, L_t, \{\eta_{t+k}\}_{k=0}^{\tau}\Big] = \\ &= E_t\Bigg[\left(\prod_{k=0}^{\tau-1}\right)^j H_t^j(\bar{\eta}_{jt} - E\eta_{jt}) + \left(\prod_{k=0}^{\tau-2} A_{t+\tau-k}\right)^j \cdot L_{t+1}^j(\bar{\eta}_{jt+1} - E\eta_{jt+1}) + \ldots \\ &\quad \ldots + \left(\prod_{k=0}^{\tau-m-1} A_{t+\tau-k}\right)^j \cdot L_{t+m}^j(\bar{\eta}_{jt+m} - E\eta_{jt+m})\Bigg] \end{aligned} \tag{42}$$

where the superscript j refers to the jth column of the matrix. It is easy to see that, when $A_t = A$, $B_t = B$ and $C_t = C$, $\forall t$, the equation (42) reduces to standard impulse responses and, when E_t and υ_t are correlated (that is both the sign and the size of the shocks matter a shock in υ_t), may induce changes in A_t, B_t or C_t.

Given 37, responses in the NLR model can be computed as follows:

1. Choosing t, τ, and J_t. Drawing $\Omega^\delta = L_t^\delta(L_t^\delta)', (\sigma^2)'$ from their posterior distribution and u_t^δ from $N(0, (\sigma^2)^2 I \otimes L_t^\delta(L_t^\delta)')$. Computing $y_t^\delta = \chi_t\beta_t + L_t\bar{\eta}_t + X_t u_t^\delta$.

2. Drawing $\Omega = L_{t+1}^\delta(L_{t+1}^\delta)', (\sigma^2)^\delta, \mathcal{H}_{t+1}^\delta, \phi^\delta$. Drawing η_{t+1}^δ from their posterior distribution. Using the law of motion of the factors to compute β_{t+1}^δ, $\delta = 1, 2, \ldots, \Delta$, and the definition of D to compute χ_{t+1}. Drawing u_{t+1}^δ from $N(0, (\sigma^2)^\delta I \otimes L_{t+1}^\delta(L_{t+1}^\delta)')$ and computing $y_{t+1}^\delta = \chi_{t+1}\beta_{t+1} + L_{t+1}\bar{\eta}_{t+1} + X_{t+1}u_{t+1}$, $\delta = 1, 2, \ldots, \Delta$.

3. Repeating Step 2 and computing β_{t+k}^δ, y_{t+k}^δ, $k = 1, 2, \ldots, \tau$.

4. Repeating Steps 1 - 3 by setting $\eta_{t+K} = E(\eta_{t+1})$, $k = 1, 2, \ldots, m$ and using the draws for the shocks in 1 - 3.

Responses to structural shocks to the law of motion of the factors can be computed in the same way. An impulse in $\upsilon_t = \bar{\upsilon}$ lasting *(m+1)* periods implies from (41) that:

$$E(\bar{\gamma}_{t+\tau} - \gamma_{t+\tau}) = D \sum_{k=0}^{m} H_{t+k}(\bar{\eta}_{t+\tau-k} - E\eta_{t+\tau} - k) \tag{43}$$

and

$$\begin{aligned} IR_{t,t+\tau} \quad = \quad & E_t \Big[\prod_{k=0}^{\tau} \Big(\bar{A}_{t+1,\tau-k} - A_{t+\tau-k} \Big) Y_{t+1} + \sum_{l=1}^{\tau} \prod_{k=0}^{l-1} \Big(\bar{A}_{t+1,\tau-k} - A_{t+\tau-k} \Big) \cdot \\ & \cdot B_{t+\tau-l-1} \cdot C_{t+\tau-l-1} + \sum_{l=1}^{\tau} \prod_{k=0}^{l-1} \Big(\bar{A}_{t+1,\tau-k} - A_{t+\tau-k} \Big) \cdot \\ & \cdot L_{t+\tau-l} \eta_{t+\tau-l} \Big] \end{aligned} \tag{44}$$

B Conditional Forecasts

There are two types of conditional forecasts one can compute in this framework. Those involving displacement of the endogenous variables W_t and Z_t from their unconditional path, and those involving a particular path for a subset of the endogenous variables from their conditional path. Both types of conditional forecasts can be constructed using the output of the Gibbs sampler routine.

Consider first displacing the endogenous variables from their expected future path for *m+1* periods. Calling the new path $\bar{W}_{t+k}$ and $\bar{Z}_{t+k}$, $k = 0, 1, \ldots, m$. Then, the response of $Y_{t+\tau}$ is:

$$\begin{aligned} IR_{t,t+\tau} \quad = \quad & E_t \Big[\Big(\prod_{k=0}^{\tau-2} A_{t+\tau-k} \Big) B_{t+1} \Big(\bar{W}_{jt} - W_{jt} \Big) C_{t+1} \Big(\bar{Z}_{jt} - Z_{jt} \Big) + \Big(\prod_{k=0}^{\tau-3} A_{t+\tau-k} \Big) \cdot \\ & \cdot B_{t+2} \Big(\bar{W}_{jt+1} - W_{jt+1} \Big) \cdot C_{t+2} \Big(\bar{Z}_{jt+1} - Z_{jt+1} \Big) + \ldots + \Big(\prod_{k=0}^{\tau-2-m} A_{t+\tau-k} \Big) \cdot \\ & \cdot B_{t+m+1} \Big(\bar{W}_{jt+m} - W_{jt+m} \Big) \cdot C_{t+m+1} \Big(\bar{Z}_{jt+m} - Z_{jt+m} \Big) \Big] \end{aligned} \tag{45}$$

Thus, to compute conditional forecasts of this type in the NLR model, one need to:

1. Choosing t, τ, and a path $\{\bar{W}_{t+k}, \bar{Z}_{t+k}\}_{k=0}^{m}$. Drawing Ω^{δ}, $(\sigma^2)^{\delta}$ from their posterior, drawing $E_t^{\delta} + X_t u_t^{\delta}$ and computing y_t^{δ}.
2. Drawing $(H_t)^{\delta}$, Ψ^{δ} from their posterior distribution; drawing υ_{t+1}^{δ} and using the law of motion of the factors to draw β_{t+1}^{δ}, $\delta = 1, 2, \ldots, \Delta$ and the definition of D to compute χ_{t+1}. Then, $E_{t+1}^{\delta} + X_{t+1}U_{t+1}^{\delta}$ are drawn to compute $y_{t+1}^{\delta} = \chi_{t+1}\beta_{t+1}^{\delta} + (E_{t+1}^{\delta} + X_{t+1}u_{t+1}^{\delta})$, $\delta = 1, 2, \ldots, \Delta$.
3. Repeating Step 2 in order to compute β_{t+k}^{δ}, y_{t+k}^{δ}, $k = 1, 2, \ldots, \tau$.
4. Repeating Steps 1 - 3. In this way, it sets $W_{t+k} = E(W_{t+k})$, $Z_{t+k} = E(Z_{t+k})$, $k = 0, 1, \ldots, m$, using the draws for the shocks in 1 - 3.

Finally, considering the case in which the future path of a subset of Y_t's is fixed. For example, in a system with real and financial linkages, one would like to work out on a given path for the future growth spillovers. Hence, partioning $Y_t = A_t Y_{t-1} + B_t W_{t-1} + C_t Z_{t-1} + E_t$ in two blocks, let $Y_{2t+k} = \bar{Y}_{2t+k}$ be the fixed variables and Y_{1t+k} those allowed to adjust, the Impulse Responses are:

$$\begin{aligned} IR_{t,t+\tau} &= E\Bigg[L_t^1\Bigg(\prod_{k=0}^{t-1} A_{t+\tau-k}\Bigg)^1(\bar{\eta}_{2t} - \eta_{2t}) + L_{t+1}^1\Bigg(\prod_{k=0}^{t-2} A_{t+\tau-k}\Bigg)^1(\bar{\eta}_{2t+1} - \eta_{2t+1}) + \ldots \\ &\quad \ldots + L_{t+m}^1\Bigg(\prod_{k=0}^{t-1-m} A_{t+\tau-k}\Bigg)^1(\bar{\eta}_{2t+m} - \eta_{2t+m})\Bigg] \end{aligned} \tag{46}$$

where $\bar{\eta}_{2t+k} = \bar{Y}_{2t+k} - A_{21,t+k}Y_{1t-k-1} - A_{22,t+k}Y_{2t-k-1} - B_{2t+k}W_{t+k-1} - C_{2t+k}Z_{t+k-1}$ and the super-script *1* refers to the first row of the matrix. Hence, to compute this type of conditional forecasts one need to:

1. Partitioning $y_t = (y_{1t}, y_{2t})$, choosing t and a path $\{y_{2t+k}\}_{k=0}^{\tau}$. Using the model to solve for the $\bar{\eta}_{2t}$ that gives $y_{2t} = \bar{y}_{2t}$, backing out the implied y_{1t}^{δ} once draws for E_{1t}^{δ}, and computing u_t^{δ} from their posterior distribution. Thus, υ_{t+1}^{δ} can de drawn using the law of motion of the factors to obtain β_{t+1}^{δ}, with $\delta = 1, 2, \ldots, \Delta$, and the definition of D to compute χ_{t+1}.

2. Using the model to solve for $\bar{\eta}_{2t}$ that gives $y_{2t+1} = \bar{y}_{2t+1}$, backing out the implied y^{δ}_{1t+1} once draws for E^{δ}_{1t+1}, and computing u^{δ}_{t+1} as above. Hence, once can draw υ^{δ}_{t+2}, using the law of motion of the factors to compute β^{δ}_{t+2}, with $\delta = 1, 2, \ldots, \Delta$, and the definition of D to compute χ_{t+2}.

3. Repeating Step 2 and computing β^{δ}_{t+k}, y^{δ}_{t+k}, $k = 2, 3, \ldots$.

4. Repeating Steps 1 - 3, once can set $\eta^{\delta}_{2t+k} = E(\eta_{t+k})$, $\forall k$ using the draws for the shocks in 1 - 3.

In Step 2 of all algorithms, it has implicitly assumed that selecting a path for the shocks does not alter neither the law of motion of the factors nor the beliefs about the true structural shocks.

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