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Applying the Fock spaces-based tools to the modeling of RBF neural networks: a quantum RBF neural network approach

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Abstract

Radial Basis Function (RBF) neural networks are multi-layer feed forward networks widely used for pattern recognition, function approximation and data interpolation problems in areas such as time series analysis, image processing and medical diagnosis. Rather than using the sigmoid activation function as in back propagation networks, the hidden neurons in a RBF neural network use a Gaussian or some other radial basis function. As part of a theoretical and experimental study of RBF neural networks, in this paper we adopt a Fock spaces-based approach, and show how it can be applied to the modeling a RBF neural network. Specifically, we are working to explore and to understand the new theoretical and operational aspects of this kind of neural network from a quantum perspective.

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1 Introduction

Quantum theory and quantum theory based experimentation had opened the possibility that physical non-causal relationships, in the Aristotelian sense, can be modeled mathematically. These relationships are known as quantum "superposition" and "entanglement". For example, the spin of protons or electrons may be even correlated for considerable spatial distances without any interaction causal (Aristotelian type) between them -if there is a causal interaction in this phenomenon, it should operate faster than the speed of light and then would contradict the theory of relativity.

Basing on the above, we consider in this proposal neural signal processing as a process of quantum physics rather than classical physics, since quantum physics is a mechanism capable of exhibiting a lot of coherent processes at the quantum level (original discussion can be seen in [1], a study from the point of view of physics in [2] and a version with further explanation in [3]).

Quantum physics opens new possibilities for conceptualizing the mechanisms that occur at the neuronal level, these possibilities include both the conception of neural dynamics as a quantum process and as a process in high-dimensional spaces (like the space-model, currently proposed in string theory [4, 5]). Therefore, the application of a mathematical approach to the dynamic cerebral theory would allow us to use the generated quantum physics, in order to provide new tools for modeling of brain processes, which are highly dependent on chemical and ionic processes. For example, release of neurotransmitters from presynaptic terminals is controlled by the movement of calcium ions and these ions are small enough so they can be governed by deterministic laws of classical physics.

Quantum theory, in principle, could be used to describe the dynamics of ions in the brain, and once established this mechanical description, it could be used for incremental modeling of synapses, neural circuits, neural layers, and neural networks to achieve modeling an entire neuronal region. According to this quantum approach, description of the brain, from the point of view of the second quantization, is itself an expansion to a high-dimensional space (infinite dimensional Hilbert space). The second quantization is the standard quantum formulation for modeling the dynamics of a multi-particle system, resulting in significant use in both Quantum Field Theory (since a quantized field is an operator with many degrees of freedom) and the Quantum Theory of Condensed Matter (given that the study involves many particles).

A perfect scenario for exploring the use of this quantum approach in modeling neural signal processing is provided by artificial neural network models [6]. In this paper, we propose a quantum model for artificial neural networks, based on a Hilbert space through the second quantization. The mathematical constructions of such Hilbert space leads to impose the following restrictions: 1) we assume that the total capacity of neuronal space is $\Omega \subset \mathbb{R}^3$, and 2) this capacity can be decomposed into independent components Ω_1,...,Ω_n, each of them responsible for various aspects of neural signals, *i.e.*, each component representing a particular type of information in the neural signals. In this paper, we show how our Hilbert space-based approach can be applied to the modeling of a RBF neural network. Accordingly, initially we present a review of mathematical preliminaries, focused on the definition of Fock space (section 2). In section 3 we present and discuss our Fock space-based approach for modeling neural signal spaces. In section 4 the theoretical aspects of artificial networks are introduced. Then in section 5 we develop and discuss a study case the quantum RBF Neural Network Approach. Finally, section 6 concludes the paper with the discussion of the results obtained.

2 Mathematical Preliminaries

In this section we give the definition of the Fock space, which is a new Hilbert space builded from a given Hilbert space.

Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be Hilbert spaces; for each $\varphi_1 \in \mathcal{H}_1$ and $\varphi_2 \in \mathcal{H}_2$, the product $\varphi_1 \otimes \varphi_2$ denotes the bilinear form that acts on $\mathcal{H}_1 \times \mathcal{H}_2$ by the following rule:

$$(\varphi_1 \otimes \varphi_2)(\psi_1, \psi_2) = \langle \psi_1, \varphi_1 \rangle_1 \langle \psi_2, \varphi_2 \rangle_2,$$

where $\langle \cdot, \cdot \rangle_i$ is the inner product in the Hilbert space $\mathcal{H}_i$.

Let $\mathcal{E}$ be the set of all finite linear combinations:

$$\mathcal{E} = \{\sum_{i=1}^{n} \alpha_i(\varphi_{i_1} \otimes \varphi_{i_2}) : \ \forall i = 1, ..., n \quad \varphi_{i_1} \in \mathcal{H}_1, \ \varphi_{i_2} \in \mathcal{H}_2, \ \alpha_i \in \mathbb{C}\}.$$

We define an inner product $\langle \cdot, \cdot \rangle$ over $\mathcal{E}$, as follows:

$$\langle \varphi \otimes \psi, \eta \otimes \mu \rangle = \langle \varphi, \eta \rangle_1 \langle \psi, \mu \rangle_2.$$

This definition extends to all $\mathcal{E}$ by the bilinearity property and it can be proved that $\langle \cdot, \cdot \rangle$ is positive definite and well defined.

Definition 2.1. *Let be the space $\mathcal{H}_1 \otimes \mathcal{H}_2$ as the completation of $\mathcal{E}$ under the inner product $\langle \cdot, \cdot \rangle$ defined above. $\mathcal{H}_1 \otimes \mathcal{H}_2$ will be called the tensor product of $\mathcal{H}_1$ and $\mathcal{H}_2$; $\mathcal{H}_1 \otimes \mathcal{H}_2$ is a Hilbert space.*

We have the following two propositions whose proofs can be found in [7].

Proposition 2.2. *If $\{\varphi_k\}$ and $\{\psi_l\}$ are orthonormal basis of $\mathcal{H}_1$ and $\mathcal{H}_2$ respectively, $\{\varphi_k \otimes \psi_l\}$ is an orthonormal basis of $\mathcal{H}_1 \otimes \mathcal{H}_2$.*

Proposition 2.3. *Let (M_1, μ_1) and (M_2, μ_2) be measure spaces, and let consider the spaces $L^2(M_1, d\mu_1)$ and $L^2(M_2, d\mu_2)$. Then, there exists a unique isomorphism from $L^2(M_1, d\mu_1) \otimes L^2(M_2, d\mu_2)$ to $L^2(M_1 \times M_2, d\mu_1 \otimes d\mu_2)$ such that, $f \otimes g \mapsto f \cdot g$.*

Definition 2.4. *If $\{\mathcal{H}_n\}_{n=1}^{\infty}$ is a sequence of Hilbert spaces, let us define the direct sum space as the set*

$$\mathcal{H} = \bigoplus_{n=1}^{\infty} \mathcal{H}_n = \left\{ \{x_n\}_{n=1}^{\infty} \ : \ x_n \in \mathcal{H}_n, \quad \sum_{n=1}^{\infty} \|x_n\|_{\mathcal{H}_n}^2 < \infty \right\}.$$

Again, the set $\mathcal{H}$ is a Hilbert space under the inner product $\langle \cdot, \cdot \rangle : \mathcal{H} \times \mathcal{H} \longrightarrow \mathbb{C}$ given by

$$\langle \{x_n\}_{n=1}^{\infty}, \{y_n\}_{n=1}^{\infty} \rangle_{\mathcal{H}} = \sum_{n=1}^{\infty} \langle x_n, y_n \rangle_{\mathcal{H}_n}.$$

Definition 2.5. *Let $\mathcal{H}$ be a Hilbert space, we denote by $\mathcal{H}^{\otimes^n}$ the tensorial product of n-th order, i.e, $\mathcal{H}^{\otimes^n} = \mathcal{H} \otimes \mathcal{H} \otimes \cdots \otimes \mathcal{H}$. Let $\mathcal{H}^0 = \mathbb{C}$, and we introduce*

$$\Gamma(\mathcal{H}) = \bigoplus_{n=0}^{\infty} \mathcal{H}^{\otimes^n}.$$

$\Gamma(\mathcal{H})$ is called the Fock space over $\mathcal{H}$; this space will be separable if $\mathcal{H}$ is separable.

If we consider $(\Omega, d\mu)$ as the measure space with Lebesgue measure restricted to $\Omega \subset \mathbb{R}^3$, then we can form the Hilbert space $\mathcal{H} = L^2(\Omega)$, in which case, an element $\psi \in \Gamma(L^2(\Omega))$ is a sequence of functions that is of the form

$$\psi = \{\psi_0, \psi_1, \psi_2, \cdots\}$$

where $\psi_0 \in \mathbb{C}$, and for all k, $\psi_k \in L^2(\Omega^k)$. From Definition 2.4 we have that the inner product and the norm in $\Gamma(\mathcal{H})$ are:

$$\langle \psi, \varphi \rangle_{\Gamma(\mathcal{H})} = \overline{\psi_0}\varphi_0 + \sum_{k=1}^{\infty} \langle \psi_k, \varphi_k \rangle_{L^2(\Omega^k)},$$

$$\|\psi\|^2_{\Gamma(\mathcal{H})} = |\psi_0|^2 + \sum_{k=1}^{\infty} \langle \psi_k, \psi_k \rangle_{L^2(\Omega)^{\otimes k}}$$

$$= |\psi_0|^2 + \sum_{k=1}^{\infty} \int_{\Omega^k} |\psi_k(x_1, x_2, \cdots, x_k)|^2 dx_1 \cdots dx_k < \infty.$$

To conclude this section we define for each $g \in \mathcal{H}$ the exponential vector $\psi(g)$, of argument g, whose components are given by:

$$\psi_0 = 1, \quad \psi_k(g) = \frac{1}{\sqrt{k!}} g \otimes \cdots \otimes g; \tag{1}$$

it can be shown [8] that the set of exponential vectors over the Hilbert space $\mathcal{H}$ is dense on $\Gamma(\mathcal{H})$.

3 Brain Signal Space from a Quantum Perspective

Magnetoencephalography (MEG) and electroencephalography (EEG) directly measure the magnetic field or electric potentials caused by neural activation. The major sources of both the EEG and MEG are widely accepted to be localized current sources in the cerebral cortex. It is now well established that signals in the brain are processed by cells known as neurons. Much of the related literature uses a set of equivalent current dipoles or random point fields to represent these localized sources [2]. It is known that measurements concerning the brain activity such as MEG or EEG and other techniques are based on quantum principles. For this reason it would seem obvious that a quantum approach should be more suitable than a classical (non quantum) approach for studying the interaction of neural signals. The human brain contains about 100 billion neurons. Therefore, a lot of signals could be described in terms of a classical probabilistic model according to the fact that the signals are represented by populations of excited neurons. However, there are many well-known facts that are in contradiction with the classical models. For instance, establishing the fact that changes in some region of the brain have immediate consequences concerning the other regions is in contradiction to classical models. Moreover, there is no special region where the memory is localized, namely, one cannot distinguish neurons supporting signals and neurons supporting the memory (neurons permanently change their purpose without any loss of information).

In our quantum approach, for describing interactions between neural signals, we will represent the activity in different parts of the brain using certain beam splitters well-known in quantum optics. Hereby, a collapse of a certain state can occur (like in the case of a quantum measurement). This collapse causes a rapid decrease of the density of the excited neurons, after a short period of increase of that density caused by the accumulation of some elementary quanta produced by the neural cells. Both events (*i.e.*, decrease and increase) could be measurable considering an intrinsic uncertainty as a result of this quantization. Denoting by Ω the volume occupied by the brain in three dimensional space $\mathbb{R}^3$, then we consider, from a quantum perspective, the space of particles inside Ω as the Hilbert space $L^2(\Omega, \mu)$, where μ is the usual Lebesgue measure.

Given that a neuron is excited if its post-synaptic potential exceeds a certain threshold, then we can state, as part of the quantum approach, the following postulate:

A neuron is excited if and only if a quantum particle is located inside the neuron.

Thus, we can consider the space of indistinguishable particles, which, in mathematics, is the symmetrization of the Fock space, and which we denote by $\Gamma(L^2(\Omega,\mu))$, or just by $\Gamma(L^2(\Omega))$, *i.e.*,

$$\mathcal{H} = \Gamma(L^2(\Omega)) = \{\eta : \eta \text{ is an excited neuron within } \Omega\}.$$

In addition, if we make the decomposition of a neural region into subregions (measurable subspaces), then we can consider our space of excited neurons as:

$$\mathcal{H} = \Gamma(L^2(\Omega)) = \bigotimes_{k=1}^{n} \Gamma(L^2(\Omega_k)).$$

This decomposition is consistent with both the fact that a neural region is subdivided into areas intended to perform different tasks and with the possibility that a signal can be decomposed in terms of other elemental signals. To describe the interaction between two signals will use the unitary operator Θ acting on the space:

$$\mathcal{H} \otimes \mathcal{H} = \bigotimes_{k,l=1}^{n} \{\Gamma(L^2(\Omega_k)) \otimes \Gamma(L^2(\Omega_l))\}$$

If we consider that the brain is characterized by a parallel and distributed processing [1], then the operator Θ can be expressed as:

$$\Theta = \bigotimes_{k,j=1}^{n} \Theta_{k,l}$$

where each $\Theta_{k,l}$ is an unitary operator acting on: $\Gamma(L^2(\Omega_k)) \otimes \Gamma(L^2(\Omega_l))$. A proposal for the explicit form of Θ acting on exponential vectors (note that the exponential vectors form a dense subset in $\Gamma(L^2(\Omega))$) is given by:

$$\Theta_r(\psi(\eta_1) \otimes \psi(\eta_2)) = \psi(r_1\eta_1 + r_2\eta_2) \otimes \psi(r_3\eta_1 + r_4\eta_2)$$

where $\eta_1, \eta_2 \in L^2(\Omega)$ and $r_i \in \mathbb{R}$, with $|r_1|^2 + |r_2|^2 = 1$ and $|r_3|^2 + |r_4|^2 = 1$.

To fulfill the condition (unitarity), $\Theta\Theta = I_{\mathcal{H}\otimes\mathcal{H}}$ taking into account points in the Bloch sphere [9], we can take:

$$r_j = \frac{1}{\sqrt{2}}, \quad j = 1, 2, 3; \quad r_4 = -\frac{1}{\sqrt{2}}$$

or

$$r_j = -\frac{1}{\sqrt{2}}, \quad j = 1, 2, 3; \quad r_4 = \frac{1}{\sqrt{2}}.$$

So if we choose the first possibility, the interaction of two neural signals η_1, η_2 would be modeled by the unitary operator Θ on $\mathcal{H} \otimes \mathcal{H}$ by:

$$\Theta(\psi(\eta_1) \otimes \psi(\eta_2)) = \psi(\frac{1}{\sqrt{2}}(\eta_1 + \eta_2)) \otimes \psi(\frac{1}{\sqrt{2}}(\eta_1 - \eta_2)). \tag{2}$$

4 Theoretical Aspects of Artificial Networks

Artificial neural networks (ANN) are mathematical models inspired by the structure and functioning of biological neural networks [6]. These models are conceived to emulate a number of functions performed by the brain, such as pattern recognition, memory, learning and generalization or abstraction of knowledge, among others.

4.1 The Artificial Neuron

Artificial neurons are the constitutive units in an artificial neural network. An artificial neuron can be seen as a processing unit which receives one or more inputs and combines them to produce an output.

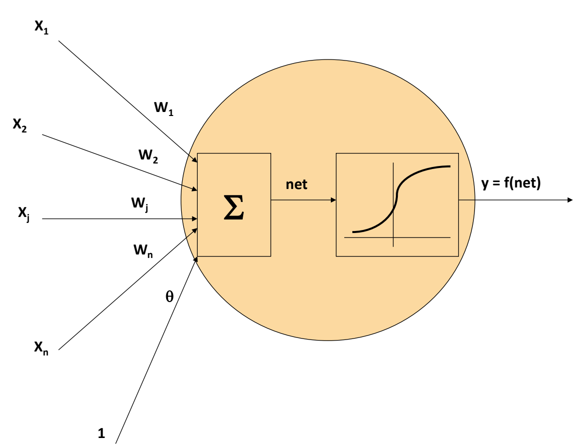

Figure 1: Artificial Neuron Model

Figure 1 shows one of the artificial neuron models widely reported in the literature. The meaning of the terms and symbols used in this figure are explained below.

$X_1, X_2, ..., X_n$, 1 are the $n+1$ input signals.

$W_1, W_2, ..., W_n$, Θ are the weights, representing the strength of the synaptic connection.

The function *net* determines how the network inputs $\{X_j : 1 \leq j \leq N\}$ are combined inside the neuron.

In almost all artificial neuron models all inputs X_j are weighted by the synaptic transmission efficiency $\{W_j : 1 \leq j \leq N\}$ and are summed. In this case, the neuron input consists of the value.

$$net = \sum_{j=1}^{N} W_j X_j + \Theta$$

The quantity Θ is called the *bias* and is used to model the threshold. The resulting value net meaning the degree in which the artificial neuron is activated as a result of inputs received. On the biological level, this is analogous to the values of the Resting Membrane Potential (RMP) as a result of the spatiotemporal summation of postsynaptic potentials.

The function $f(net)$ models the axon of a biological neuron and its value is propagated and received as an input by another neuron, through a synapse. This function is known as transfer function. There are a lot of transfer functions reported in literature [6].

A commonly used criterion in selecting a transfer function is to ensure both the best generalization and the fast learning of the neural model (*i.e.*, single artificial neuron as perceptron model or multilayer neural network). Among the transfer functions, commonly used in neural models are: Identity, Hard-limited threshold, Linear threshold, Sigmoid function, Hyperbolic tangent and Gaussian. As seen below, Gaussian function - expression (3) - is a radial basis function commonly used in Radial Basis Function (RBF) Neural Networks; as will be seen in the next section.

$$f(net) = \frac{1}{\sqrt{2\pi\sigma}} exp(-\frac{1}{2}(\frac{net-\mu}{\sigma})^2). \tag{3}$$

4.2 Radial Basis Function (RBF) Networks

Radial Basis Function (RBF) neural networks [10, 11] are multi-layer feed forward networks, composed of an input layer; one hidden layer and an output layer (see Figure 2). Neurons in the hidden layer perform local processing, using Radial Basis Function (RBF) as activation functions.

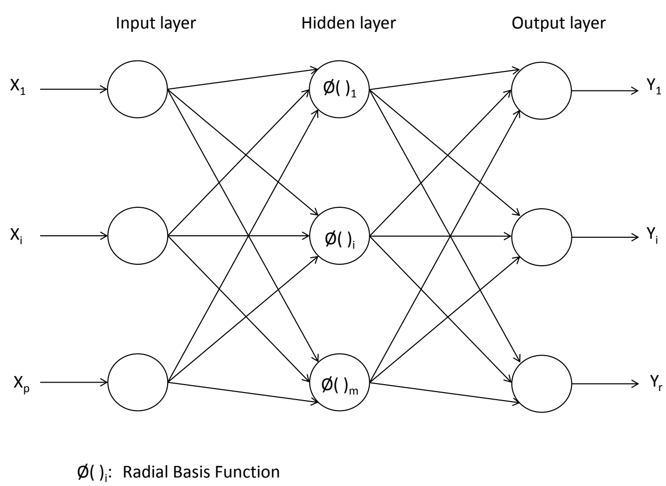

Figure 2: Structure of a Typical RBF Neural Network

RBF Neural Networks define hiperelipses or hyperspheres that divide the input space. This kind of neural network has found its main applications in areas such as time series analysis, image processing and medical diagnosis.

A radial basis function is a real-valued function whose value depends only on the distance to the origin or to some other point C, named center, as shown in the equations (4)

$$\phi(X) = ||X|| \quad \text{either} \quad \phi(X, C) = ||X - C|| \tag{4}$$

where $||X - C||$ is the Euclidean distance between points X, C.

Gaussian, Inverse quadratic and Inverse multiquadratic functions are examples of radial basis functions commonly used in RBF neural networks. As already mentioned, in a RBF neural network, radial basis functions determine the activation of the hidden layer neurons based on the received input vector expression (5)

$$\phi_i(n) = \phi\Big(\frac{||X(n) - C_i||}{d_i}\Big), \quad \text{for every } \ i = 1, 2, \ldots, m. \tag{5}$$

where, ϕ is a radial basis function, $C_i = (C_{i1}, C_{i2}, \dots, C_{ip})$ are the vectors or points representing the centers of the radial basis functions, d_i are real numbers, diviations of the functions and $||\cdot||$ is the Euclidean distance from the input vector to the center of the function, in our case:

$$||X(n) - C_i|| = \sqrt{\sum_{j=1}^{p}[x_j(n) - c_{ij}]^2}.$$

Finally, using Gaussian radial basis function we obtain the following expression:

$$\phi(n) = exp\big(-\frac{||X(n) - C_i||^2}{2d_i^2}\big).$$

4.3 Toward a Hilbert Spaces-based RBK Neural Network

Taking into account the mathematical definitions and principles introduced in sections 2 and 3, as well as the traditional model of RBF artificial neural network presented above, in the next section we propose a novel model of a RBF artificial neuron based on the mathematical tools provided by Fock spaces. The main idea behind the proposed model is to explore and understand new theoretical and operational aspects of this neural model from a quantum perspective.

5 The Quantum RBF Neural Network Approach

In this section we begin by modeling a RBF artificial neuron belonging to the hidden layer of RBF neural network. We choose the Gaussian function as the activation function of this neuron.

Applying the definition of exponential vector given in (1) the exponential vectors, which are a dense in Fock space $\Gamma(L^2(\mathbb{R}))$, for two Gaussian functions given in (6), are the vectors ψ in the Fock space $\Gamma(L^2(\mathbb{R}))$ given by (7).

$$\phi_1(x_1) = exp(-\frac{x_1^2}{2}) \quad \phi_2(x_2) = exp(-\frac{x_2^2}{2}) \tag{6}$$

where

$$\psi_0(\phi_i) = 1, \;\; \psi_k(\phi_i) = \frac{1}{\sqrt{k!}}\phi_i \otimes \cdots \otimes \phi_i, \quad i = 1, 2; \quad \forall k \geq 1, \tag{7}$$

Moreover, due to isomorphism of Definition 2.4, the exponential vectors for Gaussian functions are:

$$\psi_0(\phi_i) = 1, \;\; \psi_k(\phi_i) = \frac{1}{\sqrt{k!}}\prod_{l=1}^{k}\phi_{i,l}, \quad i = 1, 2; \quad \forall k \geq 1,$$

Using algebraic properties of Gaussian functions we obtain:

$$\psi_k(\phi_1) = \frac{1}{\sqrt{k!}}exp(-\frac{kx_1^2}{2}), \;\; \psi_k(\phi_2) = \frac{1}{\sqrt{k!}}exp(-\frac{kx_2^2}{2}); \tag{8}$$

As can be seen in the above expressions, each of the components of an exponential vector, whose argument is a Gaussian RBF, is also a Gaussian function.

Based on the previous statement, the interaction of two signals given by the operator Θ is, as defined in (2):

$$\Theta(\psi(\phi_1) \otimes \psi(\phi_2)) = \psi(\frac{1}{\sqrt{2}}(\phi_1 + \phi_2)) \otimes \psi(\frac{1}{\sqrt{2}}(\phi_1 - \phi_2)),$$

According to (8), the k-th component of this new exponential vector is:

$$\Theta_k(\psi(\phi_1) \otimes \psi(\phi_2)) = \psi_k(\frac{1}{\sqrt{2}}(\phi_1 + \phi_2)) \otimes \psi_k(\frac{1}{\sqrt{2}}(\phi_1 - \phi_2)).$$

Using again both Definition 2.4 and the algebraic properties of Gaussian functions, we have:

$$\psi_k(\frac{1}{\sqrt{2}}(\phi_1 \pm \phi_2)) = \frac{1}{\sqrt{2k!}}(\phi_1(x_1) \pm \phi_2(x_2))^k \tag{9}$$

Hence (9) can be expressed as:

$$\begin{aligned} &\Theta_k(\psi(\phi_1(x_1)) \otimes \psi(\phi_2(x_2))) \\ &= \psi_k(\frac{1}{\sqrt{2k!}}(\phi_1(x_1) + \phi_2(x_2))^k) \otimes \psi_k(\frac{1}{\sqrt{2k!}}(\phi_1(x_1) - \phi_2(x_2))^k). \end{aligned} \tag{10}$$

Taking into account both Newtons binomial expansion and the properties of exponential functions, we obtain that the components of these new exponential vectors are also radial basis functions (RBF). Hence, the obtained quantized version describing the expression for the activation of a hidden neuron in a RBF neural model is consistent with the expression of activation described in the traditional RBF neural network, as discussed in section 4.2.

6 Conclusion

Artificial neural networks, as mathematical models inspired by the structure and functioning of the biological neural networks, have proven to be a robust and flexible scenario for the study and exploration of aspects of brain dynamics. In this paper we have presented the quantum version of an artificial neural network model whose activation functions are the radial basis functions (RBF). Considering as an example of RBF the Gaussian function, which belongs to the Hilbert space $L^2(\Omega)$, we built the bosonic Fock space based on the Hilbert space. The bosonic Fock space is a useful tool for modeling the dynamics of multiparticle clusters with spatial symmetry in quantum mechanics. Thus, we have proposed a theoretical neural network model in which the interaction between neurons is given by tensor mechanisms of the Fock spaces, obtaining a quantum model of an artificial neural network. The main advantage of the proposed model is that the patterns of interaction between neurons, given by equation (10), are virtually endless, and this is a very important feature in exploring the dynamics of brain neural networks. In the future, we plan to consider other artificial neural network models to demonstrate the effectiveness of the Fock spaces-based approach for modeling the dynamics of such systems.

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