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Real options model in green energy investment

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Abstract

We proposed the study on the timing of investment and option value for green energy company. Considering various paremeters that can affect this investment, such as the discount rate, the is investment cost, the mean reversion speed, the mean reversion level, the uncertainty parameter, etc; the simulation study was conducted and the optimal investment time for the company was found and it is better to invest just at the time where the price of fuel becomes low even when considering the subsidies and lower taxes that can be made dependind on regime switching.

Keywords: Investment; Real options; Optimal stopping; Option value; Ambiguity

1 Introduction

The financial assessment of any investment is done by asking yourself whether and when the later should be implemented. The real options model

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can be used to estimate the investment value for green energy investment that can be solved by three methods such as partial differential equation approach, dynamic programming approach and simulation method [5]. In terms of the uncertain factors, we consider standard fuel price which evolves stochastically over time and the biological fuel price that can be used by an asset which produces the green energy. The biological fuel is driven by subsidies and lower taxes and is cheaper than the price of standard fuel.

Since the value of energy project depends on many uncertain factors in its lifetime, it is good to express the project value by its expectation $\mathbb{E}[.]$. If the investment on an energy project starts in year t, the total net present value of the project will be represented as follows

$$V_t = \mathbb{E}\left[\sum_{i=t}^{t+L} e^{-r(i-t)}\Pi_i - I_t\right]$$

where r is the discount rate, I_t is investment cost and Π_i denotes the cash flow in year i which comprises the returns from selling energy, operation and maintenance cost, and tax expenditure [6],[7].

The cash flow Π_t in year t can be written as follows

$$\Pi_t = p_t^e q_t^e - OMCq_t^e - tc_t$$

where p^e is the price of green energy, q^e is the quantity of energy sold, OMC stands for the operation and maintenance cost per unit of energy output, and tc denotes the tax expenditure [9].

It is known that the generating capacity of a green energy power generation system gradually decreases with the natural aging of equipment and other factors. The reason why we can get

$$q_t^e = q_{t-1}^e(1 - dr)$$

where dr is the annual declining rate. The investment environment of green energy is closer to a stochastic scenario that is characterized by uncertainty and managerial flexibility where the Net Present Value (NPV) method may lead to the suboptimal investment decisions. The real options method here can create the optimal investment strategies [4],[8].

Stochastic process is an appropriate technique for describing the uncertain factors. To characterize the uncertainties in market and technological development we can use the Geometric Brownian Motion (GBM) as follows

$$dS_t = \alpha S_t dt + \sigma S_t dZ$$

where S_t is an uncertain variable, α and σ are drift and volatility parameters and dZ is an independent increment of Wiener process [3].
The expected value of S is $\mathbb{E}[S_t] = S_0 e^{\alpha t}$
and the sum of the discounted expected future cash-flows if the investment is made at time t is given by

$$\mathbb{E}_t \left[\int_t^{\infty} e^{-r(u-t)} S_u du \right] = \frac{S_t}{r - \alpha}$$

where r is the constant risk-free rate and $\alpha < r$.

The remainder of the paper is structured as follows: in Sections 2, we present the price model; in Sections 3, we present the optimal stopping under ambiguity in either discrete time and continuous time, then in section 4, we present the option value. Section 5 deals with the simulation approach used to estimate the investment timing and option value. We offer concluding remarks in Section 6.

2 Model

We consider a risk-neutral company that discount with a risk-free interest rate $r > 0$. The company own an asset that at time $t_0 = 0$ have the finite life-time of $\tau > 0$ and that consume x units of fuel per time unit. The price $p(t)$ of fuel evolves stochastically over time and follows the Ornstein-Uhlenbeck process

$$dp(t) = k\,(m - p(t))\,dt + \sigma dW_t, \quad p(0) = p_0 \geq 0 \tag{1}$$

where $k > 0$ is the mean reversion speed which is the assumption that a price of stock will tend to move to the average price over time, m is the mean reversion level, $\sigma > 0$ is the uncertainty parameter and dW_t is the increment of Weiner process with a normal distribution [3].
During the life-time of the asset the company has at any time the opportunity

(option) to adjust its asset in a way that it can also tolerate biologic fuel. Mainly driven by subsidies and lower taxes the price $p_B(t)$ of biologic fuel is cheaper than the price of standard fuel and depends on the time for which the option is bought. In particular, we assume that

$$p_B(t) = (1-\xi)\, p(t)$$

where $0 \leq \xi < 1$.
From (1) we can derive

$$p(t) = e^{-kt} p_0 + \left(1 - e^{-kt}\right) m + \sigma \int_0^t e^{ks} dW(s)$$

and

$$\mathbb{E}\left(p(t_2) | p(t_1) = p_{t_1}\right) = e^{-k(t_2-t_1)} p_{t_1} + \left(1 - e^{-k(t_2-t_1)}\right) m + \frac{\sigma^2 \left(1 - e^{-k(t_2-t_1)}\right)}{k}$$

for every $t_2 \geq t_1 \geq 0$.
If the company invests at a time $t > 0$, therefore, it expects discounted savings of

$$V\left(p(t), t\right) = \mathbb{E} \int_t^\tau x \left(p(s) - p_B(s)\right) e^{-r(s-t)} ds = x\xi \int_t^\tau \mathbb{E}\left(p(s) | p(t)\right) e^{-r(s-t)} ds$$

Thus, investing at time t generates an expected profit of

$$\pi\left(p(t), t\right) = V\left(p(t), t\right) - I(t)$$

where $I(t)$ is the investment costs.
The cashflow process Y evolves as follows:

$$dY(t) = Y(t) \left(\alpha dt + \zeta \left(\rho dW(t) + \sqrt{1-\rho^2} dW^0(t)\right)\right) \tag{2}$$

where $W^0(t)$ is a Wiener process, $\rho^2 < 1$ is the correlation coefficient between market uncertainty and the cashflow process uncertainty, and α, ζ are all constants.
Following [7], the possibility to invest can be regarded as a real option. Hence, the company should not invest immediately but wait with the investment until the price of standard fuel reaches the time-depending optimal threshold $p^*(t)$.

3 Optimal Stopping under Ambiguity

We can consider the optimal stopping time problem in both discrete and continuous time models with ambiguity.

Discrete Time

The state space and filtration are given by $(\Omega, \{\mathcal{F}_t\}\, t \in \mathbb{N})$ and we assume that $\mathcal{F}_0$ is trivial in that it contains only events for probability 0 or 1. Consider $\mathcal{F}$ to be the σ-field generated by the union of all $\mathcal{F}_t$, $t \in \mathbb{N}$ and P is a subjective probability measure for an agent which is used as a reference probability measure. Denote $\mathcal{P}$ as a set of probability measures on $(\Omega, \{\mathcal{F}_t\}\, t \in \mathbb{N})$. The decision maker has option to make decision at any stopping time τ with values in $\mathbb{N} \cup \{\infty\}$ in the market. The realized payoff $\{X_t\}\, t \in \mathbb{N}$ on $(\Omega, \{\mathcal{F}_t\}\, t \in \mathbb{N}, P)$ is an adapted process [1],[2].
X is $\mathcal{P}$-uniformly integrable if it satisfies $\lim\limits_{K \to \infty} \sup\limits_{Q \in \mathcal{P}} \mathbb{E}_Q[|X| 1_{\{}|X| > K\}] = 0$ and we set $X_\tau(\omega) = 0$ if $\tau(\omega) = \infty$.
We can have here 2 possibilities, finite and infinite time horizon respectively.

Finite Time Horizon

For an ambiguity-averse decision maker, the problem of optimal stopping is

$$\max \inf_{Q \in \mathcal{P}} \mathbb{E}_Q[X_\tau]$$

for any stopping time $\tau \leq T < \infty$.
For each $Q \in \mathcal{P}$, set
$V_T^Q = X_T$ in the terminal time and $V_t^Q = \max\{X_t, \mathbb{E}_Q[V_{t+1}|\mathcal{F}_t]\}$,$t < T$.
The process $\{V_t^Q\}t \in \mathbb{N}$ is the smallest Q-supermartingle that dominates X, i.e., V^Q is the Snell envelope of X under Q and $\tau_* = inf\{t \geq 0 : V_t^Q = X_t\}$ is an optimal stopping time in the classical setting when Q is the only one probability measure under consideration [1].
Define the multiple prior Snell envelope of X with respect to $\mathcal{P}$ recursively by

$$V_t = \max\{X_t, ess \inf_{Q \in \mathcal{P}} \mathbb{E}_Q[V_{t+1}|\mathcal{F}_t]\}, t = 0, 1, \cdots, T-1$$
$$V_T = X_T$$

Then we have the following results:

- V is the smallest multiple prior supermartingle with respect to $\mathcal{P}$ that dominates X
- V is the value process of the optimal stopping problem under ambiguity, that is,
  $V_t = ess\sup_{\tau \geq t} \inf_{Q \in \mathcal{P}} \mathbb{E}_Q[X_\tau | \mathcal{F}_t]$
- $\tau_* = inf\{t \geq 0 : V_t = X_t\}$ is an optimal stopping time.

Infinite Time Horizon

For the infinite time horizon, let us define the value function

$$V_t^\infty = ess\sup_{\tau \geq t} \inf_{Q \in \mathcal{P}} \mathbb{E}_Q[X_\tau | \mathcal{F}_t]$$

Then we have the following results:

- V^∞ is the smallest multiple prior supermartingle with respect to $\mathcal{P}$ that dominates X, and V^∞ is bounded by a $\mathcal{P}$-uniformly integrable random variable
- The value process V^∞ satisfies the Bellman principle (each subpolicy of an optimum policy must itself be an optimum policy with regard to the initial and the terminal states of the subpolicy. Here, the terms policy,subpolicy, optimum policy and state are primitive and are not given specific meanings. In each application, these terms are understood in certain ways comparable to the situation of interest), for all $t \geq 0$,
  $V_t^\infty = \max\{X_t, ess \inf_{Q \in \mathcal{P}} \mathbb{E}_Q[V_{t+1}^\infty | \mathcal{F}_t]\}$
- $\tau_* = inf\{t \geq 0 : V_t^\infty = X_t\}$ is an optimal stopping time if it is finite.

Continuous Time

Let us consider the continuous time framework for a finite time horizon with terminal time T. Here the agent has option to make decision at any time $t \in [0,T]$ in the market. The realized payoff at time t is X_t and the payoff process is $\{X_t\}_{0 \le t \le T}$ on $(\Omega, \{\mathcal{F}_T\}, P)$.
The classical stopping time problem is to characterize the following value process
$V_t^a := ess \sup_{\tau \in \varphi_t} \mathbb{E}_P[X_\tau | \mathcal{F}_t]$, for $0 \le t \le T$,
where φ the class of $\{\mathcal{F}_t\}$-stopping times with values in $[0,T]$ [10],[11].
For a stopping time $\nu \in \varphi$, we set $\varphi_\nu := \{\tau \in \varphi; \tau \ge \nu \text{a.s}\}$.
The optimal stopping problem under consideration is to characterize the following value process and the corresponding optimal stopping time simultaneously

$$V_\nu := ess \sup_{\tau \in \varphi_\nu} \varepsilon_g[X_\tau | \mathcal{F}_\nu], \tag{3}$$

for all $\nu \in \varphi$
where $\varepsilon_g[X_\tau | \mathcal{F}_\nu] := y_\nu$ is the conditional g-evaluation of X_τ related to (g, ν, τ); where y_ν is the solution of
$y_t = X_\tau - \int_{t\wedge\tau}^{\tau} g\left(c_s, y_s, z_s, \omega, s\right) ds - \int_{t\wedge\tau}^{\tau} z_s dW_s$
and g is an aggregator or an ambiguity level. The value function (3) is equivalent to a solution of the reflected Backward Stochastic Differential Equations(BSDEs)[1]

$$V_t = X_t - \int_t^T g\left(V_s, z_s, s\right) ds + A_T - A_t - \int_t^T z_s dW_s$$

and together with

1. $V, A \in \varphi^2$ and $z \in \mathcal{L}^2\left(0, T; \mathbb{R}^d\right)$. A is non-decreasing and $A_0 = 0$
2. $V_t \ge X_t$, for $0 \le t \le T$ and if $A = A^c + A^d$ where A^c (respecivement A^d) is the continuous (respectivement purely discontinuous) part of A with $\int_0^T \left(V_t - X_t\right) dA_t^c = 0$
3. $V_t - V_{t-} = -\left(X_{t-} - V_t\right)^+$

Hence, the optimal investment time for the company is determined by

$$t = inf\{t : Y_y(t) \ge y_1\}$$

where $Y_y(t)$ is given by (2), y_1 is the smooth matching point representing the optimal threshold.

4 Option value

The value $F(p(t), t)$ of the option to invest is the solution of the differential equation

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 F(p,t)}{\partial^2 p} + k(m-p)\frac{\partial F(p,t)}{\partial p} + \frac{\partial F(p,t)}{\partial t} = rF(p,t)$$

and also meets the following conditions:

1. Zero is an absorbing barrier of the price process, i.e
   $\lim_{p(t)\to 0} F(p(t), t) = 0,\ \forall t \geq 0$

2. The investment opportunity has no value if the asset is no longer in use,
   $F(p(t), t) = 0,\ \forall t \geq \tau$

3. Continuity-condition, i.e
   $F(p^*(t), t) = \Pi^*(p(t), t)$

4. Smooth-pasting condition, i.e
   $\dfrac{\partial F(p^*(t), t)}{\partial p} = \dfrac{\partial \Pi^*(p(t), t)}{\partial p}$

The following results give us the option value, once exercised and still not exercised.

Proposition 1. *Suppose a decision maker is considering a real option to invest in a project at sunk cost I with a payofffunction $g = V_t - F_t$ at maurity T. Suppose the project demand is shocked multiplicatively by an ambiguous volatility model driven by a Choquet-Brownian motion with parameter $c \in]0,1[$ given by:*

$$dZ_t = (2c-1)dt + 4c(1-c)dB_t$$

The constant conditional capacity c plays a key role and represents the decision maker's attitude towards policy ambiguity due to legislative regime switching.

Then, the project value V_t at time t , once exercised , is given by the sum of the current profit and the discounted continuation value, that is;

$$V_t(S_t, Z_t) = S_t D(T-t) + e^{-r(T-t)}\mathbb{E}_\mu \left[g(T, S_T, Z_T)|F_t^S\right] \tag{4}$$

where the values of S_t and Z_t are assumed to be independent of D and $\mathbb{E}_\mu$ denotes the Choquet expectation.

Proof. The project value V_t at time t , with expiration time T , is equal, once exercised, to the expected value $\mathbb{E}^P$ of the discounted cash flows with respect to the probability measure P conditional on the filtration F_t, such that:

$$V_t(S_t) = \mathbb{E}^P\left[\int_t^T e^{-r(u-t)} S_u du | F_t\right] \tag{5}$$

Suppose initially that the value of the project if it were to be sold, V_t, is stochastic but that the value of the project in place, F, is constant. The firm receives $V_t - F$ when it invests. The investment timing problem consists of finding a number a_t^*, for every time t, such that if $\dfrac{V_t}{F} \geq a_t^*$, the investment is undertaken, and otherwise not undertaken. This investment decision schedule $\{a_t^*\}$ is chosen so as to maximize the time zero expected present value of the payoff $V_t - F$. For an arbitrary boundary $\left\{a_t^{'}\right\}_0^T$, the value of the investment opportunity is the expected present value of the payo given by

$$X(T) = \mathbb{E}_0\left\{e^{-\mu t'}\left[V_{t'} - F\right]\right\} \tag{6}$$

where t' is the date at which $\dfrac{V}{F}$ first reaches the boundary $a_t^{'}$ and $X(T)$ is the time zero value of an investment opportunity that expires at T. The expectation is taken over the first passage times t' and μ, is the appropriate given discount rate.

In the special case where the investment opportunity is infinitely lived, it is possible to solve explicitly for the maximized value of (6). When $T = \infty$, we have $a^{'}$ for all t. Maximizing (6) reduces to the following problem,

$$\max_{a'} F(a'-1)\mathbb{E}_0\left\{e^{-\mu t'}\right\}$$

Now consider the same problem, except that F_t is also random. The problem involves choosing a boundary B to maximize

$$\mathbb{E}_0\left\{(V_t - F_t)e^{-\mu t}\right\}$$

subject to

$$\begin{cases} dV(t) = V(t)(\alpha_v dt + \sigma_v dZ_v) \\ dF(t) = F(t)(\alpha_f dt + \sigma_f dZ_f) \end{cases}$$

where Z is a standard Wiener process.

By letting $V' = kV$ and $F' = kF$, where k is an arbitrary positive number, we can consider the problem involves choosing a boundary B' to maximize

$$\mathbb{E}_0 \left\{ ({V'}_t - {F'}_t) e^{-\mu t} \right\}$$

subject to

$$\begin{cases} dV'(t) = V'(t) \left\{ \alpha_v dt + \sigma_v dZ_v \right\} \\ dF'(t) = F'(t) \left\{ \alpha_f dt + \sigma_f dZ_f \right\} \end{cases}$$

Those two problems are identical, so the boundaries B and B' must be the same and hence independent of k. Thus, the correct rule is to invest when the ratio $\dfrac{V}{F}$ reaches or exceeds a fixed boundary and wait otherwise and the expected present value of the payoff is

$$\mathbb{E}_0 \left\{ F_{t'} (a' - 1) e^{-\mu t'} \right\} = (a' - 1) \mathbb{E}_0 \left\{ F_{t'} e^{-\mu t'} \right\}$$

where the expectation is taken over the joint density of F_t and the first passage times for $\dfrac{V_t}{F_{t'}}$.

Let $L = \mathbb{E}_0 \left\{ F_{t'} e^{-\mu t'} \right\}$, using that L is dierentiate with respect to v and f satisfies the partial differential equation

$$\mu L = \frac{1}{2} \left\{ L_{vv} V^2 \sigma_v^2 + L_{ff} F^2 \sigma_f^2 \right\} + 2 L_{vf} V F \sigma_{vf} + L_v \sigma_v V + L_f \sigma_f F \tag{7}$$

Its solution must satisfy certain boundary conditions:

(i) $L = F$ when $a = \dfrac{V}{F} = a'$ and (ii) $L \longrightarrow 0$ as $\dfrac{V}{F} \longrightarrow 0$.

Guess that the form of L is

$$L = k F^s a^t$$

and using the boundary condition (i), we must have $k = a^{-t}$ and $s = 1$.

With these constraints (7) can be written as

$$\mu = \frac{1}{2} t (t-1) \sigma^2 + t \sigma_v V + (1-t) \sigma_f$$

where $\sigma^2 = \sigma_v^2 + \sigma_f^2 - 2\sigma_{vf}$ This equation will have both a positive and a negative root. Boundary condition (ii) requires that $t > 0$, so the positive solution is the correct one.

Then the value of the opportunity is

$$X = (a' - 1)\mathbb{E}_0 \left\{ F_{t'} e^{-\mu t'} \right\} = (a' - 1) F_0 \left(\frac{V_0}{a'} \right)^{\epsilon}$$

where

$$\begin{cases} \epsilon = \sqrt{\left(\frac{\sigma_v - \sigma_f}{\sigma^2} - \frac{1}{2} \right)^2 + 2\frac{\mu - \alpha_f}{\sigma^2}} + \left(\frac{1}{2} - \frac{\sigma_v - \sigma_f}{\sigma^2} \right) \\ a'(t) = \dfrac{\epsilon}{\epsilon - 1} \\ \sigma^2 = \sigma_v^2 + \sigma_f^2 - 2\rho_{vf}\sigma_v\sigma_f \end{cases}$$

and ρ_{vf} is the instantaneous correlation between the rates of increase of V and F.

Let us consider that a project's profit flow given by

$$dS_t = (\mu + m\sigma)S_t + s\sigma S_t dB_t \tag{8}$$

whith dB_t a Wiener process with mean $m = 2c - 1$ and variance $S^2 = 4c(1 - c)$. Solving (8) and rewrite (5), we will have

$$S_h = S_t exp \left(\int_t^h \left(\mu + m\sigma - \frac{1}{2} s^2 \sigma^2 \right) du + \int_t^h s\sigma dBu \right)$$

$$\mathbb{E}^P \left[\int_t^{t+\tau} e^{-r(h-t)} S_h dh | F_t \right] = \int_t^{t+\tau} \mathbb{E}^P \left[e^{-r(h-t)} S_h | F_t \right] dh$$

$$\begin{aligned} &= S_t \textstyle\int_t^{t+\tau} e^{-r(h-t)} exp \left(\int_t^h \left(\mu + m\sigma - \frac{1}{2}s^2\sigma^2 \right) du \mathbb{E}^P \left[exp \left(\int_t^h s\sigma dBu \right) | F_t \right] dh \right. \\ &= S_t \textstyle\int_t^{t+\tau} e^{-r(h-t)} e^{\left(\mu + m\sigma - \frac{1}{2}s^2\sigma^2 \right)(h-t)} \mathbb{E}^P exp \left(s\sigma \left(B_h - B_t \right) | F_t \right) dh \\ &= S_t \textstyle\int_t^{t+\tau} e^{\left(-r + \mu + m\sigma - \frac{1}{2}s^2\sigma^2 \right)(h-t)} exp \left(\dfrac{1}{2} s^2 \sigma^2 \left(h - t \right) \right) dh \\ &= S_t \textstyle\int_t^{t+\tau} e^{(-r+\mu+m\sigma)(h-t)} dh \\ &= S_t \dfrac{1 - e^{-(r-(\mu+m\sigma))\tau}}{r - (\mu + m\sigma)} \end{aligned}$$

The decision maker has to determine the optimal moment t', $t' \in [t, T]$ to exercise the option to invest. This F_t optimal stopping time is the one which maximizes the value of the project, over the whole period considered (principle of optimality), taking into account the discounted cost of investing, at discount rate r. The stopping time is a random variable that described the exercise date of the option. □

Proposition 2. *The Option value,* V_t *at time* t*, while still not exercised, is given by,*

$$V_t = \max\left\{S_t - I, e^{-rdt}\mathbb{E}^P\left[dV_t|F_t\right]\right\}$$

This means that the decison maker wishes to maximize expected project value at time t *by choosing an optimal stopping time* τ*.*

Proof. The option value function is derived by splitting the decision between the immediate investment and waiting for a short time interval $t + dt$,

$$V_t = \max_{t' \geq t} \mathbb{E}^P\left[\int_{t'}^{t'+\tau} e^{-r(s-t)}S_h dh - e^{-r(t'-t)}I|F_t\right]$$

$$= \max\left\{\mathbb{E}^P\left[\int_{t}^{t+\tau} e^{-r(s-t)}S_h dh|F_t\right] - I, \max_{t' \geq t+dt} \mathbb{E}^P\left[\int_{t'}^{t'+\tau} e^{-r(s-t)}S_h dh - e^{-r(t'-t)}I|F_t\right]\right\}$$

Using the value of S_h found in Proposition 1 and applying the tower property of conditional expectation, we have

$$V_t = \max\left\{S_t - I, e^{-rdt}\max_{t' \geq t+dt} \mathbb{E}^P\left[\mathbb{E}^P\left[\int_{t'}^{t'+\tau} e^{-r(s-t-dt)}S_h dh - e^{-r(t'-t-dt)}I|F_{t+dt}\right]|F_t\right]\right\}$$

Therefore

$$V_t = \max\left\{S_t - I, e^{-rdt}\mathbb{E}^P\left[dV_t|F_t\right]\right\}$$

□

If we account for the option to invest we find that increased risk aversion erodes option value and increases the required investment threshold, increased risk aversion facilitates investment by reducing the amount of installed capacity. The higher risk aversion, the incentive to avoid exposure to unfavourable market conditions by decreasing the amount of installed capacity is more profound than the incentive to delay investment due to the decrease of the project′s expected utility.

To determine the optimal investment threshold and capacity of the project,

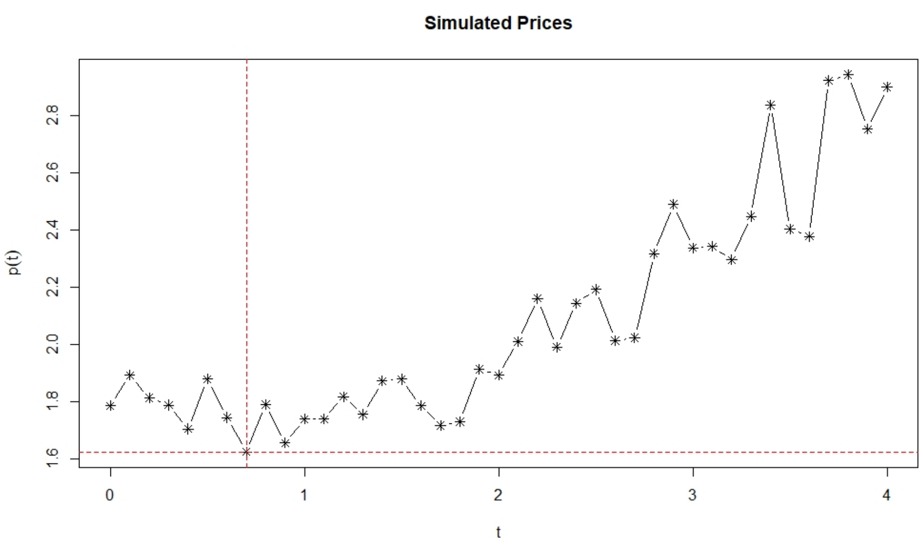

Figure 1: Plot of the similated prices

we assume that investment occurs immediately, which implies knowledge of the output price at investment and enables the calculation of the corresponding optimal capacity by maximising the value of the now-or-never investment opportunity.

5 Simulation Study

Using the Smooth-pasting condition, $F(p^*(t), t) = \Pi^*(p(t), t)$, we can find the optimal investment time t^* for the company and the option value to invest $F(t^*)$.

The simulation was done using $\pi\left(p(t),t\right) = V\left(p(t),t\right) - I(t)$ where $I(t) = I_f + I_v e^{-\eta t}$ is the investment costs; and also

$$\begin{aligned} V\left(p(t),t\right) &= x\xi \int_t^\tau \left\{ e^{-k(s-t)} p_t + (1-e^{-k(s-t)})m + \frac{\sigma^2(1-e^{-k(s-t)})}{k} \right\} ds \\ &= x\xi \Bigg\{ p(t)e^{-kt}\left(-\frac{e^{-k\tau}}{k} + e^{-kt}\right) + m(\tau - t) + m\frac{e^{-kt}}{k}\left(e^{-k\tau} - e^{-kt}\right) \\ &+ \frac{\sigma^2}{k}(\tau - t) + \frac{\sigma^2}{k}\frac{e^{-kt}}{k}\left(e^{-k\tau} - e^{-kt}\right)\Bigg\} \end{aligned}$$

The following parameters were used: $k = 0.5$; $p_0 = 1.5$; $m = ln(1.5)$; $\sigma = 0.2$; $k = 0.5$; $\tau = 10$; $\eta = 0.5$; $\xi = 0.3$; $I_f = 175$; $I_v = 100$ and $x = 100$.

Using those parameters, we have got that the optimal investment time for the company which is determined by

$$t = inf\{t : Y_y(t) \geq y_1\}$$

is obtained after 0.7 year as it can be seen on Figure 1 and at that time the coresponding option value is 66.73.

6 Conclusion

The investment in green energy requires complex decisions before making it. We proposed that the price $p(t)$ of fuel evolves stochastically over time and follows the Ornstein-Uhlenbeck process and from this we get the optimal time for investment together with the option value for the company.

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