

The Nonlinear Saving Growth Model

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Abstract

A financial crisis may be described by large decreases in the prices of stocks, real estate, or other assets. Namely, in an open economy, government budget deficit, as a negative public saving, raises real interest rates, crowds out domestic investment, decreases net capital outflow, decreases the level of asset prices, causes the domestic currency to appreciate. A decrease in aggregate demand causes output and prices to fall. The recession may further increase budget deficit. The basic aim of this paper is to provide a relatively simple chaotic saving growth model that is capable of generating stable equilibria, cycles, or chaos. A key hypothesis of this work is based on the idea that the coefficient $\pi = \beta / (g + \beta + n - 1)$ plays a crucial role in explaining local growth stability of the saving, where, n - net capital outflow as a percent of the real gross domestic product; g - government consumption as a percent of the real gross domestic product; β - the accelerator.

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1 Introduction

The global financial crisis (GFC) or global economic crisis developed with remarkable speed starting in the late summer of 2008 with the credit crunch, when a loss of confidence by US investors in the value of sub-prime mortgages caused a liquidity crisis. This crisis has damaged a large part of the world's financial system. A typical financial crisis is described by declines in asset prices and the failures of financial system. A series of effects then leads to a fall in output, which reinforces the causes of the crisis. The mechanism of the financial crises is presented in Fig.1. Asset prices depend not only on expectations of earnings, but also on interest rates, which are used to determine the present value of asset income. Government budget deficit raises real interest rates. When

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the government spends more money than it spends, then the government runs a budget deficit, and public saving becomes a negative number.

Economic systems are inherently acyclical. In this context, it is important to analyze the role of the saving rates in amplifying the effects of financial system on the real economy. In this view, relatively decreasing saving and / or relatively increasing budget deficits in the advanced countries have important impact on economic stability. When savings is eroded, banks become more reluctant to lend leading to sharper economic downturns.

Chaos theory attempts to reveal structure in aperiodic, unpredictable dynamic systems. The type of linear analysis used in the theory of economic growth presumes an orderly periodicity that rarely occurs in economy. In this sense, it is important to construct deterministic, nonlinear economic dynamic models that elucidate irregular, unpredictable economic behavior.

When the government runs a budget deficit, it reduces the national saving. The interest rate rises. The level of asset prices decrease. Further, the higher interest rate reduces net foreign investment (net capital outflow). Reduced net foreign investment (net capital outflow), in turn, reduces the supply of domestic currency in the market for foreign-currency exchange, which causes the real exchange rate of domestic currency to appreciate.

National saving is the source of the supply of loanable funds. Domestic investment and net capital outflow are the sources of the demand for loanable funds. At the equilibrium interest rate, the amount that people want to save exactly balances the amount that people want to borrow for the purpose of buying domestic capital and foreign capital. At the equilibrium interest rate, the amount that people want to save exactly balances the desired quantities of domestic investment and net capital outflow.

The supply and demand for loanable funds determine the real interest rate. The interest rate determine net capital outflow, which provides the supply of dollars in the market for foreign –currency exchange. The supply and demand for dollars in the market for foreign-currency exchange determine the real exchange rate.

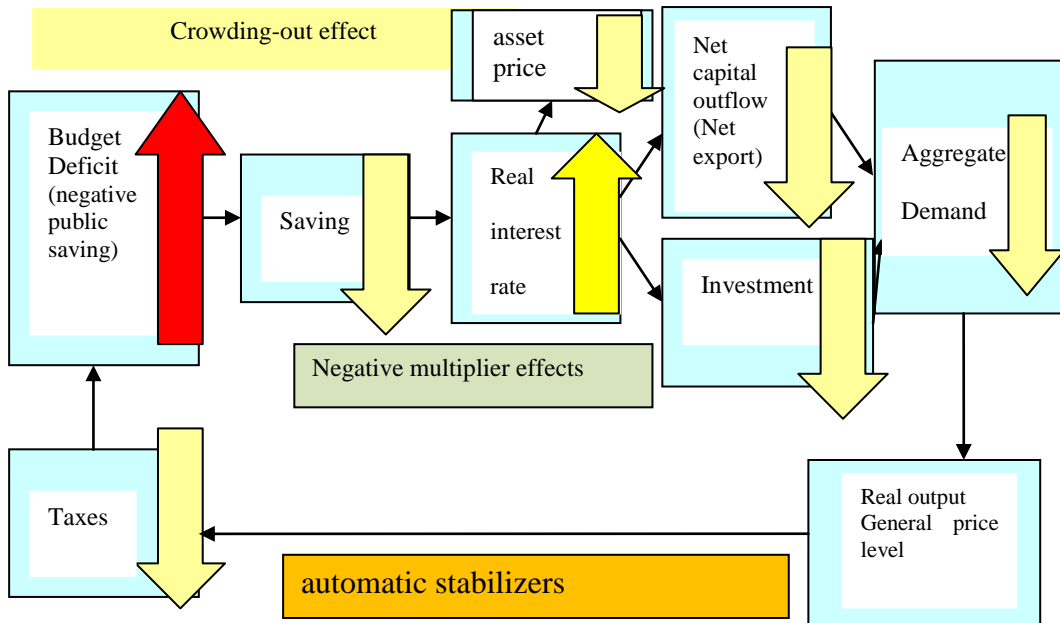


Figure 1: Causes of the financial crisis or the effects of a government budget deficit

A financial crisis may be described by large decreases in the prices of stocks, real estate, or other assets. Namely, in an open economy, government budget deficit raises real interest rates, crowds out domestic investment, decreases net capital outflow, decreases the level of asset prices, causes the domestic currency to appreciate. However this appreciation makes domestic goods and services more expensive compared to foreign goods and services. In this case, exports fall, and imports rise. Namely, net exports fall. Further, the real gross domestic product falls as a consequence of the negative open economy multiplier effects

Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (i) extreme sensitivity to initial conditions ; (ii) cause and effect are not proportional; and (iii) nonlinearity.

2 Methodology

Chaos theory is used to prove that erratic and chaotic fluctuations can indeed arise in completely deterministic models. Chaos theory reveals structure in aperiodic, dynamic systems. The number of nonlinear business cycle models use chaos theory to explain complex motion of the economy. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change. Chaos theory started with Lorenz's [11] discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and

Yorke [10] discovered that the simple logistic curve can exhibit very complex behaviour. Further, May [13] described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day [1,2], Day [3,4], Grandmont [6], Goodwin [5], Medio [14], Lorenz [12], Jablanovic [7, 8, 9], among many others.

3 Results

Irregular movement of the saving can be analyzed in the formal framework of the chaotic saving growth model :

$$S_t = I_t + NCO_t \quad (1)$$

$$S_t = Y_t - C_t - G_t \quad (2)$$

$$S_t = S_{p,t} - Bd_t \quad (3)$$

$$NCO_t = n Y_t \quad (4)$$

$$S_p = \gamma Y_t \quad (5)$$

$$I_t = \beta (Y_t - Y_{t-1}) \quad \beta > 1 \quad (6)$$

$$Bd_t = b Y_t \quad (7)$$

$$C_t = \alpha Y_{t-1} \quad (8)$$

$$G_t = g Y_t \quad (9)$$

Where : S – national saving; Y- the real gross domestic product; NCO – net capital outflow (net foreign investment); I- investment; S_p – private saving; Bd – budget deficit; C - consumption; G_t – government consumption; n - net capital outflow as a percent of the real gross domestic product; b – budget deficit as a percent of the real gross domestic product; γ – private saving as a percent of the real gross domestic product; g – government consumption as a percent of the real gross domestic product; α - average propensity to consume; β – the accelerator.

By substitution one derives:

$$S_t = \left[\frac{\beta}{(g + \beta + n - 1)} \right] S_{t-1} - \left[\frac{\alpha}{(g + \beta + n - 1)(\gamma - b)} \right] S_{t-1}^2 \quad (10)$$

Further, it is assumed that the current value of the saving is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the saving growth rate depends on the current size of the saving, S, relative to its maximal size in its time series S^m . We introduce s as $s = S / S^m$. Thus y range between 0 and 1.

Again we index s by t , i.e., write s_t to refer to the size at time steps $t = 0,1,2,3,\dots$. Now growth rate of the saving is measured as:

$$s_t = \left[\frac{\beta}{(g + \beta + n - 1)} \right] s_{t-1} - \left[\frac{\alpha}{(g + \beta + n - 1)(\gamma - b)} \right] s_{t-1}^2 \tag{12}$$

This model given by equation (10) is called the logistic model. For most choices of α , β , γ , b , g , and n there is no explicit solution for (11). This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (11) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of s_t . This difference equation (11) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point s_0 the solution is highly sensitive to variations of the parameters α , β , γ , b , g , and n ; secondly, given the parameters α , β , γ , b , g , and n , the solution is highly sensitive to variations of the initial point s_0 . In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

4 The Logistic Equation

The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst, a Belgian mathematician interested in the modeling of human populations. He applied his logistic equation for population growth to demographic studies. The model was created to produce solutions for a single population that grows from a small initial number of individuals to a limited population with an upper bound implicitly set by factors such as food supply, living space or pollution (the effect of a "carrying capacity" that would limit growth). His formula is called the "logistic model" or the Verhulst model.

It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t) \quad , \quad \pi \in [0, 4] \quad , \quad z_t \in [0, 1] \tag{12}$$

is equivalent to the iteration of growth model (10) when we use the identification

$$z_t = \frac{\alpha}{\beta(\gamma - b)} s_t \tag{13}$$

and

$$\pi = \left[\frac{\beta}{(g + \beta + n - 1)} \right] \tag{14}$$

Using (13) and (11) we obtain

$$\begin{aligned} z_t &= \frac{\alpha}{\beta(\gamma-b)} s_t = \\ &= \left[\frac{\alpha}{\beta(\gamma-b)} \right] \left\{ \left[\frac{\beta}{(g+\beta+n-1)} \right]^{s_{t-1}} - \left[\frac{\alpha}{(g+\beta+n-1)(\gamma-b)} \right]^{s_{t-1}^2} \right\} \\ &= \left[\frac{\alpha}{(\gamma-b)(g+\beta+n-1)} \right]^{s_{t-1}} - \left[\frac{\alpha^2}{\beta(g+\beta+n-1)(\gamma-b)^2} \right]^{s_{t-1}^2} \end{aligned}$$

On the other hand, using (11), (12) and (13) we obtain

$$\begin{aligned} z_t &= \pi z_{t-1} (1 - z_{t-1}) = \left[\frac{\beta}{(g+\beta+n-1)} \right] \left[\frac{\alpha}{\beta(\gamma-b)} \right]^{s_{t-1}} \left\{ 1 - \left[\frac{\alpha}{\beta(\gamma-b)} \right]^{s_{t-1}} \right\} \\ &= \left[\frac{\alpha}{(\gamma-b)(g+\beta+n-1)} \right]^{s_{t-1}} - \left[\frac{\alpha^2}{\beta(g+\beta+n-1)(\gamma-b)^2} \right]^{s_{t-1}^2} \end{aligned}$$

Thus we have that iterating (11) is really the same as iterating $z_{t+1} = \pi z_t (1 - z_t)$ using (13) and (14).

It is important because the dynamic properties of the logistic equation (12) have been widely analyzed (Li & Yorke [10], May [13]).

It is obtained that :

- For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;
- (ii) For $1 < \pi < 3,57$ there exist fixed points the number of which depends on π ;
- (iii) For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1) / \pi$;
- (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;
- (v) For $3 < \pi < 4$ all solutions will continuously fluctuate;
- (vi) For $3,57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period.

This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

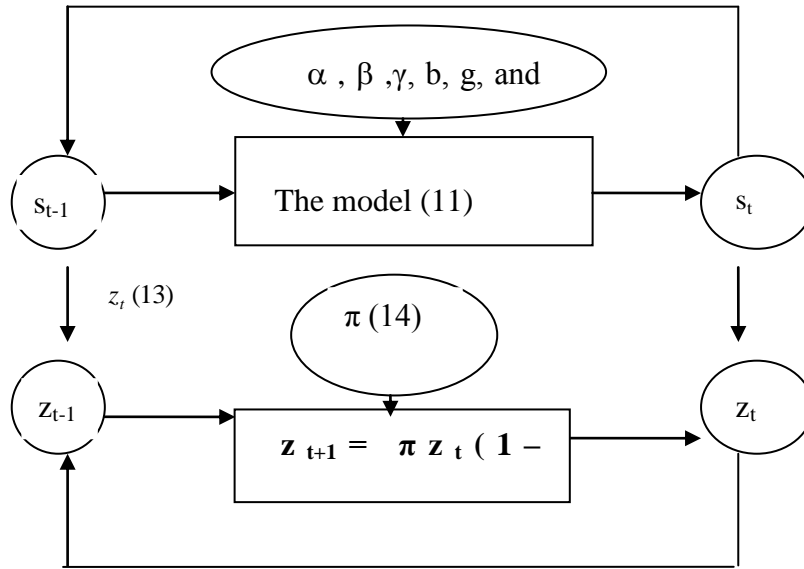


Figure 2: Two quadratic iterations running in phase are tightly coupled by the transformations indicated

5 Conclusion

This paper creates the simple chaotic saving growth model . The model (11) has to rely on specified parameters $\alpha , \beta , \gamma , b , g ,$ and n , and initial value of saving s_0 . This difference equation (11) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point s_0 the solution is highly sensitive to variations of the parameters $\alpha , \beta , \gamma , b , g ,$ and n ; secondly, given the parameters $\alpha , \beta , \gamma , b , g ,$ and n , the solution is highly sensitive to variations of the initial point s_0 . In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

A key hypothesis of this work is based on the idea that the coefficient $\pi = \beta / (g + \beta + n - 1)$ plays a crucial role in explaining local growth stability of the saving, where, n - net capital outflow as a percent of the real gross domestic product; g - government consumption as a percent of the real gross domestic product; β - the accelerator.

It is important to increase national saving, private saving, net outflow capital; and to decrease budget deficit through fiscal and monetary reforms.

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