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# A Generalized ideal based-zero divisor graphs of Noetherian regular δ-near rings (GIBDNR- δ-NR)

**N.V. Nagendram[1], T.V. Pradeep Kumar[2] and Y. Venkateswara Reddy[3]**

## Abstract

A near-ring N is called a δ-Near – Ring if it is left simple and $N_0$ is the smallest non-zero ideal of N and a δ-Near – Ring is a non-constant near ring. A Commutative ring N with identity is a Noetherian Regular δ-Near Ring if it is Semi Prime in which every non-unit is a zero divisor and the Zero ideal is Product of a finite number of principle ideals generated by semi prime elements and N is left simple which has $N_0 = N$, $N_e = N$. In this paper, we introduce the generalized ideal-based zero divisor graph structure of Noetherian Regular δ- near-ring $N$, denoted by $\Gamma_I(N)$. It is shown that if $\Gamma$ is a completely reflexive ideal of $N$, then every two vertices in $\Gamma_I(N)$ are connected by a path of length at most 3, and if $\Gamma_I(N)$ contains a cycle, then the core $K$ of $\Gamma_I(N)$ is a union of triangles and rectangles. We have shown that if $\Gamma_I(N)$ is a bipartite graph for a completely semi-prime ideal $I$ of $N$, then $N$ has two prime ideals whose intersection is $I$.

[1] Lakireddy Balireddy College of Engineering, L. B. Reddy Nagar, Mylavaram, Krishna District, AP., India, email: nvn220463@yahoo.co.in
[2] ANU college of Engineering and Technology, Acharya Nagarjuna University, Nagarjuna Nagar, AP., India, email: pradeeptv5@gmail.com
[3] ANU College of Engineering and Technology, Acharya Nagarjuna University, Nagarjuna Nagar, AP., India.

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# 1 Introduction

Throughout this paper, N denotes a zero-symmetric Noetherian Regular δ-Near-Ring not necessarily with identity unless otherwise stated. For any vertices x, y in a graph G, if x and y are adjacent, we denote it as x ¼ y. By the concept of a zero-divisor graph of a commutative ring with identity, but this work was mostly concerned with coloring of Noetherian Regular δ- Near-Rings and associate to a commutative ring with identity a (simple) graph $\Gamma_I$ (N), whose vertex set is Z(N)* = Z(N) \ {0}, the set of nonzero-divisor of N, in which two distinct x, y ∈ Z(N)* are joined by an edge if and only if xy = 0. They investigated the interplay between the ring-theoretic properties of N and the graph-theoretics properties of $\Gamma_I$ (N) .The zero-divisor graph has also been introduced and studied for semi-groups.

The generalized the notion of the zero-divisor graph for structure of Noetherian Regular δ- near-ring N, denoted by $\Gamma_I$(N). For given ideal I of a commutative ring N, he defined an undirected graph $\Gamma_I$ (N) with vertices {x ∈ R \ I : xy ∈ I for some y ∈ R \ I }, where distinct vertices x and y are adjacent if and only if x.y ∈ I. We, extended this graph structure to Noetherian Regular δ-Near-Ring. Let I be a completely reflexive ideal (i.e., ab ∈ I implies ba ∈ I for a, b ∈ N) of N. Then the ideal-based zero-divisor graph, denoted by $\Gamma_I$ (N), is the graph whose vertices are the set. {x ∈ N \ I : xy ∈ I for some y ∈ N \ I} with distinct vertices x and y are adjacent if and only if xy ∈ I.

In this paper, we introduced a generalized ideal-based zero-divisor graph structure of the Noetherian Regular δ- near-ring *N*. Let *N* be a Noetherian

Regular δ- near-ring and $I$ be a completely reflexive ideal of $N$. We define an undirected graph $\Gamma_I(N)$ with vertices $\{x \in N \setminus I$ : there exists $y \in N \setminus I$ such that $x_1y_1 \in I$ for some $x_1 \in \langle x\rangle \setminus I$ and $y_1 \in \langle y\rangle \setminus I\}$, where distinct vertices $x$ and $y$ are adjacent if and only if $x_1y_1 \in I$ for some $x_1 \in \langle x\rangle \setminus I$ and $y_1 \in \langle y\rangle \setminus I$, where $\langle x\rangle$ denotes the ideal of Noetherian Regular δ- near-ring $N$ generated by $x$.

Let $\Gamma_I(N)$ is a induced sub-graph of $\Gamma_I(N)$, and if $I = \{0\}$, then $\Gamma_I(N)$ will be denoted imply by $\Gamma_I(N)$. Also $\Gamma_I(N) = \varphi$ if and only if $I$ is a prime ideal of Noetherian Regular δ- near-ring $N$. That is, $V(\Gamma_I(N)) = \varphi$ if and only if $V(\Gamma_I(N/I)) = \varphi$. Observe that $|V(\Gamma_I(N))| = 0$ if and only if $|V(\Gamma_I(N))| = 0$. Also $|V(\Gamma_I(N))| \cdot |V(\Gamma_I(N))|$.

**Example:** Below are the generalized zero-divisor graphs for several Noetherian Regular δ- near-ring $N$.

Note that these examples show that the graph structures $\Gamma_I(N)$ and $\Gamma_I(N)$ are not isomorphic and non-isomorphic Noetherian Regular δ- near-ring $N$ of near rings may have the isomorphic generalized zero-divisor graph.

Given a graph $G$, for distinct vertices $x$ and $y$ of $G$, let $d(x, y)$ be the length of the shortest path from $x$ to $y$. The diameter of a connected graph is the supremum of the distances between vertices. The core $K$ of $G$ is the union of all cycles of $G$. For any subset $S$ and ideal $I$ of $N$, we define $IS = \{n \in N : nS \subseteq I\}$. If $S = \{a\}$, then we denote I{a} by $Ia$. In this paper the notations of graph theory are from [5], the notations of Noetherian Regular δ- near-ring $N$.

## 2 Preliminaries

In this section we give the preliminary definitions and examples and the required literature to this paper.

**Definition 2.1** A Near – Ring is a set N together with two binary operations "+" and "." Such that

( i ) (N, +) is a Group not necessarily abelian

( ii ) (N, .) is a semi Group and

( iii ) for all $n_1$, $n_2$, $n_3$ $\in$ N, $(n_1 + n_2). n_3 = (n_1 . n_3 + n_2 . n_3)$,

i.e., right distributive law.

**Examples 2.2** Let $M_{2x2}$ ={ {(aij) / Z ; Z is treated as a near-ring}. $M_{2x2}$ under the operation of matrix addition '+' and matrix multiplication '.'. ◊

**Example 2.3** Z be the set of positive and negative integers with 0.

(Z, +) is a group. Define '.' on Z by a . b = a, for all a, b $\in$ Z. Clearly (Z, +, .) is a near-ring. ◊

**Example 2.4** Let Z12 = {0, 1, 2 ,…, 11}. (Z12, +) is a group under '+' modulo 12. Define '.' on Z12 by a . b = a, for all a$\in$ Z12. Clearly (Z12, +, .) is a near-ring. ◊

**Definition 2.5** A near-ring N is Regular Near-Ring if each element a $\in$ N, then there exists an element x in N such that a = axa.

**Definition 2.6** A Commutative ring N with identity is a Noetherian Regular δ-Near Ring if it is Semi Prime in which every non-unit is a zero divisor and the Zero ideal is Product of a finite number of principle ideals generated by semi prime elements and N is left simple which has $N_0 = N$, $N_e = N$.

**Definition 2.7** A Noetherian Regular delta Near Ring (is commutative ring) N with identity, *the zero-divisor graph* of N, denoted Γ(N), is the graph whose vertices are the non-zero zero-divisors of N with two distinct vertices joined by an edge when the product of the vertices is zero.

**Note 2.8** We will generalize this notion by replacing elements whose product is zero with elements whose product lies in some ideal I of N. Also, we determine (up to isomorphism) all Noetherian Regular delta near rings $N_i$ of N such that $\Gamma(N)$ is the graph on five vertices.

**Definition 2.9** A near-ring N is called a δ-Near – Ring if it is left simple and $N_0$ is the smallest non-zero ideal of N and a δ-Near – Ring is a non-constant near ring.

**Definition 2.10** A δ-Near-Ring N is isomorphic to δ-Near-Ring and is called a Regular δ-Near-Ring if every δ-Near-Ring N can be expressed as sub-direct product of near-rings {Ni}, Ni is a non-constant near-ring or a δ-Near-Ring N is sub-directly irreducible δ-Near-Rings Ni.

**Defintion 2.11** Let N be a Commutative Ring. Let N be a Noetherian Regular δ-Near-Ring if each P ∈ A ($N_N$) is strongly prime i.e., P is a δ-Near – Ring of N.

**Example 2.12** Let N = $\begin{bmatrix} F & F \\ 0 & F \end{bmatrix}$ where F is a field. Then P(N) = $\begin{bmatrix} 0 & F \\ 0 & 0 \end{bmatrix}$

Let , σ : N → N be defined by , σ((a b = a 0 0 c)) 0 c

It can be seen that a σ endomorphism of N and N is a σ(*)-Ring or Noetherian Regular δ-Near– Ring. ◊

**Definition 2.13** Let (N, +, •) be a near-ring. A subset L of N is called a ideal of N provided that 1. (N, +) is a normal subgroup of (N, +), and 2. m.(n + i) — m.n ∈L for all i ∈L and m,n ∈ N.

**Definition 2.14** *Cycle* : An *algebraic cycle* of an algebraic variety or scheme *X* is a formal linear combination $V = \sum n_i \cdot V_i$ of irreducible reduced closed sub-schemes. A coefficient $n_i$ is called multiplicity of $V_i$ in *V*. Ad hoc, the coefficients are integers, but rational coefficients are also widely used.

**Example 2.15** Under the correspondence {*irreducible reduced* closed sub-schemes V ⊂ X} implies and implied by the set {points of X} (V maps to its *generic point* (with respect to the Zariski topology), conversely a point maps to its closure (with the reduced sub-scheme structure)) an algebraic cycle is thus just a formal linear combination of points of X. The group of cycles naturally forms a group Z*(X) graded by the dimension of the cycles. The grading by co-dimension is also useful, then the group is usually written Z*(X). ◊

**Note 2.16** On the term cycle may refer to several closely related objects.

(i) A closed walk, with repeated vertices allowed. See path (graph theory). (This usage is common in computer science. In graph theory it is more often called a closed walk.)

(ii) A closed (simple) path, with no other repeated vertices or edges than the starting and ending vertices. (This usage is common in graph theory, see "Cycle graph") This may also be called a simple cycle, circuit, circle, or polygon.

(iii) A closed directed walk, with repeated vertices allowed. (This usage is common in computer science. In graph theory it is more often called a closed directed walk.)

(iv) A closed directed (simple) path, with no repeated vertices other than the starting and ending vertices. (This usage is common in graph theory.) This may also be called a simple (directed) cycle.

(v) The edge set of an undirected closed path without repeated vertices or edges. This may also be called a circuit, circle, or polygon.

(vi) An element of the binary or integral (or real, complex, etc.) cycle space of a graph. (This is the usage closest to that in the rest of mathematics, in particular algebraic topology.) Such a cycle may be called a binary cycle, integral cycle, etc.

**Note 2.17** Cycle

(i) An interval of space or time in which one set of events or phenomena is completed.

(ii) A complete rotation of anything.

(iii) A process that returns to its beginning and then repeats itself in the same sequence.

(iv) A series of poems, songs or other works of art

(v) A programme on a washing machine, dishwasher, or other such device.

(vi) A pedal-powered vehicle, such as a unicycle, bicycle, or tricycle; or, motorized vehicle that has either two or three wheels, such as a motorbike, motorcycle, motorized tricycle, or motortrike.

(vii) A single, a double, a triple, and a home run hit by the same player in the same game.

(viii) A closed walk or path, with or without repeated vertices allowed.

## 3 Main results

**Theorem 3.1** Let I be a completely reflexive ideal of Noetherian Regular δ-Near Ring N. Then $\Gamma_I(N)$ is a connected graph and diam.$(\Gamma_I(N)) \leq 3$.

**Proof.** Let $a \approx x \approx b$ be a path in $\Gamma_I$ (N),). Then there exist x1, x2 $\in \langle x \rangle \setminus I$ ; $a_1 \in \langle a \rangle \setminus I$ and b1 $\in \langle x \rangle \setminus I$ such that $a \approx x \approx b \in I$ is contained in a cycle length $\leq$ 3. Let us assume that $a_1 b_1 \notin I$ for all $a_1 \in \langle a \rangle \setminus I$ and $b_1 \langle b \rangle \setminus I$.

Case (i) Let $x_1 = x_2$. Then either $Ia_1 \cap Ib_1 = I \cup \{x_1\}$ or there exists $c \in Ia_1 \setminus Ib_1$ such that $c \notin I \cup \{x1\}$. Then $ca_1$, $cb_1 \in I$. In the first case, $I \cup \{x1\}$ is an ideal. In the second case $a \approx x \approx b \approx c \approx a$ is contained in a cycle of length $\leq 3$.

Case (ii) Let $x_1 \neq x_2$. Then clearly, $\langle a1 \rangle \cap \langle b1 \rangle \not\subset I$. Then for each $z \in \langle a1 \rangle \cap \langle b1 \rangle \setminus I$, we have $zx_1 \in \langle a_1 \rangle \langle x_1 \rangle \subseteq I$ and $zx_2 \in I$.

Clearly either $x_1 \neq x$ or $x_2 \neq x$. Say $x_1 \neq x$. Then we have a path $a \approx x1 \approx b$ and hence $a \approx x \approx b \approx x_1 \approx a$ is contained in a cycle of length $\leq 3$. $\square$

**Lemma 3.2** Let I be a completely reflexive ideal of Noetherian Regular δ-Near Ring N. For any x, y∈ $\Gamma_I$ (N), if x ≈ y is an edge in $\Gamma_I$ (N), then for each n ∈ N \ I, either n ≈ y or x ≈ y 0 is an edge in \ $\Gamma_I$ (N) for some y' ∈ (y) \ I.

**Proof.** Let *x, y* ∈ *N* \ *I* with *x* ≈ *y* be an edge in $\Gamma_I$ (*N*) and suppose that *n* ≈ *y* is not an edge in $\Gamma_I$ (*N*) for any *n* ∈ *N* \ *I*. Then $x_1y_1 \in I$ for some $x_1 \in hxi \setminus I$;
$y_1 \in hyi \setminus I$ and $ny_1 \in I$. But $(ny_1)x_1 \in I$. So *x* ≈ *y 0* is an edge in $\Gamma_I$ (*N*) for some *y* ' ∈ $(y_i) \setminus I$. □

**Example 3.3** Let *N* =( *F F* 0 *F* ) , where *F* = {0, 1} is the field under addition and multiplication modulo 2. Then it's prime radical *P* = {( 0 0, ( 0 1 0 0 ) , 0 0 )} is a completely reflexive ideal of the near-ring N and its generalized zero-divisor graph $\Gamma_P$ (N) is easy to verify that P [ {a} is not an ideal of Noetherian Regular δ-Near Ring N for any a in (a2 ) \ P and $a_4 \approx a_2 \approx a_6$ is not contained in cycle of length 4. ◊

**Corollary 3.4** Let I be a completely reflexive ideal of Noetherian Regular δ-Near Ring N and |V ( $\Gamma_I$ (N))| > 2. If I ∪ {x} is not an ideal of N for any x ∈ N \ I, then every edge in $\Gamma_I$ (N) is contained in a cycle of length ≤ 4, and therefore $\Gamma_I$ (N) is a union of triangles and squares.

**Lemma 3.5** Let I be a completely reflexive ideal of Noetherian Regular δ-Near Ring N. Then, $\Gamma_I$ (N) can be neither a pentagon nor a hexagon.

**Proof.** Suppose that $\Gamma_I$ (N) is a ≈ b ≈ c ≈ d ≈ e ≈ a, a pentagon. Then by Corollary 3.4, for one of the vertices (say b), I ∪ $\{b_1\}$ is an ideal of Noetherian Regular δ-Near Ring N for some b1 ∈ <b> \ I. Then in the pentagon, there exist $d_1$ ∈ <d> \ I and $e_1$ ∈ <e> \ I such that $d_1e_1 \in I$. Since I ∪ $\{b_1\}$ is ideal, $b_1d_1 = b_1 = b_1e_1$. But $b_1(d_1e_1) \in I$, then $b_1 \in I$, a contradiction. The proof for the hexagon is the same. □

**Theorem 3.6** Let I be a completely reflexive ideal of Noetherian Regular δ-Near Ring N. Then $\Gamma_I$ (N) is connected graph with diam ( $\Gamma_I$ (N)) ≤ 3.

**Proof.** Let x, y ∈ $\Gamma_I$ (N). If $x_1y_1 \in$ I for some $x_1 \in$ <x> \ I and $y_1 \in$ <y> \ I, then d(x, y) = 1. Let us assume that $x_1y \notin$ I for all $x_1 \in$ <x> \ I and for all $y_1 \in$ <y> \ I. Then $x_1^2 \notin$ I and $y_1^2 \notin$ I for all $x_1 \in$ <x> \ I and for all $y_1 \in$ <y> \ I. Since x, y ∈ $\Gamma_I$ (N), there exist $x_2 \in$ <x> \ I; $y_2 \in$ <y> \ I and $a_1, b_1 \in N \setminus (I \cup \{x_2, y_2\})$ such that $a_1x_2 \in$ I and $b_1y_2 \in$ I.

If $a_1 = b_1$, then $x \approx a_1 \approx y$ is a path of length 2. So assume that $a_1 \neq b_1$. If $a_1b_1 \in$ I, then $x \approx a_1 \approx b_1 \approx y$ is a path of length 3. Otherwise $a_1b_1 \notin$ I. Then <a1> ∩ <b1> ∉ I. Now for every d ∈ <$a_1$> ∩ <$b_1$> (I ∪ {x2, y2}), we have $Dx_2 \in$ <$dx_2$> ⊆ <$a_1x_2$> ⊆ I and $dy_2 \in$ <$b_1y_2$> ⊆ I. Thus x ≈ d ≈ y is a path of length 2 and hence $\Gamma_I$ (N) is connected and diam($\Gamma_I$ (N) ≤ 3. □

**Theorem 3.7** Let I be a completely reflexive ideal of Noetherian Regular δ-Near Ring N. Then the following holds good:

(i) If N has identity, then $\Gamma_I$ (N) has no cut-vertices. (ii) If N has no identity and if I is non-zero ideal of Noetherian Regular δ-Near Ring N, then $\Gamma_I$ (N) has no cut-vertices.

**Proof.** Suppose that the vertex x of $\Gamma_I$ (N) is a cut vertex. Let u ≈ x ≈ w be a path in $\Gamma_I$ (N). Since x is a cut-vertex, x lies in every path from u to w.

Case (i) Assume that N is a Noetherian Regular δ-Near Ring with identity. For any u, v ∈ $\Gamma_I$ (N), there exist a path u ≈ 1 ≈ w which shows x(≠ 1) in $\Gamma_I$ (N) is not a cut vertex. Suppose x = 1. Then there exist $u_1 \in$ <u> \ I;$w_1 \in$ <w> \ I and $t_1, t_2 \in$ N \ I such that $u_1t_1, w_1t_2 \in$ I which implies $u_1,w_1 \in \Gamma_I$ (N).

Since $\Gamma_I$ (N) is connected, there exist n, $n_1 \in N \setminus (I \cup \{x\})$ such that $u_1 \approx n \approx w_1$ or $u_1 \approx n \approx n_1 \approx w_1$ is a path in $\Gamma_I$ (N) which implies u ≈ n ≈ w ≈ 1 ≈ u or u ≈ n ≈ $n_1$ ≈ w ≈ 1 ≈ u is a cycle in $\Gamma_I$ (N), contradicting x = 1 is a cut-vertex.

Case (ii) Let N be a Noetherian Regular δ-Near Ring without identity and I be a non-zero ideal of Noetherian Regular δ-Near Ring N.

Since $u \approx x \approx w$ is a path from u to w, then there exist $u_1 \in \langle u\rangle \setminus I$; $w_1 \in \langle w\rangle \setminus I$ and for all $x_1, x_2 \in \langle x\rangle \setminus I$ such that $u_1x_1 \in I$ and $w_1x_2 \in I$.

Let us suppose that $x_1 = x_2$.

If $u_1+I = x_1+I$, then $u_1w_1 \in I$ which implies u is adjacent to w. Similarly, if $x2 + I = w1 + I$, u is adjacent to w. So assume that $u_1 + I \neq x_1+ I$ and $x_2 + I \neq w_1 + I$. Let $0 \neq i \in I$. Then $u_1x_1 \in I$ and $w_1x_2 \in I$ which imply that $u_1(x1 +i), w_1(x_1 +i) \in I$. If $x = x_1 +i$, then $x \neq x_1$ which implies $u \approx x_1 \approx w$ is a path in $\Gamma_I$ (N). Otherwise $u \approx (x1 +i) \approx w$ is a path in $\Gamma_I$ (N). Thus there exist a path from u to w not passing through x, a contradiction.

Let us suppose, if either $x_1 = x$ or $x_2 = x$.

Without loss of generality, let us assume that $x_1 = x$ and $x_2 \neq x$. Then $u_1x \in I$ and $x_2w_1 \in I$ which implies $u_1x_2 \in I$ and $x_2w_1 \in I$, and so we have a path $u \approx x_2 \approx w$, a contradiction.

Let us suppose, that Neither $x_1$ nor $x_2$ equal to x.

If $x_1x_2 \in I$, then we have a path $u \approx x_1 \approx x_2 \approx w$, a contradiction.

Otherwise $x_1x \notin I$.

If $x_1x_2 = x$, then $u_1x \in I$ and $w_1x \in I$. By above , we have a contradiction.

So assume that $x_1x_2 \neq x$, then we have a path $u \approx x_1x_2 \approx w$, a contradiction.

Thus x can not be a cut-vertex.

Here we have the following question. If N is a Noetherian Regular δ-Near Ring without identity and {0} is a completely reflexive ideal of N, then whether $\cup\, [\Gamma_I$ (N)] has a cut-vertex.

Hence we have the theorem. □

**Theorem 3.8** Let I be a completely reflexive ideal of N. If $\Gamma_I$ (N) contains a cycle, then the core K of $\Gamma_I$ (N) is a union of triangles and rectangles. Moreover, any vertex in $\Gamma_I$ (N) is either a vertex of the core K of $\Gamma_I$ (N) or else is an end vertex of $\Gamma_I$ (N).

**Proof.** Let a $\in$ K and assume that a is not in any square or rectangle in $\Gamma_I$ (N). Then a is part of a cycle a $\approx$ b $\approx$ c $\approx$ d $\approx \cdots \approx$ a which implies $c_1d_1 \in$ I for some $c_1 \in$ <c> \ I and $d_1 \in$ <d> \ I. Also, by Lemma 2.4, I $\cup$ {$a_1$} is an ideal of N for some $a_1 \in$ <a> \ I. Then $d_1a_1 = a_1 = c_1a_1$ and $a_1(d_1c_1) \in$ I which implies $a_1 \in$ I, a contradiction.

Let use assume that $\Gamma_I$ (N) ¸ 3. If x is a vertex in $\Gamma_I$ (N), then one of the following is true:

1. x is in the core;
2. x is an end vertex of $\Gamma_I$ (N);
3. a $\approx$ x $\approx$ b is a path in $\Gamma_I$ (N) where a is an end vertex and b $\in$ K;
4. a $\approx$ x $\approx$ y $\approx$ b or a $\approx$ y $\approx$ x $\approx$ b is a path in $\Gamma_I$ (N), where a is an end vertex and b$\in$ K.

In the first two cases, we are done. Let us assume that a $\approx$ x $\approx$ b is a path with b $\in$ K. Then by Lemma 2.4, I $\cup${x1} is an ideal of a Noetherian Regular Delta Near ring N for some $x_1 \in$ <x> \ I and x $\approx$ b $\approx$ c $\approx$ d $\approx$ b or x $\approx$ b $\approx$ c $\approx$ d $\approx$ e $\approx$ b is a path in $\Gamma_I$ (N) which implies $c_1d_1 \in$ I for some $c_1 \in$ <c> \ I and $d_1 \in$ <d> \ I. Since, x $\notin$ K, we have $x_1c_1 = x_1$ and so x is a vertex in the cycle x $\approx$ b $\approx$ c $\approx$ d $\approx$ x, a contradiction.

Without loss of generality, let us assume a $\approx$ x $\approx$ y $\approx$ b is a path in $\Gamma_I$ (N). Since b $\in$ K, there is some c $\in$ K such that c $\neq$ b and b $\approx$ c is part of a cycle. Then a $\approx$ x $\approx$ y $\approx$ b $\approx$ c is a path in $\Gamma_I$ (N). But the distance from a to c is four, a contradiction unless y $\approx$ c or x $\approx$ c is an edge. However, if y $\approx$ c is an edge, then y $\in$ K. By case 3, x is also in the core. If instead, x $\approx$ c is an edge, then

x $\approx$ y $\approx$ b $\approx$ c $\approx$ x is a cycle. Thus, x, y $\in$ K. Hence it must be the case that any vertex x of $\Gamma_I$ (N) is either an end or in the core.

Hence we proved the theorem. □

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