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Lending Values and Liquidity Risk

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Abstract

In this paper we show how to derive a liquidity adjusted lending value in the case where the collateral is given by a single stock. Following [12] and [7], the collateral market value is adjusted as a function of the position size based on the existence of a one-parameter exponential supply curve. The lending value is then determined as usual, i.e. such that the probability that after a margin call the collateral value falls below the client exposure is at most $\epsilon > 0$. The curve parameter for a specific stock can be estimated from intraday data by means of a simple regression. Furthermore, we show that an affine model where the liquidity parameter characterizing the exponential supply curve is assumed to be a function of the Average Daily Trading Volume (ADTV) has an excellent predictive power. This implies that the ADTV can be used for a simple and direct computation of the liquidity parameter avoiding the use of the intraday data. Concrete examples highlight the impact of liquidity risk on the lending value.

Mathematics Subject Classification: C1, G1
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1 Introduction

Lombard Lending is the standard terminology used in Switzerland for collateralized lending against liquid assets such as publicly traded stocks, bonds, etc., and is usually characterized by a variable credit limit expressed as a fixed percentage of the collateral market value: the so called *lending value*. Briefly speaking, at the beginning of the contract the client is assigned a fixed percentage called lending value which depends on the "quality" of the pledged assets. The lending value multiplied by the market value of the collateral determines the (variable) lending limit up to which the client is allowed to draw money from the bank. Usually, a Lombard loan contract states that the bank is allowed to liquidate (part of) the assets as soon as the running haircut, which is given by the relative difference between the assets value and the lent amount, falls below some fixed threshold. The latter typically equals 75% of the initial (or required) haircut, i.e. 75% of the running haircut at the time of the beginning of the contract.

However, since also normal market fluctuations may determine a running haircut erosion exceeding the required margin, the bank, instead of applying the contract terms zealously, adopts a different control policy. For example, as soon as the haircut erosion lies between 0% and 25%, the client comes into a warning stage where his positions are monitored carefully but no measure to reestablish the required haircut is taken. A so called margin call stage is entered when the haircut erosion exceeds 25% of the required haircut. In this case the bank asks the client to reestablish the required margin bringing new collateral or reducing exposure creating transactions within an imposed time, typically 10 business days. If the client does not react or the required haircut is not reestablished through the price movement of the collateral itself, then the bank can start liquidating (part of) the assets. Otherwise, the whole control process restarts. The time interval from the last margin call to the liquidation instant is called closeout period. It is clear that at the end of the closeout period, the bank still realizes a loss if the assets value has decreased rapidly enough so that the income due to the assets liquidation does not cover the banks exposure. The Lombard credit risk has therefore two components: a market risk component stemming from the movements of the assets value and a purely obligor specific risk component due to the fact that the obligor may not react on margin calls. Thus, the default time, intended as the time of exiting the closeout period, can be seen as a combination of a market default time (equal to some margin call time plus the length of the closeout period) and of a client default time occurring if the obligor does not react on that margin call. Besides this, there is also a timing-risk issue that has to be considered. Indeed, because both the magnitude and the occurrence of a Lombard loss are contingent on the default time, which is random, we have that all occurring cash-flows are not known with certainty ex-ante.

In summary, the Lombard credit risk consists of a market risk component stemming from the movements of the collateral's value, and of a purely obligor specific risk component due to the fact that the obligor may not react on margin calls. If the obligor specific risk component is neglected, then the Lombard risk becomes a pure market risk with a derivative character since the form of the Lombard loss looks like the payoff of a random maturity put option with stochastic strike.

1.1 Literature review

There is a large literature on liquidity. For instance [1] propose a liquidity adjusted capital asset pricing model where the expected return of a security is increasing in its expected illiquidity and its net beta, which is proportional to the covariance of its return net of its exogenous illiquidity costs with the market portfolios net return. The net beta can be decomposed into the standard market beta and three betas representing different forms of liquidity risk. These liquidity risks are associated with: commonality in liquidity with the market liquidity, return sensitivity to market liquidity, and liquidity sensitivity to market returns. [1] measure the liquidity by means of β coefficients defined using the covariances between returns and the return variance. In other words they do not propose a structural model taking liquidity into account.

Another liquidity paper is [5]. They provide a model that links an assets market liquidity and traders' funding liquidity. Traders provide market liquidity, and their ability to do so depends on their availability of funding. Conversely, traders' funding, depends on the assets' market liquidity. [5] show that margins are destabilizing and market liquidity and funding liquidity are mutually reinforcing, leading to liquidity spirals. The model explains the empirically documented features that market liquidity can suddenly dry up, has commonality across securities, is related to volatility, is subject to “flight to quality”, and co-moves with the market. The model provides new testable predictions, including that speculators capital is a driver of market liquidity and risk premiums. [5] define their measure of market illiquidity as the absolute deviation of the price from its fundamental value. However, they do not formulate explicitly a liquidity component in the asset price model.

The work of [2] surveys reviews the literature that studies the relationship between liquidity and asset prices. They review the theoretical literature that predicts how liquidity affects a securitys required return and discuss the empirical connection between the two. However, they do not examine the literature on liquidity and asset pricing, i.e. they do not assess the large literature on market microstructure, which studies trading mechanisms and the origins of illiquidity. This is done e.g. by [15], [14], [4] and [9].

[15] provides a detailed survey of the theoretical literature and considers the standard reference for the economic theory of market microstructure. [14] surveyed the literature on the microstructure studies, building on empirical, theoretical and experimental studies relating to markets and trading. [9] provides a detailed conceptual overview about trading, the people who trade securities and contracts, the marketplaces where they trade, and the rules that govern trading; his focus is on the practitioners not on the academic literature. [8] surveyed the studies on the microstructure regarding the microstructure factors and asset price dynamics. [4] provide a comprehensive review analyzing the price formation and trading process, interrelation between institutional structure, strategic behavior, prices and welfare. Finally, [10] provides a detailed integrated introduction to the most important models of empirical market microstructure studies. Additional liquidity references can be found in [11].

In summary, our paper although it does not provide new methodological steps regarding liquidity modeling, highlights the effect of liquidity contained in the model of [12] and [7] in the Lombard lending business in view of lending values.

2 Modeling Lombard Loans

The mathematical modeling of Lombard lending requires the analysis of essentially three components: a model for the banks exposure to the client, i.e. a description of the way the client makes use of his lending limit; a formalization of the default triggers; a model for the market value of the pledged assets.

We start introducing some notation. Consider a probability space $(\Omega, \mathcal{G}, P)$ equipped with the natural filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ of a process $V = (V_t)_{t\geq 0}$ describing the market value of the collateral over time. Further, let $X = (X_t)_{t\geq 0}$ be an $\mathbb{F}$-adapted right-continuous process describing the banks exposure to the client over time and denote by $\lambda \in (0,1]$ the *lending value*.

At the beginning of the contract, i.e. at time $t = 0$, the amount of money at clients disposal is bounded from above by λV_0. The quantity $1-\lambda$ is called the *initial* or *required haircut*, whereas the process $(V_t - X_t)/V_t$ is called the *running haircut at time* t. In other words, the initial haircut $1-\lambda$ multiplied by the initial market value of the collateral V_0 represents the excess collateral the lender wants at inception. Finally, for $s < t$ denote by $V^*_{s,t} := \sup_{s<u\leq t} V_u$ the running maximum of V over the time interval $(s,t]$. In order to simplify the notation, we write V^*_t for $V^*_{0,t}$.

2.1 Margin calls and client behavior

Suppose that the bank applies a control policy triggered by margin calls as described in the introduction. Formally, for a fixed threshold $\alpha \in (0,1)$, a margin call occurs as soon as the running haircut erosion exceeds α, i.e. as soon as $X/V > \lambda/\beta$, where $\beta := 1-(1-\lambda)\alpha > \lambda$ is called *margin call trigger*. Formally, consider the $\mathbb{F}$-stopping times $(\eta_n)_{n\geq 1}$ defined by $\eta_1 := \inf\{t > 0 \,|\, V_t/X_t < \beta/\lambda\}$ and, for $n > 1$,

$$\begin{aligned} \eta_{2n} &:= \inf\{t > \eta_{2n} \,|\, V_t/X_t > \beta/\lambda\}, \\ \eta_{2n+1} &:= \inf\{t > \eta_{2n} \,|\, V_t/X_t < \beta/\lambda\}. \end{aligned} \tag{2.1}$$

For example, assuming that the client draws up to his limit (see Assumption 2.3), we have for $t < \eta_1$ that $X_t = \lambda V^*_t$ and that η_1 is precisely is the first time t for which $V_t < V^*_t$.

Once a margin call occurs, the bank asks the client to reestablish the required haircut bringing new collateral or reducing the exposure within a imposed time of $\delta > 0$ time units. Denoting by $T > 0$ the loan maturity, a natural question that arises is: what happens if a margin call occurs within $(T-\delta, T]$? One possibility is to assume that the contract terminates at T the latest meaning that for a margin call occurring within $(T-\delta, T]$ the amount of time given to the client to readjust his position equals the remaining lifetime of the contract. Another possibility is stated in the following assumption which is assumed to hold throughout the rest of the paper.

Assumption 2.1. *If a margin call occurs within $(T-\delta, T]$, then the maturity of the contract is artificially prolonged so that the client still has δ time units to react on that margin call.*

The next step in the formalization of our model consists in specifying the lending mechanism, i.e. the way the client makes use of the lending limit given by the lending value and represented by the process X. The first assumption we make is the following.

Assumption 2.2 (Client creditworthiness). *There is a non-negative random variable τ_C such that, prior to τ_C, the obligor is willing to reduce its exposure if a margin call occurs whereas from τ_C onward he is not.*

Further, we assume in the sequel that the client's behavior is driven by the following mechanism, which is particularly in line with credit practice in Wealth Management for clients leveraging their assets portfolio using the borrowed amount for the purchase of new assets.

Assumption 2.3 (Speculative client).

(i) An obligor always draws up to his limit as long as the market value of the collateral increases and he sticks to the current exposure otherwise.

(ii) If a margin call occurs at the time η and over $[\eta, \eta+\delta)$ the required haircut is not reestablished by the movements of the collaterals market value itself, i.e. $V^*_{\eta,\eta+\delta} = V_\eta$, then the obligors exposure remains constant over $[\eta; \eta+\delta)$, i.e. $X_s = X_\eta$ for all $s \in [\eta, \eta+\delta)$.

(iii) *If the obligor reacts on a margin call occurring at time η, then he reduces the exposure to exactly reestablish the required haircut δ time units after the margin call time, i.e. $X_{\eta+\delta} = \lambda V_{\eta+\delta}$.*

The impact of Assumptions 2.2 and 2.3 on the different process involved in the modeling of the Lombard business is illustrated schematically in Figure2.1 where τ denotes the default time defined in Definition 2.2. Assumptions 2.2 and 2.3 together with Definition 2.1 allow us to determine the exposure process X explicitly.

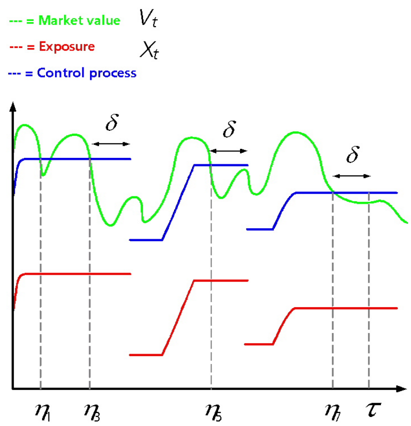

Figure 2.1: Lombard processes.

Definition 2.1 (Critical margin call times). *Consider the $\mathbb{F}$-stopping times*

$$\tilde{\tau}_n := \inf\{t > \tilde{\tau}_{n-1} + \delta \mid V^*_{t-\delta,t} < \beta V^*_{\tilde{\tau}_{n-1},t}\}, \quad n \geq 1, \tag{2.2}$$

where $\tilde{\tau}_0 := 0$. We call the random times $\tilde{\tau}_n$, $n \geq 1$ critical margin call times.

Remark 2.1.

(i) *Observe that $\{\tilde{\tau}_n\}_{n\geq 1} \subset \{\eta_{2n-1}\}_{n\geq 1}$.*

(ii) *If the obligor behaves accordingly to Assumption 2.3 and $\tau_C \geq \tilde{\tau}_n$, then the random time $\tilde{\tau}_n$ is precisely the n-th margin call time for which the haircut erosion exceeds the level α for all δ time units from $\tilde{\tau}_n$ onward.*

Using this notation, we have that the exposure process $(X_t)_{t\geq 0}$ takes the form

$$X_t = \lambda \sum_{n=1}^{\infty} V^*_{\tilde{\tau}_{n-1}} 1_{\{\tilde{\tau}_{n-1} \leq t < \tilde{\tau}_n\}} \quad \text{on} \quad \{\tau_C > t\}. \tag{2.3}$$

2.2 The default time and the Lombard loss

The next model component is given by the default time, denoted by τ, which we identify with the time at which the bank liquidates (part of) the pledged assets.

Definition 2.2 (Default time). *The default time τ of the Lombard loan is the time of the first jump time of $\tilde{N}_t := \sum_{n\geq 1} 1_{\{\tilde{\tau}_n \wedge \tau_C \leq t\}}$ given by the critical margin call times $(\tilde{\tau}_n)_{n\geq 1}$, i.e.*

$$\tau := \inf\{t \geq 0 \,|\, \tilde{N}_t \geq 1\} = \inf\{\tilde{\tau}_n \,|\, \tilde{\tau}_n \geq \tau_C \quad n \geq 1\}. \tag{2.4}$$

The interpretation behind the above definition is that $\tau - \delta$ is the last critical margin call time prior to closeout or, in other words, the first time where the client is asked to readjust his position but either he does not nor the market value of the collateral increases sufficiently to reestablish the required haircut.

Finally, assuming that it is possible to instantaneously liquidate the assets at default, we have, provided that a default occurs, that the bank incurs a loss L equal to the positive part of the difference between the exposure at default and the market value of the collateral at liquidation. By definition, we have that the the default time $\tau = \tilde{\tau}_n$ for some n and that the exposure cannot increase between $\tau - \delta$ and τ. Thus, $X_\tau = X_{\tau-\delta} = \lambda V^*_{\tilde{\tau}_{n-1},\tau-\delta} = \lambda\beta^{-1}\beta V^*_{\tilde{\tau}_{n-1},\tau-\delta} = \lambda\beta^{-1}V_{\tau-\delta}$. We obtain,

$$L = (X_\tau - V_\tau)^+ 1_{\{\tau \leq T+\delta\}} = (\lambda\beta^{-1}V_{\tau-\delta} - V_\tau)^+ 1_{\{\tau \leq T+\delta\}}. \tag{2.5}$$

Remark 2.2. *(a) If we replace Assumption 2.1 with the assumption that the Lombard contract terminates at T the latest, then L takes the form $L = (X_\tau - V_\tau)^+ 1_{\{\tau \leq T\}} + (X_T - V_T)^+ 1_{\{T < \tau \leq T+\delta\}} = (X_{\tau'} - V_{\tau'})^+ 1_{\{\tau' \leq T\}}$ where $\tau' := \tau - \delta + (T - \tau + \delta) \wedge \delta$.*

(b) We want to stress that in Definition 2.2 we did not postulate that the time τ_C is a $\mathbb{F}$-stopping time.

2.3 Neglecting the obligor specific risk component

In order to simplify our model, we can assume that the client never reacts on margin calls, i.e. that $\tau_C = 0$. In this case, we have that the default time τ occurs δ time units after the first critical margin call time $\tilde{\tau}_1$ defined in (2.2), i.e.

$$\tau = \tilde{\tau}_1 + \delta = \inf\{t \geq \delta \mid V^*_{t-\delta,t} < \beta V^*_{0,t}\} + \delta. \tag{2.6}$$

In particular, we have that τ is a $\mathbb{F}$-stopping time.

2.4 A model choice for the asset value process

The last model ingredient is given by a model for the market value process V. We restrict ourselves to the situation where the pledged assets consist of a single stock portfolio and we model V as the solution of the stochastic differential equation (SDE)

$$\begin{aligned} dV_t &= V_t(\mu dt + \sigma dB_t), \quad t \geq 0, \\ V_0 &= v_0. \end{aligned} \tag{2.7}$$

where B is a standard Brownian motion, $(\mu, \sigma) \in \mathbb{R} \times (0, \infty)$ and $v_0 > 0$. The constants μ, σ and v_0 denote respectively the drift and the volatility of V as well as the initial market value of the collateral. By Itô's Lemma, the solution of this SDE is the geometric Brownian motion $V_t = v_0 \exp((\mu - \sigma^2/2)t + \sigma B_t), \quad t \geq 0$.

3 Standard Lending Values

In principle, the lending value can be chosen as any number between zero and one. However, in order to control the Lombard risk, it is common practice to define the lending value as the largest number in $(0,1)$ such that the probability that δ time units (the length of the so called closeout period[2]) after any margin call the collateral value falls below the client exposure is at most $\epsilon > 0$. Typically ϵ is set to 1%. Assuming that after a margin call the client is not allowed anymore to draw money from the bank, it follows that the lending value is characterized by

$$P[V_{\tau_n+\delta} \leq X_{\tau_n}] = P[V_{\tau_n+\delta} \leq \lambda\beta^{-1}V_{\tau_n}] \leq \epsilon \quad \text{for all } n \geq 1.$$

In order to determine the lending value, we need to specify a model for V. We assume from now on that the asset value process V follows a geometric Brownian motion with parameters μ and σ. Since $V_{\tau_n+\delta} = V_{\tau_n}Z_\delta$ for a random variable Z_δ, which is lognormally distributed with parameters $(\mu - \sigma^2/2)\delta$ and $\sigma^2\delta$, we obtain

$$\begin{aligned} P[V_{\tau_n+\delta} \leq X_{\tau_n}] &= P[V_{\tau_n+\delta} \leq \lambda\beta^{-1}V_{\tau_n}] = P[V_{\tau_n}Z_\delta \leq \lambda\beta^{-1}V_{\tau_n}] = P[Z_\delta \leq \lambda\beta^{-1}] \\ &= \Phi\left(\frac{\log(\lambda/\beta) - (\mu - \sigma^2/2)\delta}{\sigma\sqrt{\delta}}\right). \end{aligned}$$

Thus, the probability that δ time units after any margin call the collateral value falls below the client exposure is at most ϵ if

$$\lambda \leq \beta \exp\left((\mu - \sigma^2/2)\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon)\right),$$

whence

$$\lambda \leq \frac{(1-\alpha)\exp\left((\mu - \sigma^2/2)\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon)\right)}{1 - \alpha\exp\left((\mu - \sigma^2/2)\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon)\right)}, \tag{3.1}$$

so that the right hand side of (3.1) defines the lending value.

Remark 3.1. *The $1-\epsilon$ worst case scenario for the collateral value within the closeout period conditioned on the information up to a margin call time, i.e. the lowest value reached by the process within $[\tau - \delta, \tau]$ provided that $V_{\tau-\delta} = v$ for some $v > 0$, is given by*

$$vF_{Z_\delta}^{-1}(\epsilon) = v\exp\left((\mu - \sigma^2/2)\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon)\right).$$

[2] In this context the closeout period has to be intended as the time given to the client to react to a margin call, and has not to be confused with the time needed to liquidate (part of) the collateral.

Observe that $F_{Z_\delta}^{-1}(\epsilon)$ *is different from the right hand side of* (3.1)*. Setting the lending value equal to* $F_{Z_\delta}^{-1}(\epsilon)$ *would be incorrect since this expression does not take into account that at a margin call time the* α *percentage of the haircut has already been eroded.*

4 Liquidity Adjusted Lending Values

The derivation of the lending value formula of the previous section was based on the assumption that immediate liquidation of the assets at the end of the closeout period was possible without affecting the transaction price. This assumption becomes questionable if the number of collateral units held is large as for instance in the case when it exceeds by far the average daily trading volume for the asset at hand. In this situation, the price one would receive would be significantly smaller than the one one would get for a much smaller transaction, or, in the extreme case, it could be even impossible to liquidate the entire position. The effects of the transaction size on the transaction price are commonly referred to as one aspect of liquidity risk, so that the lending value determination for a large collateral size must take into account the potential liquidity costs. In the sequel, we will assume that liquidation of arbitrary large amounts of the collateral is always possible, so that liquidity risk reduces to a decrease in the amount earned by selling the desired amount of collateral.

The idea of [7] and [12] is to include liquidity costs in the collateral market value directly by means of an exponential supply curve taking into account the transaction size. Formally, for $x \in R$ denoting the order flow bought ($x > 0$) or sold ($x < 0$) by the trader, where the zeroth order ($x = 0$) corresponds to the *marginal trade*[3], the price per share payed/rceived for the order flow x, denoted by $V_t(x)$, is assumed to be given by

$$V_t(x) = e^{\gamma x} V_t, \tag{4.1}$$

[3]That is to say the trade of infinitesimally small quantities of the collateral. Observe that x should be an integer in principle. However, the number of outstanding shares is typically very large, (of the order 10^6 or more) so that trading a single share can be interpreted as a marginal trade, and the classical price process V can be seen as the marginal trade price process.

where V_t is the classical collateral value per share, i.e. a geometric Brownian motion, and $\gamma \geq 0$ is a constant, which we call the *liquidity parameter*. The term $e^{\gamma x}$ captures the quantity impact on the asset price and the idea behind (4.1) is that the more liquid an asset, the smaller γ and thus the more horizontal its supply curve. In other words, the less significant the trade size the more the traded price equals the classical market value.

Suppose now that at some time t we want to liquidate $\theta \in [0,1]$ percent of a (bulk) position of size $x > 0$ we are holding. [12] show that the position value at time t including liquidity costs is the classical value less the liquidity costs L_t, which can be expressed as

$$L_t = -\theta x(V_t(-\theta x) - V_t) = -\theta x(e^{-\gamma\theta x} - 1)V_t \geq 0, \tag{4.2}$$

where the last equality follows from (4.1). Denoting the classical position value without liquidity costs by $U_t := xV_t$ and the one including liquidity costs by $\tilde{U}_t$, it follows that

$$\tilde{U}_t = U_t - L_t = xV_t + \theta x(e^{-\gamma\theta x} - 1)V_t = (1 - \theta + \theta e^{-\gamma\theta x})U_t \leq U_t. \tag{4.3}$$

Note that the decline in value is greater when γ is larger or when the percent of the position θ that is liquidated is larger.

We can now derive a lending value formula using the same ideas as before but including in the computation the transaction size, i.e. replacing V by $\tilde{U}$. It is now crucial to observe that, in our situation, the minimal liquidation percentage θ needed to avoid a loss (provided that this is possible) is random and is known only at the liquidation time τ, which is of the form $\tau_n + \delta$ for some $n \geq 1$. Indeed, at liquidation, the ratio between the (reduced) exposure $X_\tau - \theta V_\tau$, and the market value of the retained collateral $(1-\theta)V_\tau$ should, if possible, be equal to the lending value, so that θ must satisfy

$$\frac{X_\tau - \theta V_\tau}{(1-\theta)V_\tau} = \lambda.$$

Since $X_\tau - \theta V_\tau = X_{\tau-\delta} - \theta V_\tau = \lambda\beta^{-1}V_{\tau-\delta} - \theta Z_\delta V_{\tau-\delta}$, and because θ must be within $[0,1]$, we obtain

$$\theta = \frac{\lambda}{1-\lambda}\left(\frac{1}{\beta Z_\delta} - 1\right) 1_{\{\lambda\beta^{-1} < Z_\delta \leq \beta^{-1}\}} + 1_{\{Z_\delta < \beta^{-1}\}}, \tag{4.4}$$

which is a $\mathcal{F}_\tau$-measurable random variable (where $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ denotes the natural filtration of V) meaning that only at time τ we know how much to liquidate.

In order to keep the model tractable, we take a conservative approach by assuming that we liquidate the entire position, i.e. we set $\theta = 1$, so that the lending value λ is determined from

$$\epsilon \geq P[e^{-\gamma x} x V_\tau \leq \lambda x V^*_{\tau-\delta}] = P[e^{-\gamma x} Z_\delta \leq \lambda \beta^{-1}], \tag{4.5}$$

where, as before, Z_δ is lognormally distributed with parameters $(\mu - \sigma^2/2)\delta$ and $\sigma^2\delta$. Rearranging the terms, we get

$$\lambda \leq \beta \exp\left(-\gamma x + (\mu - \sigma^2/2)\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon)\right),$$

and thus

$$\lambda \leq \frac{(1-\alpha)\exp\left(-\gamma x + (\mu - \sigma^2/2)\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon)\right)}{1 - \alpha\exp\left(-\gamma x + (\mu - \sigma^2/2)\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon)\right)}. \tag{4.6}$$

As before the right hand side of (4.6) defines the lending value.

Remark 4.1.

(a) *The difference between the liquidity adjusted lending value formula* (4.6) *and the standard one* (3.1) *is given by the additional term* $-\gamma x$ *in the exponential.*

(b) *The exponential supply curve introduced above is just one possible choice among many different ones. For example, [12] propose a linear supply curve, i.e.* $V_t(x) = (1 + \gamma x)V_t$ *for some* $\gamma \geq 0$ *which, up to a first order approximation, equals the exponential supply curve. In this case, the liquidity costs are given by* $L_t = \gamma x^2 \theta^2 V_t \geq 0$*, i.e. by* $\gamma x^2 V_t$ *for* $\theta = 1$*. The lending value formula becomes*

$$\lambda = \left[\frac{(1-\alpha)(1-\gamma x)\exp\left((\mu - \sigma^2/2)\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon)\right)}{1 - \alpha(1-\gamma x)\exp\left((\mu - \sigma^2/2)\delta + \sigma\sqrt{\delta}\Phi^{-1}(\epsilon)\right)}\right]^+. \tag{4.7}$$

5 Estimation of the Liquidity Parameter

Daily estimates for γ can be obtained by means of a simple regression starting from tick data. Assume that in a given day we observe the trading sequence (t_i, x_i, v_i), $i = 1, \ldots, n$; i.e. the first transaction during that day occurs at time t_1, has size x_1, is executed at a price v_1 and is followed by another transaction at time t_2 having size x_2, price v_2 and so on. Because of (4.1), and since $V_t = V_0 \exp((\mu - \sigma^2/2)t + \sigma B_t)$ for some standard P-Brownian motion B, we have that

$$\begin{aligned}\log\left(\frac{v_{i+1}}{v_i}\right) &= \log\left(\frac{V_{t_{i+1}}(x_{i+1})}{V_{t_i}(x_i)}\right) \\ &= \gamma(x_{i+1} - x_i) + (\mu - \sigma^2/2)(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i} \cdot \epsilon_i,\end{aligned} \tag{5.1}$$

where $\epsilon_1, \ldots, \epsilon_n$ are iid standard normal random variables. The above expression can be rewritten as

$$y_i := \frac{\log(v_{i+1}/v_i)}{\sqrt{t_{i+1} - t_i}} = \gamma \cdot \frac{x_{i+1} - x_i}{\sqrt{t_{i+1} - t_i}} + (\mu - \sigma^2/2)\sqrt{t_{i+1} - t_i} + \sigma\epsilon_i, \tag{5.2}$$

i.e. the parameter γ can be estimated by means of a linear regression for the response variables y_i and the predictors $w_i := (x_{i+1} - x_i)/\sqrt{t_{i+1} - t_i}$ and $z_i := \sqrt{t_{i+1} - t_i}$, $i = 1, \ldots, n-1$. Observe that the regression implicitly yields an estimate for μ through an estimate for σ.

One problem related to the implementation of (5.2) is due to the fact that, usually, intraday data are not signed, i.e. we do not know whether a transaction of size x occurring at time t was a buy or a sell since the data only report the absolute value of the transaction size. Nevertheless, intraday data can be classified into market buy/sell orders using the so called *tick test* (see [13] for a discussion). A trade is referred to as an *uptick*, a *downtick*, or a *zerotick* depending on whether the price of the previous trade was lower, higher, or the same than the price of the trade at hand. Zeroticks are further classified into *zero-upticks* and *zero-downticks* according to the last price change, i.e. a zerotick is a zero-uptick if the last price change was an uptick, and it is a zero-downtick otherwise. The tick test classifies a trade as market buy order if it is either an uptick or a zero-uptick. The other trades are classified as sell orders[4].

[4] As it can be found in [13], by convention, the first trade of a new issue is deemed an

A second issue arising during the estimation procedure is given by the fact that trades are often executed simultaneously, i.e. there are series of tick data having the same trading time. This phenomenon is due to the very complicated nature of stock exchanges, but essentially can be reduced to the fact that buy orders have to match sell orders on the order book. For more details about the mechanisms governing stock exchanges see e.g. the SWX webpage or [16]. The problem in this case is that both the response variables y_i and the predictors w_i, z_i are undefined. For each of these series of simultaneous trades, we will assume therefore that the relevant time step $t_{i+1}-t_i$ in (5.2) is given by the last non zero time step. This is motivated by the fact that a trade within a series of simultaneous trades, and in particular its price and size, is influenced (as any other trade in general) by the market situation at the time immediately before that trade[5]. Formally, consider the situation where a trade at time s, is followed by $k \geq 2$ simultaneous trades at time $t > s$ which are followed by some other trade(s) at time $u > t$:

$$\begin{array}{rll} t_i = s & x_i & v_i \\ t_{i+1} = t & x_{i+1} & v_{i+1} \\ \cdots & \cdots & \cdots \\ t_{i+k} = t & x_{i+k} & v_{i+k} \\ t_{i+k+1} = u & x_{i+k+1} & v_{i+k+1} \\ \cdots & \cdots & \cdots \end{array}$$

In this case, the relevant time step for all ticks $i+1, \ldots, i+k$ entering (5.2) is $t - s > 0$. Finally, we censor any series of simultaneous trades occurring at the beginning of day by taking only the last trade of that series. In other words, if a trading day starts with k simultaneous trades occurring at the times $t_1 = \cdots = t_{k-1} = t_k < t_{k+1}$, $k \geq 2$, then the $k-1$ ticks occurring at the times

uptick, perhaps because there is a usually a ceremonial buy transaction by the president of the company. However, the trade history data we will use for the estimation of γ will not go back to the first trade. Thus, we will adopt the convention that the first trade of our data history is an uptick, i.e. a buy.

[5]Note that it would have been incorrect to collapse such a series into a single transaction since the resulting (cumulated) size would distort the results. As an alternative to this procedure, we could fit a generalized linear model starting from (5.1).

$t_1, \ldots, t_{k-1}$ are excluded.

Remark 5.1. *As stated by [7], their method utilizes transaction prices instead of bid-ask spreads for three reasons. First, trades may be executed "inside the spread" which implies that quotes potentially overestimate liquidity costs. Second, the bid-ask spread is a commitment to a specific volume which may change according to market conditions. Third, quotes may be "stale" for infrequently traded stocks.*

6 Empirical Results

In this section we investigate the impact of liquidity on lending values for the 15 selected stocks of Table 6 traded at the SWX. For each stock, we collected five years of tick data over the period from 06/01/2001 to 05/31/2006. The first group of five stocks (ABB, UBS, Swiss Re, Nestle, and Kudelski) are commonly referred to as highly liquid equities, whereas the second group (Lonza Grp,...,Charles Voegele), and the third one (Kaba Hldg,..., Lindt & Sprüngli) are usually seen as medium liquid, respectively illiquid assets. Of course, in this context, we are intentionally vague about the definition of liquidity so that some people might disagree about our classification. However, as we will see next the intuitive rank ordering of Table 6 is almost perfectly reflected in the estimated liquidity parameters.

The results for the daily γ estimates are collected in Tables A.1 to A.5, which contain the summary statistics for the daily γ estimates generated by (5.2). The first three rows of each table contain the median or the mean of the daily γ values and the corresponding 95%-confidence intervals boundaries, whereas row four to six detail some percentiles of the estimated γ values. The fact that $\widehat{\gamma}$ varies over time, although γ was assumed to be constant, should not surprise since this is definitely due to the inappropriateness of the geometric Brownian motion as a model for the stock price. Indeed, one cannot expect constant γ estimates if already the basic model without the liquidity term $e^{\gamma x}$ is not able to produce constant estimates for μ and σ.

Row seven displays the percentage of days for which a non-negative estimate was obtained. We see that there are few days (13%–20% of the cases for

Table 6.1: Summary table for the selected stocks as of June 1, 2006. The reported volume is the average daily volume over May 2006, the market capitalization (MC) expressed in Swiss Francs, and the last column details the daily average number of transactions.

Name	Bid	Ask	Volume	MC	#
ABB (ABBN)	15.20	15.25	1.7898×10^{7}	3.14680×10^{10}	2112
UBS (UBSN)	138.30	138.40	3.479×10^{6}	1.51930×10^{11}	3205
Swiss Re (RUKN)	84.80	84.85	1.951×10^{6}	2.74730×10^{10}	2086
Nestle (NESN)	362.25	362.50	1.182×10^{6}	1.48091×10^{11}	2750
Kudelski (KUD)	31.25	31.30	5.306×10^{5}	1.51300×10^{9}	793
Lonza Grp (LONN)	82.00	82.15	3.272×10^{5}	4.26500×10^{9}	690
Swatch (UHR)	194.60	194.70	1.650×10^{5}	1.23830×10^{10}	792
Swisscom (SCMN)	394.25	394.50	1.557×10^{5}	2.45000×10^{10}	947
SERONO (SEO)	768.50	769.50	5.900×10^{4}	8.53900×10^{9}	945
Charles Voegele (VCH)	96.50	97.00	2.862×10^{4}	8.84000×10^{8}	155
Kaba (KABN)	307.25	308.00	1.162×10^{4}	1.08600×10^{9}	119
OZ Holding (OZI)	94.05	94.70	9.821×10^{3}	4.86000×10^{8}	21
Publigroup (PUBN)	398.50	399.00	3.118×10^{3}	1.00300×10^{9}	77
Bank Sarasin (BSAN)	3257.00	3275.00	6.880×10^{2}	2.12200×10^{9}	37
Lindt & Sprüngli (LISN)	23695.00	23800.00	1.250×10^{2}	5.18500×10^{9}	33

illiquid titles and almost none for the very liquid ones) for which the estimated γ value is negative which, according to the model, should not be the case. Due to their sporadic occurrence, the reasons behind negative γ values have not been investigated in detail.

The last four rows record the percentages of days for which various estimates are significant at a 95%-confidence level. Specifically, we distinguish between the significance of the full model (γ and μ) and the one of the γ estimate in isolation from μ, where the two subcases of negative and non-negative γ values are also considered. For example, we see from column one and row nine in Table A.1 that, at a 95%-confidence level, the test hypothesis $H_0 : \gamma = 0$ for LISN is rejected in favor of the alternative $H_1 : \gamma \neq 0$ in 46.65% of the cases (days) meaning that the γ predictor has an explanation power in 571 out of 1224 days[6]. For the other stocks, the percentage of days for which $\widehat{\gamma}$ is significant ranges from 36.81% (OZI) to almost 100% for highly liquid stocks

[6]Observe that this is the results of a two-sided t-test and that the result of a one-sided t-test (due to the constraint $\gamma \geq 0$) would produce an even higher figure.

like UBSN or RUKN. With the exception of LISN, BSAN, and OZI belonging to the group of more illiquid stocks, $\widehat{\gamma}$ is significant in more than 55% of the cases.

On the contrary, although the results are not reproduced here, the estimate for $\eta := \mu - \sigma^2/2$, and thus for μ, are not significant in most of the cases. This means that the null-hypothesis $H_0 : \eta = 0$ cannot be rejected in many cases, i.e. that the second predictor $\sqrt{t_{i+1} - t_i}$ in (5.2) has a poor explanation power for the observed data. This is however of secondary importance since we are mainly interested in $\widehat{\gamma}$, but can be used to support the choice $\mu = \sigma^2/2$ for the computation of lending values in Section 7.

As it can be seen from Figures B.1 to B.4, the γ estimates appear to be a bit "noisy" whereas, as expected, the 30–, the 60–days moving average time series, and a smoothed average process (see Appendix B for the details), show a much more regular behavior. We also observe that the number of negative values for any of the averages is much smaller than the one we have for daily γ estimates. For UHR, RUKN, UBSN, and especially for LONN, there seems to exist time points at which some event affected the liquidity parameter substantially. Looking at the data, we see that on April 3, 2003 there was a stock split for LONN with a ratio 10:1, which, intuitively, increased the stock liquidity. This event is immediately reflected in the γ estimate, which, exactly on the same day, approximately decreases by a factor 10. The same happened for UHR, RUKN, and UBSN on August 10, August 17, and July 16, 2001, with split ratios of 10, 20, and 3 to 1, respectively. Again, the γ estimates decrease exactly at this dates approximately by a factor of 10, 20, and 3, respectively. The latter would suggest that after a stock split the liquidity parameter must be divided by the split ratio. Detailed information about the stock splits can be found e.g. at `www.eurexchange.com`.

Finally, the bottom right picture of Figure B.4 combines the smoothed averages for all stocks. With a few exceptions like UBSN and ABBN, or LONN and SCMN, we see that the curves do not cross each other, at least over sufficiently short time periods. In particular, highly liquid stocks (i.e. the ones having small (averaged) γ values tend to remain highly liquid over the entire sample period. The same can be stated for illiquid assets like LISN or OZI. Finally, since most of the curves tend to show a slight downward trend, we can conclude that, besides the effects induced by stock splits, there has been a moderate general trend towards a more liquid market.

To summarize, it seems that the proposed model is able to differentiate between different degrees of liquidity because the $\widehat{\gamma}$ smooth average is of the order 10^{-7} for highly liquid stocks like ABBN or UBS, 10^{-6} for medium liquid titles like UHR or KUD, and 10^{-4} for very illiquid assets like BSAN or LISN. Additionally, although not reproduced here, the corresponding (smoothed) 95%-confidence intervals do not usually overlap, at least for stocks belonging to different liquidity groups and for not too long time periods, implying that the estimates can be seen as significantly different from each other (bottom right picture of Figure B.4).

6.1 The financial crisis

We extend the results obtained for ABB and UBS to the period 2006 – 2011 in order to see the effects of the financial crisis of 2007–2008 on the liquidity parameter. Below in Figure 6.1 we have the results relative to the γ estimates for ABB and and in Figure 6.2 the results for UBS.

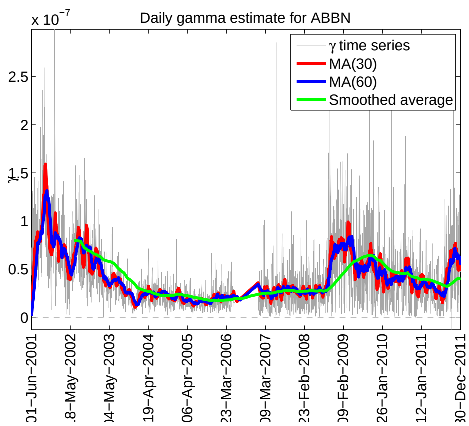

Figure 6.1: Displayed are the updated results for ABB over 2001 – 2011.

We see in Figures 6.1 and 6.2 that for both tickers ABB and UBS there is an increase in the smoothed average values (green line) over the period starting around the second half of 2008. Further, again for both stocks, the smoothed γ average decreases around the end of 2011 and reaches more or less the same level it had in 2007.

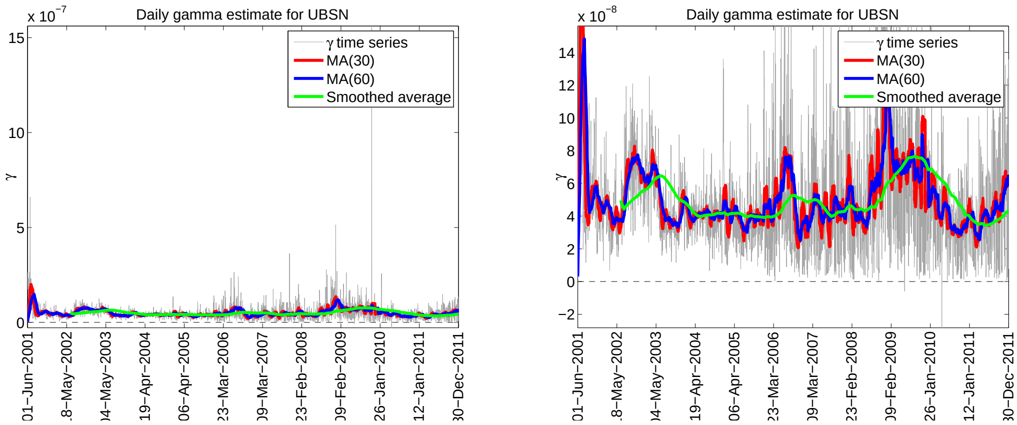

Figure 6.2: Displayed are the updated results for UBS over 2001 – 2011. The picture on the right is simply the same as the picture on the left but with a different scale on the y-axis.

The interpretation of this phenomenon is the following: the 2008 financial crisis was catched by the model more or less instantly.

7 Liquidity Effects on Lending Values

We illustrate the impact of the liquidity parameter γ on the lending value at hand of two examples, where the collateral is given by UBSN and LISN, which represent two extremes of our liquidity spectrum. The lending values computations are based on the assumption that the drift parameter μ of the underlying GBM equals $\sigma^2/2$. The volatilities are assumed here to be 15% for UBSN, and 21% for LISN. Further, we assume in this section that the minimum margin erosion α triggering a margin call is 25%, that the closeout period δ equals 10 days for a 250 days long year, and that the confidence level ϵ for lending values is 1%. Finally, we take the mean of the γ smoothed average as an input in the lending value formula, i.e. 4.672949×10^{-8} for UBSN, and 3.985406×10^{-4} for LISN. Table 7 shows the impact of the transaction size x on the lending values.

Since the stock volatilities are comparable, it is not surprisingly that the impact on the lending value is primarily governed by the product γx, and that

Table 7.1: Effects of the liquidity parameter on the lending value. The holding volume figure in Swiss Francs is given by the product of x and the mid price derived from columns two and three of Table 6. Observe that the standard lending value is a special case of the liquidity adjusted one for $x = 0$.

	LISN		UBSN	
x (# stocks)	100	600	100'000	1'000'000
Holding Vol. (CHF)	2'374'750	14'248'500	13'835'000	138'350'000
Holding Vol. (%MC)	0.0458%	0.2748%	0.00911%	0.0911%
Holding Vol. (ADTV mult.)	0.80	4.80	0.029	0.29
γx	3.99×10^{-2}	2.39×10^{-1}	4.67×10^{-3}	4.67×10^{-2}
λ liq.adj.	83.65%	65.25%	90.61%	85.81%
λ standard	88.05%	88.05%	91.22%	91.22%

the decrease in the lending values is almost the same for both stocks as long as γx is more or less the same (first and fourth column of Table 7). It is also interesting to notice that a massive reduction of the LISN lending value (from 88.05% to 65.25%) occurs when $x = 600$, i.e. approximative five times the ADTV, which defines in this case the so called *bulk risk indicator*. The latter is defined as the minimum between the number of stocks corresponding to five times the ADTV, and the number of stocks corresponding to 3% of the market capitalization. Usually, five times the ADTV is relevant for determining the bulk risk indicator. The question at hand is now whether the bulk risk indicator is able to distinguish between situations where liquidity risk is more or less relevant. We plot in Figure 7.1 the lending value curves as function of the number of stocks (left picture), and of the exposure (right). The circles on the curves correspond to the bulk risk indicator levels.

With the only exception of OZI, we see that within each of the three liquidity groups the circle heights, i.e. the lending values corresponding to the bulk risk indicator, are more or less the same. Again, for the more illiquid stocks (LISN, BSAN, PUBN, KABN) and for exposures equal to the bulk risk indicator, the lending value reduces significantly from about 85% to approximately 50%. Very small lending values are obtained instead for the other titles as soon as the size approximates the bulk risk indicator. If the latter would be a good indicator for liquidity risk, then one would expect the circles in Figure 7.1

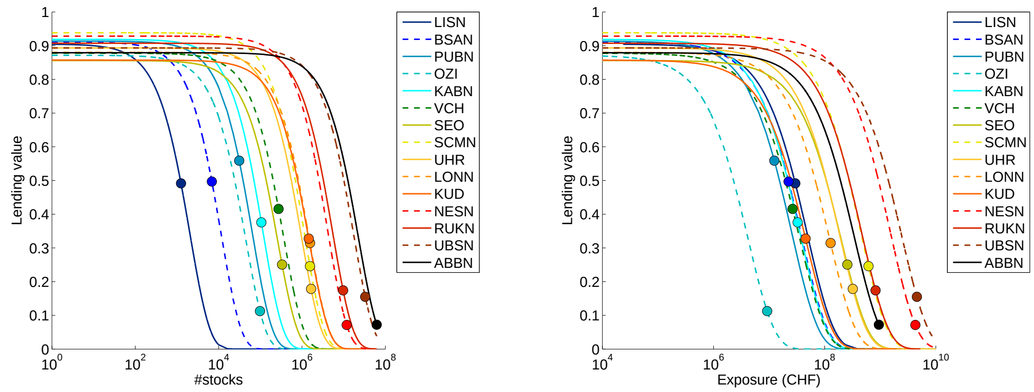

Figure 7.1: Lending value curves as a function of the number of stocks, and of the exposure. The circles on the curves are the lending values obtained for a position size equal to the bulk risk indicator.

to be at least aligned on the same horizontal line. Indeed, if we believe that the liquidity parameter, whence the liquidity adjusted lending value, properly reflects liquidity risk, then we should get the same lending value for the same amount of risk[7]. It follows that the current lending value computations, where the standard lending value is taken, i.e. the size effect is neglected, and liquidity risk is somehow controlled by not entering transactions exceeding the bulk risk indicator, becomes questionable.

8 Implementation

The major problem of the approach based on the (exponential) supply curve that arises as soon as the methodology has to be applied to a large number of stocks is that the estimation of the γ parameters requires a huge amount of (very expensive) data, so that a large scale implementation of the methodology becomes a formidable task. This problem can be solved observing that there is a clear relationship between the smoothed γ averages and the ADTV, and exploiting the fact that the latter is a quantity which is easily available. Indeed, performing a linear regression for the 10-th logarithm of

[7]The effect of different volatilities on the lending values is negligible compared to the size effect, at least for the selected stocks. Assuming that the stocks all have the same volatility produces very similar pictures, which, for the sake of brevity, have not been included here.

the smoothed γ averages as a function of the 10-th logarithm of ADTV (see Table 6, column four) yields the results illustrated in Figure 8.1. The estimated intercept is $\widehat{a} = -1.87096$, and the estimated slope equals $\widehat{b} = -0.794554$. The R^2-statistic, the F-statistic, the P-value, and the estimate for the error variance are respectively 0.97718, 556.671, 4.677814×10^{-12}, and 0.0324404, meaning that a linear model seems to be appropriate in this case. It follows that a good approximation for the (to be estimated and unknown) γ parameter can be obtained from

$$\widehat{\gamma} \approx 10^{\widehat{a}} \cdot \text{ADTV}^{\widehat{b}},$$

i.e. $\widehat{\gamma}$ can be accurately predicted from the average daily trading volume.

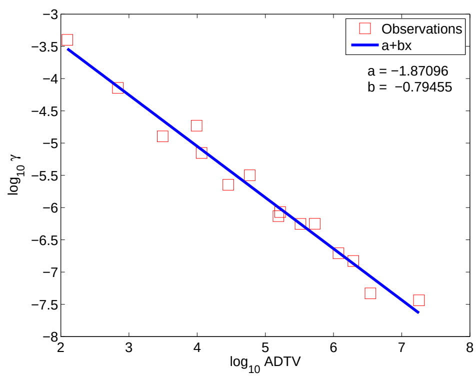

Figure 8.1: Linear regression output.

9 Conclusions

The proposed model relies on a single new parameter (the liquidity parameter γ), which has an intuitive and clear interpretation. Both the liquidity adjusted lending value formula and the estimation of γ have a sound theoretical basis. Moreover, despite its simplicity, the model is able to differentiate between different degrees of liquidity, and the effect of the liquidity parameter on the lending value seems to be appropriate for the transaction size at hand. We want to emphasize that the lending values resulting from the liquidity adjusted lending value formula have been obtained without any ad hoc adjustment. Finally, the liquidity parameter can be derived easily from the

ADTV based on the γ-estimates obtained for a reference set of stocks, so that the implementation of the liquidity adjusted lending value formula becomes straightforward. Moreover, averages of the daily γ estimates show a stable behavior over time, so that, the linear regression parameters have not to be updated frequently. Finally, our model was able to capture the financial crisis of 2007 – 2008 as shown in Figures 6.1 and 6.2.

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A Tables

Each of the tables in this section contains the following information. Rows one to six detail the median of the (daily) lower bound of a 95%-confidence interval for the parameter $\widehat{\gamma}$, the mean of the daily $\widehat{\gamma}$, the median of the (daily) upper bound of a 95%-confidence interval for the parameter $\widehat{\gamma}$, as well as the 25%-, 50%-, and 75%-percentile of the daily $\widehat{\gamma}$ values. We excluded from the computations those days yielding a negative $\widehat{\gamma}$. The 7-th row, shows the percentage of days (over the sample period) for which a non-negative $\widehat{\gamma}$ was produced. Row number 8 records the percentage of days for which the full model estimates (i.e. both $\widehat{\gamma}$ and $\hat{\eta}$, where $\eta := \mu - \sigma^2/2$) are significant at a 95%-confidence level. Rows 9 to 11 display similar information.

Table A.1: Summary statistics for LISN, BSAN, and PUBN.

	LISN	BSAN	PUBN
95%-lower bound (median)	3.21×10^{-6}	-2.83×10^{-6}	1.28×10^{-6}
mean	4.73×10^{-4}	9.59×10^{-5}	1.68×10^{-5}
95%-upper bound (median)	5.09×10^{-4}	9.46×10^{-5}	1.60×10^{-5}
25%-percentile	8.71×10^{-5}	1.26×10^{-5}	3.22×10^{-6}
50%-percentile	2.44×10^{-4}	4.11×10^{-5}	8.47×10^{-6}
75%-percentile	5.59×10^{-4}	1.12×10^{-4}	2.01×10^{-5}
days (%) : $\gamma \geq 0$	87.42%	82.71%	88.18%
days (%) : full model significant (@95%)	44.20%	37.66%	56.70%
days (%) : γ significant (@95%)	46.65%	40.07%	56.30%
days (%) : γ significant (@95%) and $\gamma \geq 0$	44.61%	36.91%	53.13%
days (%) : γ significant (@95%) and $\gamma < 0$	2.04%	3.16%	3.17%

Table A.2: Summary statistics for OZI, KABN, and VCH.

	OZI	KABN	VCH
95%-lower bound (median)	-5.20×10^{-7}	1.05×10^{-6}	3.81×10^{-7}
mean	2.47×10^{-5}	8.24×10^{-6}	2.75×10^{-6}
95%-upper bound (median)	2.03×10^{-5}	7.77×10^{-6}	2.73×10^{-6}
25%-percentile	1.76×10^{-6}	1.82×10^{-6}	6.64×10^{-7}
50%-percentile	7.26×10^{-6}	4.25×10^{-6}	1.50×10^{-6}
75%-percentile	2.33×10^{-5}	9.71×10^{-6}	3.20×10^{-6}
days (%) : $\gamma \geq 0$	80.47%	89.04%	91.12%
days (%) : full model significant (@95%)	37.22%	59.57%	61.06%
days (%) : γ significant (@95%)	36.81%	61.80%	62.25%
days (%) : γ significant (@95%) and $\gamma \geq 0$	33.64%	59.57%	59.87%
days (%) : γ significant (@95%) and $\gamma < 0$	3.17%	2.22%	2.38%

Table A.3: Summary statistics for SEO, SCMN, and UHR.

	SEO	SCMN	UHR
95%-lower bound (median)	1.96×10^{-6}	4.72×10^{-7}	5.06×10^{-7}
mean	3.18×10^{-6}	7.00×10^{-7}	9.83×10^{-7}
95%-upper bound (median)	3.22×10^{-6}	7.55×10^{-7}	8.31×10^{-7}
25%-percentile	1.54×10^{-6}	3.78×10^{-7}	4.17×10^{-7}
50%-percentile	2.57×10^{-6}	6.12×10^{-7}	6.62×10^{-7}
75%-percentile	4.34×10^{-6}	9.48×10^{-7}	1.10×10^{-6}
days (%) : $\gamma \geq 0$	99.45%	99.92%	99.68%
days (%) : full model significant (@95%)	98.18%	98.89%	98.18%
days (%) : γ significant (@95%)	98.81%	99.13%	98.42%
days (%) : γ significant (@95%) and $\gamma \geq 0$	98.42%	99.13%	98.34%
days (%) : γ significant (@95%) and $\gamma < 0$	0.40%	0.00%	0.08%

Table A.4: Summary statistics for LONN, KUD, and NESN.

	LONN	KUD	NESN
95%-lower bound (median)	2.17×10^{-7}	3.27×10^{-7}	1.48×10^{-7}
mean	8.21×10^{-7}	5.84×10^{-7}	1.93×10^{-7}
95%-upper bound (median)	3.96×10^{-7}	5.98×10^{-7}	2.01×10^{-7}
25%-percentile	1.85×10^{-7}	2.70×10^{-7}	1.08×10^{-7}
50%-percentile	3.07×10^{-7}	4.53×10^{-7}	1.74×10^{-7}
75%-percentile	5.36×10^{-7}	7.91×10^{-7}	2.53×10^{-7}
days (%) : $\gamma \geq 0$	99.13%	99.13%	100.00%
days (%) : full model significant (@95%)	94.06%	96.35%	100.00%
days (%) : γ significant (@95%)	95.01%	96.91%	100.00%
days (%) : γ significant (@95%) and $\gamma \geq 0$	95.01%	96.59%	100.00%
days (%) : γ significant (@95%) and $\gamma < 0$	0.00%	0.32%	0.00%

Table A.5: Summary statistics for RUKN, UBSN, and ABBN.

	RUKN	UBSN	ABBN
95%-lower bound (median)	1.04×10^{-7}	3.68×10^{-8}	2.40×10^{-8}
mean	2.11×10^{-7}	4.88×10^{-8}	3.97×10^{-8}
95%-upper bound (median)	1.41×10^{-7}	4.76×10^{-8}	3.23×10^{-8}
25%-percentile	8.20×10^{-8}	3.19×10^{-8}	1.62×10^{-8}
50%-percentile	1.23×10^{-7}	4.24×10^{-8}	2.80×10^{-8}
75%-percentile	1.83×10^{-7}	5.73×10^{-8}	5.19×10^{-8}
days (%) : $\gamma \geq 0$	100.00%	100.00%	100.00%
days (%) : full model significant (@95%)	100.00%	100.00%	99.84%
days (%) : γ significant (@95%)	100.00%	100.00%	99.84%
days (%) : γ significant (@95%) and $\gamma \geq 0$	100.00%	100.00%	99.84%
days (%) : γ significant (@95%) and $\gamma < 0$	0.00%	0.00%	0.00%

B Figures

This section illustrates graphically the results obtained for the parameter γ. Beyond the daily $\widehat{\gamma}$ values (grey line), the red, and the blue line show the behavior of a 30- and of a 60-days moving averages of $\widehat{\gamma}$. The green line is a smoothed average of the daily $\widehat{\gamma}$, which is defined as follows. For a fixed time, consider the $\widehat{\gamma}$ monthly averages over the last 12 months, denoted by $\overline{\gamma}_1, \dots, \overline{\gamma}_{12}$. The smoothed average is then defined as $(1/6)\sum_{i=0}^{5} \overline{\gamma}_{(6+i)}$, i.e. it is the arithmetic average of the 6 highest monthly γ averages.

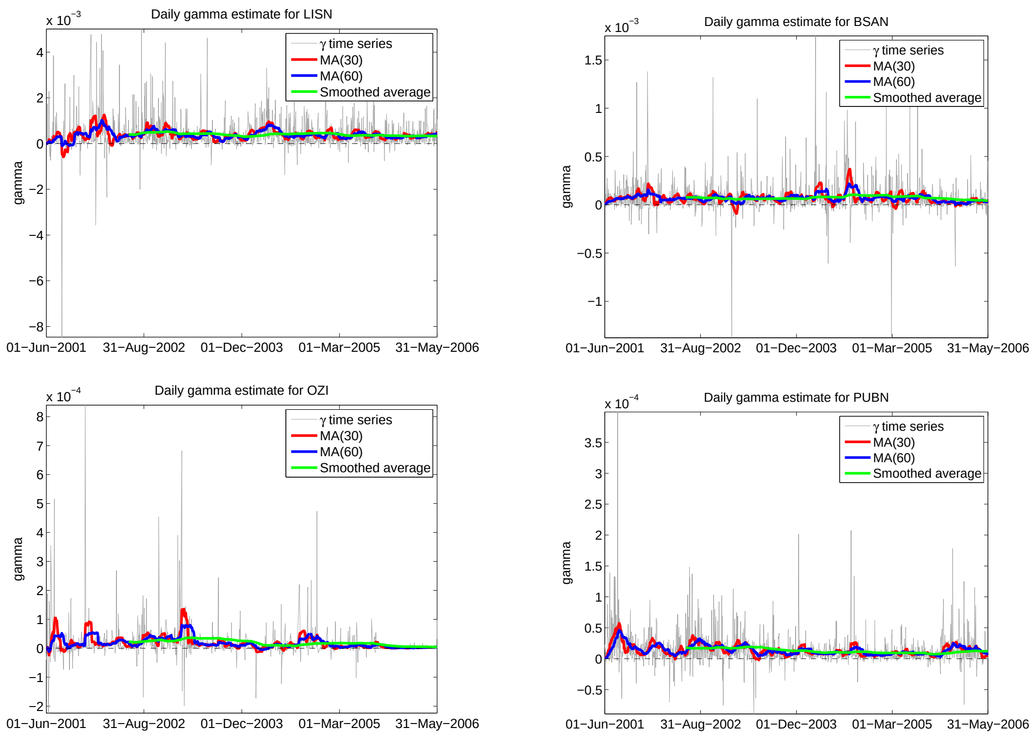

Figure B.1: Daily γ estimates for LISN, BSAN, OZI, and PUBN.

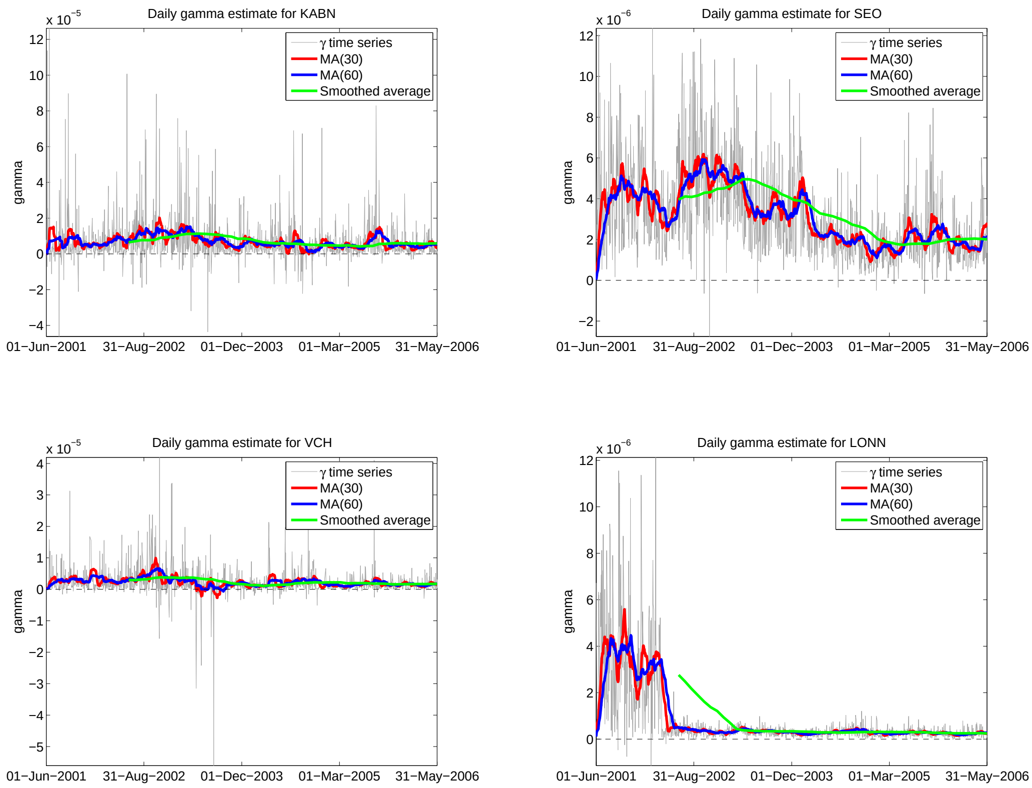

Figure B.2: Daily γ estimates for KABN, SEO, VCH, and LONN.

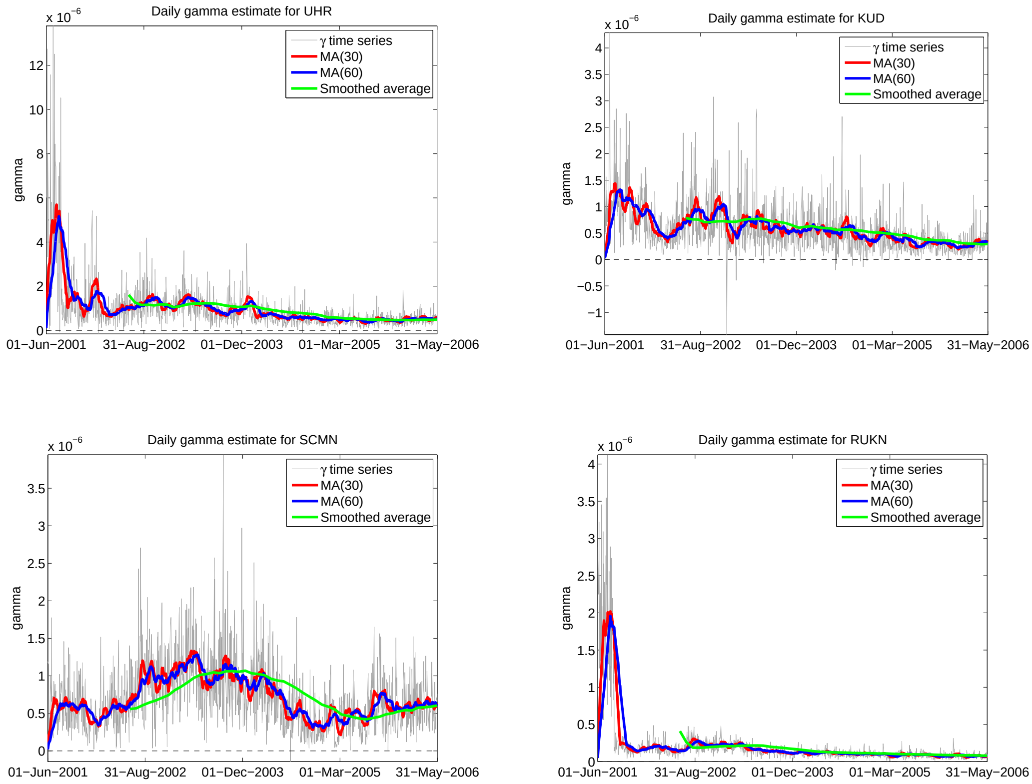

Figure B.3: Daily γ estimates for UHR, KUD, SCMN, and RUKN.

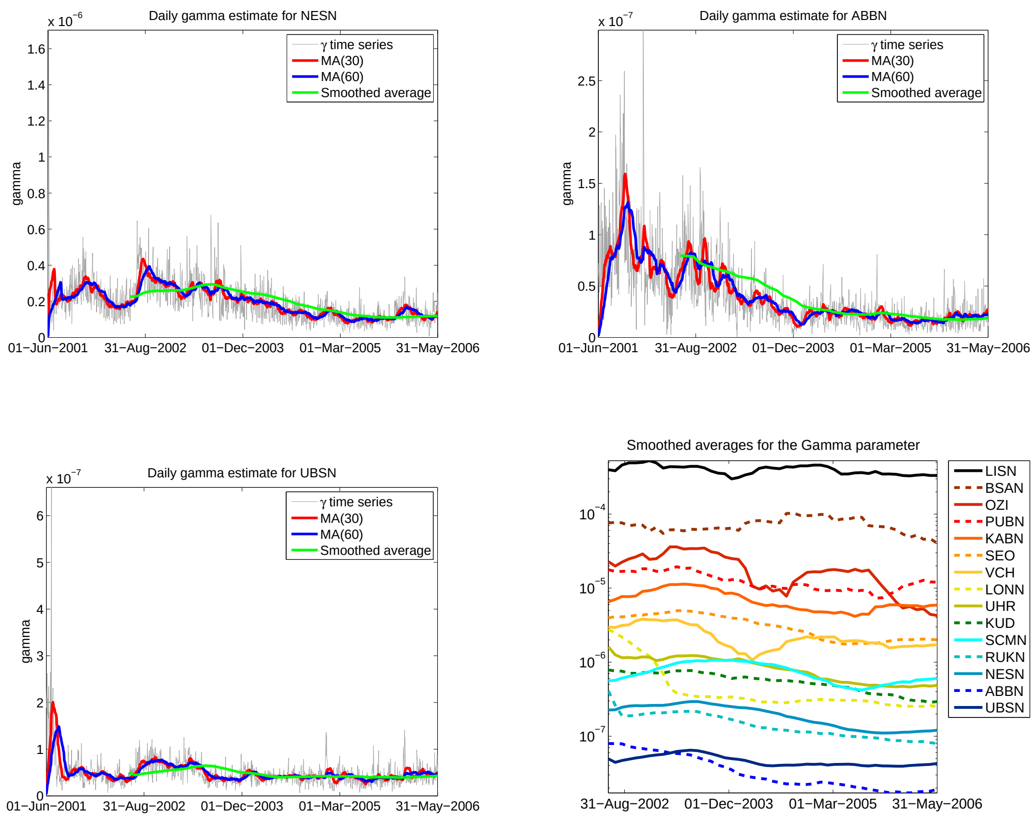

Figure B.4: Daily γ estimates for NESN, ABBN, UBS. The bottom right picture shows the smoothed averages for all selected stocks together.

C New Figures

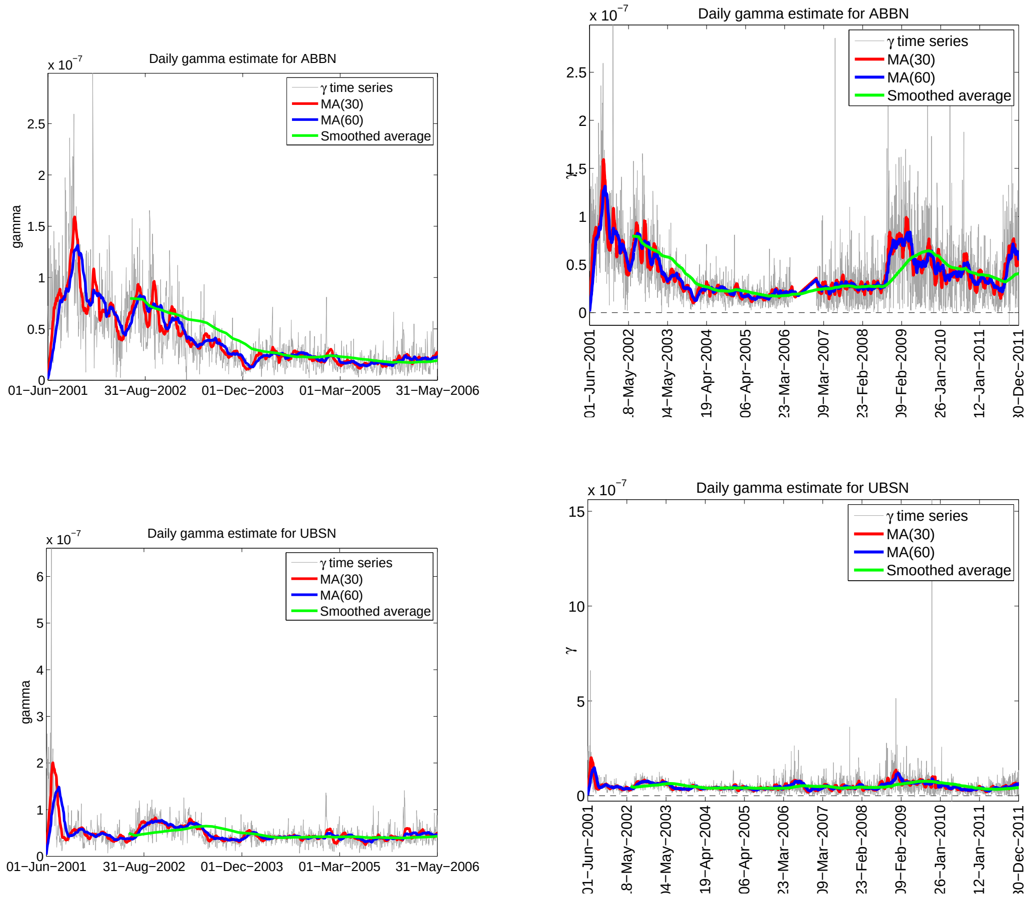

Figure C.1: Daily γ estimates for ABBN, UBS over the period 2001–2011.