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An Improved Composite Forecast For Realized Volatility

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Abstract

The purpose of this study is to determine whether a superior forecast for security volatility can be derived by finding a balance between historical data, implied volatility and an empirical implied distribution. Data are evaluated from option contracts and historical prices sampled on the first trading day of every month over a five year period from 2007 to 2012. These data are analyzed to determine the value of a weighted combination of the three sources of information and to uncover if this approach provides a forecast with a higher correlation to realized volatility. A linear optimization solution is formulated to determine the best possible composite volatility forecast. The results of the test show that there is statistically significant evidence in which the composite volatility forecast is preferred at a 95% confidence level over individual forecasts. With a better predictor for security volatility, this optimization process could be applied to the creation of portfolios that better meet investor risk preference.

JEL Codes: C12, C14, C58, G17

Keywords: Implied Volatility, Empirical Implied Distribution, Realized Volatility

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1 Introduction

Modern technical analysis forecasts of volatility are achieved both by leveraging historical data (statistical methods) and forward-looking data (implied volatility from derivative prices). In this study these two sources combined with a novel approached called an empirical implied distribution (EID) are used in order to create a composite forecast. The hypothesis is that a certain combination of volatility predictions will create an improved forecast. Linear optimization will be used to determine this composite value and uncover which of the three forecast contributes most (or least) to better predictions. The forecast will be back tested for accuracy using a standard hypothesis test of the correlation between predictions and the realized volatility (i.e. what the model predicts and what actually happens). This area of financial engineering is relatively unexplored to date; leaving the potential for valuable information towards current methods of risk management and related future work. The assets used in this study consists of four Exchange Traded Funds (ETFs) including the SPDR S&P 500 (SPY), the iShares S&P SmallCap 600 Index Fund (IJR), the United States Oil Index Fund (USO) and the SPDR Gold Trust (GLD). These four were chosen as they represent a large portion of the market and will assist in verifying the results of this study.

2 Background

2.1 Forecasting Volatility Using Forward Looking Market Data

The primary method technical investors use to predict how the markets perform in the future is by analyzing the past performance of a financial asset. By observing the changes in prices and volume of stocks traded, technical analysts often assume that the log adjusted returns of a security are normally distributed and that the expected return is interpreted as the mean, symbolized by [1]. The risk of an asset is expressed the standard deviation of the returns about the mean, symbolized by σ (referred to as volatility). These two parameters, along with the assumption that the returns follow a random walk described by Brownian Motion (or Weiner Process), gave technical analysts a framework to evaluate the performance of a particular asset or group of assets. The classical approach of asset evaluation is to estimate these two parameters using standard statistical methods on historical data.

However, other methods for generating forecasts of these parameters have become popular. An example is that volatility can also be forecast by analyzing the trading prices for options contracts. In 1973, Fischer Black and Myron Scholes developed an innovative model for pricing a European-style call option. The popular calculation for determining volatility is derived from this model known as the Black-Scholes (B-S) equation:

$$C(S,t) = N(d_+)S - N(d_-)Ke^{-r(T-t)}$$
$$d\pm = \frac{\ln(\frac{S}{K}) + (r \pm \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \quad (1)$$

The equation has five parameters, which include the strike price (K), the underlying price at time t (S), the risk free rate (r), the contract time to maturity (T-t), and the volatility of the underlying asset (σ) [2]. Equation 1 can only evaluate the price for a call option (C). In order to price a put option the use of put-call parity can used with arbitrage assumptions as shown in equation (2).

$$P(S,t) = Ke^{(-r(T-t))} - S + C(S,t) \quad (2)$$

One of the reasons this equation is used as a basis for estimation is that all of the parameters except the volatility of the underlying asset can be directly observed. So given a market price you can back out the volatility value and interpret what the price is implying about it (volatility), hence 'implied volatility' (IV). These equations have flaws in reality as they fail to account for significant changes in the market such as the crashes experienced in 1984 and 2008. As such actual market prices deviate from those predicted by the model. One primary reason for these inaccuracies stems from normality assumption that favors returns closer to the mean instead of the tails of the distribution. The B-S equation assumes that the variance remains constant across all strike prices in a given chain. Therefore, it would be expected for the resulting implied volatilities to form a straight line when plotted, in reality, they form a curved line known as the 'volatility smile' [4]. However some gains have been made by attempting to adjust for this skew with more complex models and assumptions [5].

Despite these facts, the model and its derivative work provide useful information for predictive purposes in many scenarios. Because implied volatility is so readily interpreted from a market price; simply solving for this lone parameter yields a prediction. That is to say that a given price implies a future volatility as expressed through B-S. However uncovering IV is not trivial, there is no closed from solution for solving this parameter so estimation methods must be employed. For example, one of the most common approaches is to apply the Newton-Raphson method (a recursion approach) for each strike price in an options chain [3].

2.2 Empirical Implied Distributions

An empirical implied distribution (EID) is an interpretation of the price premiums of an options chain for a given time to expiration. It is calculated by looking at the price dynamics of out of the money premiums within a given chain. It is important to point out that in this context the EID is not a formal probability distribution but a range of premiums interpreted by a potential realization of return (if the price reaches a given strike).

Implied probability distribution literature is important, for example, Shiratsuka [6] determined that an implied distribution does contain information useful in creating forecasts for future price movements, however, the accuracy of these predictions rely heavily on choice of sample period and are not as powerful as the historical data. An excellent approach to developing implied distributions is given by Rubinstein and Jackwerth (1996). The focus in these works is to uncover probability (hence a formal probability distribution) where as an EID is only concerned with price dynamics. Empirical implied distribution volatility (EIDV), can be extracted from the options premiums by using equations (3), (4) and (5).

$$\mu_{\mathrm{IDV}} = \frac{\sum_{i=1}^{n} (\ln \frac{K_i}{S(t)})(C_i)}{\sum_{i=1}^{n} C_i} \tag{3}$$

$$\sigma^2{}_{\mathrm{IDV}} = \frac{\sum_{i=1}^{n} (\ln \frac{K_i}{S(t)} - \mu_{IDV})^2 (C_i)}{\sum_{i=1}^{n} C_i} \tag{4}$$

$$\sigma_{\mathrm{IDV}} = \sqrt{\frac{\sum_{i=1}^{n} (\ln \frac{K_i}{S(t)})(C_i)}{\sum_{i=1}^{n} C_i}} \tag{5}$$

As is obvious from the above expressions the EID is constructed by standard statistical first and second moment estimations. An example of how these distributions are constructed is detailed in the following section.

3 Methodology

The example for this section will use the SPDR S&P 500 (SPY) which is an ETF designed to track the performance of the S&P 500. The first trading day of each month was used as a reference point for the following 30 calendar days. All of the returns were calculated as the logarithmically adjusted rate of the close price on the respective dates. Finally, all of the estimates and realized volatility were annualized so that they could be compared for analysis.

First, a historical forecast for the volatility of SPY was made using a 24-month moving average. A 24-month warm up period was included in the data set to allow for a statistical baseline. The square root of this variance was used as for the predicted standard deviation of the returns using historical data. For forward looking volatility estimates, only out-of-the-money options (both calls and puts) were used because they are more liquid and also result in a unimodal distribution of prices. Any options that displayed arbitrage opportunities were eliminated. Midpoint prices where used and options chains where truncated after two sequential identical prices less than $.05 where observed.

The implied volatility estimation was gained through the use of the Newton-Raphson method on the B-S model averaging the first four out of the money values for calls and puts for the first trading day of each month. Since the first trading day of each month does not always occur exactly 30 days later, the EIDV and IV were interpolated as a 30-day average. The two options chains to be used (near and next) are the two chains expiring between 0-30 days and 30-60 days, respectively. The following calculation was used:

$$Var = Var_{near} + (Var_{next} - Var_{near})\left(\frac{30 - TTM_{near}}{TTM_{next} - TTM_{near}}\right) \qquad (6)$$

where TTM represents the time to maturity in days. A demonstration of this calculation will use data from December 1st 2010 (Figure 1). On this particular day, the variance of the near ending option chain was .000271 and was set to expire in 29 days while the variance for the next options chain was .001425 and set to expire in 61 days. After annualizing the calculated variance, the result was .01031. Figure 1 shows the price distribution used for one of the data points used in the estimation with puts being represented with returns lower than zero and calls with returns higher. As expected, the distribution has a heavy, negative skew towards puts as recent investors generally value insurance on their holdings over the potential gains from calls.

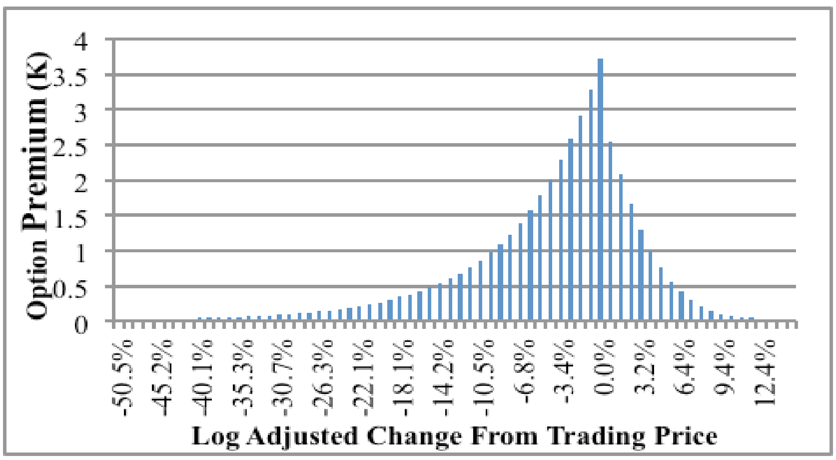

Figure 1: SPY Implied Distribution (DEC 1st 2010)

Finally, the realized volatility was calculated by analyzing daily data rather than monthly data in order to calculate the actual standard deviation of the next 30 (calendar) days so that it could be used to compare with the other two estimates.

For example, the realized volatility for March 2009 was calculated as the standard deviation of the returns from the 21 trading days in March. Finally, the implied volatility, empirical implied distribution volatility, historical data, and realized volatility were plotted on the Figure 2 for observation and comparison.

As shown by the Figure 2 above, we can see the how both the implied volatility and the empirical implied distribution are much more accurate at predicting the realized volatility than the historical data using the 24-month moving average. Despite this observation, it is important to note that historical data has been found to be a good predictor of future performance [7]. The 24-month moving average has a large smoothing effect due to its relatively long time period and nonweighted characteristics. The time frame of this study also encompasses the massive financial crisis the U.S. experienced in late 2008.

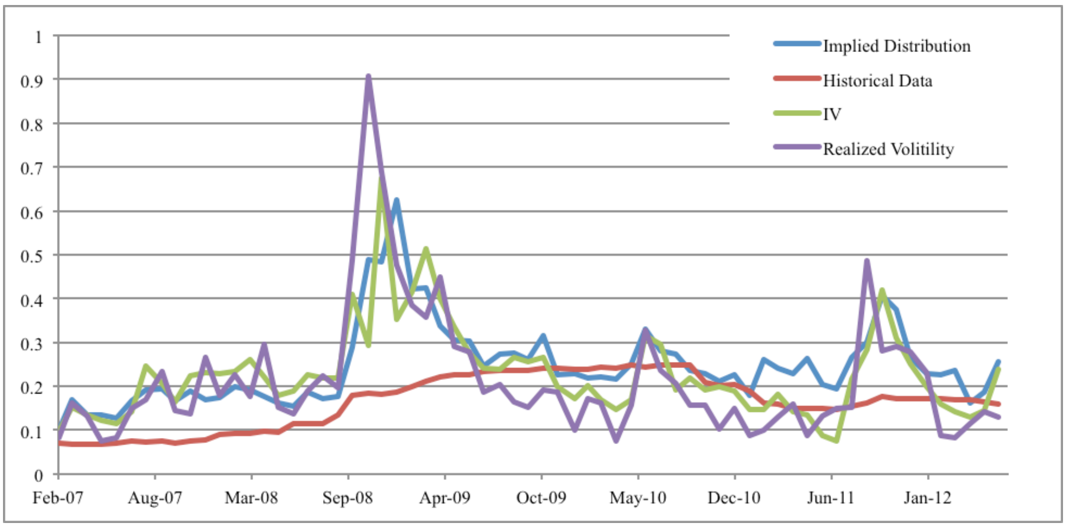

Figure 2: SPY Volatility Chart Comparing the Three Separate Estimates vs. Realized Volatility.

The coefficient of correlation, r, was calculated to measure the linear association between the realized volatility and each estimate as seen in Table 1. The next step was to conduct a hypothesis test for correlation to determine if the results were statistically significant.

The hypotheses and test statistic were formulated as follows.

$$H_0 : r = 0$$
$$H_1 : r \neq 1 \tag{7}$$
$$Tstat = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Table 1: Hypothesis Test for Correlation

	Historical Data	***IV***	***EIDV***
r	0.1669	0.7364	0.739947
t		2.000	
T stat	1.344	8.639	8.731
Results	FTR	Reject	Reject

The value for t was obtained with an α = .05 and 60 degrees of freedom. The degrees of freedom varied from security to security as the number of data points changed, but the α value remained constant. While all three correlations were positive, the implied volatility and empirical implied distribution correlations were much stronger. The test statistic for the implied volatility and implied distribution were much greater than the t value meaning that we should reject the null hypothesis with statistical significance.

The next step was to construct a linear optimization model to predict for the ideal weighted forecast between the three estimates for a composite estimate of the three approaches. A composite volatility estimate, λ, was used as a new parametric defined by equation (8) so long as the weights (w_i) met the constraint in equation (9).

$$\lambda = w_{IV}\ \sigma_{IV} + w_{IDV}\ \sigma_{IDV} + w_{HV}\ \sigma_{HV} \tag{8}$$

$$\text{s.t.:}\ \ w_{IV} + w_{IDV} + w_{HV} = 1 \tag{9}$$

In equations (8) and (9), the weights of each respective estimate are varied when looking for the optimal combination. Similarly, the same weights can be used to determine the mean of the composite volatility estimate.

$$\mu_\lambda = w_{IV}\ \mu_{IV} + w_{IDV}\ \mu_{IDV} + w_{HV}\ \mu_{HV} \tag{10}$$

The next step was to calculate the standard deviations for the realized volatility and the composite volatility estimate.

$$\sigma_{real} = \sqrt{\frac{\sum (x-\mu)^2}{n-1}} \tag{11}$$

$$\sigma_\lambda = \sqrt{w_{IV}^{\ 2}\ \sigma^2_{\ IV} + w^2_{\ IDV}\ \sigma^2_{\ IDV} + w^2_{\ HV}\ \sigma^2_{\ HV}} \tag{12}$$

The objective function was set to maximize the correlation between the realized volatility and the newly created λ.

$$Max: \rho_{real,\lambda} = \sqrt{\frac{\sigma_{real,\lambda}}{\sigma_{real}\sigma_\lambda}} \tag{13}$$

where $\sigma_{real,\lambda} = E(real,\lambda) + \mu_{real}\mu_\lambda$

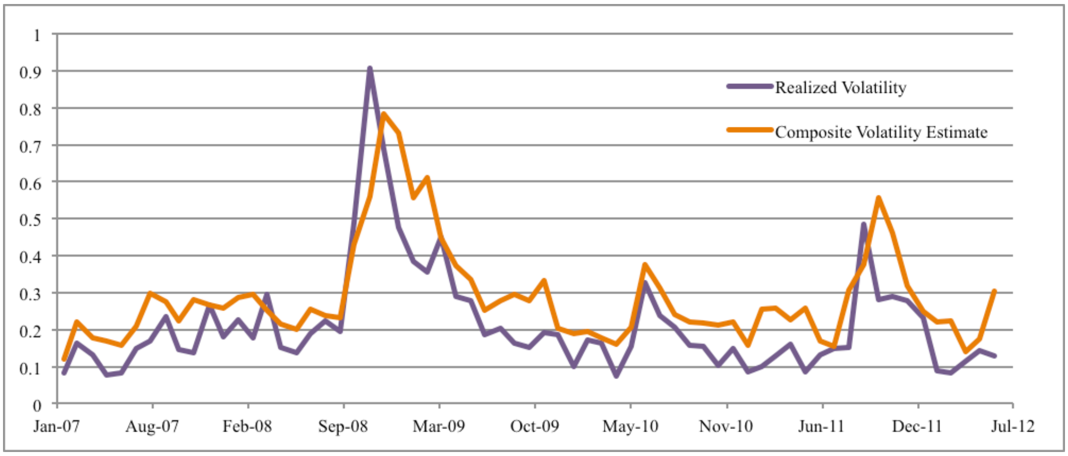

Figure 3: The Composite Volatility Estimate vs. Realized Volatility.

Table 2: Hypothesis Test for the Composite Volatility Estimate for SPY

Value	*λ*
r	0.811717432
T	2.000
T stat	11.03125338
Results	FTR

Figure 3 depicts the composite volatility estimate compared to the realized volatility for SPY. Simple observation shows that the composite volatility estimate tends to overestimate the realized volatility. This is true at every analyzed time frame except for three occasions, two being times when the stock market was going through the crash in 2008 and subsequent effects experienced in late 2011.

One of the most promising conclusions that can be drawn from this graph is that the composite volatility estimate should make for a good forecasting parametric of future volatility. The peaks and valleys of the composite volatility closely mirror the realized volatility across the observed data. Further analysis should be done to determine the predictive capabilities of this approach across various asset domains. Additionally, the correlation between the two was over 81% and had a test statistic of 11.031 as shown in Table 2, both strong values.

4 Results and Conclusion

The same process explained in the previous section was repeated for three additional securities to include IJR, GLD, and USO. The compiled summary can be seen in Figure 4 and Table 3.

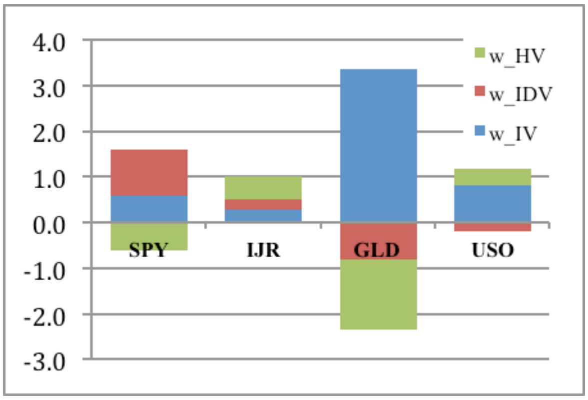

Figure 4: Stacked Bar Chart of the Resultant Weights

Figure 4 provides insight as to what average weight each ETF forecast consisted of. Values could be negative because the weight simply is the coefficient by which the original respective estimate is multiplied by as equation (8) showed. SPY preferred the implied volatility and empirical implied distribution while it discounted the historical data. IJR, on the other hand, did not discount any of the estimates using roughly 50% historical data, 30% implied volatility and 20% implied distribution volatility. The GLD ETF highly favored the implied volatility estimation. Finally, USO utilized roughly 83% of the implied volatility and 36% historical data in exchange for subtracting about 18% of the empirical implied distribution estimate.

Table 3: Summary Correlations and Improvement.

	SPY	*IJR*	*GLD*	*USO*
Previous Best Correlation Estimate	0.739 IDV	0.613 IV	0.704 IV	0.482 IV
λ Correlation	0.812	0.633	0.733	0.508
Improvement	9.70%	3.23%	4.14%	5.38%

It was found that the correlation can be improved significantly by using this composite forecast method. All four ETFs experienced an increase in correlation after the methodology was applied. On average, this method improved the correlation 5.61%. At a 95% confidence level, a paired t test shows that these improvements are statistically significant. This confirms our original hypothesis that certain combinations of volatility estimations can be found through the use of linear optimization to create an improved estimate for the volatility of a security's returns. The composite volatility estimate represents a statistically superior estimate. More accurate volatility forecasts will allow investors, particularly risk and portfolio managers, to make better investments decisions.

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