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The orbit spaces of linearly ordered systems on continuums

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Abstract

In this paper, the concept of an orbit space is generalized from a discrete dynamical system (X, f) to a linearly ordered system $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$, and it is shown that a general orbit space is a continuum if each X_α is a continuum in a linearly ordered system $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$. As a special case, it is obtained that the orbit space of any discrete dynamical system (X, f) is a continuum if X is a continuum.

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1 Introduction

In recent years, the inverse limits has been widely used in Dynamical Systems, a series of good results were obtained (cf. [1]-[6]). But the applications of inverse limits in Dynamical Systems have been encountered many difficulties because the direction of the chain maps of inverse limits is exactly the opposite

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to the tracks of the relative dynamical system. In this paper, we introduce the concepts of the orbit spaces by reversing the direction of chain maps of inverse limit space, and discuss to preserve the nature of continuums in their orbit space.

Let $\{X_\alpha\}_{\alpha\in\Gamma}$ be a family of topological spaces, where Γ is linearly ordered set[7], we denote by $\Pi_{\alpha\in\Gamma}X_\alpha$ the product space of $\{X_\alpha\}_{\alpha\in\Gamma}$, and assume that $\pi_\alpha^\beta : X_\alpha \to X_\beta$ is a continuous mapping when $\alpha \le \beta$ for any $\alpha, \beta \in \Gamma$. The triples $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$ is called to be a linearly ordered system if the following two conditions are satisfied:

(A1) $\pi_\beta^\gamma \pi_\alpha^\beta = \pi_\alpha^\gamma$ when $\alpha \le \beta \le \gamma$ for any $\alpha, \beta, \gamma \in \Gamma$,

(A2) $\pi_\alpha^\alpha = id_{X_\alpha}$ for any $\alpha \in \Gamma$, where id_{X_α} is an identity mapping from X_α to X_α.

Usually, the subspace $\{x = (x_\alpha)_{\alpha\in\Gamma} \in \Pi_{\alpha\in\Gamma}X_\alpha : \pi_\alpha^\beta(x_\alpha) = x_\beta, \alpha \le \beta(\forall \alpha, \beta \in \Gamma)\}$ of the product space $\Pi_{\alpha\in\Gamma}X_\alpha$ is called to be the orbit space of the linearly ordered system $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$ and is denoted by $\mathcal{O}\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$, where each point $x \in \mathcal{O}\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$ is called to be an orbit(or,a direct limit) of $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$, each mapping π_α^β is called to be a link mapping of $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$.

Assume that $f : X \to X$ is a continuous mapping where X is a topological space. In Dynamical Systems, we call that (X, f) is a discrete dynamical system,and for every $x_0 \in X$, the sequence $\{x_0, f(x_0), f^2(x_0), ...\}$ is called to be an orbit of (X, f) and is denoted by $O_f^+(x_0)$. The set of all orbits of (X, f) is a subspace of the product space $\Pi_{n=0}^\infty X_n$ where each $X_n = X$, we call it an orbit space of (X, f) and denote it by $\mathcal{O}_f^+(X)$.

As a special case of a linearly ordered system, when $\Gamma = Z^+$ (the set of all non-negative integers),let us put $X_n = X$ and $\pi_n^{n+1} = f$ for each $n \in Z^+$. It is easy to see that the linearly ordered system $\{X, f^{m-n}, Z^+\}$ is the discrete system (X, f) and $\mathcal{O}\{X, f^{m-n}, Z^+\}$=$\mathcal{O}_f^+(X)$.

As a discrete dynamical system (X, f), the following two questions should be fundamental questions :

Question 1.1. *Is $\mathcal{O}_f^+(X)$ a compact space if X is a compact space?*

Question 1.2. *Is $\mathcal{O}_f^+(X)$ a continuum space if X is continuum?*

In this paper, we first study orbit spaces of a linearly ordered system $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$, several topological properties are obtained. By using them, the following theorem is proved:

MAIN THEOREM Let $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$ be a linearly ordered system, if each link map π_α^β is a continuous and onto map, the following two conclusions hold:

(B1) $\mathcal{O}\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$ is a compact space if each X_α is a compact space.

(B2) $\mathcal{O}\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$ is a continuum if each X_α is a continuum.

As a special case of these results, the above two questions will be given certainly answer.

In this paper, X denotes a topological space, ϕ denotes empty set, $\mathcal{N}_Y(x)$ denotes the open neighborhood system of a point x in a subspace Y of X and is denoted by $\mathcal{N}(x)$ when $Y = X$. $\overline{A}$ denotes the closure of a subset A of X. Let $X = \mathcal{O}\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$, we denote by p_α the mapping of the projection from $\Pi_{\alpha'\in\Gamma} X_{\alpha'}$ onto X_α, and put $\pi_\alpha = p_\alpha|_X : X \to X_\alpha$ for each $\alpha \in \Gamma$.

Throughout this paper, all concepts and terminologies on topological spaces are from [7] and all topological spaces are Hausdorff spaces.

2 The fundamental proposition of orbit spaces

Proposition 2.1. *The orbit space of a linearly order system $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$ is a closed set of the product space $\prod_{\alpha\in\Gamma} X_\alpha$*

PROOF. Let $X = \mathcal{O}\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$. For any $\alpha, \beta \in \Gamma$ satisfying $\alpha \leq \beta$, let's pick

$$M_\alpha^\beta = \{x = (x_\alpha)_{\alpha\in\Gamma} \in \Pi_{\alpha\in\Gamma} X_\alpha : \pi_\alpha^\beta(x_\alpha) = x_\beta\}.$$

Because $\pi_\alpha^\beta \pi_\alpha(x) = \pi_\alpha^\beta(x_\alpha) = x_\beta = \pi_\beta(x)$,i.e.,

$$M_{\alpha\beta} = \{x = (x_\alpha)_{\alpha\in\Gamma} \in \Pi_{\alpha\in\Gamma} X_\alpha : \pi_\alpha^\beta \pi_\alpha(x) = \pi_\beta(x)\},$$

then $M_{\alpha\beta}$ is a closed subset of the product space $\Pi_{\alpha'\in\Gamma} X_{\alpha'}$ by[7,Theorem 1.5.4]. Thus,

$$X = \bigcap_{\alpha\leq\beta} M_{\alpha\beta}$$

is closed in $\Pi_{\alpha\in\Gamma} X_\alpha$.

It is known that a closed subset of a compact space is compact. Therefore, the following corollary holds trivially by Tychonoff Theorem ([7,Theorem 3.2.4]),

Corollary 2.2. *Let $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$ be a linearly ordered system, then $\mathcal{O}\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$ is a nonempty compact subset of the product space $\Pi_{\alpha\in\Gamma}X_\alpha$ if X_α is a compact space and $X_\alpha \neq \phi$ for every $\alpha \in \Gamma$.*

PROPOSITION 2.3 *Assume that X is the orbit space of a linearly ordered system $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$, then the following family of open subsets of X:*

$$\mathcal{B}=\{\pi_\alpha^{-1}(U_\alpha)\text{: } U_\alpha \text{ is an open subset of } X_\alpha, \alpha \in \Gamma\}$$

is a base of X.

PROOF *Let $X = \mathcal{O}\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$. For any $x \in X$ and for any $U \in \mathcal{N}_X(x)$, there exists an open subset V of the product space $\Pi_{\alpha\in\Gamma}X_\alpha$ such that $U = X \cap V$. Thus, there are $W_{\alpha_1}, W_{\alpha_2}, ..., W_{\alpha_k}$ which are respectively open subsets of $X_{\alpha_1}, ..., X_{\alpha_k}$ such that*

$$x \in \bigcap_{i=1}^{k} p_{\alpha_i}^{-1}(W_i) \subset V.$$

where k is a nature number. Put $\alpha = min\{\alpha_i : 1 \leq i \leq k\}$, then $\alpha \leq \alpha_i$ and $(\pi_\alpha^{\alpha_i})^{-1}(W_i)$ is an open subset of X_α for $i = 1, 2, ..., k$, then $U_\alpha = \bigcap_{i=1}^k(\pi_\alpha^{\alpha_i})^{-1}(W_i)$ is an open subset of X_α. We can readily check that $x \in \pi_\alpha^{-1}(U_\alpha)=\pi_\alpha^{-1}(\bigcap_{i=1}^k(\pi_\alpha^{\alpha_i})^{-1}(W_i))=\bigcap_{i=1}^k(\pi_\alpha^{-1}(\pi_\alpha^{\alpha_i})^{-1}(W_i))= X\bigcap(\bigcap_{i=1}^k(p_{\alpha_i}^{-1}(W_i)) \subset X\bigcap V = U$.

Thus, $\mathcal{B}$ is a topological base of X.

Proposition 2.3. *Let $X = \mathcal{O}\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$, $A \subset X$ and $A_\alpha = \pi_\alpha(A)$ for every $\alpha \in \Gamma$. If $\widehat{\pi}_\alpha^\beta = \pi_\alpha^\beta|_{\overline{A}_\alpha}$ when $\alpha \leq \beta$ for any pair $\alpha, \beta \in \Gamma$, then $\{\overline{A}_\alpha, \widehat{\pi}_\alpha^\beta, \Gamma\}$ is a linearly ordered system and $\mathcal{O}\{\overline{A}_\alpha, \widehat{\pi}_\alpha^\beta, \Gamma\} = \overline{A}$, where $\overline{A}$ and $\overline{A}_\alpha$ are respectively closures of A and A_α in X.*

PROOF. For any $\alpha, \beta \in \Gamma$ satisfying $\alpha \leq \beta$, it is easy to check that

$$\widehat{\pi}_\alpha^\beta(\overline{A}_\alpha) = \widehat{\pi}_\alpha^\beta(\overline{\pi_\alpha(A)}) \subset \overline{\widehat{\pi}_\alpha^\beta\pi_\alpha(A)} = \overline{\pi_\beta(A)} = \overline{A}_\beta,$$

then $\widehat{\pi}_\alpha^\beta : \overline{A}_\alpha \rightarrow \overline{A}_\beta$ is a well continuous map. Thus $\{\overline{A}_\alpha, \widehat{\pi}_\alpha^\beta, \Gamma\}$ is a linearly ordered system.

Let us put $Y = \mathcal{O}\{\overline{A}_\alpha, \widehat{\pi}_\alpha^\beta, \Gamma\}$, we assert first that Y is a closed set of X.

For any $x = (x_\alpha)_{\alpha\in\Gamma} \in X-Y$, there exists $\alpha_0 \in \Gamma$ such that $x_0 \in X_{\alpha_0}-\overline{A}_{\alpha_0}$, i.e., $\pi_{\alpha_0}^{-1}(X_{\alpha_0} - \overline{A}_{\alpha_0}) \in \mathcal{N}(x)$ and $\pi_{\alpha_0}^{-1}(X_{\alpha_0} - \overline{A}_{\alpha_0})\bigcap Y = \phi$. In fact, if there is some $y \in \pi_{\alpha_0}^{-1}(X_{\alpha_0} - \overline{A}_{\alpha_0})\bigcap Y$, i.e., $y \in \pi_{\alpha_0}^{-1}(X_{\alpha_0} - \overline{A}_{\alpha_0})$ and $y \in Y \subset \Pi_{\alpha\in\Gamma}\overline{A}_\alpha$, then $\pi_{\alpha_0}(y) = y_{\alpha_0} \in (X_{\alpha_0} - \overline{A}_{\alpha_0}) \bigcap \overline{A}_{\alpha_0} = \phi$. This is a contradiction. So, Y is a closed set of X.

Moreover, we show that $\overline{A} = Y$.

In fact, for any $x \in Y$ and for any $V \in \mathcal{N}_Y(x)$, there exist $\alpha \in \Gamma$ and $U_\alpha \in \mathcal{N}_{X_\alpha}(x_\alpha)$ such that $x \in \pi_\alpha^{-1}(U_\alpha) \bigcap Y \subset V$ since $\mathcal{B}_Y(x) = \{\pi_\alpha^{-1}(U_\alpha) \bigcap Y : U_\alpha \in \mathcal{N}(x_\alpha), \alpha \in \Gamma\}$ is a neighborhood base of a point x in Y, then $\pi_\alpha(x) = x_\alpha \in U_\alpha \bigcap \pi_\alpha(Y) \subset U_\alpha \bigcap \overline{A}_\alpha$. Therefore,

$$V \bigcap A \supset (\pi_\alpha^{-1}(U_\alpha) \bigcap Y) \bigcap A = (\pi_\alpha^{-1}(U_\alpha)) \bigcap A \neq \phi,$$

i.e., $x \in \overline{A}$. Thus $Y \subset \overline{A}$. On the other hand, it is obvious that $\overline{A} \subset \overline{Y} = Y \subset X$ holds because $A \subset Y$. Thus, $Y = \overline{A}$. The proof is completed.

3 The proof of main theorems

Theorem 3.1. *Let $\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$ be a linearly ordered system, if each link map π_α^β is an onto map, then $\mathcal{O}\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$ is a continuum if and only if each X_α is a continuum.*

Proof Take $X = \mathcal{O}\{X_\alpha, \pi_\alpha^\beta, \Gamma\}$, it has been shown that X is compact if each X_α is compact in Corollary 2.2.

Now we assert that X is a connected space.

Assume that there exist two closed subsets A_1, A_2 of X such that $X = A_1 \bigcup A_2$ and $A_1 \bigcap A_2 = \phi$, now we show that it is inevitable that either $A_1 = \phi$ or $A_2 = \phi$.

For every $\alpha \in \Gamma$, let us pick $A_{i\alpha} = \pi_\alpha(A_i)$ for $i = 1, 2$ and let $B_\alpha = A_{1\alpha} \bigcap A_{2\alpha}$, where π_α is the projection mapping from X to X_α. Then three subsets $A_{1\alpha}$, $A_{2\alpha}$, B_α of X_α are compact and connected, i.e., they are subcontinuums of X_α.

Let's put $\widehat{\pi}_\alpha^\beta = \pi_\alpha^\beta|_{B_\alpha}$ for any pair α, $\beta \in \Gamma$ satisfying $\alpha \leq \beta$, it is obvious that $\widehat{\pi}_\alpha^\beta$ is well defined as a map from B_α to B_β since $\widehat{\pi}_\alpha^\beta(B_\alpha) = \pi_\alpha^\beta(B_\alpha) = \pi_\alpha^\beta(A_{1\alpha} \bigcap A_{2\alpha}) \subset \pi_\alpha^\beta(A_{1\alpha}) \bigcap \pi_\alpha^\beta(A_{2\alpha}) = \pi_\alpha^\beta \pi_\alpha(A_1) \bigcap \pi_\alpha^\beta \pi_\alpha(A_2) = \pi_\beta(A_1) \bigcap \pi_\beta(A_2) = A_{1\beta} \bigcap A_{2\beta} = B_\beta$. So, $\{B_\alpha, \widehat{\pi}_\alpha^\beta, \Gamma\}$ be a linearly ordered system.

Similarly, it is easily seen that each $\{A_{i\alpha}, \pi_i^\beta, \Gamma\}$ is also a linearly ordered system when we put each $\pi_{i\alpha}^\beta = \pi_\alpha^\beta|_{A_i}$, since $\pi_{i\alpha}^\beta(A_{i\alpha}) = \pi_\alpha^\beta \pi_\alpha(X_i) = \pi_\beta(A_i) =$

$A_{i\beta}$ for $i = 1, 2$. By using Proposition 2.3, we have $\mathcal{O}\{A_{i\alpha}, \pi_{i\alpha}^{\beta}, \Gamma\} = A_i$ for $i = 1, 2$.

Let $B = \mathcal{O}\{B_\alpha, \widehat{\pi}_\alpha^\beta, \Gamma\}$, it is obvious that $B \subset \mathcal{O}\{A_{i\alpha}, \pi_{i\alpha}^{\beta}, \Gamma\} = A_i$ since each $B_\alpha \subset A_{i\alpha}$ and $\widehat{\pi}_\alpha^\beta = \pi_\alpha^\beta|_{B_\alpha} = (\pi_\alpha^\beta|_{A_{i\alpha}})|_{B_\alpha} = \pi_{i\alpha}^\beta|_{B_\alpha}$ for $i = 1, 2$. So, $B = \phi$ because $A_1 \bigcap A_2 = \phi$. Moreover, there exists $\alpha_0 \in \Gamma$ such that $B_{\alpha_0} = A_{1\alpha_0} \bigcap A_{2\alpha_0} = \phi$ since each $\pi_\alpha^\beta : X_\alpha \to X_\beta$ is an onto mapping for $\alpha \le \beta$.

Since $\pi_{\alpha_0}(X)$ is a normal subspace of X_{α_0}, there exist two open subsets $U_{1\alpha_0}$, $U_{2\alpha_0}$ of $\pi_{\alpha_0}(X)$ such that

(1) $A_{1\alpha_0} \subset U_{1\alpha_0}$, $A_{2\alpha_0} \subset U_{2\alpha_0}$ and $U_{1\alpha_0} \bigcap U_{2\alpha_0} = \phi$.

For any $\alpha \in \Gamma$, we put $Z_\alpha = X_\alpha - (U_{1\alpha} \bigcup U_{2\alpha})$ where $U_{i\alpha} = (\pi_\alpha^{\alpha_0})^{-1}(U_{i\alpha_0})$ when $\alpha \le \alpha_0$ and $i = 1, 2$, and let $Z_\alpha = X_\alpha$ when $\alpha > \alpha_0$. Then

(2) $\{Z_\alpha, \overline{\pi}_\alpha^\beta, \Gamma\}$ is a linearly order system, where $\overline{\pi}_\alpha^\beta = \pi_\alpha^\beta|_{Z_\alpha}$ for any $\alpha \in \Gamma$.

In fact, for any pair $\alpha, \beta \in \Gamma$ satisfying $\alpha \le \beta$, if $\beta \le \alpha_0$, then $\overline{\pi}_\alpha^\beta(Z_\alpha) = \overline{\pi}_\alpha^\beta[X_\alpha - (\pi_\alpha^{\alpha_0})^{-1}(A_{1\alpha_0} \bigcup A_{2\alpha_0})] = \overline{\pi}_\alpha^\beta(\pi_\alpha^{\alpha_0})^{-1}[X_{\alpha_0} - (A_{1\alpha_0} \bigcup A_{2\alpha_0})] = \overline{\pi}_\alpha^\beta(\pi_\beta^{\alpha_0}\pi_\alpha^\beta)^{-1}[X_{\alpha_0} - (A_{1\alpha_0} \bigcup A_{2\alpha_0})] = \overline{\pi}_\alpha^\beta(\pi_\alpha^\beta)^{-1}(\pi_\beta^{\alpha_0})^{-1}[X_{\alpha_0} - (A_{1\alpha_0} \bigcup A_{2\alpha_0})] \subset X_\beta - (U_{1\beta} \bigcup U_{2\beta}) = Z_\beta$. If $\alpha_0 < \beta$, then $\overline{\pi}_\alpha^\beta(Z_\alpha) \subset \pi_\alpha^\beta(X_\alpha \subset X_\beta = Z_\beta$. I.e., (2) is true.

Put $Z = \mathcal{O}\{Z_\alpha, \overline{\pi}_\alpha^\beta, \Gamma\}$, since $Z \subset X$ and $\pi_{\alpha_0}(X) = \pi_{\alpha_0}(A_1) \bigcup \pi_{\alpha_0}(A_2) = A_{1\alpha_0} \bigcup A_{2\alpha_0} \subset U_{1\alpha_0} \bigcup U_{2\alpha_0}$, then $\pi_{\alpha_0}(X) \bigcap Z_{\alpha_0} = \phi$. If $Z \ne \phi$, we can put $x = (x_\alpha)_{\alpha\in\Gamma} \in Z \subset \Pi_{\alpha\in\Gamma} Z_\alpha$, then $x_{\alpha_0} = \pi_{\alpha_0}(x) \in \pi_{\alpha_0}(X) \bigcap Z_{\alpha_0}$. This contradicts to $\pi_{\alpha_0}(X) \bigcap Z_{\alpha_0} = \phi$. So, we have $Z = \phi$.

Then, there exists some $\alpha^* \in \Gamma$ such that $Z_{\alpha^*} = \phi$. It holds obviously that $\alpha^* \le \alpha_0$, since $Z_\alpha = X_\alpha \ne \phi$ for every $\alpha > \alpha_0$. Then

$$\pi_{\alpha^*}(X) = A_{1\alpha^*} \bigcup A_{2\alpha^*} \subset U_{1\alpha^*} \bigcup U_{2\alpha^*} \subset \pi_{\alpha^*}(X),$$

i.e., $\pi_{\alpha^*}(X) = U_{1\alpha^*} \bigcup U_{2\alpha^*}$, therefore it is true that either $U_{1\alpha^*} = \phi$ or $U_{2\alpha^*} = \phi$ holds because $\pi_{\alpha^*}(X)$ is connected, and

$U_{1\alpha^*} \bigcap U_{2\alpha^*} = (\pi_{\alpha^*}^{\alpha_0})^{-1}(U_{1\alpha_0}) \bigcap (\pi_{\alpha^*}^{\alpha_0})^{-1}(U_{2\alpha_0})$
$= (\pi_{\alpha^*}^{\alpha_0})^{-1}(U_{1\alpha^*} \bigcap U_{2\alpha_0}) = \phi$.

By (1), it is true that either $A_{1\alpha^*} = \phi$ or $A_{2\alpha^*} = \phi$ holds. So, $A_1 = \phi$ or $A_2 = \phi$.

This proof is completed.

Corollary 3.2. *Let X be continuum, $f : X \to X$ is a continuous map from X onto self, then $\mathcal{O}_f^+(X)$ is a continuum.*

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