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On the predictive ability of GARCH and SV models of volatility: An empirical test on the SENSEX index

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Abstract

We examined and compared forecasting ability of GARCH and Stochastic Volatility models represented in the state space form using Kalman Filter as an estimator for the models. The models are applied in the context of Indian stock market. For estimation purpose, daily values of Sensex from Bombay Stock Exchange (BSE) are used as the input. The results confirmed the volatility forecasting capabilities of both models. Finally, we interpreted that which model performs better in the out-of-sample forecast for h-step ahead forecast. Forecast errors of the volatility were found in favour of SV model for a 30-day ahead forecast. This also shows that Kalman filter can be used for better estimates and forecasts of the volatility using state space models. Finally the numerical results make evident the effectiveness and relevance of the proposed state space estimation.

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1 Introduction

Volatility in financial markets has been a growing area of interest of researchers seeking to understand market dynamics. It has attracted the attention of policy makers and market traders, investors and risk managers especially during the past two decades because it can be used as a measurement of risk. Primarily the volatility receives a great deal of concern from financial market participants because greater volatility in the stock, bond and foreign exchange markets raises important public policy issues about the stability of financial markets.

Volatility forecasting is important for option pricing, portfolio management and risk management. Nowadays, volatility has become the subject of trading. There are now exchange-traded contracts written on volatility. Financial market volatility also has a wider effect on financial regulation, monetary policy and macro economy. Therefore there is a need of research in financial market volatility modeling and hence forecasting.

The two main sources of volatility forecasts are time-series models and implied volatilities which is calculated from observed prices. The time-series models remain the major source of volatility forecasting which can also be used to predict the time-varying beta. Accurate forecasting of time-varying beta is important for several reasons. Firstly, the prediction of the beta value helps investors to make their investment decisions easier. Secondly, for corporate financial managers, forecasts of the conditional beta not only benefit them in the capital structure decision but also in investment appraisal.

Sensex is the most followed market index in the Indian stock market and consists of the 30 largest and most actively traded stocks and representative of various sectors on the BSE. The Indian stock market has been an active area of research during the last few decades along with markets of other emerging economies. In India, different variants of GARCH model have been considered to study volatility forecasting and a large volume of literature focuses on modeling volatility using these models. The present paper tests an alternate modeling technique for the estimation of the volatility in the Indian stock market by using State Space (SS) model. Control engineers and physicists were the first to introduce the State Space models for modeling of continuously changing unobserved state variable. The unknown model parameters in such models were estimated by the Kalman filter (KF, popularly named after Kalman (1960). The Kalman Filter algorithm plays an integral role in the modeling, estimating and further predicting the states of State Space models[1].

Several studies have reported applications of SS Model in estimating price volatility [2]. These models are becoming more popular because they are able to represent complex dynamics equations through a simple structure of matrices. Further, it is easier to apply the Kalman Filter to SS Models because they allow for certain relevant analysis that would otherwise be very difficult, such as modeling samples with missing data or observation errors [3]. This approach has been applied for stochastic volatility (SV) Model where the time-varying variance as a stochastic process was modeled in state space form and further estimated using Quasi Maximum Likelihood methods [4, 5, 6] and [7].

However, GARCH type modeling has been an popular area of research. These models are also widely used because their properties are somewhat similar to the observed properties of empirical financial data which can capture various stylized facts [8]. Additionally, they are simple to model, gives a better fit to the empirical financial data and are estimated by commonly used estimation procedures like Maximum Likelihood (MLE) approaches. The concern while using the GARCH models in forecasting is the errors occurring when the forecasts are computed [9]. To overcome this limitation, an alternate parameter estimation approach proposed by (Jerez *et.al.*)[10] is used in this paper. The state space GARCH-KF model is obtained by treating mean and variance equations internally in its equivalent state-space formulation suitable for Kalman Filter estimation. This approach allows certain computations and analysis that would otherwise not be possible by traditional estimation methods.

Many researchers have compared different time series data, daily exchange rate and stock prices using variants of GARCH models to forecast the volatility. Their findings showed that most of the advanced models did not provide better forecasts than GARCH (1,1) model[11]. Therefore, in this paper we have selected GARCH(1,1) model for forecasting using the framework of E^4 toolbox, referred in Spanish as '*Estimacin de modelos Economtricos en Espacio de los*

Estado '. This toolbox allows for observation errors, missing data and GARCH error modeling in state space form.

The study described in present paper can be justified on following ideas. The evidence from other developing markets provide mixed evidence of forecasting performances in volatility models[12, 13, 14]. However, there has been no comprehensive study of GARCH based State Space Modeling of volatility in India. As a result, this paper aims to compare stochastic volatility, GARCH(1,1) and GARCH(1,1)-KF model which is a state space approach, which substantially differ in terms of their computational model from the traditional estimation methods.

The paper uses the daily closing values of BSE-Sensex for the period 01 January 2006 to 22 August 2013. The daily index values of Sensex are collected from the official websites of BSE[15]. The data set considered, has enough number of observations to perform time-series analysis on the models to get meaningful results. We have modeled the stochastic volatility (SV) forecasting models using a state space modeling (SSM) toolbox for Matlab [16]. The GARCH models are represented in state space form and the estimation is done by E^4 toolbox for MATLAB. The estimation results of SV model are presented and compared with out-of-sampling forecast of GARCH(1,1) model.

The rest of paper is organized as follows. Section 2 gives a brief overview of the GARCH and Stochastic volatility models along with the state space representation. Then, we estimate and analyze the models by presenting the main results of the paper. In section 3, out-of-sample forecast of the estimators are discussed. Section 4 explores the comparative performance of the various forecasting models and a 30-day ahead forecast is given in tabular and graphical representation. Section 5 gives the concluding remarks.

2 Models Overview & State Space Representation

State space methods are tools which allows one to estimate the unknown parameters along with the time varying states of linear models. It can also be used to forecast future states and observations even if there is some missing data. The following section will review the basic model representation of State Space. The results of model parameters estimation are briefly discussed in subsequent sections.

2.1 Linear Gaussian State Space Model

This section provides a brief review of linear Gaussian state space model. Let y_t denote an $p \times 1$ observation vector related to an $m \times 1$ vector of unobservable components α_t (states sequences), by the so-called measurement equation eq:1,

$$y_t = Z_t\alpha_t + \epsilon_t, \quad \epsilon_t \sim N(0, H_t) \tag{1}$$

$$\alpha_{t+1} = c_t + T_t\alpha_t + R_t\eta_t, \quad \eta_t \sim N(0, Q_t) \tag{2}$$

The evolution of the states is governed by the process or state equation (2). Thus, the matrices Z_t, c_t, T_t, R_t, H_t, Q_t, a_1, P_1 are required to define a linear Gaussian state space model [17, 2]. The matrix Z_t is the state to observation linear transformation matrix, for univariate models it is a row vector $m \times 1$. The matrix c_t is the same size as the state vector, and it is constant in the state update equation, although it can be dynamic or dependent on model parameters. The square matrix T_t $[m \times m]$ defines the time evolution of states. The matrix R_t $[m \times r]$ transforms general disturbance into state space and exists to allow for more varieties of models. H_t $[p \times p]$ and Q_t $[r \times r]$ are Gaussian variance matrices governing the disturbances, and a_1 and P_1 are the initial conditions[18]. The specification of the state space model is completed by the initial conditions concerning the distribution of $\alpha_1 \sim N(a_1, P_1), \forall t$.

2.2 GARCH Model

The GARCH(1,1)-model includes one lag of the conditional variance given by following equation:

$$y_t = \epsilon_t h_t \tag{3}$$

$$h_t^2 = \phi + \alpha y_{t-1}^2 + \beta h_{t-1}^2 \tag{4}$$

ϵ_t is an IID process with zero mean and variance of unity. In most applications ϵ_t is assumed to be NID(0,1). To ensure the existence of the conditional variance and for avoiding the degeneration of the process $\phi > 0$ and $\alpha, \beta \geq 0$ must hold, $y_t|\Omega_{t-1} \approx iidN(0, h_t^2)$ such that the conditional variance h_t^2 , follows a GARCH(1,1) equation:

Defining $v_t = y_t^2 - h_t^2$, it follows that previous equation can be rewritten as

$$(1 - \alpha B - \beta B)y_t^2 = \phi + (1 - \beta B)v_t \tag{5}$$

where B is the backshift operator. This equation is analogous to an ARMA equation

$$y_t^2 = \sigma^2 + n_t \tag{6}$$

$$[1 - (\alpha + \beta)B]\, n_t = (1 - \beta B)v_t \tag{7}$$

with parameters $(\alpha + \beta)$ and β. Here σ^2, the variance, is given by[19]:

$$\sigma^2 \equiv E(y_t^2) = \frac{\phi}{(1 - (\alpha + \beta))}$$

hence the estimation procedure for time series models based on the state space KF can be used. It is shown in [10] that for GARCH processes, it is possible to have a state space formulation i.e. GARCH-KF. In order to obtain an efficient estimation for the parameters and hence predictions, the approach is based on the Kalman Filter.

2.3 Stochastic Volatility Model

Stochastic volatility is a popular modeling technique for non-linear/ non-Gaussian state-space models and hidden Markov models. It is probably an important volatility model for daily currency and equity indices returns, with forecast properties similar to ARCH or GARCH.

The stochastic volatility(SV) model in discrete time for a observation asset log-returns y_t is formulated in this section. The Stochastic volatility can be appropriately represented by the unobserved state variable as shown in equation (8) and (9).

$$y_t = \mu + \sigma_* exp\left(\frac{1}{2}h_t\right)\epsilon_t, \qquad \epsilon_t \sim IID(0, 1), \tag{8}$$

with

$$h_{t+1} = \phi h_t + \eta_t, \qquad \eta_t \sim IID(0, \sigma_\eta^2), h_1 \sim N(0, \sigma_\eta^2/(1-\phi^2)), \tag{9}$$

for $t = 1, \ldots, T$. The parameter μ denotes the unconditional expectation of the return process y_t. The scaling parameter σ_* is the average standard deviation with $\sigma_* > 0$. The unobserved log-volatility process is denoted by $h_t = log(\sigma_t^2)$ [20].

Here, the logarithm ensures positivity of (σ_t^2). The regression parameter ϕ is $0 < \phi < 1$ [20] and is usually reported to take on values greater than 0.8. The constant μ will be treated as fixed and set to zero as shown by [21]. Here, h_t is modeled as a first-order autoregressive process. For details see [2]. The disturbances ϵ_t and η_t are Gaussian white noise[22] where, ϵ_t represents the new information ; η_t reflects the shocks to the newsflow's intensity.

The SV model in (8) and (9) are commonly referred as the log-normal SV model. It represents a state space model where the observation equation describes the relationship between the univariate vector of observations, $y = (y_1, \ldots, y_T)'$, and the state vector. The hidden volatility process $\theta = (h_1, \ldots, h_T)'$ is specified in the state equation, which models the dynamic properties of h_t. As ϵ_t and h_t, are stochastic (both enter the multiplication in the mean equation), the basic SV model is nonlinear, hence linear approach cannot be used.

In the estimation problem there is no closed expression for the likelihood function. Therefore, the parameters of the SV model cannot be estimated by directly applying standard maximum likelihood techniques. The estimation has to be conducted by approximation of Likelihood or using simulation-based techniques based on observable past information[23]. There are methods proposed [7, 2] to linearize the SV model by squaring the returns and taking logarithms as shown in equation (10) and (11):

$$logy_t^2 = h_t + log\sigma_*^2 + log\epsilon_t^2, \tag{10}$$

$$h_{t+1} = \phi h_t + \eta_t, \tag{11}$$

where the disturbance term, η_t, is assumed to be uncorrelated in the transformed model. Taking logarithm leads to a heavily skewed distribution of log ϵ_t^2 with a long left-hand tail.

Our model-based estimated actual volatility is fitted well, by the Gaussian distribution using Quasi Maximum Likelihood. The estimates of volatility [24] are obtained by

$$\begin{aligned} &\sigma^2_{t|T} \equiv Var(y_t|Y_T) \\ &E(exp(h_t)|Y_T) = exp(\mu_{t|T} + s^2_{t|T}/2) \end{aligned} \tag{12}$$

with variance

$$\begin{aligned} Var(\sigma^2_{t|T}) &= E(exp(2h_t)|Y_T) - \{E(exp(h_t)|Y_T)\}^2 \\ &= exp(2\mu_{t|T} + 2s^2_{t|T})\left\{1 - exp(-s^2_{t|T})\right\} \end{aligned} \tag{13}$$

The smoothing estimates of the square root of volatiltiy are also calculated by

$$\sigma^*_{t|T} \equiv E(exp(h_t/2)|Y_T) = exp(\mu_{t|T}/2 + s^2_{t|T}/8) \tag{14}$$

with variance

$$= exp(\mu_{t|T} + s^2_{t|T}/2)\left\{1 - exp(-s^2_{t|T}/4)\right\} \tag{15}$$

2.4 Kalman Filter

The general form of a state space model is defined by an observation (or measurement) equation and a state transition (or state) equation. The structure and dynamics are similar to linear state space gaussian model with noise [1]. The measurement equation describes the relation between observed sequences and unobserved (hidden) state variables. State transition equation describes the dynamics of the state variables based on information from the past. The forecast of the next state can be completely described by the knowledge of the present state and the future input.

The Kalman filter method, is an iterative computational algorithm which is used to calculate forecasts and forecast variances for state space models. The algorithm is as follows: First, in each step of the process, the next observation is forecasted based on the previous observation and the estimate of the previous observation. Second, one step ahead forecast is computed by updating

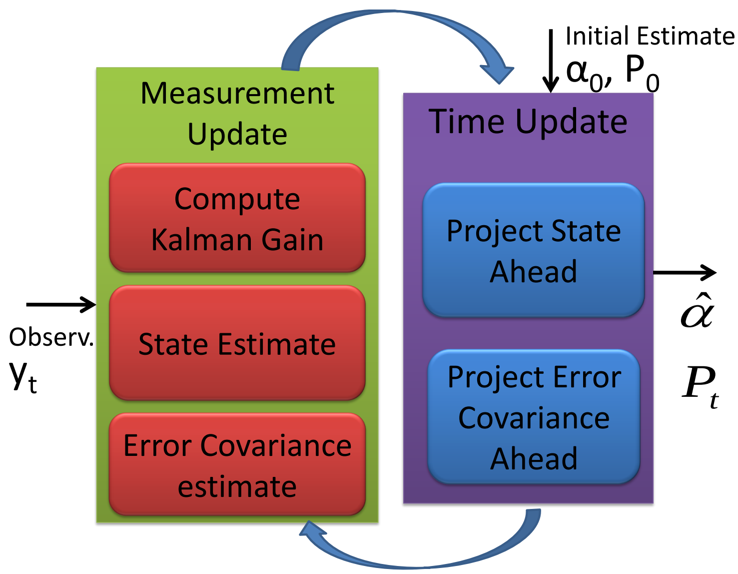

Figure 1: Kalman Filter Recursion

the previous forecast. Finally, the above process is repeated again. Figure 1 explains the recursion process in an easy and convenient flowchart form [25].

The updates for each forecast are weighted averages of the previous observation and the previous forecast error. The weights are also known as the Kalman gain. In the algorithm, the kalman gain is chosen such that the forecast variances are minimized in least square sense.

The KF recursively computes the states predictions of y_t, which is conditional on past information and also on the variance of their prediction error. The vector, v_t, is the time t innovation. i.e. the new information in y_t that could not be predicted from knowledge of the past, is the one-step-ahead prediction error.

The normal Kalman filter recursion are as follows

Measurement Update:

$$
\begin{aligned}
v_t &= y_t - Z_t a_t, \\
F_t &= Z_t P_t Z_t^T, \\
K_t &= T_t P_t Z_t^T F_t^{-1}, \\
L_t &= T_t - K_t Z_t.
\end{aligned}
$$

Time Update:

$$
\begin{aligned}
a_{t+1} &= c_t + T_t a_t + K_t v_t, \\
P_{t+1} &= T_t P_t L_t^T + R_t Q_t R_t^T.
\end{aligned}
$$

The matrix K_t is referred as the Kalman gain and the models uses it extensively for forecasting. Kalman filter can additionally be used to improve upon the measurement of current and past volatility estimates using filtering and state smoothing which are not used in this paper.

3 Estimation and Model Analysis

This study uses daily closing value of SENSEX from the time period starting from 01 January 2006 to 22 August 2013. All the stock market index data are collected from the official website of BSE. The daily returns are calculated for each series shown in equation (16).

$$
r_t = (log(P_t) - log(P_{t-1})) * 100 \tag{16}
$$

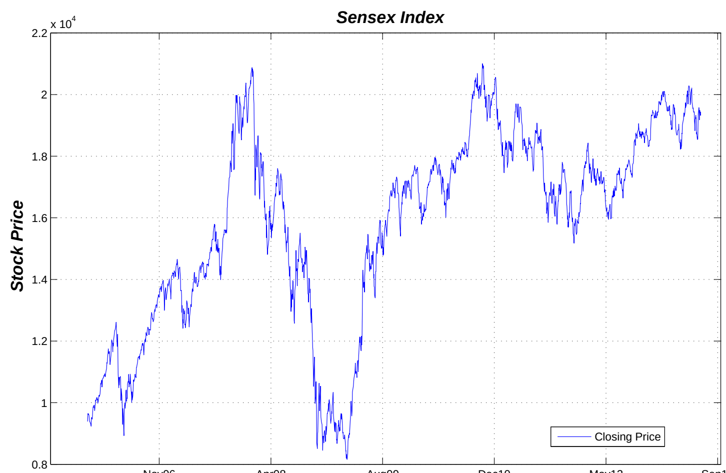

Figure 2: Sensex Closing Index

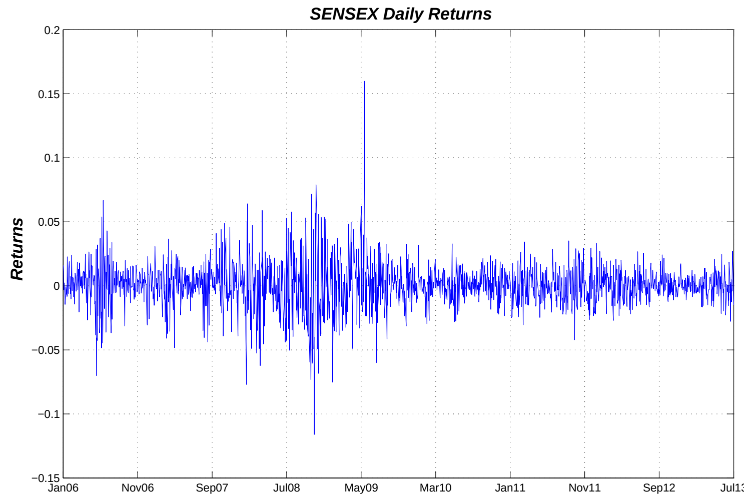

Figure 3: Sensex Return

where r_t is the daily return series, P_t is the current stock price and P_{t-1} is the stock price of the previous period. Figures 2 and 3 respectively plot the price and the index returns over the sample period. Our final working sample consists of 1900 data points for Sensex. In order to make forecasts, the whole sample is divided into two parts comprising of 1870 in-sample observations from 01 January 2006 to 07 July 2013 and 30 out-of-sample observations from 08 July 2013 to 22 August 2013 which are used for model performance evaluation.

Descriptive Statistics(SENSEX)	
Period	1/1/2006 to 22/8/2013
Mean	0.00039114
Median	0.00097652
Maximum	-0.1160
Minimum	0.1599
Kurtosis	10.1986
Skewness	0.1321
Std. Dev	0.0171

Table 1: Descriptive Statistics

Descriptive statistics for the Sensex returns series are shown in Table 1. It shows the mean, median, maximum, kurtosis, skewness and standard deviation of the series. As expected for a time series of returns, the mean is close to zero. The mean daily return is 0.00039114. The sample maximum is 0.1599 which happened on 18 May 2009. The volatility (measured as a standard deviation) is 0.0171. The returns are positively skewed (skewness= 0.1321) which indicates that there are more positive than negative outlying returns in Indian Stock Market. The kurtosis coefficient is positive, having high value for the return series(Kurtosis = 10.1986) which indicates leptokurtosis or fat taildness in the underlying distribution.

3.1 Unit root test

Augmented Dickey Fuller(ADF) test is used to test for the presence of unit root in the returns series. The ADF test statistics is tested for the null hypothesis of unit root at 1% level of significance. A formal application of ADF test on log returns, rejects the null hypothesis of a unit root in the return series. The value of ADF statistics is much lower than the critical value for the model *with trend (-2.5691)* and *without trend (-1.9416)* at all the 4-lags shown in Table 2. Hence, the hypothesis that the daily volatility in the Sensex index over the period from January 2006 to July 2013 has a unit root is rejected.

ADF Test on SENSEX			
	With Trend	**Without Trend**	
Lags	stat value	stat value	p-value
0	-40.3126	-40.3126	0.001
1	-40.3146	-30.2715	0.001
2	-40.2924	-25.3026	0.001
3	-40.2708	-22.4195	0.001
4	-40.2496	-20.3959	0.001

Table 2: ADF Test

3.2 Model Estimation

Maximum Likelihood estimation is performed for identification of parameters in both the models. Log likelihood is maximized numerically using the kalman covariance, details are given by Durbin and Koopman(2001) [2] and [26]. Recently, full asymptotic solver based on maximum likelihood estimation of state space models has been provided by Aston and Peng(2011) [18] in SSM Toolbox of Matlab. The *estimate* and *kalman* functions of SSMODEL class were used to perform the estimate and 1-step ahead prediction for SV Model. The GARCH models are estimated using $E4$ toolbox which is based on exact maximum likelihood optimization. Model specification and forecasting was also performed using the $E4$ toolbox [10]. The model estimates of **GARCH(1,1)** model after estimation are:

GARCH(1,1) model Parameter Estimates			
Parameter	Estimate	Std. Error	t-test
ϕ	0.027385	0.00593	4.6180
α	0.10127	0.0091045	11.1231
β	0.89225	0.008909	100.1522

Table 3: Garch(1,1) model Estimation

The model estimates of **GARCH(1,1)-KF** model in state space form after estimation are:

GARCH(1,1)-KF model Parameter Estimates				
Parameter	Estimate	Std. Dev.	t-test	Gradient
σ_t	2.6820	0.8377	3.2015	0.0000
$(\alpha + \beta)$	0.9898	0.0047	-210.7738	0.0053
β	0.8971	0.0122	-73.4885	-0.0029

Table 4: Garch model Estimation with Kalman Filter

The model becomes $y_t^2 = 2.6820 + n_t$ with $[1 - .9898B]\, n_t = (1 - 0.8971B)v_t$

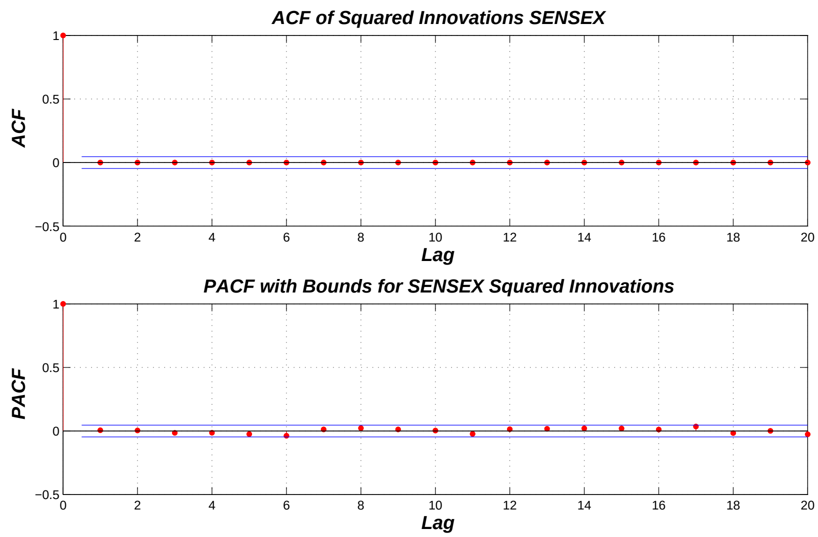

Figure 4: ACF & PACF

The ACF and PACF are used as identification tools as they provide some indication of the broad correlation characteristics of the returns [23]. Figure 4 shows graphically the ACF and PACF of squared innovations of GARCH model, there is little indication of correlation in the residuals, which is clear as there are no significant spikes. To conclude, the residuals of the GARCH models are consistent with white noise. The figure shows that ACF die out quickly, indicating that the GARCH process innovations are close to stationary. This is an indication that GARCH models is adequate model class for this data set.

The QML estimation method is used to estimate the model parameters of SV. This is implemented by combining predefined observation Gaussian noise with constant and autoregressive model using model concatenation in SSMODEL class in the toolbox. Hence, the estimates are obtained by the Kalman filter by treating ϵ_t and η_t as though they were normal and by minimizing the prediction-error. To solve the QML estimates, *estimate* and *kalman* function operations are performed on return series with the specified SV Model parameters.

Table 5 shows the estimates of **Stochastic Volatility** model in state space form:

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 0.9887 \end{bmatrix}, R = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, Z = \begin{bmatrix} 1 & 1 \end{bmatrix}, H_t = [4.834], Q = [0.02138] \quad (17)$$

SV Model Parameters Estimates	
Variables	**SV Model**
$\phi 1$	0.9887
ζ	0.02138
ϵ	4.834

Table 5: SV Model Parameter Estimate

3.3 Evaluation Measures

Four measures are used to evaluate the forecast accuracy of the models, namely, the mean square error (MSE), the root mean square error (RMSE), the mean absolute error (MAE) and mean absolute percentage error (MAPE). They are defined by:

$$\begin{aligned} MSE &= \frac{1}{n}\sum_{t=1}^{n}(\hat{\sigma_t} - \sigma_t)^2 \\ RMSE &= \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{\sigma_t} - \sigma_t)^2} \\ MAE &= \frac{1}{n}\sum_{t=1}^{n}|\hat{\sigma_t} - \sigma_t| \\ MAPE &= \frac{1}{n}\sum_{t=1}^{n}(|\hat{\sigma_t} - \sigma_t)/\sigma_t| \end{aligned} \quad (18)$$

where, $\hat{\sigma_t}$ is the forecast value and σ_t is the actual value calculated using equation(19). Statistically, actual volatility is often estimated as the sample standard deviation

$$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \mu)^2} \quad (19)$$

where r_t is the return on day t, and μ is the average return over the T -day period. In this context, the model which has minimum forecast error using the evaluation measure techinques mentioned above, is considered to be the best volatility forecasting model. Table 6 clearly shows that both the models has less forecast error values by using all four evaluation measures.

Forecast Error Statistics			
	GARCH(1,1)	**GARCH-KF**	**SV Model**
MSE	0.0985	0.0790	0.0340
RMSE	0.3138	0.2811	0.1843
MAE	0.2828	0.2487	0.1552
MAPE	0.3017	0.2695	0.1619

Table 6: Forecast Error Statistics

The result of the estimated volatility values by using Garch(1,1) and SV model is shown in figure 5

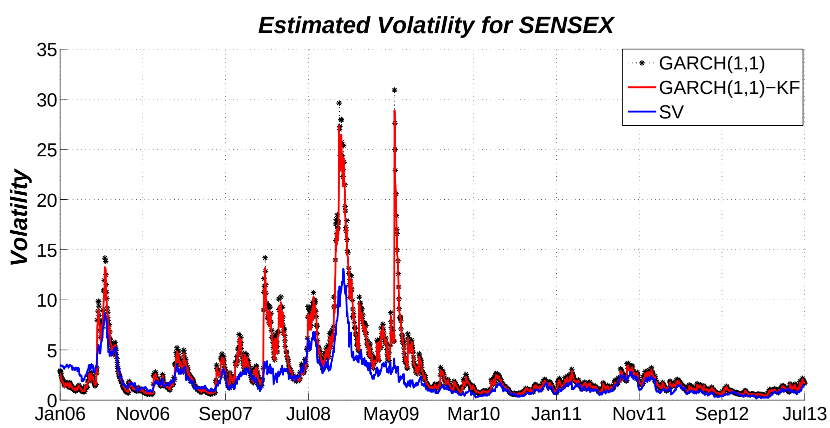

Figure 5: In Sample Volatility estimate

4 Comparison of Out of Sample forecast

After obtaining the daily volatility series, the 1-day ahead forecasts are chosen for the forecasting horizon of 30 days. Furthermore, a period has to be

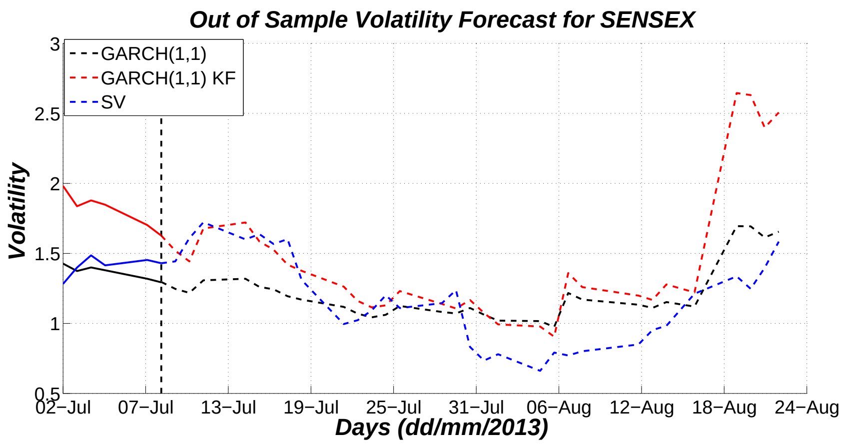

Figure 6: Out of Sample Forecast

chosen for estimating parameters and a period for predicting volatility. The data series of period starting from 1/1/2006 to 7/7/2013 is used to estimate the models. Therefore, the first day for which an out-of sample forecast is obtained is 08/07/2013.

Using the estimated models, sequential 1-day ahead forecasts are made. Hence, in total 30 daily volatility values are forecasted. With this setup, the models are required to predict volatility for the above mentioned period. The out of sample forecast for GARCH(1,1) and Stochastic Volatility models are shown in figure 6.

Figure 7 shows the 30-day ahead point forecast of both the models plotted along with the actual volatility, which is used as benchmark calculated using equation (19). The graph shows that stochastic volatility has more appropriate forecast as it has lesser residual errors when both models are estimated using maximum-likelihood based technique.

5 Conclusion

This paper examined the performance of state space methods for forecasting stock market volatility of the Sensex index. The important models considered here are the GARCH(1,1), GARCH(1,1)-KF and SV models. GARCH(1,1)-KF is an alternative estimation of GARCH(1,1) model using the

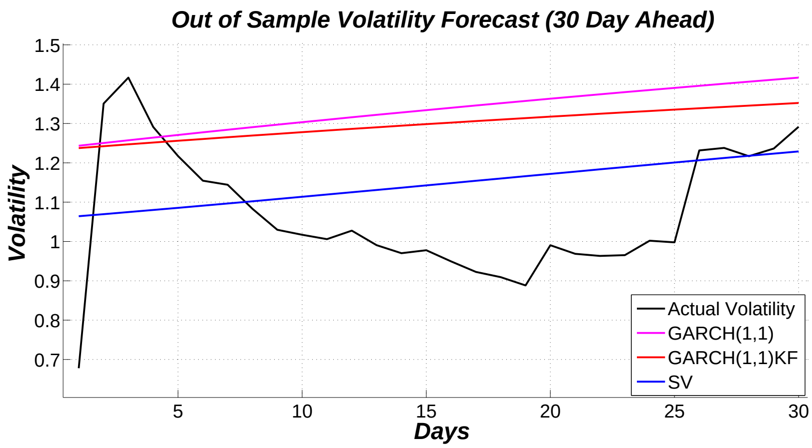

Figure 7: Volatility Forecast Comparison

Kalman filter approach with the model representation in state space form.

To investigate the forecasting ability of the models we fit the models to daily closing returns and estimate the models using SSM and E^4 toolbox in Matlab. Forecast errors based on return forecasts of the estimates and the actual volatility are used to evaluate the out-of-sample forecasting ability of the three models. It was found that the SV model forecasts are more precise based on the evaluation measures for 30-day ahead forecast. The results also show evidence in favour of the Kalman filter approach as compared to GARCH(1,1) estimation methods.

The empirical results of this paper provide strong support for the application of the state space model in volatility forecasting. Finally, we have presented a simple forecasting application of the Kalman Filter, which has proved useful to forecast the stock market data. For future research, it would be interesting to explore different fat tailed distribution and time varying KF model to fit the above models in state space form.

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