

# Effect of trajectory optimization parameters on energy consumption and CO<sub>2</sub> emissions for a gasoline powered vehicle

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## Abstract

One of the most important research interests in the automotive sector is energy consumption and pollutant emissions reduction. Usage developments are continuing in parallel to technological evolution to maximize the efficiency of the system. One of the recent methods in usage field is eco-driving. It refers to a fuel economic driving behaviour which is the result of a trajectory optimization. In this research a parametric study on the impact of some optimization constraints on the optimal operation of a gasoline powered vehicle is presented. A vehicle model is described and the optimization algorithm based on dynamic programming is introduced. The optimization results on a defined trajectory under different road and trip constraints are discussed in this paper. These constraints include trip time and road grade, as well as the impact of state discretization. With the results, impact of these constraints on fuel consumption and CO<sub>2</sub> emissions is presented and the trade-off between fuel consumption and trip time is discussed. The results show that optimal vehicle operation vary depending on road and trip constraints.

**Keywords:** Trajectory optimization, eco-driving, dynamic programming, fuel consumption.

## 1 Introduction

The transport sector is one of the major contributors of non-renewable fuel consumption and greenhouse gas emissions (GHG). Higher transport demand,

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increased ownership of private cars, growing settlement and urban spreading with longer distances and changes in lifestyle have resulted in increasing energy consumption, Eurostat (2013). With this in mind, energy consumption and air pollutant emissions reduction has become one of the most important research subjects in automotive sector.

Regarding the worldwide growing demand for transportation, there is not a single major development that can solve the problems of fuel consumption and pollutant emissions alone. Therefore, developments in different technological and usage fields should continue in parallel to resolve these issues. The available options for reducing fuel consumption are developing more efficient drive train systems, (i.e. engines and transmissions), reducing vehicle weight and drag by using lighter materials and redesigning vehicles, using alternative fuels and developing new technologies such as electric and hybrid vehicles.

By developing more efficient powertrain systems, the components are designed to operate with the highest efficiency, respecting the performance demand by customer. However, since driving consists of highly complex tasks, the driver using this powertrain is an uncertain parameter and his behaviour has a direct impact on its efficiency. Therefore, another category of developments to reduce fuel consumption of vehicles is to adapt the driver behaviour such that vehicle operation is optimized. This is referred to as eco-driving. Eco-driving is the concept of changing driving behavior and vehicle maintenance practices to improve fuel efficiency and reduce greenhouse gas emissions of existing vehicles. First it was shown by Schwarzkopf (1977) that the fuel consumption depends not only on drive train efficiency, but also on vehicle's operation. Since then many studies have shown that eco-driving has a great potential in reducing fuel consumption and emissions, Monastyrsky (1993), Martin (2012) and Mensing (2013).

In addition to driving behaviour, the concept of eco-driving may also include trip planning, idling avoidance and vehicle maintenance. Ericsson (2000) identified acceleration, deceleration, speed, and gear change behaviour as the main indicators of an eco-driving style. Besides these factors, Beusen (2009) included coasting distance, i.e. the distance drivers let the car roll without pressing the gas or brake pedals in their analyses. In comparison to other technological solutions for fuel consumption reduction, eco-driving is less expensive and contributes to high gains in fuel consumption. Constructors try to develop fuel economic solutions while respecting the constraints on pollutant emissions in order to respect emission regulations.

There are different approaches in implementing the results of trajectory optimization in real vehicle operation. An interface is needed in order to transfer information about optimal vehicle operation to the driver.

In-vehicle driver support systems give continuous feedback and guidance to driver in order to drive in optimal vehicle operation. Several such systems have been presented and evaluated in the literature. In these eco-driving support systems, the optimal operation of vehicle is determined. Generally, these systems can be

grouped in two categories: advisory systems and active systems. Advisory tools consist of a display that gives advice to the driver for reducing his fuel consumption. In active systems, some limitations are imposed to the driver so that he does not have access to all engine performance zones. Or an active accelerator pedal is used which influences the acceleration rate by applying some resistance on the gas pedal, Varhelyi (2004).

In this research we have performed a parametric study on the impact of several road and trip constraints on optimal trajectory. For this purpose, the trajectory optimization algorithm developed by Mensing (2013) in her PhD thesis was used. The algorithm is based on dynamic programming method for finding the optimal velocity and gear choice for a conventional passenger vehicle. By applying the numerical trajectory optimization method on an inverse vehicle model, the best vehicle operation for a given trip is determined. The impact of time constraint, road grade and state discretization size on optimization results were investigated and optimal vehicle operation under these circumstances was compared. By taking into account the road grade in the optimization process, engine cut-off and free roll strategies were compared.

In the following section the vehicle modeling for optimization is presented. In the third part the optimization method, our approach in this work and constraints application will be presented. In the fourth section results of optimization under these constraints and their analysis is discussed.

## VEHICLE MODEL

The vehicle studied in this work is a conventional passenger car with a weight of 1470 kg. Its drive train architecture is modeled as presented schematically in Figure 1. It consists of the engine, the clutch, a five speed gear box and the final drive reduction. Auxiliary losses are also added to the model. The engine is a 1.6 liters gasoline engine.

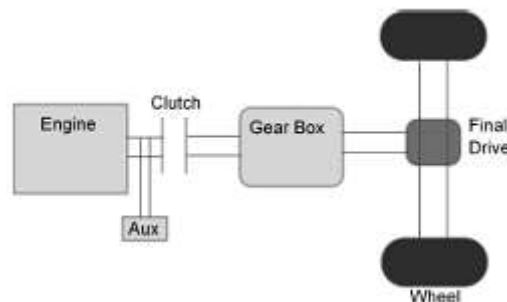


Figure 1: Drive train schematic of a conventional vehicle

For optimization goals, an inverse vehicle model is used. In direct modeling, the system inputs are inserted by driver, which are acceleration and brake pedals, gear choice and auxiliary use. Using these inputs, the drive train components operation

is calculated and the required drive torque and engine speed are determined. The output of this model is the vehicle speed and acceleration. However, in the inverse approach, a desired vehicle speed and acceleration is defined as input. The drive train components operation is calculated in backward to identify the required engine operation points for satisfying the desired vehicle operation. Therefore, the energy consumption of vehicle can be determined for a given vehicle mission. With this approach the human driver is eliminated from the calculations, as well as uncertainties due to human behaviour. By using an inverse model in the optimization process, the necessary drive train output, which minimizes fuel consumption, can be determined as a function of specified vehicle speed and acceleration.

The required force at wheels to reach desired velocity and acceleration can be calculated by considering resistance forces and drive train force ( $F_{drive}$ ). Resistance forces are consisted of aerodynamic drag ( $F_{aero}$ ), rolling resistance ( $F_{roll}$ ) and road grade resistance ( $F_{grade}$ ).

By considering longitudinal motion of vehicle and writing Newton's second law for chassis, we have:

$$M_{veh} \frac{dV_{veh}}{dt} = F_{drive} - F_{res}$$

$$M_{veh} \frac{dV_{veh}}{dt} = F_{drive} - F_{aero} - F_{roll} - F_{grade}$$

$$F_{aero} = \frac{1}{2} \rho C_d A V^2$$

$$F_{roll} = C_r M_{veh} g \cos(\alpha)$$

$$F_{grade} = M_{veh} g \sin(\alpha)$$

Where  $M_{veh}$  is the vehicle mass,  $g$  the gravitational constant,  $\rho$  the air density and  $A$  the vehicle's frontal surface.  $C_d$  and  $C_r$  represent coefficients of drag and rolling resistance respectively and  $\alpha$  is the road grade.

By calculating the vehicle speed ( $V_{veh}$ ) and knowing the tire radius ( $R_{tire}$ ) we can determine wheel operation as:

$$\omega_{wheel} = V_{veh} / R_{tire}$$

By modeling the losses in the gear box, the final drive, the clutch and the auxiliaries, we are able to determine the engine operation. Constant efficiency for the final drive and gears with respect to operation torque and speed is assumed. Losses in the clutch exist if the engine speed is below idle and the clutch is slipping. The auxiliary components are assumed to absorb a constant power of 300W. The gasoline engine is modeled using an engine efficiency map with maximum efficiency of 40%. Knowing the engine speed and torque the instantaneous fuel consumption can then be calculated using a look up table. Using this model the instantaneous fuel consumption can be computed as a function of vehicle velocity ( $V_{veh}$ ) and vehicle acceleration ( $a_{veh}$ ):

$$\dot{m}_{fuel} = f(V_{veh}, a_{veh})$$

This inverse model is used in the optimization process to determine the fuel consumption for a given vehicle speed and acceleration. The optimal velocity profile and gear choice are determined to minimize the instantaneous fuel consumption.

## **TRAJECTORY OPTIMIZATION**

In this part the trajectory optimization method for calculating the optimal vehicle operation is presented. We have applied the algorithm developed by Mensing (2014) in her PhD studies. The algorithm is based on dynamic programming method for finding the optimal velocity and gear choice for a conventional passenger vehicle. By applying the numerical trajectory optimization method on an inverse vehicle model, the best vehicle operation for a given trip is determined.

The idea of using an optimization algorithm to find the optimal velocity profile for road vehicles was first studied by Schwarzkopf by using Pontryagin's Maximum Principle, Schwarzkopf (1977). In general, optimization methods applied to trajectory optimization problems are the dynamic programming method or the Pontryagin's Maximum Principle. In Dynamic programming method usually complex system models and constraints are easily implemented, therefore it has a high computational cost. Dynamic programming is based on the Bellman principle of optimality, which states: "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.", Bellman (1957).

In dynamic programming the optimal result is found by calculating the cost and indexes backwards, i.e. from the final state to the first state. Then the optimal trajectory is retraced by using the optimal indexes in forward, Mensing (2013).

The objective of this part is to minimize the vehicle's energy consumption. For this purpose, the optimization objective function and constraints have to be defined.

By defining the vehicle motion in discrete form we have:

$$d_{k+1} = d_k + V_k \Delta t + \frac{1}{2} a_k \Delta t^2$$

$$V_{k+1} = V_k + a_k \Delta t$$

Where  $d$  is the distance,  $V$  and  $a$  are the longitudinal velocity and acceleration of vehicle respectively and  $\Delta t$  is the time step for discretization. By considering distance ( $d$ ) and vehicle velocity ( $V$ ) as two states of the system, the constraints on initial and final states will be:

$$d(0) = d_0 \quad d(f) = d_f$$

$$v(0) = v_0 \quad v(f) = v_f$$

$$t_f = T$$

The objective function to be minimized is represented as:

$$J_1 = \sum_{t=0}^T \dot{m}_{fuel}(t)$$

In order to respect the time constraint, Mensing (2013) developed a two dimensional dynamic programming algorithm with a weighting factor integrated in the cost function. This approach reduces the computational time about 80% comparing to a three dimensional dynamic programming and considering time as the third state, Mensing (2013). Therefore the objective function for minimizing fuel consumption while respecting a desired arrival time can be presented as:

$$J = J_1 + \beta \sum_{t=0}^T \Delta t$$

In order to respect a final arrival time,  $T$ , the factor  $\beta$  should be determined. For a vehicle mission defined on a distance with velocity and time constraints, a  $\beta$  value can be associated to the final time. To fix the time constraint, an advanced root-

finding method, Brent (1971), and an iterative process of  $\beta$  update is used. By minimizing the error function the appropriate  $\beta$  is determined for respecting the desired arrival time,  $T$ :

$$error = T - t_f(\beta)$$

Using the dynamic programming optimization method and applying constraints to study vehicle mission under different conditions, we are able to determine the impact of several road and trip constraints on fuel consumption and CO<sub>2</sub> emissions.

## 2 Results

In this section the results of trajectory optimization with constraints on trip time and road grade will be discussed. Also the impact of discretization step on optimal results will be presented.

### 2.1 Arrival time vs. fuel consumption, a trade-off

As demonstrated by previous research (Mensing, 2013), fuel consumption is highly related to the duration tolerated by the driver to achieve his trip. In order to study the trade-off between fuel consumption and final arrival time, the Pareto front is computed. This method is usually used in multi-objective optimization problems. In this study, since the objective functions are fuel consumption and arrival time, the Pareto front could be illustrated in a 2-D plot as in Figure 2.

Figure 2 illustrates the results of two dimensional dynamic programming with the arrival time on x-axis and fuel consumption on y-axis for different  $\beta$  values. On this figure the trade-off between these two objectives is presented. For  $\beta=0$ , no weight is associated to arrival time in the objective function, therefore higher arrival time is achieved while reducing the fuel consumption. By increasing the value of  $\beta$ , arrival time is reduced and naturally the fuel consumption is increased.

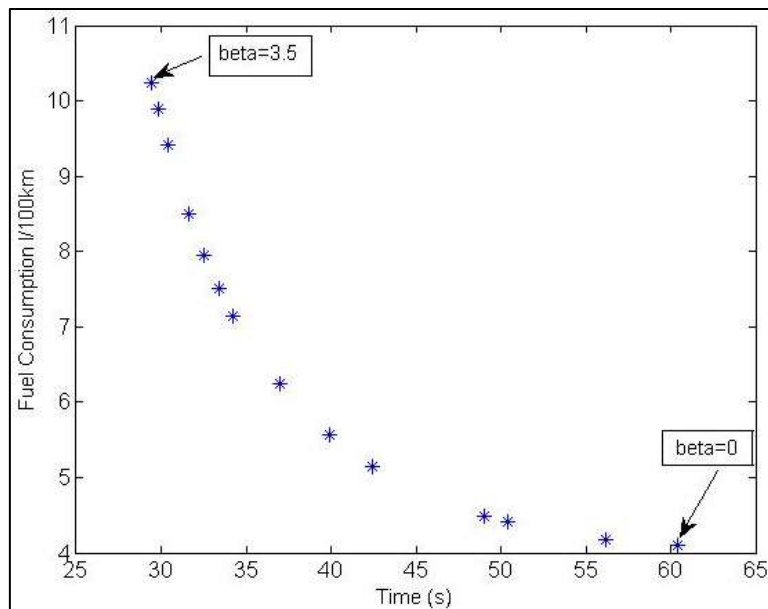


Figure 2: Pareto front

## 2.2 Time constraint effect

In order to determine the impact of time constraint on the optimal velocity profile and gear choice, we applied the two dimensional dynamic programming algorithm with a weighting factor on time on a 500 meters trajectory.

Discretization steps in distance and velocity are  $\Delta d=10$  m and  $\Delta v=0.1$  m/s. The final arrival time is set to 30 s.

Figure 3 illustrates the impact of time constraint on optimal trajectory. On the figure on top, we can observe the optimal velocity profile with imposed arrival time (blue) and without time constraint (red) according to time. We have the same comparison for optimal gear choice and fuel rate for both cases on the middle and bottom figures respectively. As we can observe from the figure, time constraint results in driving with higher velocities. It is seen that the arrival time when no time constraint is set, is twice the final desired time (60 seconds).

By comparing gear choices, it is seen that with time constraint, reaching a high velocity and passing to neutral gear (sailing idle) is more favorable for fuel consumption. However when the arrival time is set free, driving with lower velocities and passing to injection cut off is more favorable. This can be observed on the bottom figure, where the fuel injection rate is around zero from  $t=30$  seconds to the end of trip (red curve).



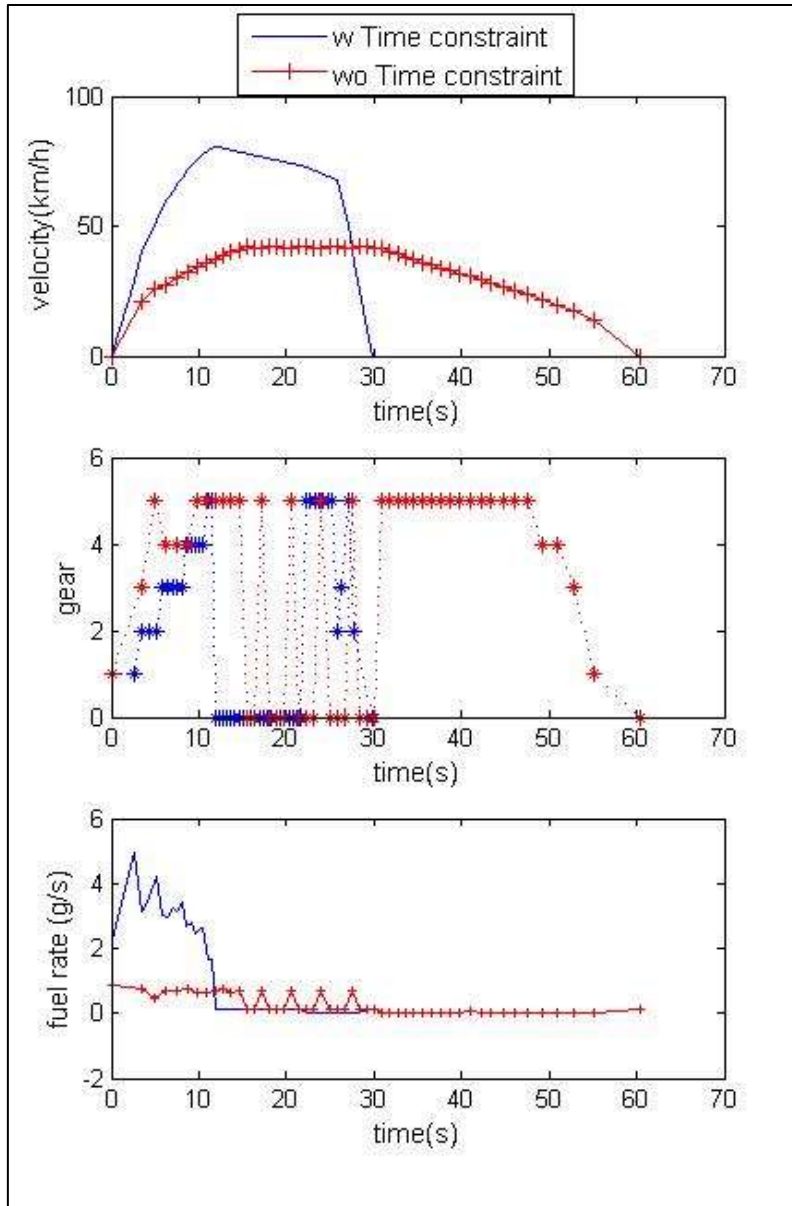


Figure 3: impact of time constraint on optimal trajectory on a flat road

For a better comparison of two optimal velocity profiles with and without time constraint, both profiles are illustrated in Figure 4. It can be seen that in case of no time constraint, the optimal solution between  $t=15$  to 30 seconds of trip, is an oscillation of nearly 2 km/h around the cruising speed by engaging and disengaging the neutral gear (sailing idle).

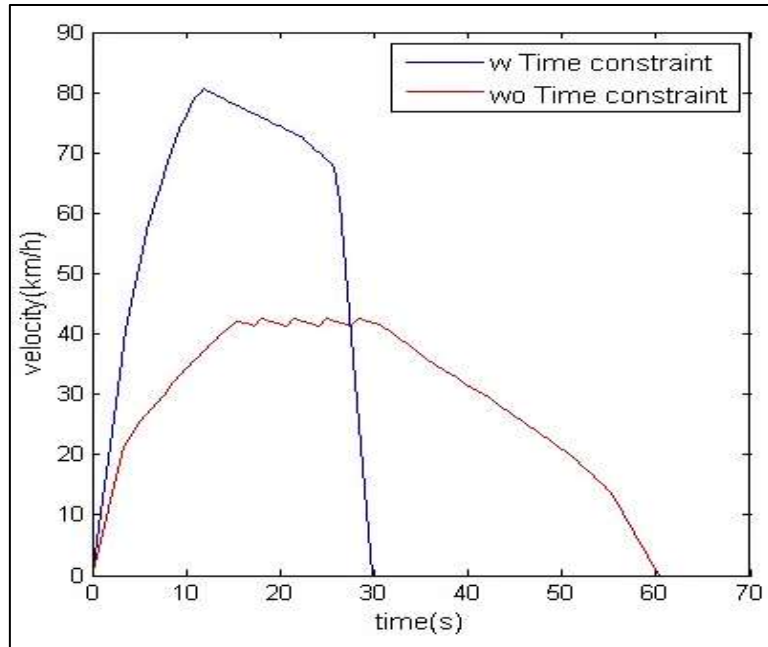


Figure 4: impact of time constraint on optimal velocity profile on a flat road

### Grade effect

Next we have studied the impact of time constraint on a non-flat road. The travelled distance is 500 m long, with a maximum grade of 4% along the road, as illustrated in Figure 5.

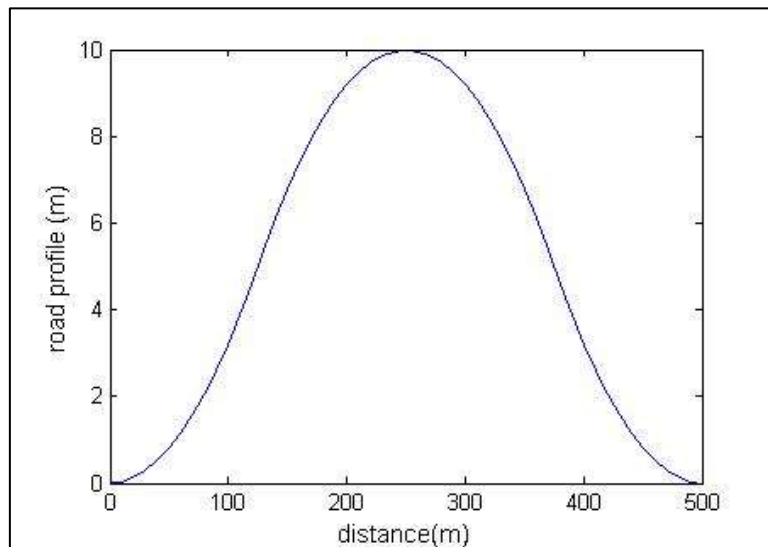


Figure 5: road profile with a maximum grade of 4%

From Figure 6 it can be seen that similar to the flat road, when no time constraint is set, lower velocities and acceleration rates are achieved. However, as shown in the middle figure comparing gear selection, on downhill section of road (from 250 m), the neutral gear cannot be engaged for not exceeding maximum velocity limits and the optimal solution found is the injection cut-off.

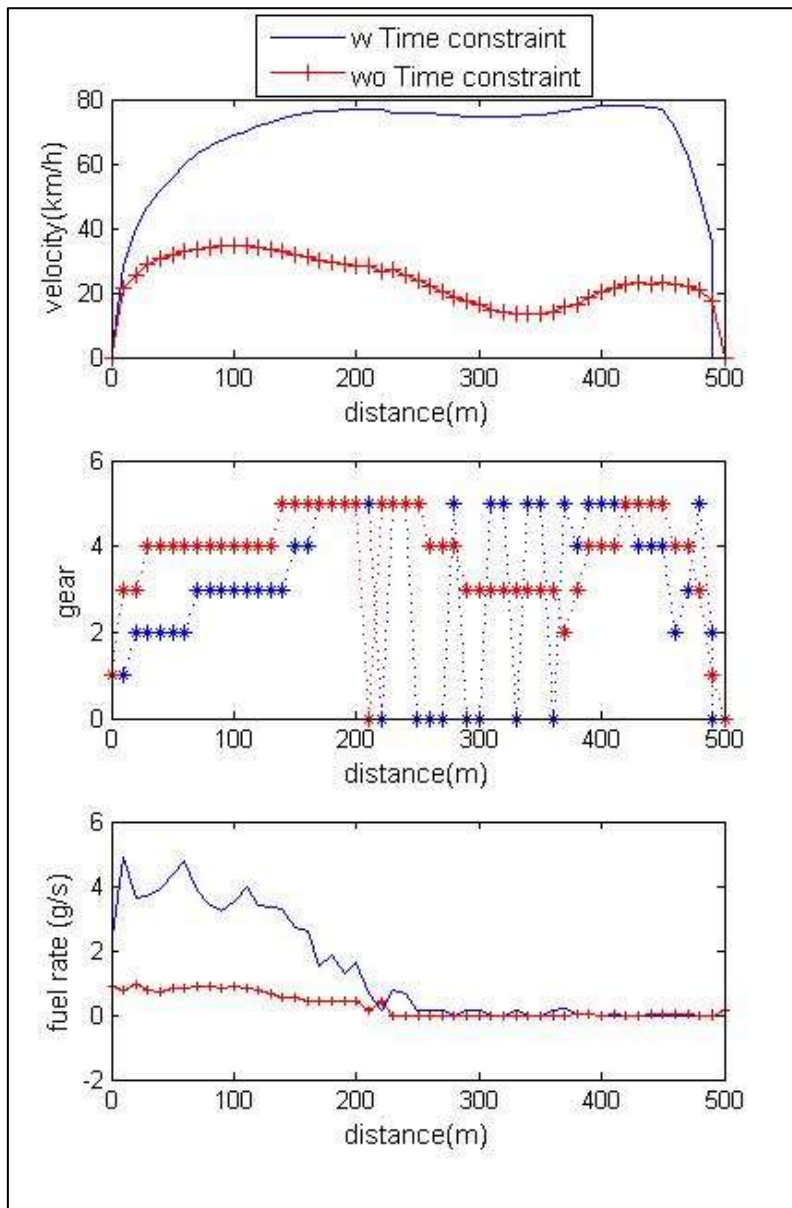


Figure 6: impact of time constraint on optimal trajectory on a non-flat road

### Discretization effect

Next we have studied the impact of state discretization step size on optimal solution. For this purpose, we have compared the results of trajectory optimization on a 500 m ride on a flat road without time constraint. We have compared the velocity discretization steps of 0.1 m/s and 0.2 m/s.

As illustrated on Figure 7, the optimal solution form depends on the discretization step. By reducing the step size in dynamic programming, more possibilities for optimal velocity appear in velocity edges. This influences also the optimal gear choice, as illustrated on the middle figure. By setting the  $\Delta v=0.1$  m/s in dynamic programming, the system has the possibility of choosing the neutral gear and showing the oscillating sailing idle behaviour as described above (blue curves in Figure 7).

However, when a bigger discretization step size is set in the optimization process ( $\Delta v=0.2$  m/s), all velocity edges do not appear for the optimal solution (red curves) and the sailing idle behaviour does not appear.

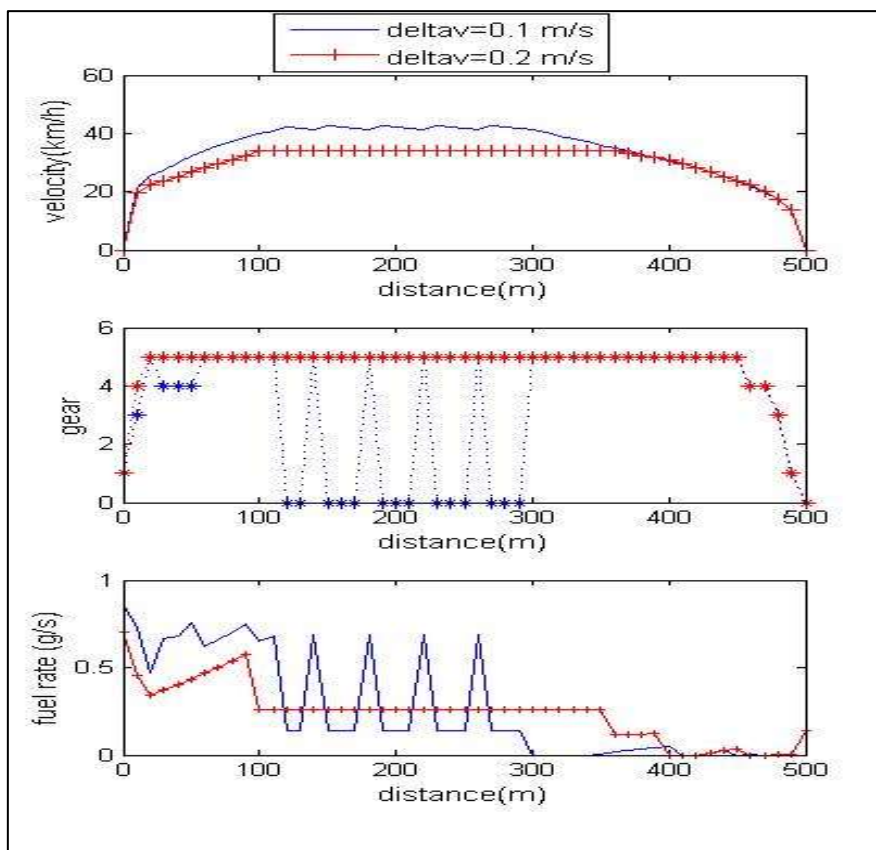


Figure 7: impact of velocity discretization size on optimal trajectory on a flat road

In **Error! Reference source not found.** we have compared the fuel consumption (l/100km) and CO<sub>2</sub> emission (g/km) for the studied vehicle missions described above.

Table 1: fuel consumption and CO<sub>2</sub> emission comparison with time and road grade constraints

Trip	Fuel consumption (l/100km)	CO <sub>2</sub> emission (g/km)
Flat with time constraint	9.9	232.4
Flat without time constraint	4.1	96.6
Max. 4% grade with time constraint	12.5	293.2
Max. 4% grade without time constraint	5.4	126.9

As it can be seen on the above table, by doubling the arrival time for the same trip, the CO<sub>2</sub> emission is reduced by 58%. The road grade of 4% has a less important impact on fuel consumption and CO<sub>2</sub> emissions. In case with arrival time constraint of  $T=30$  s, road grade causes an increase of nearly 26% in the CO<sub>2</sub> emissions. The increase is about 31% in case of no time constraint.

### 3 Conclusion

In this work we have performed a parametric study on the impact of some road and trip constraints on the optimal trajectory. A two dimensional dynamic programming method was used for trajectory optimization. By using an inverse vehicle model, the optimal vehicle operation was determined for a short simple trajectory of 500 m, under specific constraints on final arrival time and road grade. We have found that optimal trajectory solution depends significantly on these constraints, as well as numerical limitations such as state discretization step size. An important theoretical gain of 58% in fuel consumption was observed by increasing the trip time. Therefore, for minimizing a vehicle's fuel consumption and pollutant emissions, it would be essential to take into account the road and trip characteristics in finding the optimal velocity profile and gear choice. We have observed that according to the road grade and time constraint conditions, sailing idle strategy could be preferred to injection cut-off. We have also observed that the form of optimal solution in dynamic programming depends on the discretization size. By reducing the velocity step, sailing idle behaviour appears in the optimal solution on some parts of the trip. A perspective for the future work would be to use real driving cycles for trajectory optimization and validate the

results on an experimental setup. Furthermore, these information could be used in advance driving assistance systems for having more accurate eco-driving support tools, which take into account infrastructure's information in calculating the best vehicle operation.

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