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Estimating the VaR (Value-at-Risk) of Brazilian stock portfolios via GARCH family models and via Monte Carlo Simulation

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Abstract

The objective this work is to calculate the VaR of portfolios via GARCH family models with normal and t-student distribution and via Monte Carlo Simulation. We used three portfolios composite with preferential stocks of five Ibovespa companies. The results show that the t distribution adjusts better to data, because the violation ratio of the VaR calculated with t distribution is less than the violation ratio estimated with normal distribution.

Keywords: VaR; GARCH; Monte Carlo Simulation
JEL Classification: G17; C53

1 Introduction

The risk management has been passed by several changes in last decades. The financial deregulamentation with end of Bretton Woods system provided a higher investment diversity , as well as greater likelihood both profits and

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losses. Therefore, the agents problem consists in minimize risk maximizing return. The risk measure more employed by financial market is the VaR (Value at Risk) due the same to be simple because is showed in monetary value. The VaR arises after big losses that investors had in the begin of 90s. The VaR also is a of risk measure more used by Accord Basel to regulate the bank system.

The VaR is the higher loss probable whether the worst scenario occurs. The VaR is calculated by several methods, as parametrics, non-parametrics and semi-parametrics. The VaR also can be estimated via Monte Carlo Simulation or by historical simulation of returns.

The literature about the VaR is huge, including important works as Jorion (2007); Chela, Abrahão and Kamagawa (2011);Gaglione, Lima and Linton (2008); Manganelli and Engle (2001); Glasserman, Heildelberger and Shahabuddin (2000); Taylor (2005) among others. The works above disscus the VaR of several ways, like the CAViaR of Manganelli and Engle (2001) and the VQR backtesting of Gaglione, Lima and Linton (2008).

This paper propose to calculate the VaR of portfolios composed by Brazilian Companies Stocks employing GARCH family models following t and normal distribution, and through Monte Carlo Simulation for, to verify how distribution adjusts better to empirical data. The paper also will realize backtestings with the goal of to know whether the models perform well same for sample used, that comprehend high volatility periods, as subprime crisis in 2008. Beyond this introduction and conclusion the paper have three more sections. The second one review the literature about VaR and the third discuss the methods. The fourth section show the research results.

2 Literature Review

Bezerra (2001) estimates the VaR of Petrobras Stock employing Monte Carlo Simulation to compare the result with outcomes obtained through parametric models. The author found empirical evidence which the evaluation obtained by Monte Carlo Simuation overcomes the parametric method estimate. Still according to Bezerra (2001) the Monte Carlo Simulation Method is better due your ability of to capture non-linearity effects of financial assets.

Chela, Abrahão and Kamogawa (2011) estimated the VaR of three portfolios through GARCH-DCC and CCC, O-GARCH and EWMA models. The authors reduced the dimension of portfolio composed by interest rate, exchange rate, stock index and high volatility asset as CDS for instance, with the methodology denominated of main components. As evaluation criterium the researches employed the Kupiec test, the worst relative loss and the average VaR. The first one measures the cover efficiency, the second the cover in the worst scenario and the third the cover cost. The paper conclusion is that the better models according to criterium of the weighing among risk control in the frequency and in the worst loss average VaR cost were the traditional VaR estimated by EWMA and the VaR estimated by O-GARCH.

According to Jorion (2007) Mr. Till Guldimann of J. P. Morgan created the expression "Value-at-Risk" in the final of 80s. However, the VaR models started to be developed in the begin of 90s as response to financial crisis in this period.

Gaglione, Lima and Linton (2008) calculated the VaR via quantile regression, with the goal to check whether the risk exposure increase in the assets. The authors did Monte Carlo Simulation to show that your model have more power than another backtesting models.

Still according to Gaglione, Lima and Linton (2008) the VaR is a statistical measure that summarizes in a single number the worst loss in a time horizon given reliance interval and also is the main risk measure utilized by financial market. However, a author research problem is how calculate of the better way a VaR model. The VQR(VaR Quantile Regression) test shown in the paper finds evidences which the VaR underestimates the risk in some periods. To prove the VQR test efficiency the researchers did Monte Carlo Simulation and compared the results with other tests results. The VaR is estimated by Riskmetrics e with GARCH (1,1) model with normal errors. In some Monte Carlo experiments the Kupiec test (1995) and Christoffersen (1998) obtained a better performance than VQR test.

The simulation made with data of the S & P 500 by Gaglione, Lima and Linton (2008) shows that the GARCH (1,1) is a good VaR estimate according to the backtestings performed, despite of assumption of normality. Therewith, The authors don't model the stylized fact of heavy tail, making which likely the VaR violation is greater than significance level. The Riskmetrics model VaR (99%) hadn't a good fit for data, according to VQR test.

Cordeiro (2009) employs copula methodology to calculate Ibovespa VaR because, according author, copula function provides higher flexibility to risk agregation when compared with traditional approachs of risk measure. In the research is demonstrated several ways of to evaluate the VaR, for instance Monte Carlo Simulantion and GARCH family models. Cordeiro (2009) calculates the VaR using copulas, historical simulation, and delta-normal method.

Forward, The author performs the backtesting, aiming to check whether the estimated VaR from copulas have a better performance. The portolios used by Cordeiro (2009) are composed from Ibovespa index and exchange rate R$/U$$. The results indicate which for VaR 99% the better model was Frank Copulas and to VaR 95% delta-normal model performed better. In the conclusions Cordeiro (2009) stated that the main critic to use of delta-normal method is, it inability in to replicate fat tails of financial data.

Araújo (2009) demonstrates that a optimal portfolio composed by Brazilian multimarket funds is more efficient when the risk measure employed is the Conditional Value-at-Risk (CVaR). The portfolio was obtained through Markowitz efficient frontier. According to author, the CVaR measures the expected loss conditioned to expected loss which equal or higher than VaR. One of research conclusions is that portfolio funds selected from CVaR method generates a greater cover to investor. However, one of research fail is because it don't make backtesting to verify the efficiency of VaR and CVaR models.

Manganelli, and Engle (2001) evaluates the VaR through various methods, as GARCH models and Monte Carlo Simulation. There are in the paper two original contributions to epoch:the first one introduction of extreme value theory in the conditional Value-at-Risk and the second is the estimation of expected shortfall with a simple regression. The researchers emphasize which GARCH model and Riskmetrics underestimates the VaR when is assumed normal distribution in the errors. However, the EWMA and GARCH advantage cited by authors in relation to non-parametric and semi-parametric models is the absence of misspecification. The performance of models was evaluated through Monte Carlo Simulation. The conclusions shows that the CAViaR produces better estimate to heavy tails of financial data.

Taylor (2005) estimates the risk of stock indices and individual shares via CAViaR. One of Taylor (2005) conclusions is which the asymmetric CAViaR performs better than GARCH models estimated with t distribution. Taylor (2005) also defends the thesis of a improves of heavy tails modeling with the employ of CAViaR.

Glasserman, Heildelberger and Shahabuddin (2000) describes, analyses and evaluates a algorithm which estimates the probability of loss in a portfolio employing Monte Carlo Simulation. According to the authors the Monte Carlo Simulation can have a huge computational cost, mainly when We have a great number of assets in the portfolio or a high number of path simulations. Then, for diminish the path simulations, the authors use the variance reduction method for thus, to solve the problem of high computational cost.

Jorion (2002) detaches the VaR importance in comparison among risk profile of diverse banks. The paper estimates the relationship between disclosed VaR by banks and your revenues. This relationship is important , because shows how much the bank needs exposure itself to risk for increase your revenue. The research conclusion is which the banks with low exposure present a lower VaR and revenue volatility than the banks more exposed to risk.

3 Methodology

3.1 Data

The sample researched is composed from preferential stock prices of the following Brazilian Companies: Petrobras, Vale, Bradesco, Eletrobras and Pão de Açúcar. The period surveyed goes of January, 01, 2000 to March, 14, 2012. Next, We obtained the composed returns for all assets. Forward, We construct three portfolios from this assets. The percentage allocated in each stock was choosed through of optmization of returns according to Markowitz' efficient frontier and It will be presented in the results.

3.2 Value-at-Risk (VaR)

Danieíelsson (2011) defines the VaR as:

$$Pr\left[Q \leq -VaR(p)\right] = p \tag{1}$$

$$p = \int_{-\infty}^{-VaR(p)} f_q\left(x\right) dx \tag{2}$$

Where Q is the profit and loss function and p is the VaR probability. Therewith, the VaR can be easily calculated through of the expression:

$$VaR\left(p\right) = -\sigma\gamma\left(p\right)\vartheta \tag{3}$$

Where ϑ is the portfolio value and $\gamma\left(p\right)$ is the inverse of distribution used.

3.3 Family GARCH Models

Define ε_t the portfolio return without structure in the mean, the equation for conditional volatility is:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{4}$$

This is the GARCH model developed by Bollerslev (1986). With the goal of to model asymmetry of financial asset, Nelson (1991) creates the EGARCH model and Glosten, Jaganathan and Runkle (1993) developes the TGARCH model. The EGARCH specification is:

$$ln\left(\sigma_t^2\right) = \omega + \sum_{j=1}^{p} \beta_j ln\left(\sigma_{t-j}^2\right) + \sum_{i=1}^{q} \alpha_i \left[\left(\frac{|\varepsilon_{t-i}|}{\sigma_{t-i}} - E\frac{|\varepsilon_{t-i}|}{\sigma_{t-i}}\right)\right] + \sum_{i=1}^{q} \theta_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \tag{5}$$

The model captures basically if a negative shock in the return causes a higher impact in the volatility than positive shocks. Whether the parameter $\theta_j = 0$, not exists asymmetry.

The TGARCH model is defined by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \gamma_i I\left[\varepsilon_{t-i} < 0\right] \varepsilon_{t-i}^2 \tag{6}$$

The shock asymmetry is captured by binary variable which is 1 for negative returns and 0 otherwise. When θ is positive the model is asymmetric and negative returns impact more the volatility.

3.4 Monte Carlo Simulation

We follow the algorithm of Huynh, Lai and Soumaré (2008) and the data generator process for stock prices is:

$$\frac{dpetr4\left(t\right)}{petr4\left(r\right)} = \mu_{petr4} dt + \sigma_{petr4} \sqrt{dt} dZ_1\left(t\right) \tag{7}$$

$$\frac{dvale5\left(t\right)}{vale5\left(r\right)} = \mu_{vale5} dt + \sigma_{vale5} \sqrt{dt} dZ_2\left(t\right) \tag{8}$$

$$\frac{bbdc4\left(t\right)}{bbdc4\left(r\right)} = \mu_{bbdc4} dt + \sigma_{bbdc4} \sqrt{dt} dZ_3\left(t\right) \tag{9}$$

$$\frac{elet6\left(t\right)}{elet6\left(r\right)} = \mu_{elet6} dt + \sigma_{elet6} \sqrt{dt} dZ_4\left(t\right) \tag{10}$$

$$\frac{dpcar4\left(t\right)}{pcar4\left(r\right)} = \mu_{pcar4} dt + \sigma_{pcar4} \sqrt{dt} dZ_5\left(t\right) \tag{11}$$

The variables Z are generated through Cholesky decomposition method employing the correlation obtained in the descriptive statistics.

4 Results

The first portfolio constructed, whose We'll call portfolio 1, is a equal-weight portolio, where We allocatted 20% to each stock. Next, We optimized a portfolio with short sale permitted, whose We'll call portfolio 2. We detach which the procedure to optimize the portfolio is via maximization of the portfolio return. The third portfolio was obtained maximizing the portfolio return without short sale, i.e., no negative weights. We emphasize that all portfolios had smaller standard deviation than any individual stock, showing the power of diversification. Moreover, the VaR of less risk stock ELET6 is R$ 5127,50, when calculated with normal distribution. The VaR of more risky portfolio is 4888,90, lower than ELET6 VaR. We calibrate the portfolio value in R$ 100.000,00. All portfolios have daily average return higher than Ibovespa index, whose return is 0.04%. Table 1 presents the descriptive statistics for the portfolios and the stocks.

The next step was to calculate the $VaR^{1}\%$ to portfolios employing historical data, using normal and t-student distribution. We hope that the t distribution control better than normal the heavy tails of financial data. We expect which the portfolio optimization reduces the portfolio risk, i.e, portfolios 2 and 3 can be less risky than 1. However this situation didn't occur, because portfolio 2 had high return and risk than portfolio 1, indicating same failures in optimization through efficient frontier.

The portolio 2, where short-sale is allowed, presents a return of 271.88%, higher than return of porfolio 1, which is 176.1%. However, the VaR of portfolio 2 is greater than portolio 1 VaR in all simulations. Regarding the portolio 2 weights, We shall do a short-sale of Eletrobras share in a ration of 18.56%. The results found are according to the theory, showing which a increase in the return increases the risk. Making a numerical exercise with portfolio 1 and 2, we observe which a increase in 1% in the expected return causes a high in R$ 9.57 in the portfolio VaR.

The portolio 3, optimized without short-sale, shows that we shall to invest in Vale, Bradesco and Pão e Açúcar only. The portfolio is more risky and have higher return than portofolio 1. Table 2 presents the ratio allocated in each share. The risk return trade-off among portfolios 1 and 3 is of R$7.81, showing that a rise in 1% of return increases the VaR in this amount. The increase in R$1.00 in the VaR of portfolio 2 rises the return in 0.10, while in the portfolio 3 this increase is 0.12%, concluding then which the risk premiun of portfolio 3 is higher than porfolio 2.

Forward, We check whether the portfolio optimization reduced the time-varying VaR, since which the VaR estimated by historical simulation using normal and t distribution aren't smaller in the optimized portfolios. Therefore, We estimate GARCH, EGARCH and GJR models to three portofolios and we use the conditional variance to estimate the VaR. The goal is to verify if the average VaR of portfolio 2 and 3 is lower than VaR of porfolio 1. To estimate the GARCH is need to check whether exists structure in the mean equation. The Q test, that is in table 1, indicates that there isn't structure in the average for any portfolio to 5%. Therefore, We utilize the own serie without structure in the mean in the model estimation.

We estimate nine family GARCH models, with normal and t distribution for each portfolio. The estimated parameters are in table 5. Next, We calculate the VaR for all portfolios evolving at time, for calculate your average, as shown in table 4. For to calculate the VaR we use the variance forecast one step ahead estimated by each model. The period in that the portfolios had higher VaR were in the 2008 sub prime crisis. However, the analysis of figures 3,4 and 5 shows which portfolio 1 was less risky in this period. The $VaR^{1\%}$ of portfolio 1 didn't attain R$15.000.00, while in other portfolios this measure reached close of R$20.000,00. The parameters signals of asymmetry evaluated from EGARCH and GJR models are according to theory, denoting which occurs increases in volatility for negative shocks in the return. Another peak of volatility identified by models was in 2011, reflex of Euro Zone crisis. The portfolios which have a average VaR close of R$4000,00 in all period, It had a VaR close of R$10.000,00 in this period.

The $VaR^{1\%}$ to the three portfolios was calculated admitting t-student distribution, with the objective of to replicate the stylized fact of heavy tails. We observe which there was a increases in the VaR for all portfolios, denoting what was expected, that t distribution improves the adjust of model, because the extreme value of tail of the portfolio return was represented. Analysing the figures 6,7 and 8 We conclude which there was periods during 2008 crisis in that the expected loss of agents for the portfolio value of R$100.000,00 exceeded R$20.000,00. This value corresponds more three times the average VaR observed of R$6682,00.

Tables 8 and 9 bring the one step ahead forecast for VaR with daily horizon.

We note that asymmetry models, as EGARCH and GJR forecast higher risk for all portfolios, both with normal and t distribution. This fact occurs due the model to capture risk aversion of agents, indicating greater volatility when the return is negative. The values evaluated from model with t distribution were superior than VaR estimated using normal distribution, which corroborates the thesis that t distribution replicates better the stylized facts of financial data and computes a value more reliable for the risk.

The VaR of portolios were estimated through Monte Carlo Simulation. Multivariate normal variables were generated by Cholesky decomposition method of correlation matrix. The prices of May, 14, 2012 were used as initial values. The results obtained are in the table 10. We conclude which values obtained from Monte Carlo Simulation are close to the VaR evaluated through t-student distribution, in comparison with values computed by normal distribution, which fortifies the hypothesis that t-student distribution fits better to data. We observe also which when We increase the number of trajectories the daily VaR reduces, indicating convergence to average VaR of the model. For the annual horizon of the VaR We admit 250 trading days in the year. The values computed are very high, showing a loss probability of 70% of investment for one year. However, as standard deviation composes the data generator process, this high estimate is justified because in the sample exists high volatility periods, as in 2008 at sub-prime crisis and 2011 at Euro Zone crisis.

Next, We did backtesting for all portfolios, aiming to test the violation ratio of VaR. The violation occurs when the loss exceed the VaR calculated. According to Daníelsson (2011) a VaR model is considered inaccurate if the violation ratio of VaR is smaller than 0.5 or higher than 1.5. When the violation ratio is 1, the VaR is inside the significance level. The results of Bernoulli cover test and independence test of Christoffersen (1998) are in the table 12. We verify which had, for the GARCH, a violation number of VaR greater than significance level of 1%, given that We reject null hypothesis to the Bernoulli test for all portfolios. When We employ t-student distribution in the GARCH estimation, there is a improves in the violation ration and We become to accept the null hypothesis in entire portfolios. The independence test indicates acceptation of null hypothesis is most of simulations for three portfolios, denoting that a violation in the VaR today don't indicates violation tomorrow. This results found aren't in accordance with result of Gaglione, Lima and Linton (2008), because for our portfolios studied there were a VaR violation superior to expected. A fact which can justify the difference among results is that the paper cited above used sample which don't include 2008 crisis.

The improvement in the estimates when the VaR is evaluated with t-student distribution emphasizes the thesis of Cordeiro (2009) about the inability of normal distribution in to replicate heavy tails of data. Unlike of Glasserman, Heildelberger and Shahabuddin (2000) We don't need utilize variance reduction technique to reduce computational effort, because the simulations of portolios were made with only 5 shares.

5 Conclusion

The research proposed to calculate the VaR through GARCH family models and via Monte Carlo Simulation. We conclude which the VaR estimated through GARCH models with errors following t distribution are a better risk measure than when calculated by normal distribution. This occurs because t-distribution replicates better fat tails of financial data. This conclusion is proved because the values obtained with monte carlo simulation are close to values estimated with t-student distribution. Another evidence of better fit of t is obtained by backtesting, given that the violation ratio of VaR calculated with t was smaller.

Regarding to portfolios used, entire got a average return greater than the Ibovespa index and also a lower risk than the individual asset less risky The portfolios more risky are more profitable, according to the theory of risk aversion. The time-varying VaR shows moments in which the loss probability attained near of 1/5 of portfolio value in some periods of high volatility, as 2008 crisis. Thus, the research satisfies the objective proposed e have like main contribution the comparative analysis among VaR estimated with monte carlo simulation and via familu GARCH models to Brazilian stock data.

References

[1] Araújo, L.M.B., 2009. *Composição de fundo Multimercado - Otimizao de carteira pelo método de média - CVaR.* Master Dissertation. FGV. São Paulo-SP.

[2] Bezerra, F. O., 2001. *Avaliação da Estimativa do Risco de Mercado pela Metodologia Value at Risk (VaR) com Simulação de Monte Carlo.* Master Dissertation in Management, Recife, Brazil.

[3] Bollerslev, T., 1986. Generalized Autoregressive conditional heteroskedasticity. *J. Econometrics* 31 307-327.

[4] Chela, J.L., Abrahão, J.C., Kamogawa, L.F.O., 2011. Modelos Ortogonais para Estimativa Multivariada de VaR (Value at Risk) para risco de mercado: Um estudo de caso comparativo. *Revista de Economia do Mackenzie* 9 p.70-93.

[5] Cordeiro, F.N.B., 2009. *Aplicaç ao da teoria de Cópulas para o cálculo do Value at Risk.* Master Dissertation. FGV. São Paulo-SP.

[6] Christoffersen, P. F. Evaluating interval forecasts. *International Economic Review*, v. 39, p. 841-862, 1998.

[7] Danelsson, D., 2011. *Financial risk forecasting*, Wiley Finance.

[8] Glasserman, P., Heidelberger, P., Shahabuddin, P., Variance Reduction Techniques for Estimating Value-at-Risk. *Management Science*, Vol. 46, No. 10 (Oct., 2000), pp. 1349-1364.

[9] Gaglianone, W.P.,Lima,L.R. e Linton,O., 2008. *Evaluating Value-at-Risk Models via Quantile Regressions.* Working Paper Series 161. Banco Central do Brasil.

[10] Glosten, L.R., Jaganathan, R., Runkle, D. On the relation between the expected value and the volatility of the normal excess return on stocks. *Journal of Finance*, 48, p.1779-1801, 1993.

[11] Huynh, H. T., Lai, V. S., Soumare, I., 2008. *Stochastic Simulation and Applications in Finance with MATLAB Programs*, Wiley Finance.

[12] Jorion, P., 2007. *Value-at-Risk: The new benchmark for managing financial risk.* McGraw Hill 3rd edition.

[13] Jorion, P., 2002. How Informative Are Value-at-Risk Disclosures? *The Accouting Review* 77 911-931

[14] Kupiec, P., 1995. Techniques for Verifying the Accuracy of Risk Measurement Models. *Journal of Derivatives* 3, 73-84.

[15] Manganelli, S., Engle, R., 2001. *Value-at-Risk Models in Finance.* Working Paper 75. European Central Bank.

[16] Markowitz, H. Portfolio selection. *Journal of Finance*, junho, pp. 77 - 91, 1952.

[17] Nelson, D. B. 1991. Conditional heteroskedasticity in asset returns: A new approach. *Econometrica* 59 347-370.

[18] Taylor, J.W., 2005. Generating Volatility Forecasts from Value at Risk Estimates. *Management Science*, Vol. 51, No. 5 (May, 2005), pp. 712-725

Appendix

A Tables

Table 1: Descriptive statistics

Portfolio	Mean	Std.	Min	Max	Asymmetry	Kurtosis	AC(One lag) returns	AC(One lag) of squared returns	p-value $Q_1(1)$	p-value $Q_2(1)$
1	0.0006	0.0171	-0.1045	0.1245	-0.0736	7.1890	0.0330	0.1878	0.0615	0
2	0.0008	0.0210	-0.1452	0.1637	0.0353	8.0076	0.0340	0.1363	0.054	0
3	0.0008	0.0197	-0.1385	0.1459	-0.0367	7.8613	0.0267	0.13	0.1305	0

Source:Authors

Table 2: Weights of the stocks in the portfolios

Stock/Portfolio	1	2	3
PETR4	20.00%	1.09%	0.00%
VALE5	20.00%	68.08%	68.22%
BBDC4	20.00%	46.94%	30.84%
ELET6	20.00%	-18.56%	0.00%
PCAR4	20.00%	2.45%	0.94%

Source:Authors

Table 3: VaR of portfolios

Portfolio	Historical VaR (R$)	VaR Normal (R$)	VaR t-student (R$)
1	4489.30	3981.10	4512.30
2	5406.50	4888.90	5661.00
3	5063.00	4590.70	5348.70

*Source:*Authors

Table 4: Average VaR Normal Distribution

Portfolio	GARCH			EGARCH			TGARCH		
	(1,1)	(1,2)	(2,2)	(1,1)	(1,2)	(2,2)	(1,1)	(1,2)	(2,2)
1	3766.40	3768.90	3766.90	3742.30	3739.30	3736.60	3740.20	3741.20	3739.40
2	4577.70	4585.20	4585.50	4551.20	4551.00	4528.60	4555.30	4543.50	4542.90
3	4303.90	4304.00	4310.70	4277.50	4361.10	4287.70	4284.80	4265.50	4274.60

*Source:*Authors

Table 5: Average VaR Normal Distribution

Portfolio	GARCH			EGARCH			TGARCH		
	(1,1)	(1,2)	(2,2)	(1,1)	(1,2)	(2,2)	(1,1)	(1,2)	(2,2)
1	5657.70	5654.40	5652.00	5618.90	5614.10	5613.50	5614.40	5630.20	5635.30
2	7397.10	7403.90	7405.80	7347.50	7347.00	7387.20	7351.70	7351.30	7349.90
3	7058.80	7084.60	7079.10	7009.50	7009.60	7041.70	7019.00	7041.70	7056.60

Source:Authors

Table 6: Estimated parameters of GARCH Models with normal errors

			Portfolio 1					
$\omega * 10^4$	α_1	α_2	β_1	β_2	γ_1	γ_2	θ_1	θ_2
0.080	0.069		0.900					
(0.010)	(0.006)		(0.009)					
0.0090	0.0041	0.033	0.891					
(0.012)	(0.016)	(0.017)	(0.011)					
0.130	0.037	0.071	0.324	0.517				
(0.026)	(0.015)	(0.015)	(0.285)	(0.263)				
-0.256	0.119		0.968				-0.070	
(0.035)	(0.012)		(0.004)				(0.007)	
-0.233	0.036	0.081	0.971				-0.161	0.090
(0.034)	(0.034)	(0.036)	(0.034)				(0.023)	(0.022)
-0.035	0.081	-0.054	1.691	-0.696			-0.174	0.161
(0.013)	(0.021)	(0.022)	(0.072)	(0.070)			(0.019)	(0.017)
0.099	0.002		0.904		0.112			
(0.010)	(0.007)		(0.009)		(0.012)			
0.100	0.000	0.012	0.897		0.117	-0.012		
(0.001)	(0.001)	(0.023)	(0.011)		(0.036)	(0.034)		
0.175	0.000	0.016	0.139	0.684	0.097	0.089		
(0.038)	(0.001)	(0.016)	(0.324)	(0.296)	(0.024)	(0.036)		

Portfolio 2

0.120	0.082		0.887					
(0.017)	(0.007)		(0.010)					
0.111	0.076	0.012	0.896					
(0.016)	(0.016)	(0.007)	(0.009)					
0.013	0.085	0.068	0.000	0.789				
(0.011)	(0.012)	(0.031)	(0.0313)	(0.279)				
-0.299	0.166		0.961				-0.081	
(0.039)	(0.014)		(0.005)				(0.008)	
-0.209	0.161	-0.017	0.972				-0.149	0.085
(0.003)	(0.004)	(0.032)	(0.032)				(0.022)	(0.022)
0.078	0.197	-0.177	1.741	-0.743			-0.149	0.144
(0.030)	(0.025)	(0.024)	(0.053)	(0.052)			(0.016)	(0.017)
0.165	0.025		0.876		0.122			
(0.021)	(0.007)		(0.012)		(0.013)			
0.168	0.001	0.028	0.868		0.151	-0.028		
(0.033)	(0.017)	(0.017)	(0.013)		(0.030)	(0.030)		
0.168	0.002	0.028	0.818	0.046	0.155	-0.028		
(0.103)	(0.017)	(0.021)	(0.619)	(0.541)	(0.030)	(0.084)		

Portfolio 3								
0.100	0.079		0.892					
(0.012)	(0.007)		(0.009)					
0.100	0.079	0.000	0.892					
(0.015)	(0.017)	(0.017)	(0.010)					
0.174	0.071	0.067	0.032	0.779				
(0.055)	(0.012)	(0.036)	(0.547)	(0.492)				
-0.302	0.164		0.961				-0.080	
(0.039)	(0.013)		(0.004)				(0.007)	
-0.255	0.137	0.017	0.967				-0.143	0.072
(0.036)	(0.031)	(0.032)	(0.004)				(0.022)	(0.021)
-0.012	0.180	-0.164	1.778	-0.780			-0.149	0.145
(0.005)	(0.022)	(0.021)	(0.042)	(0.042)			(0.016)	(0.015)
0.139	0.025		0.878		0.119			
(0.016)	(0.006)		(0.011)		(0.013)			
0.159	0.000	0.036	0.857		0.156	-0.036		
(0.002)	(0.019)	(0.019)	(0.014)		(0.032)	(0.032)		
0.148	0.000	0.030	0.866	0.000	0.155	-0.030		
(0.085)	(0.019)	(0.023)	(0.565)	(0.493)	(0.032)	(0.078)		

Source:Authors. ( )=Std.

Table 7: Estimated parameters of GARCH Models with t-student errors

Portfolio 1								
$\omega * 10^4$	α_1	α_2	β_1	β_2	γ_1	γ_2	θ_1	θ_2
0.080	0.069		0.900					
(0.010)	(0.006)		(0.009)					
0.094	0.045	0.0311	0.888					
(0.013)	(0.016)	(0.017)	(0.011)					
0.140	0.041	0.074	0.245	0.585				
(0.027)	(0.015)	(0.014)	(0.260)	(0.239)				
-0.256	0.119		0.968				-0.075	
(0.035)	(0.012)		(0.004)				(0.007)	
-0.232	0.036	0.082	0.971				-0.163	0.094
(0.034)	(0.034)	(0.035)	(0.004)				(0.023)	(0.022)
-0.036	0.080	-0.051	1.680	-0.685			-0.173	0.161
(0.014)	(0.021)	(0.022)	(0.074)	(0.073)			(0.019)	(0.017)
0.099	0.002		0.902		0.109			
(0.010)	(0.007)		(0.009)		(0.012)			
0.084	0.000	0.015	0.899		0.119	-0.015		
(0.018)	(0.030)	(0.029)	(0.015)		(0.044)	(0.043)		
0.088	0.001	0.015	0.850	0.044	0.125	-0.015		
(0.139)	(0.030)	(0.034)	(1.619)	(1.458)	(0.044)	(0.181)		

Portfolio 2

0.122	0.082		0.887					
(0.017)	(0.007)		(0.010)					
0.106	0.081	0.000	0.893					
(0.027)	(0.023)	(0.025)	(0.016)					
0.122	0.102	0.000	0.505	0.362				
(0.070)	(0.026)	(0.064)	(0.622)	(0.557)				
-0.296	0.167		0.961				-0.077	
(0.039)	(0.014)		(0.005)				(0.008)	
-0.201	0.163	-0.020	0.974				-0.148	0.089
(0.031)	(0.032)	(0.031)	(0.004)				(0.022)	(0.022)
-0.568	0.191	0.163	-0.015	0.943			-0.078	-0.087
(0.111)	(0.024)	(0.026)	(0.026)	(0.025)			(0.014)	(0.014)
0.160	0.028		0.874		0.115			
(0.020)	(0.007)		(0.012)		(0.013)			
0.155	0.017	0.017	0.868		0.135	-0.017		
(0.033)	(0.024)	(0.025)	(0.018)		(0.042)	(0.043)		
0.205	0.003	0.034	0.577	0.259	0.153	-0.007		
(0.085)	(0.019)	(0.018)	(0.517)	(0.453)	(0.030)	(0.074)		

Portfolio 3

0.100	0.079		0.892					
(0.012)	(0.007)		(0.009)					
0.074	0.078	0.011	0.901					
(0.023)	(0.025)	(0.026)	(0.014)					
0.088	0.094	0.013	0.638	0.244				
(0.079)	(0.027)	(0.091)	(1.011)	(0.912)				
-0.297	0.163		0.962				-0.075	
(0.039)	(0.013)		(0.004)				(0.007)	
-0.240	0.139	0.011	0.969				-0.143	0.079
(0.034)	(0.031)	(0.032)	(0.004)				(0.022)	(0.021)
-0.012	0.180	-0.164	1.781	-0.783			-0.145	0.141
(0.005)	(0.021)	(0.021)	(0.042)	(0.041)			(0.016)	(0.015)
0.137	0.027		0.877		0.113			
(0.016)	(0.007)		(0.011)		(0.013)			
0.124	0.005	0.028	0.873		0.146	-0.028		
(0.026)	(0.024)	(0.024)	(0.017)		(0.043)	(0.044)		
0.123	0.004	0.025	0.826	0.045	0.155	-0.025		
(0.118)	(0.023)	(0.033)	(0.975)	(0.859)	(0.044)	(0.133)		

Source:Authors. ( )=Std.

Table 8: Forecast one step ahead of VaR with Normal errors

Portfolio	GARCH			EGARCH			TGARCH		
	(1,1)	(1,2)	(2,2)	(1,1)	(1,2)	(2,2)	(1,1)	(1,2)	(2,2)
1	3255.10	3188.60	3181.90	3961.20	3977.80	4052.90	3750.90	3708.20	3664.90
2	3685.50	3685.40	3694.50	4329.20	4262.30	4294.60	4153.90	4155.80	4151.80
3	3599.80	3599.90	3597.40	4231.10	4195.10	4234.80	4119.30	4142.10	4139.80

Source:Authors

Table 9: Forecast one step ahead of VaR with t errors

Portfolio	GARCH			EGARCH			TGARCH		
	(1,1)	(1,2)	(2,2)	(1,1)	(1,2)	(2,2)	(1,1)	(1,2)	(2,2)
1	4833.80	4782.70	4764.70	5967.40	6009.60	6088.70	5615.60	5559.70	5578.40
2	5909.00	5909.50	5893.10	7020.30	6862.70	6974.60	6713.70	6710.80	6727.30
3	5828.30	5828.00	5822.40	7004.10	6823.00	6993.80	6763.50	6786.80	6778.50

Source:Authors

Table 10: VaR of Portfolios evaluated through Monte Carlo Simulation

Portfolio	1		2		3	
N	Daily	Annual	Daily	Annual	Daily	Annual
100	6503.00	64433.00	7281.70	73306.00	6424.70	71049.00
1000	5847.50	57131.00	7253.30	76696.00	6366.80	66328.00
10000	5481.20	56544.00	7108.50	75897.00	6588.60	65609.00
100000	5653.60	55781.00	6929.60	76398.00	6437.20	66154.00
1000000	5617.20	55963.00	6911.80	76677.00	6494.60	65936.00
10000000	5626.60	55929.00	6917.10	76570.00	6487.90	65856.00

Source:Authors

Table 11: Backtestings

Portfolio	1		2		3	
Method	VR	VaR Volatility	VR	VaR Volatility	VR	VaR Volatility
EWMA	1.50	0.0162	1.68	0.0209	1.81	0.0196
MA	1.95	0.0050	2.04	0.0103	1.95	0.0092
HS	1.04	0.0094	1.36	0.0174	1.40	0.0154
GARCH	1.59	0.0125	1.59	0.0179	1.59	0.0165
GARCH t	1.54	0.0130	1.45	0.0184	1.54	0.0174

Source:Authors

Table 12: Coverage and Independece test

Portfolio	1				2				3			
Test	Coverage Test		Independence Test		Coverage Test		Independence Test		Coverage Test		Independence Test	
Method	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
EWMA	4.82	0.02	0.41	0.52	8.57	0.00	2.02	0.15	11.98	0.00	1.58	0.20
MA	15.84	0.00	6.65	0.00	18.65	0.00	9.55	0.00	15.84	0.00	3.54	0.05
HS	0.05	0.83	1.38	0.23	2.64	0.10	0.63	0.42	3.30	0.06	0.55	0.45
GARCH	6.58	0.01	0.30	0.58	6.58	0.01	1.13	0.28	6.58	0.01	1.13	0.28
GARCH t	5.66	0.02	0.35	0.55	4.02	0.04	0.94	0.33	5.66	0.02	1.06	0.30

Source:Authors

B Figures

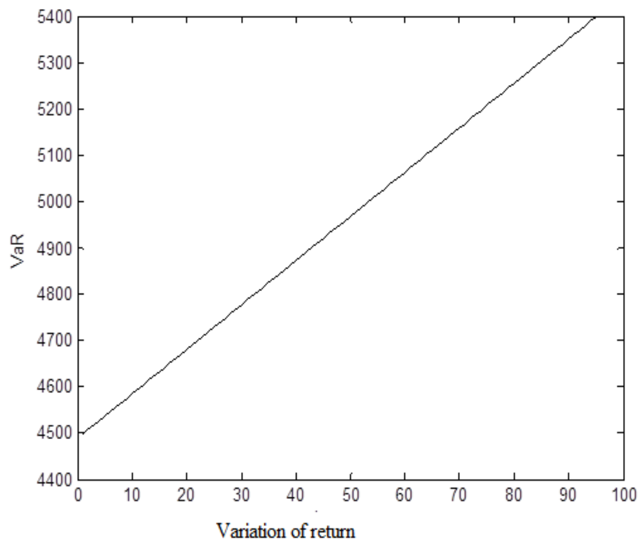

Figure 1: Risk Return relationship portfolio 1 and 2

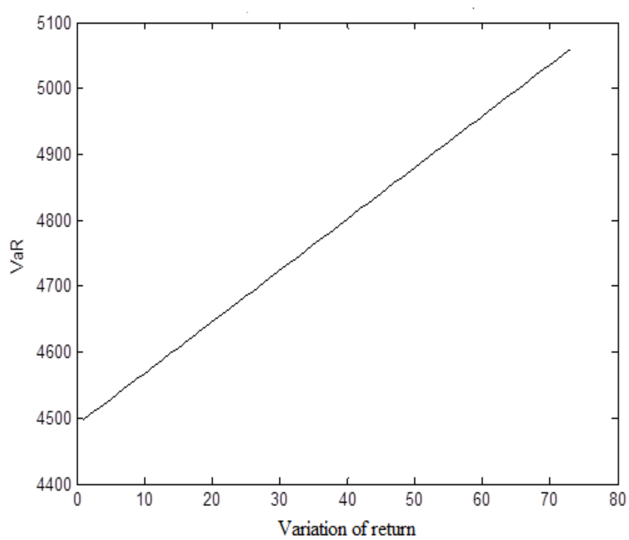

Figure 2: Risk Return relationship portfolio 2 and 3

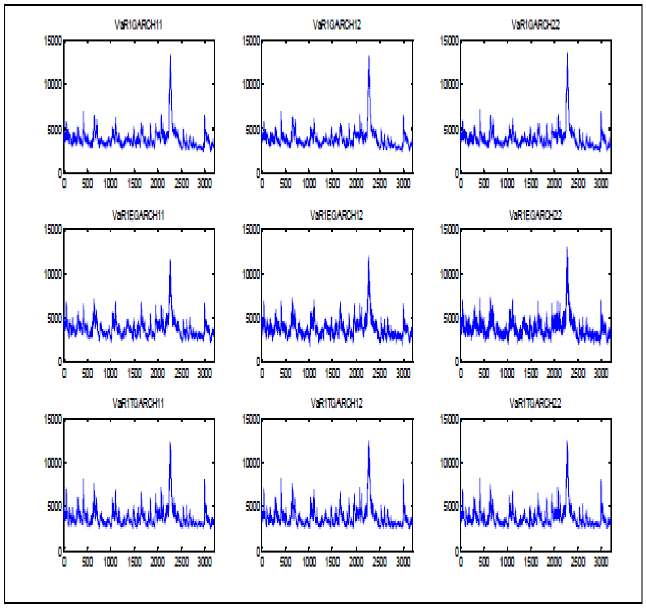

Figure 3: VaR of portfolio 1 with normal distribution

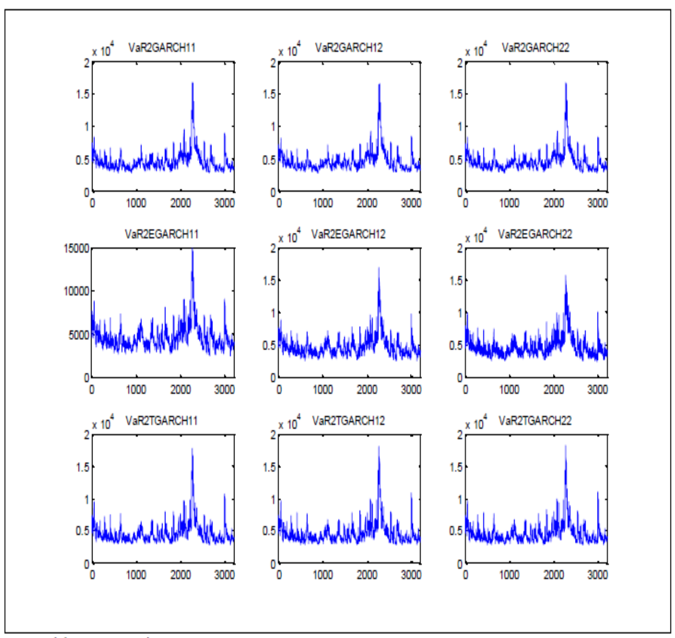

Figure 4: VaR of portfolio 2 with normal distribution

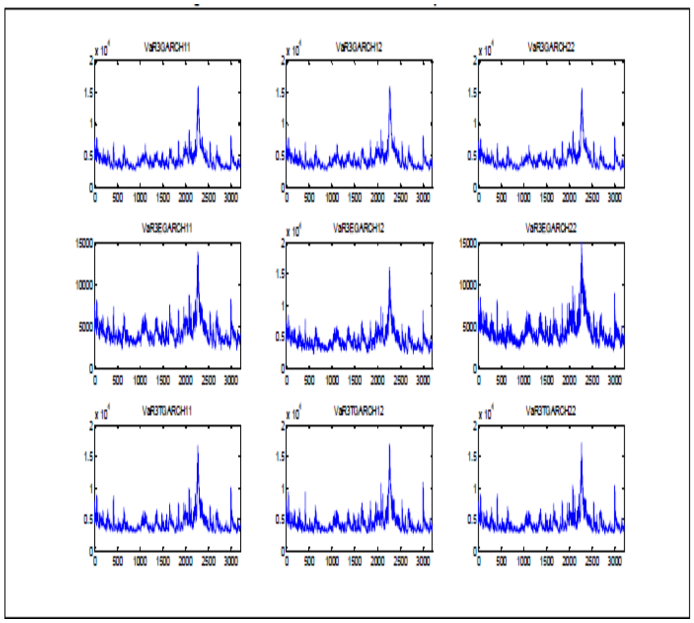

Figure 5: VaR of portfolio 3 with normal distribution

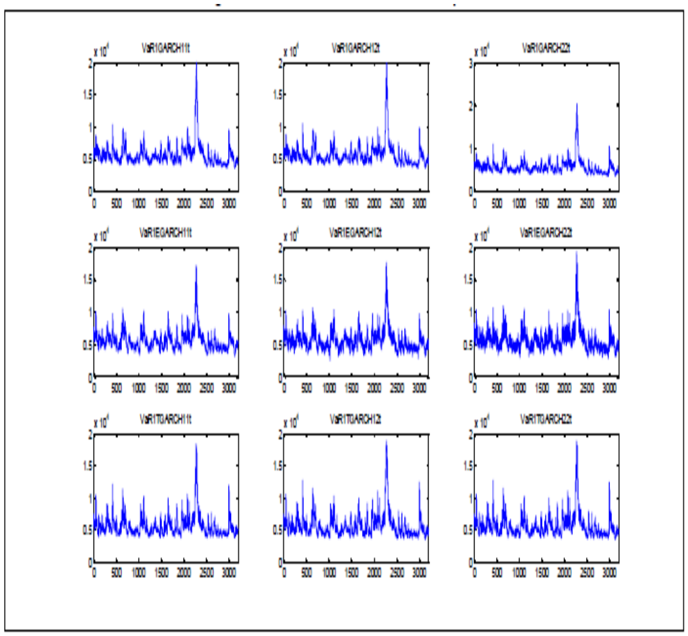

Figure 6: VaR of portfolio 1 with t distribution

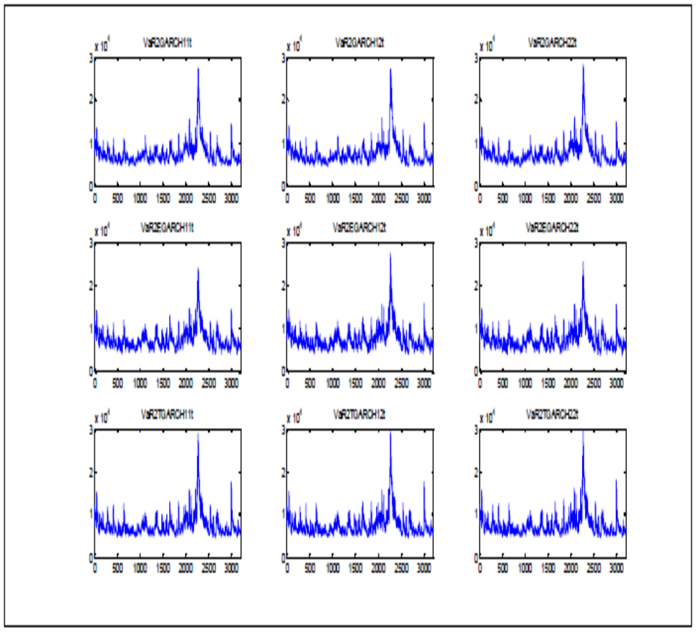

Figure 7: VaR of portfolio 2 t normal distribution

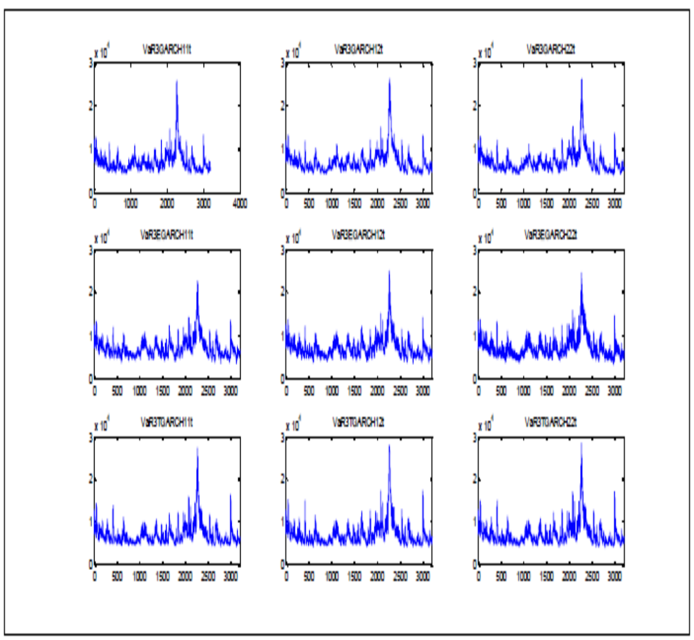

Figure 8: VaR of portfolio 3 with t distribution

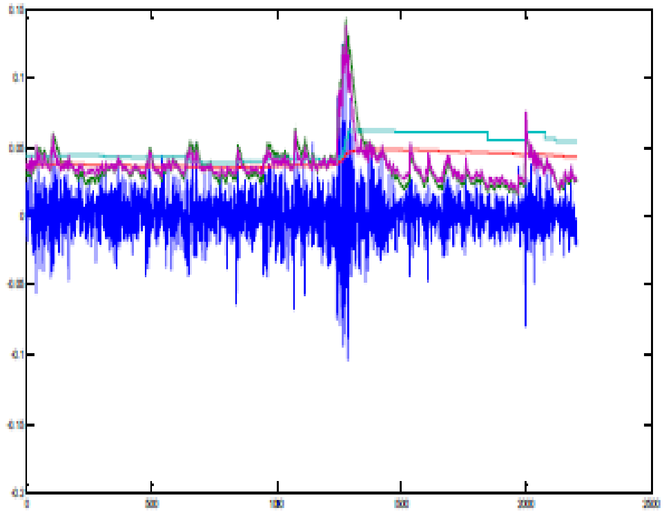

Figure 9: Backtesting portfolio 1

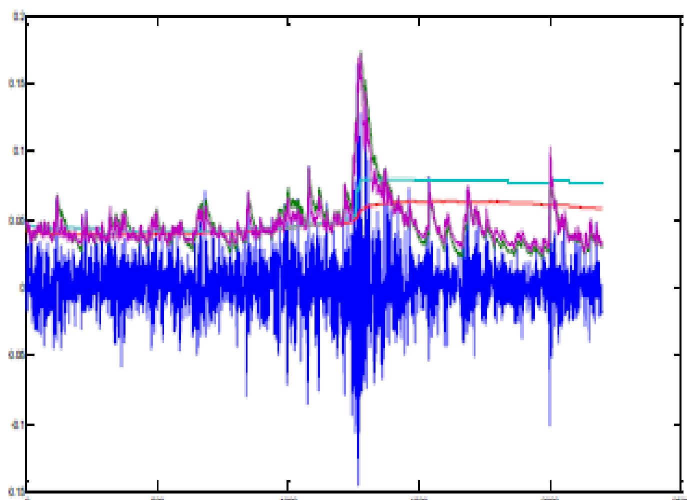

Figure 10: Backtesting portfolio 2

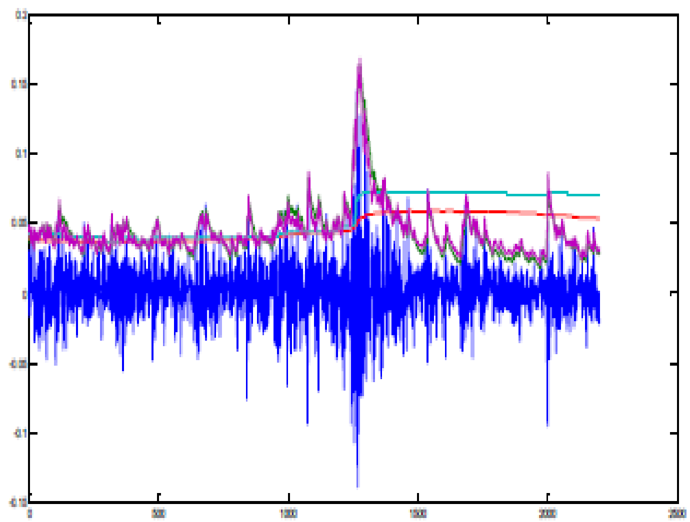

Figure 11: Backtesting portfolio 3