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Analysis of the Matrix Structure in the Preference Shift of Customer Brand Selection for Automobile

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Abstract

Consumers often buy higher ranked brand after they are bored using current brand goods. This may be analyzed utilizing matrix. Suppose past purchasing data are set input and current purchasing data are set output, then transition matrix is identified using past and current data. If all brand selections are composed by the upper shifts, then the transition matrix becomes an upper triangular matrix. The goods of the same brand group would compose the Block Matrix in the transition matrix. Condensing the variables of the same brand group into one, analysis becomes easier to handle and the transition of Brand Selection can be easily grasped. We have made a questionnaire investigation concerning automobile purchase before. In that paper, the questionnaire was carried out mainly on an urban area. In this paper, we make an investigation on a rural area. The questionnaire investigation to automobile purchasing case is conducted and above structure is confirmed. We can make forecast utilizing the estimated transition matrix. We can foresee the future purchase and the producers can make effective

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marketing plan to cope with this. Unless planner for products does not notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.

Mathematics Subject Classification: 15A23; 15A24

Keywords: brand selection; matrix structure; brand position; automobile industry

1 Introduction

It is often observed that consumers select the upper class brand when they buy the next time. Focusing the transition matrix structure of brand selection, their activities may be analyzed. In the past, there are many researches about brand selection [1-5]. But there are few papers concerning the analysis of the transition matrix structure of brand selection. In this paper, we make analysis of the preference shift of customer brand selection. Consumers often buy higher ranked brand after they are bored using current brand goods. This may be analyzed utilizing matrix. Sup-pose past purchasing data are set input and current purchasing data are set output, then transition matrix is identified using past and current data. If all brand selections are composed by the upper shifts, then the transition matrix becomes an upper triangular matrix. The goods of the same brand group would compose the Block Matrix in the transition matrix. Condensing the variables of the same brand group into one, analysis becomes easier to handle and the transition of Brand Selection can be easily grasped. Then we confirm them by the questionnaire investigation for automobile purchasing case. If we can identify the feature of the matrix structure of brand selection, it can be utilized for the marketing strategy.

We have made a questionnaire investigation concerning automobile purchase before [6]. In that paper, the questionnaire was carried out mainly on an urban area. In this paper, we make investigation on a rural area. The questionnaire an investigation to automobile purchasing case is conducted and above structure is confirmed.

We can make forecast utilizing the estimated transition matrix. We can foresee the future purchase and the producers can make effective marketing plan to cope with this.

Unless planner for products does not notice its brand position whether it is upper or lower than another products, matrix structure make it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.

A quantitative analysis concerning brand selection has been executed by [4, 5]. [5] examined purchasing process by Markov Transition Probability with the input of advertising expense. [4] made analysis by the Brand Selection Probability model using logistics distribution.

In this paper, matrix structure is analyzed for the case the upper class brand is selected compared with the past purchasing case, and extensions for various applications are performed. Such research cannot be found as long as searched.

The rest of the paper is organized as follows. Matrix structure is clarified for the selection of brand in section 2. A block matrix structure is analyzed when brands are handled in a group and condensing method of variables of the same brand group is introduced and analyzed in section 3. A questionnaire investigation to Automobile Purchasing case is examined and its numerical calculation is carried out in section 4. Section 5 is a summary.

2 Brand selection and its matrix structure

2.1 Upper Shift of Brand Selection

It is often observed that consumers select the upper class brand when they buy the next time. Now, suppose that x is the most upper class brand, y is the second upper brand, and z is the lowest brand. Consumer's behavior of selecting brand would be $z \to y$, $y \to x$, $z \to x$ etc. $x \to z$ might be few. Suppose that x is the current buying variable, and x_b is the previous buying variable. Shift to x is executed from x_b, y_b, or z_b.

Therefore, x is stated in the following equation.

$$x = a_{11}x_b + a_{12}y_b + a_{13}z_b$$

Similarly,

$$y = a_{22}y_b + a_{23}z_b$$

And

$$z = a_{33}z_b$$

These are re-written as follows.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} \tag{1}$$

Set :

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}$$

$$\mathbf{X_b} = \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

then, $\mathbf{X}$ is represented as follows.

$$\mathbf{X} = \mathbf{A}\mathbf{X_b} \tag{2}$$

Here,

$$\mathbf{X} \in \mathbf{R}^3, \mathbf{A} \in \mathbf{R}^{3\times 3}, \mathbf{X_b} \in \mathbf{R}^3$$

$\mathbf{A}$ is an upper triangular matrix.

To examine this, generating the following data, which are all consisted by the upper brand shift data,

$$\mathbf{X^i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \cdots \qquad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{X_b^i} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \cdots \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

$$i = 1, \qquad 2 \qquad \cdots \qquad N$$

parameter can be estimated using least square method.

Suppose

$$\mathbf{X}^i = \mathbf{A}\mathbf{X}_\mathbf{b}^i + \boldsymbol{\varepsilon}^i \tag{5}$$

and

$$J = \sum_{i=1}^{N} \boldsymbol{\varepsilon}^{iT} \boldsymbol{\varepsilon}^i \to Min \tag{6}$$

$\hat{\mathbf{A}}$ which is an estimated value of $\mathbf{A}$ is obtained as follows.

$$\hat{\mathbf{A}} = \left(\sum_{i=1}^{N} \mathbf{X}^i \mathbf{X}_\mathbf{b}^{iT} \right) \left(\sum_{i=1}^{N} \mathbf{X}_\mathbf{b}^i \mathbf{X}_\mathbf{b}^{iT} \right)^{-1} \tag{7}$$

In the data group of the upper shift brand, estimated value $\hat{\mathbf{A}}$ should be an upper triangular matrix.

If the following data, that have the lower shift brand, are added only a few in equation (3) and (4),

$$\mathbf{X}^i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\mathbf{X}_{\mathbf{b}}^i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\hat{\mathbf{A}}$ would contain minute items in the lower part of triangle.

3 Matrix structure when condensing the variables of the same class

Suppose the customer select Bister from Corolla (Bister is an upper class brand automobile than Corolla.) when he/she buy next time. In that case, there are such brand automobiles as bluebird, Gallant sigma and 117 coupes for the corresponding same brand class group with Bister in other companies.

Someone may select another automobile from the same brand class group. There is also the case that the consumer select another company's automobile of the same brand class group when he/she buy next time.

Matrix structure would be, then, as follows.

Suppose w, x, y are in the same brand class group. If there exist following shifts:

$$
\begin{aligned}
y, x, w, v &\leftarrow z_b \\
x, w &\leftarrow y_b \\
y, w &\leftarrow x_b \\
x, y &\leftarrow w_b \\
v &\leftarrow y_b \\
v &\leftarrow x_b \\
v &\leftarrow w_b \\
y &\leftarrow y_b \\
x &\leftarrow x_b \\
w &\leftarrow w_b
\end{aligned}
$$

then, transition equation is expressed as follows.

$$
\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} \\ 0 & a_{32} & a_{33} & a_{34} & a_{35} \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & 0 & a_{55} \end{pmatrix} \begin{pmatrix} v_b \\ w_b \\ x_b \\ y_b \\ z_b \end{pmatrix} \tag{10}
$$

Expressing these in Block Matrix form, it becomes as follow.

$$
\mathbf{X} = \begin{pmatrix} a_{11} & \mathbf{A}_{12} & a_{15} \\ \mathbf{0} & \mathbf{A}_{22} & \mathrm{A}_{23} \\ \mathbf{0} & \mathbf{0} & a_{55} \end{pmatrix} \mathbf{X}_b \tag{11}
$$

Where,

$$
\mathbf{X} \in \mathbf{R}^5, \mathbf{A}_{12} \in \mathbf{R}^{1\times 3}, \mathbf{A}_{22} \in \mathbf{R}^{3\times 3}, \mathbf{A}_{23} \in \mathbf{R}^{3\times 1}, \mathbf{X}_b \in \mathbf{R}^5
$$

As w, x, y are in the same class brand group, condensing these variables into one, and expressing it as $\overline{w}$, then Eq.(10) becomes as follows.

$$
\begin{pmatrix} v \\ \overline{w} \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & \overline{a}_{12} & a_{15} \\ 0 & \overline{a}_{22} & \overline{a}_{23} \\ 0 & 0 & a_{55} \end{pmatrix} \begin{pmatrix} v_b \\ \overline{w}_b \\ z \end{pmatrix} \tag{12}
$$

As a_{ij} satisfies the following equation :

$$
\sum_{j=1}^{5} a_{ij} = 1 \qquad \left(\forall i\right) \tag{13}
$$

Condensed version of Block Matrix $\mathbf{A}_{22}$ are as follows, where sum of each item of Block matrix is taken and they are divided by the number of variables.

$$\bar{a}_{22} = \frac{1}{3}\sum_{i=2}^{4}\sum_{j=2}^{4} a_{ij} \tag{14}$$

$$\bar{a}_{12} = \frac{1}{3}\sum_{j=2}^{4} a_{ij} \tag{15}$$

$$\bar{a}_{23} = \sum_{i=2}^{4} a_{i5} \tag{16}$$

Generalizing this, it becomes as follows.

$$\begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_r \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1r} \\ \mathbf{0} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2r} \\ \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_{rr} \end{pmatrix} \begin{pmatrix} \mathbf{X}_{1,b} \\ \mathbf{X}_{2,b} \\ \vdots \\ \mathbf{X}_{r,b} \end{pmatrix} \tag{17}$$

Where

$$\begin{aligned} &\mathbf{X}_1 \in \mathbf{R}^{p_1}, \cdots, \mathbf{X}_r \in \mathbf{R}^{p_r}, \\ &\mathbf{A}_{11} \in \mathbf{R}^{p_1 \times p_1}, \mathbf{A}_{12} \in \mathbf{R}^{p_1 \times p_2}, \cdots, \mathbf{A}_{1r} \in \mathbf{R}^{p_1 \times p_r} \\ &\mathbf{A}_{21} \in \mathbf{R}^{p_2 \times p_1}, \mathbf{A}_{22} \in \mathbf{R}^{p_2 \times p_2}, \cdots, \mathbf{A}_{2r} \in \mathbf{R}^{p_2 \times p_r} \\ &\vdots \\ &\mathbf{A}_{rr} \in \mathbf{R}^{p_r \times p_r} \end{aligned}$$

When the variables of each Block Matrix are condensed into one, transition matrix is expressed as follows.

$$\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_r \end{pmatrix} = \begin{pmatrix} \bar{a}_{11}, & \bar{a}_{12}, & \cdots & \bar{a}_{1r} \\ 0, & \bar{a}_{22}, & \cdots & \bar{a}_{2r} \\ \vdots & & & \vdots \\ 0, & 0, & \cdots & \bar{a}_{rr} \end{pmatrix} \begin{pmatrix} \bar{x}_{1,b} \\ \bar{x}_{2,b} \\ \vdots \\ \bar{x}_{r,b} \end{pmatrix} \tag{18}$$

Where

$$\begin{aligned} &\bar{a}_{ij} = \frac{1}{p_j}\sum_{k=1}^{p_i}\sum_{l=1}^{p_j} a_{kl}{}^{ij} \\ &(i = 1, \cdots, r)(j = i, \cdots, r) \end{aligned} \tag{19}$$

Here, $a_{ij}\left(1 \le k \le p_i, 1 \le l \le p_j\right)$ means the item of $\mathbf{A}_{ij}$.

Taking these operations, the variables of the same brand class group are condensed into one and the transition condition among brand class can be grasped easily and clearly. Judgment when and where to put the new brand becomes easy and may be executed properly.

4 A questionnaire investigation and numerical calculation

4.1 A questionnaire investigation

A questionnaire investigation for automobile purchasing case is executed.

<Delivery of Questionnaire Sheets>

・A questionnaire sheet : Appendix1

・Delivery Term : July to September/2012

・Delivery Place：Shizuoka and Mie Prefecture in Japan

・Number of Delivered Questionnaire sheets: 300

<Result of collected Questionnaire Sheets>

・Collected Questionnaire Sheets:98 (male:47,female:51)

・Collected sheets for Sedan Typed Automobile: 41

Fundamental statistical result is exhibited in Table 1 and Table 2. We have made a questionnaire investigation concern automobile purchase before (Takeyasu et al.,(2007)). In that paper, the questionnaire was carried out mainly on an urban area. In this paper, we make an investigation on a rural area.

Table 1: Summary for 98 sheets

Age		Sex		Occupation		Annual income (Japanese Yen)		Marriage		Kids	
Teens	10	Male	47	Student	16	0-3 million	71	Single	46	0	55
Twenties	20	Female	51	Officer	48	3-5 million	15	Married	51	1	11
Thirties	18			Company employee	6	5-7.5 million	5	Not filled in	1	2	23
Forties	20			Clerk of Organization	1	7.5-10 million	2			3	9
Fifties	15			Independents	25	10-15 million	2			4	
Sixties and over	15			Miscellaneous	0	15 million or more	0			5	0
Not filled in	0			Not filled in	2	Not filled in	3				
Sum	98		98		98		98		98		98

Table 2: Sedan Typed Summary for 41 Sheets

Age		**Sex**		**Occupation**		**Annual income (Japanese Yen)**		**Marriage**		**Kids**	
Teens	0	Male	14	Student	0	0-3 million	28	Single	17	0	10
Twenties	9	Female	27	Officer	38	3-5 million	7	Married	24	1	10
Thirties	7			Company employee	0	5-7.5 million	3	Not filled in	0	2	16
Forties	12			Clerk of Organization	0	7.5-10 million	1			3	5
Fifties	6			Independents	3	10-15 million	1			4	0
Sixties and over	7			Miscellaneous	0	15 million or more	0			5	0
Not filled in	0			Not filled in	0	Not filled in	1				
Sum	41		41		41		41		41		41

4.2 Numerical calculation

The questionnaire includes the question of the past purchasing history.

Therefore the plural data may be gathered from one sheet. For example, we can get two data such as (before former automobile, former automobile), (former automobile, current automobile).

Appendix2 shows the total ranking Table and Appendix3 shows the ranking Table for sedan type. Analyzing these sheets based on Model ranking Table (Appendix2, Appendix3), we obtained the following 202 data sets.

Analyzing collected sheets based on Model ranking Table, we obtained the data sets as follows. In order to express them in the block matrix form, we consider the case of four variable blocks $\mathbf{T}, \mathbf{N}, \mathbf{H}$ and $\mathbf{O}$ as is described in 3. The variable $\mathbf{T}, \mathbf{N}, \mathbf{H}$ and $\mathbf{O}$ stands for Toyota, Nissan, Honda and Others respectively. Each of them consists of 5 ranks.

$$\mathbf{T_n} = \begin{pmatrix} \mathbf{T_1^n} \\ \mathbf{T_2^n} \\ \vdots \\ \mathbf{T_5^n} \end{pmatrix}, \quad \mathbf{N_n} = \begin{pmatrix} \mathbf{N_1^n} \\ \mathbf{N_2^n} \\ \vdots \\ \mathbf{N_5^n} \end{pmatrix}, \quad \mathbf{H_n} = \begin{pmatrix} \mathbf{H_1^n} \\ \mathbf{H_2^n} \\ \vdots \\ \mathbf{N_5^n} \end{pmatrix}, \quad \mathbf{O_n} = \begin{pmatrix} \mathbf{O_1^n} \\ \mathbf{O_2^n} \\ \vdots \\ \mathbf{O_5^n} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{T_n} \\ \mathbf{N_n} \\ \mathbf{H_n} \\ \mathbf{O_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{A}_{13} & \mathbf{A}_{14} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{A}_{23} & \mathbf{A}_{24} \\ \mathbf{A}_{31} & \mathbf{A}_{32} & \mathbf{A}_{33} & \mathbf{A}_{34} \\ \mathbf{A}_{41} & \mathbf{A}_{42} & \mathbf{A}_{43} & \mathbf{A}_{44} \end{pmatrix} \begin{pmatrix} \mathbf{T_{n-1}} \\ \mathbf{N_{n-1}} \\ \mathbf{H_{n-1}} \\ \mathbf{O_{n-1}} \end{pmatrix} \tag{20}$$

Here,

$\mathbf{T_n} \in \mathbf{R}^5 (n = 1,2,\cdots), \mathbf{N_n} \in \mathbf{R}^5 (n = 1,2,\cdots), \mathbf{H_n} \in \mathbf{R}^5 (n = 1,2,\cdots),$
$\mathbf{O_n} \in \mathbf{R}^5 (n = 1,2,\cdots), \mathbf{A_{ij}} \in \mathbf{R}^{5\times5} (i = 1,\cdots,5)(j = 1,\cdots 5)$

We can express $\mathbf{T}, \mathbf{N}, \mathbf{H}$ and $\mathbf{O}$ as follows.

$$\mathbf{T} = \{\mathbf{T}_1, \mathbf{T}_2, \cdots\}, \quad \mathbf{N} = \{\mathbf{N}_1, \mathbf{N}_2, \cdots\}, \quad \mathbf{H} = \{\mathbf{H}_1, \mathbf{H}_2, \cdots\}, \quad \mathbf{O} = \{\mathbf{O}_1, \mathbf{O}_2, \cdots\}$$

Now, we investigate all cases. Total numbers of shifts among each blocks are as follows.

①	$\mathbf{A}_{11}$	Shift from **T** to **T** : 16		⑨	$\mathbf{A}_{31}$	Shift from **T** to **H** : 5	
②	$\mathbf{A}_{12}$	Shift from **N** to **T** : 9		⑩	$\mathbf{A}_{32}$	Shift from **N** to **H** : 5	
③	$\mathbf{A}_{13}$	Shift from **H** to **T** : 5		⑪	$\mathbf{A}_{33}$	Shift from **H** to **H** : 14	
④	$\mathbf{A}_{14}$	Shift from **O** to **T** : 9		⑫	$\mathbf{A}_{34}$	Shift from **O** to **H** : 3	
⑤	$\mathbf{A}_{21}$	Shift from **T** to **N** : 6		⑬	$\mathbf{A}_{41}$	Shift from **T** to **O** : 12	
⑥	$\mathbf{A}_{22}$	Shift from **N** to **N** : 37		⑭	$\mathbf{A}_{42}$	Shift from **N** to **O** : 15	
⑦	$\mathbf{A}_{23}$	Shift from **H** to **N** : 2		⑮	$\mathbf{A}_{43}$	Shift from **H** to **O** : 13	
⑧	$\mathbf{A}_{24}$	Shift from **O** to **N** : 7		⑯	$\mathbf{A}_{44}$	Shift from **O** to **O** : 44	

In the case of $\mathbf{A}_{11}$, the details of shifts are as follows.

1.	Shift from 5th position to 5th position of **T**	: 2	14.	Shift from 3rd position to 2nd position of **T**	: 0
2.	Shift from 5th position to 4th position of **T**	: 1	15.	Shift from 3rd position to 1st position of **T**	: 0
3.	Shift from 5th position to 3rd position of **T**	: 1	16.	Shift from 2nd position to 5th position of **T**	: 2
4.	Shift from 5th position to 2nd position of **T**	: 0	17.	Shift from 2nd position to 4th position of **T**	: 1
5.	Shift from 5th position to 1st position of **T**	: 1	18.	Shift from 2nd position to 3rd position of **T**	: 0
6.	Shift from 4th position to 5th position of **T**	: 1	19.	Shift from 2nd position to 2nd position of **T**	: 2
7.	Shift from 4th position	: 2	20.	Shift from 2nd position	: 0

	to 4th position of $\mathbf{T}$		
8.	Shift from 4th position to 3rd position of $\mathbf{T}$	:	0
9.	Shift from 4th position to 2nd position of $\mathbf{T}$	:	0
10.	Shift from 4th position to 1st position of $\mathbf{T}$	:	0
11.	Shift from 3rd position to 5th position of $\mathbf{T}$	:	0
12.	Shift from 3rd position to 4th position of $\mathbf{T}$	:	2
13.	Shift from 3rd position to 3rd position of $\mathbf{T}$	:	0

	to 1st position of $\mathbf{T}$		
21.	Shift from 1st position to 5th position of $\mathbf{T}$	:	0
22.	Shift from 1st position to 4th position of $\mathbf{T}$	:	0
23.	Shift from 1st position to 3rd position of $\mathbf{T}$	:	1
24.	Shift from 1st position to 2nd position of $\mathbf{T}$	:	0
25.	Shift from 1st position to 1st position of $\mathbf{T}$	:	0

Then, the vector $\mathbf{T}, \mathbf{T_b}$ for each case is expressed as follows.

1. $\mathbf{T} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ $\mathbf{T_b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ 2. $\mathbf{T} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ $\mathbf{T_b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ 3. $\mathbf{T} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ $\mathbf{T_b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

5. $\mathbf{T} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $\mathbf{T_b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ 6. $\mathbf{T} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ $\mathbf{T_b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ 7. $\mathbf{T} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ $\mathbf{T_b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

12. $\mathbf{T} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ $\mathbf{T_b} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ 16. $\mathbf{T} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ $\mathbf{T_b} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ 17. $\mathbf{T} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ $\mathbf{T_b} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

19. $\mathbf{T} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{T_b} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ 23. $\mathbf{T} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{T_b} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

We calculate for the cases $\mathbf{O}, \mathbf{N}, \mathbf{H}$ in the same way. Substituting these to equation (7), we obtain the following equation.

$$\mathbf{A} = \begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 2 & 1 & 0 & 1 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 2 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & 0 & 0 & 1 & 14 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 5 & 9 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 3 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
2 & 0 & 2 & 2 & 5 & 0 & 0 & 1 & 6 & 4 & 0 & 0 & 0 & 0 & 9 & 0 & 2 & 1 & 0 & 33
\end{pmatrix}$$

$$
\times \begin{pmatrix}
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 21 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 14 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 51
\end{pmatrix}^{-1}
$$

$$
=\begin{pmatrix}
0 & 0 & 0 & 0 & \frac{1}{20} & 0 & 0 & 0 & \frac{1}{34} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{7} & \frac{1}{3} & \frac{1}{34} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{5} & 0 & 0 & \frac{1}{51} \\
\frac{1}{3} & 0 & 0 & 0 & \frac{1}{20} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{6} & \frac{2}{5} & \frac{2}{5} & \frac{1}{20} & 0 & \frac{1}{7} & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{51} \\
0 & \frac{1}{3} & 0 & \frac{1}{5} & \frac{1}{10} & 0 & 0 & 0 & 0 & \frac{1}{21} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 & \frac{5}{51} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{6} & 0 & 0 & \frac{1}{20} & 0 & 0 & 0 & \frac{1}{34} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{5} & 0 & 0 & \frac{1}{51} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{34} & \frac{1}{21} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{5} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{3}{20} & 0 & 0 & \frac{1}{3} & \frac{7}{17} & \frac{2}{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{51} \\
0 & 0 & 0 & 0 & \frac{1}{20} & 0 & \frac{2}{7} & 0 & \frac{5}{34} & \frac{3}{7} & 0 & 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 & \frac{1}{51} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{10} & 0 & \frac{1}{7} & 0 & 0 & \frac{1}{21} & 0 & 0 & \frac{1}{6} & \frac{1}{10} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{17} & 0 & 0 & \frac{1}{4} & 0 & \frac{2}{5} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{5} & 0 & \frac{1}{10} & 0 & 0 & 0 & \frac{1}{34} & 0 & 0 & 0 & \frac{1}{6} & \frac{3}{10} & 0 & 0 & 0 & 0 & 0 & \frac{1}{17} \\
0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{14} & 0 & 0 & \frac{1}{5} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{20} & 0 & 0 & 0 & \frac{1}{34} & 0 & 0 & 0 & 0 & \frac{1}{10} & 0 & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{17} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{34} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{5} & 0 & 0 \\
\frac{2}{3} & 0 & \frac{2}{5} & \frac{2}{5} & \frac{1}{4} & 0 & 0 & \frac{1}{3} & \frac{3}{17} & \frac{4}{21} & 0 & 0 & 0 & 0 & \frac{9}{14} & 0 & \frac{2}{5} & \frac{1}{5} & 0 & \frac{11}{17}
\end{pmatrix} \quad (21)
$$

The number of upper level shifts were 42 and the same level shifts were 115, while the lower level shifts were 45.We can observe that rather the same level shift was dominant in this case.

Condensing each block matrix into one using the method stated in 3, we can obtain Eq.(22).

$$
\hat{\mathbf{A}} = \begin{pmatrix} 16 & 9 & 5 & 9 \\ 6 & 37 & 2 & 7 \\ 5 & 5 & 14 & 3 \\ 12 & 15 & 13 & 44 \end{pmatrix} \times \begin{pmatrix} 39 & 0 & 0 & 0 \\ 0 & 66 & 0 & 0 \\ 0 & 0 & 34 & 0 \\ 0 & 0 & 0 & 63 \end{pmatrix}^{-1}
$$

$$
= \begin{pmatrix} \frac{16}{39} & \frac{3}{22} & \frac{5}{34} & \frac{1}{7} \\ \frac{2}{13} & \frac{37}{66} & \frac{1}{17} & \frac{1}{9} \\ \frac{5}{39} & \frac{5}{66} & \frac{7}{17} & \frac{1}{21} \\ \frac{4}{13} & \frac{5}{22} & \frac{13}{35} & \frac{44}{63} \end{pmatrix} \quad (22)
$$

Based on this equation, we can clarify the shift among car makers. We consider as follows from these results.

a) The number of the shifts from “Others” to “Others” is 44. This is the greatest number and takes the share of 21.8 % of the total.

b) Next, the number of the shifts from Nissan to Nissan is 37. This has the share of 18.3% of the total.

c) There are shifts from other companies and also there are shifts to other companies. Therefore it is convenient to calculate “input-output” for the total sum.

This result is as follows.

Toyota	:	Total	0
Nissan	:	Outflow	14
Honda	:	Outflow	7
Others	:	Inflow	21

The outflow from Nissan and inflow to others consist the majority of shifts among companies.

d) Light vehicle is increasing rapidly these days. This reflects the increase of inflow to light vehicle.

5 Conclusions

Consumers often buy higher ranked brand after they are bored using current brand goods. This may be analyzed utilizing matrix. Suppose past purchasing data are set input and current purchasing data are set output, then transition matrix is identified using past and current data. If all brand selections are composed by the upper shifts, then the transition matrix becomes an upper triangular matrix. The goods of the same brand group would compose the Block Matrix in the transition matrix. Condensing the variables of the same brand group into one, analysis becomes easier to handle and the transition of Brand Selection can be easily grasped. A questionnaire investigation for automobile purchasing case was carried out and the above structure was confirmed.

We can make forecast utilizing the estimated transition matrix. We can foresee the future purchase and the producers can make effective marketing plan to cope with this. Unless planner for products does not notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand. Various fields should be examined hereafter.

In the end, we appreciate Ms. Kurumi Kawamura for her helpful support of work.

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Appendix 1

Questionnaire Sheet

1. Age

<teens ・ twenties ・ thirties ・ forties ・ fifties ・ more than sixties >

2. Sex

< Woman ・ Man >

3. Occupation

< Company employee・public official・Independent・Housewife・Miscellaneous>

4. Official position

< Chairman ・ President ・ Officer ・ General manager ・ Deputy general manager ・ Section chief ・ Deputy Section chief ・ Chief ・ Employee ・ Miscellaneous >

5. Annual Income

< Less than 3 million・5-3 million・7.5-5 million・10-7.5 million・15-10 million ・ 15 million or more >

6. Address. (City ・ Town ・ Village)

7. Marriage

< Single ・ Married >

8. Number of children

Working ( ) ,University student ( ) ,High school student ( ) ,Junior high school student ( ) ,School child ( ) ,child before entering school ( )

9. Please write the car that you own.

	Third Ahead	***Second Ahead***	***First ahead***	***Present***	***Next time***	***future***
Manufacturer Name						

	Third Ahead	***Second Ahead***	***First ahead***	***Present***	***Next time***	***future***
Model Name						
Purchase Reason						
Car Name						

Manufacturer Name :

A. Toyota Motor B. Honda Motor C. Nissan Motor D. Mitsubishi Motors
E. Mazda F. Subaru G. Isuzu H. Daihatsu Kogyo I. Suzuki J. Benz K. BMW
L. Audi M. Miscellaneous

Model Name :

a. Sedan b. Coupe (Sports car) c. One box・Minivan d. Wagon e. RV
f. Compact car・Light car g. Recreational vehicle h. Miscellaneous

Purchase Reason or Reason why you want to buy :

1.Design 2.Structure (It is possible to load with a lot of luggage.)
3.Performance (It is flexible.,The engine is good.) 4.Sales price
5.Family structure 6.Favorite Manufacturer
7.According to the lifestyle (Hobby etc.)
8.It is good for the environment. (Fuel cost etc.) 9.Area of garage
10.Present (You are presented used car.) 11.Interest rate
12.Maintenance expense (The tax is cheap.)
13. Miscellaneous (Please write in the frame.)

Appendix 2

Model Ranking Table (Total classification) (CC)

	Toyota	Nissan	Honda	Subaru	Suzuki	Mitsubishi	Mazda	BMW	Isuzu
Special C	Century (5000,4000)	President	—	—	—			Rolls-Royce Stirling	
Special B	Selshio (4300,4000)	Infiniti Shema (4500)	—	—	—			Rover 75	
Special A	Lexus SC GS Crownmajesta (4300,4000,3000)	Shema (4100 or less)	NSX						
Ⅱ	Crown (3000,2500,2000)	Fugue Cedric Gloria	Legend Inspire	Alshione				525i 528 325i	
Ⅲ	Harrier Mark X (3000,2500) Alfard High ace Cresta Chaser Progress Wyndham	Laurel Sefero Skyline Maxima	Accord Bigar Saver			Diamante			

Ⅳ	Bister (2000,1800) Premio, Camuri Ipsam RAV4 Prius (1500) Verossa Corona Bister etoile Ishis Carina	Bluebird	Integra Ascoti nova	Legacy Impreza (2000, 1800)		Gallant sigma Ransar		Megarnu	117 coupes
V	Corolla (1800,1500,1300) Sprinter bB Ractis Polte Colsa Mark Ⅱ Starlet	Sunny March Cube Pulsar Primera	Civic Fit Civichigh Bullitt Ballade	Impreza (1600, 1500)	Swift Wagon R	Mirage	Familia Capera		

Appendix 3

Model Ranking Table (Classification for Sedan Type) (CC)

	Toyota	Nissan	Honda	Subaru	Suzuki	Mitsubishi	Mazda	BMW	Isuzu
1st	Century (5000,4000) Selshio (4300,4000) Lexus SC GS Crownmajesta (4300,4000,3000)	President Infiniti Shema (4500) Shema (4100 or less)	NSX	—	—			Rolls-Royce Stirling Rover 75	
2nd	Crown (3000,2500,2000)	Fugue Cedric Gloria	Legend Inspire	Alshiorne				525i 528 325i	
3rd	Harrier Mark X (3000,2500) Alfard High ace Cresta Chaser Progress Wyndham	Laurel Sefero Skyline Maxima	Accord Bigar Saver			Diamante			

4th	Bister (2000,1800) Camuri Premio Ipsam RAV4 Prius(1500) Verossa Corona Bister etoile Ishis Carina	Bluebird	Integra Ascotinova	Legacy Impreza (2000,1800)		Gallantsigma Ransar		Megarnu	117 coupes
5th	Corolla (1800,1500,1300) Sprinter bB Ractis Polte Colsa Mark Ⅱ Starlet	Sunny March Cube Pulsar Primera	Civic Fit Civic high Bullitt Ballade	Impreza (1600,1500)	Swift WagonR	Mirage	Familia Capera		