Journal of Computations & Modelling, vol.4, no.3, 2014, 43-64
ISSN: 1792-7625 (print), 1792-8850 (online)
Scienpress Ltd, 2014

A Hybrid Method of Forecasting in the Case of the Average Daily Number of Patients

Daisuke Takeyasu[1] and Kazuhiro Takeyasu[2]

Abstract

High accuracy demand forecasting is inevitable in supply chain management. In industries, how to improve forecasting accuracy such as sales, shipping is an important issue. There are many researches made on this. In this paper, a hybrid method is introduced and plural methods are compared. Focusing that the equation of exponential smoothing method(ESM) is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method is proposed before by us which satisfies minimum variance of forecasting error. Generally, smoothing constant is selected arbitrarily. But in this paper, we utilize above stated theoretical solution. Firstly, we make estimation of ARMA model parameter and then estimate smoothing constants. Thus theoretical solution is derived in a simple way and it may be utilized in various fields. A mere application of ESM does not make good forecasting accuracy for the time series which has non-linear trend and/or trend by month. A new method to cope with this

---

[1] The Open University of Japan. E-mail: take1111@hotmail.co.jp
[2] Tokoha University. E-mail: takeyasu@fj.tokoha-u.ac.jp

Article Info: *Received* : March 28, 2014. *Revised* : May 28, 2014.
Published online : September 10, 2014.

issue is required. In this paper, combining the trend removing method with this method, we aim to improve forecasting accuracy. An approach to this method is executed in the following method. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is carried out to the sum total medical data of production and imports of the data of The average daily number of patients for two cases (The total number of patients in hospital, Outpatients number).

The weights for these functions are set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting is executed on these data. The new method shows that it is useful for the time series that has various trend characteristics and has rather strong seasonal trend. The effectiveness of this method should be examined in various cases.

Mathematics Subject Classification: 62J10; 62M10; 91B70

Keywords: minimum variance; exponential smoothing method; forecasting; trend; patient

1 Introduction

Sales forecasting is inevitable for Supply Chain Management. But in fact, it is not well utilized in industries. It is because there are so many irregular incidents therefore it becomes hard to make sales forecasting. A mere application of method does not bear good result. The big reason is that sales data or production data are not stationary time series, while linear model requires the time series as a stationary one. In order to improve forecasting accuracy, we have devised trend removal methods as well as searching optimal parameters and obtained good results. We created a new method and applied it to various time series and examined the effectiveness of the method. Applied data are sales data, production data, shipping data, stock market price data, flight passenger data etc.

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM), [1]-[4]. Among these, ESM is said to be a practical simple method.

For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend [5], utilizing Kalman Filter [6], Bayes Forecasting [7], adaptive ESM [8], exponentially weighted

Moving Averages with irregular updating periods [9], making averages of forecasts using plural method [10] are presented. For example, Maeda [6] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he could not grasp observation noise. It can be said that it does not pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii [11] pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn’t show analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before [12]. Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived. Furthermore, combining the trend removal method, forecasting accuracy was improved, where shipping data, stock market price data etc. were examined [13]－[19].

In this paper, utilizing above stated method, a revised forecasting method is proposed. A mere application of ESM does not make good forecasting accuracy for the time series which has non-linear trend and/or trend by month. A new method to cope with this issue is required. Therefore, utilizing above stated method, a revised forecasting method is proposed in this paper to improve forecasting accuracy. In making forecast such as production data, trend removing method is devised. Trend removing by the combination of linear and 2^{nd} order non-linear function and 3^{rd} order non-linear function is executed to the data of The average daily number of patients for two cases (The total number of patients in hospital, Outpatients number). The weights for these functions are set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. The new method shows that it is useful especially for the time series that has stable characteristics and has rather strong seasonal trend and also the case that has non-linear trend. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3. The Monthly Ratio is referred in section 4. Forecasting is executed in section 5, and estimation accuracy is examined.

2. Description of ESM using ARMA model, [12]

In ESM, forecasting at time $t+1$ is stated in the following equation.

$$\hat{x}_{t+1} = \hat{x}_t + \alpha\left(x_t - \hat{x}_t\right) = \alpha x_t + \left(1-\alpha\right)\hat{x}_t \tag{1}$$

Here,

$\hat{x}_{t+1}$: forecasting at $t+1$

x_t : realized value at t

α : smoothing constant $(0 < \alpha < 1)$

(1) is re-stated as

$$\hat{x}_{t+1} = \sum_{l=0}^{\infty} \alpha (1-\alpha)^l x_{t-l} \tag{2}$$

By the way, we consider the following (1,1) order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1} . \tag{3}$$

Generally, (p,q) order ARMA model is stated as

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{4}$$

Here,

$\{x_t\}$: Sample process of Stationary Ergodic Gaussian Process $x(t)$,

$t = 1, 2, \ldots, N, \ldots$

$\{e_t\}$: Gaussian White Noise with 0 mean σ_e^2 variance

MA process in (4) is supposed to satisfy convertibility condition. Utilizing the relation that

$$E\left[e_t \middle| e_{t-1}, e_{t-2}, \cdots \right] = 0$$

we get the following equation from (3).

$$\hat{x}_t = x_{t-1} - \beta e_{t-1} \tag{5}$$

Operating this scheme on $t+1$, we finally get

$$\hat{x}_{t+1} = \hat{x}_t + (1-\beta) e_t = \hat{x}_t + (1-\beta)(x_t - \hat{x}_t) \tag{6}$$

If we set $1-\beta = \alpha$, the above equation is the same with (1), i.e., equation of

ESM is equivalent to (1,1) order ARMA model, or is said to be (0,1,1) order ARIMA model because 1st order AR parameter is -1, [1], [3]. Focusing that the equation of exponential smoothing method(ESM) is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method is derived (See Appendix in detail).

Finally we get:

$$b_1 = \frac{1-\sqrt{1-4\rho_1^2}}{2\rho_1}$$
$$\alpha = \frac{1+2\rho_1-\sqrt{1-4\rho_1^2}}{2\rho_1} \tag{7}$$

Thus we can obtain a theoretical solution by a simple way.

Here ρ_1 must satisfy

$$-\frac{1}{2} < \rho_1 < 0 \tag{8}$$

in order to satisfy $0 < \alpha < 1$.

Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter.

It can be estimated only by calculating 0th and 1st order autocorrelation function.

3. Trend removal method [12]

As ESM is a one of a linear model, forecasting accuracy for the time series with non-linear trend is not necessarily good. How to remove trend for the time series with non-linear trend is a big issue in improving forecasting accuracy. In this paper, we devise to remove this non-linear trend by utilizing non-linear function.

As trend removal method, we describe the combination of linear and non-linear function.

[1] Linear function

We set

$$y = a_1 x + b_1 \tag{9}$$

as a linear function.

[2] Non-linear function

We set

$$y = a_2 x^2 + b_2 x + c_2 \tag{10}$$

$$y = a_3 x^3 + b_3 x^2 + c_3 x + d_3 \tag{11}$$

as a 2nd and a 3rd order non-linear function.

[3] The combination of linear and non-linear function

We set

$$y = \alpha_1 \left(a_1 x + b_1 \right) + \alpha_2 \left(a_2 x^2 + b_2 x + c_2 \right) \tag{12}$$

$$y = \beta_1 \left(a_1 x + b_1 \right) + \beta_2 \left(a_3 x^3 + b_3 x^2 + c_3 x + d_3 \right) \tag{13}$$

$$y = \gamma_1 \left(a_1 x + b_1 \right) + \gamma_2 \left(a_2 x^2 + b_2 x + c_2 \right) + \gamma_3 \left(a_3 x^3 + b_3 x^2 + c_3 x + d_3 \right) \tag{14}$$

as the combination of linear and 2nd order non-linear and 3rd order non-linear function. Here, $\alpha_2 = 1 - \alpha_1$, $\beta_2 = 1 - \beta_1, \gamma_3 = 1 - (\gamma_1 + \gamma_2)$. Comparative discussion concerning (12), (13) and (14) are described in section 5.

4. Monthly ratio [12]

For example, if there is the monthly data of L years as stated bellow:

$$\{x_{ij}\} \quad i = 1, \ldots, L, \quad j = 1, \ldots, 12$$

Where, $x_{ij} \in R$ in which j means month and i means year and x_{ij} is a shipping data of i-th year, j-th month. Then, monthly ratio $\tilde{x}_j, \quad j = 1, \ldots, 12$ is calculated as follows.

$$\tilde{x}_j = \frac{\frac{1}{L}\sum_{i=1}^{L} x_{ij}}{\frac{1}{L} \cdot \frac{1}{12} \sum_{i=1}^{L} \sum_{j=1}^{12} x_{ij}} \tag{15}$$

Monthly trend is removed by dividing the data by (15). Numerical examples both of monthly trend removal case and non-removal case are discussed in 5.

5. Forecasting the average daily number of patients

5.1 Analysis Procedure

Sum total data of the average daily number of patients for two cases (The total number of patients in hospital, Outpatients number) from January 2007 to December 2009 are analyzed. These data are obtained from the Annual Report of Statistical Investigation on Statistical-Survey-on-Trends-in-Pharmaceutical-Produ-ction by Ministry of Health, Labour and Welfare in Japan. First of all, graphical charts of these time series data are exhibited in Figures 1, 2.

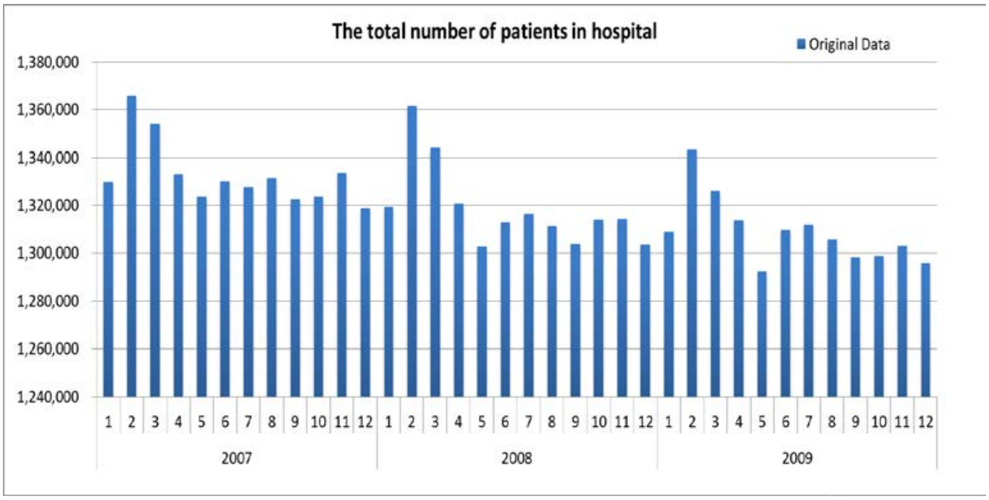

Figure 1: Sum total data of The total number of patients in hospital

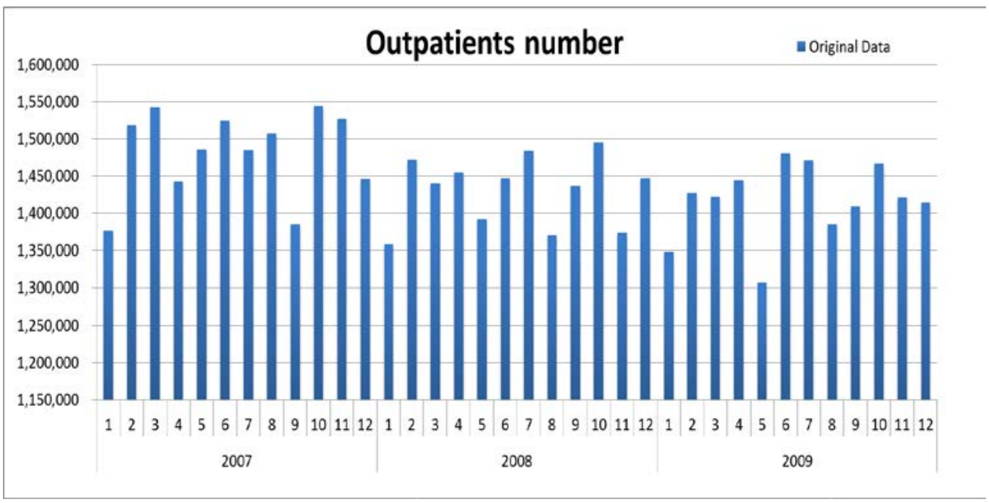

Figure 2: Sum total data of Outpatients number

Analysis procedure is as follows. There are 36 monthly data for each case. We use 24 data (1 to 24) and remove trend by the method stated in 3. Then we calculate monthly ratio by the method stated in 4. After removing monthly trend, the method stated in 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Forecasting error is expressed as:

$$\varepsilon_i = \hat{x}_i - x_i \tag{16}$$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i \tag{17}$$

Variance of forecasting error is calculated by:

$$\sigma_{\varepsilon}^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\varepsilon_i - \bar{\varepsilon}\right)^2 \tag{18}$$

5.2 Trend Removing

Trend is removed by dividing original data by (12),(13),(14). The patterns of trend removal are exhibited in Table 1.

Table 1: The patterns of trend removal

Pattern1	α_1, α_2 are set 0.5 in the equation (12)
Pattern2	β_1, β_2 are set 0.5 in the equation (13)
Pattern3	α_1 is shifted by 0.01 increment in (12)
Pattern4	β_1 is shifted by 0.01 increment in (13)
Pattern5	γ_1 and γ_2 are shifted by 0.01 increment in (14)

In pattern1 and 2, the weight of α_1, α_2, β_1, β_2 are set 0.5 in the equation (12),(13). In pattern3, the weight of α_1 is shifted by 0.01 increment in

(12) which satisfy the range $0 \le \alpha_1 \le 1.00$. In pattern4, the weight of β_1 is shifted in the same way which satisfy the range $0 \le \beta_1 \le 1.00$. In pattern5, the weight of γ_1 and γ_2 are shifted by 0.01 increment in (14) which satisfy the range $0 \le \gamma_1 \le 1.00$, $0 \le \gamma_2 \le 1.00$. The best solution is selected which minimizes the variance of forecasting error. Estimation results of coefficient of (9), (10) and (11) are exhibited in Table 2. Estimation results of weights of (12), (13) and (14) are exhibited in Table 3.

Table 2: Coefficient of (9), (10) and (11)

	1st		2nd			3rd			
	a_1	b_1	a_2	b_2	c_2	a_3	b_3	c_3	d_3
The total number of patients in hospital	-1541.4996	1345011.5	-13.263902	-1209.902	1343574.53	-7.9266716	283.98628	-4243.4392	1350530.184
Outpatients number	-2774.991739	1491104.022	-102.42681	-214.3215	1480007.78	60.0414838	-2353.982	22763.5543	1427321.382

Table 3: Weights of (12), (13) and (14)

	Monthly ratio	Pattern1		Pattern2		Pattern3		Pattern4		Pattern5		
		α_1	α_2	β_1	β_2	α_1	α_2	β_1	β_2	γ_1	γ_2	γ_3
The total number of patients in hospital	Used	0.5	0.5	0.5	0.5	1	0	1	0	1	0	0
	Not used	0.5	0.5	0.5	0.5	1	0	1	0	1	0	0
Outpatients number	Used	0.5	0.5	0.5	0.5	1	0	1	0	1	0	0
	Not used	0.5	0.5	0.5	0.5	0.41	0.59	1	0	0.41	0.59	0

Graphical chart of trend is exhibited in Figures 3, 4 for the cases that monthly ratio is used.

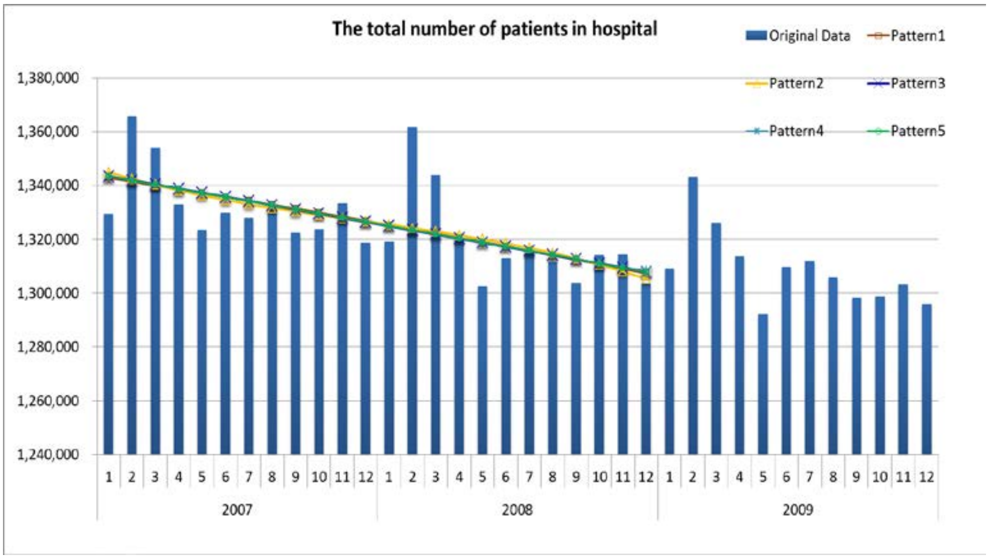

Figure 3: Trend of The total number of patients in hospital

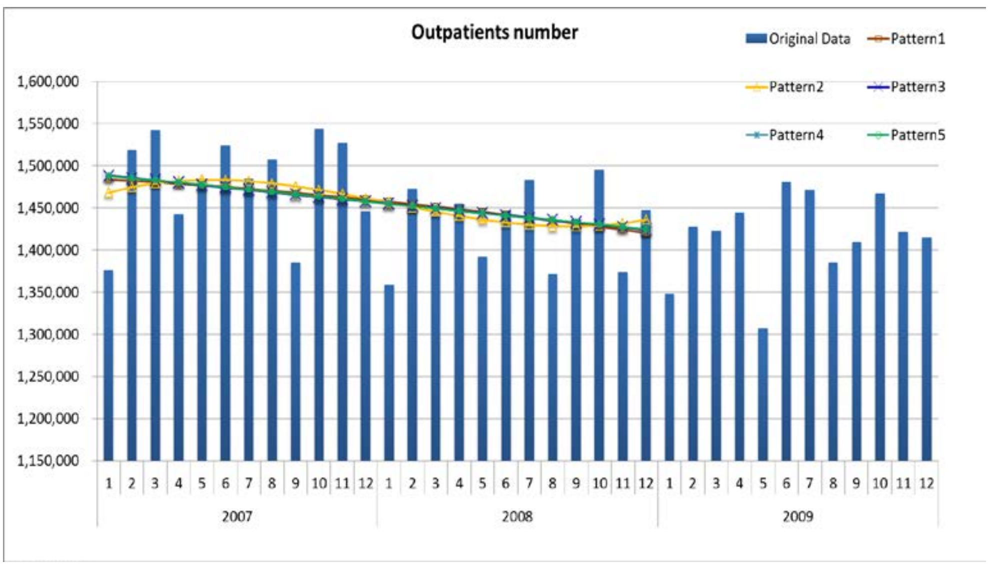

Figure 4: Trend of Outpatients number

5.3 Removing trend of monthly ratio

After removing trend, monthly ratio is calculated by the method stated in 4. Calculation result for 1st to 24th data is exhibited in Table 4 through 8.

Table 4: Monthly ratio (Pattern1)

Month	1	2	3	4	5	6	7	8	9	10	11	12
The total number of patients in hospital	0.99	1.02	1.01	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00
Outpatients number	0.93	1.02	1.02	0.99	0.98	1.02	1.02	0.99	0.97	1.05	1.00	1.00

Table 5: Monthly ratio (Pattern2)

Month	1	2	3	4	5	6	7	8	9	10	11	12
The total number of patients in hospital	0.99	1.02	1.01	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00
Outpatients number	0.94	1.02	1.02	0.99	0.98	1.02	1.02	0.99	0.97	1.05	1.00	1.00

Table 6: Monthly ratio (Pattern3)

Month	1	2	3	4	5	6	7	8	9	10	11	12
The total number of patients in hospital	0.99	1.02	1.01	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00
Outpatients number	0.93	1.02	1.02	0.99	0.98	1.02	1.02	0.99	0.97	1.05	1.00	1.00

Table 7: Monthly ratio (Pattern4)

Month	1	2	3	4	5	6	7	8	9	10	11	12
The total number of patients in hospital	0.99	1.02	1.01	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00
Outpatients number	0.93	1.02	1.02	0.99	0.98	1.02	1.02	0.99	0.97	1.05	1.00	1.00

Table 8: Monthly ratio (Pattern5)

Month	1	2	3	4	5	6	7	8	9	1 0	1 1	1 2
The total number of patients in hospital	0.99	1.02	1.01	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00
Outpatients number	0.93	1.02	1.02	0.99	0.98	1.02	1.02	0.99	0.97	1.05	1.00	1.00

5.4 Estimation of Smoothing Constant with Minimum Variance of Forecasting Error

After removing monthly trend, Smoothing Constant with minimum variance of forecasting error is estimated utilizing (7). There are cases that we cannot obtain a theoretical solution because they do not satisfy the condition of (A-9). In those cases, Smoothing Constant with minimum variance of forecasting error is derived by shifting variable from 0.01 to 0.99 with 0.01 interval. Calculation result for 1st to 24th data is exhibited in Table 9.

Table 9: Estimated Smoothing Constant with Minimum Variance

	Monthly ratio	Pattern1		Pattern2	
		ρ_1	α	ρ_1	α
The total number of patients in hospital	Used	-0.3253	0.6303	-0.2869	0.6846
	Not used	-0.0683	0.9314	-0.0691	0.9305
Outpatients number	Used	-0.4318	0.4259	-0.4560	0.3534
	Not used	-0.2566	0.7239	-0.2925	0.6769

	Monthly ratio	Pattern3		Pattern4		Pattern5	
		ρ_1	α	ρ_1	α	ρ_1	α
The total number of patients in hospital	Used	-0.3189	0.6397	-0.3189	0.6397	-0.3189	0.6397
	Not used	-0.0685	0.9311	-0.0685	0.9311	-0.0685	0.9311
Outpatients number	Used	-0.4268	0.4387	-0.4268	0.4387	-0.4268	0.4387
	Not used	-0.2561	0.7245	-0.2571	0.7233	-0.2561	0.7245

5.5 Forecasting and Variance of Forecasting Error

Utilizing smoothing constant estimated in the previous section, forecasting is executed for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Variance of forecasting error is calculated by (18). Forecasting results are exhibited in Figures 5, 6 for the cases that monthly ratio is used.

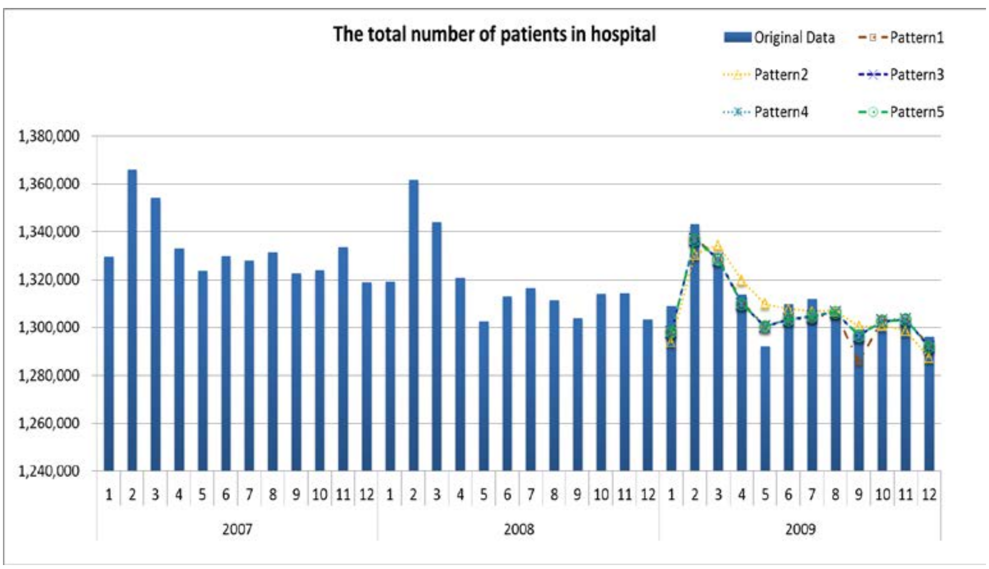

Figure 5: Forecasting Results of The total number of patients in hospital

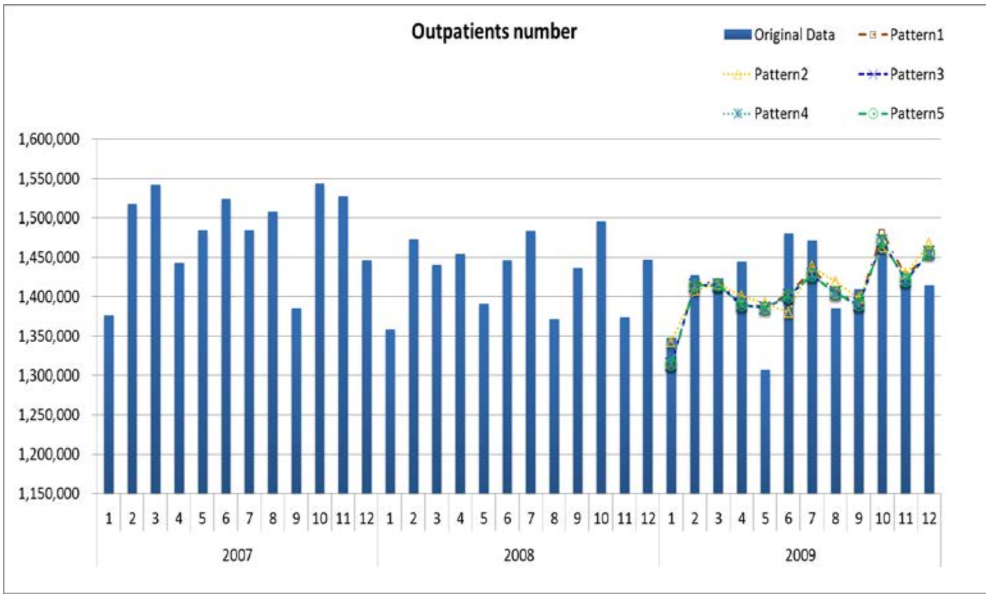

Figure 6: Forecasting Results of Outpatients number

Variance of forecasting error is exhibited in Table 10.

Table 10: Variance of Forecasting Error

	Monthly ratio	Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
The total number of patients in hospital	Used	40,792,752.20	84,713,915.84	29,506,688.24	29,506,688.24	29,506,688.24
	Not used	249,335,212.96	299,714,779.80	246,921,217.70	246,921,217.70	246,921,217.70
Outpatients number	Used	1,874,558,064.76	2,243,434,383.91	1,848,148,873.00	1,848,148,873.00	1,848,148,873.00
	Not used	5,119,066,024.32	5,768,032,698.37	5,117,541,520.00	5,174,974,513.00	5,117,541,520.00

5.6 Remarks

In all cases, that monthly ratio was used had a better forecasting accuracy. Both cases had a good result in 1st+2nd order with the case that monthly ratio was used. We can observe that monthly trend is relatively apparent in these cases.

6 Conclusion

In Supply Chain Management, Demand forecasting plays an important role. Focusing on the idea that the equation of exponential smoothing method(ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant was selected arbitrarily. But in this paper, we utilized above stated theoretical solution. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Thus theoretical solution was derived in a simple way and it might be utilized in various fields.

Furthermore, combining the trend removal method with this method, we aimed to improve forecasting accuracy. A mere application of ESM does not make good forecasting accuracy for the time series which has non-linear trend and/or trend by month. A new method to cope with this issue is required. Therefore, utilizing above stated method, a revised forecasting method is proposed in this paper to improve forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the original Sum total data of the average daily number of patients for two cases (The total number of patients in hospital, Outpatients number).

The combination of linear and non-linear function was also introduced in trend removing. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting was executed on these data. In all cases, that monthly ratio was used had a better forecasting accuracy. Both cases had a good result in 1st+2nd order with the case that monthly ratio was used. We can observe that monthly trend is relatively apparent in these cases.

Various cases should be examined hereafter in order to confirm the effectiveness of this new method.

Appendix

Estimation of smoothing constant in Exponential Smoothing Method [12]. Comparing with (3) and (4), we obtain

$$a_1 = -1,$$
$$b_1 = -\beta$$

From (1), (6),

$$\alpha = 1 - \beta .$$

Therefore, we get

$$\begin{aligned} a_1 &= -1 \\ b_1 &= -\beta = \alpha - 1 \end{aligned} \qquad \text{(A-1)}$$

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below. Let (4) be

$$\tilde{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i} \qquad \text{(A-2)}$$

$$\tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j} \qquad \text{(A-3)}$$

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (A-2), (A-3), we get the following non-linear equations which are well known [3].

$$\tilde{r}_k = \begin{cases} \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j}, & k \le q \\ 0, & k \ge q+1 \end{cases}$$

$$\tilde{r}_0 = \sigma_e^2 \sum_{j=0}^{q} b_j^2 \qquad \text{(A-4)}$$

For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only b_1, so it can be solved in the following way.

From (3) (4) (A-1) (A-4), we get

$$\begin{aligned} &q = 1 \\ &a_1 = -1 \\ &b_1 = -\beta = \alpha - 1 \\ &\tilde{r}_0 = \left(1 + b_1^2\right)\sigma_e^2 \\ &\tilde{r}_1 = b_1 \sigma_e^2 \end{aligned} \tag{A-5}$$

If we set

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0} \tag{A-6}$$

the following equation is derived.

$$\rho_1 = \frac{b_1}{1 + b_1^2} \tag{A-7}$$

We can get b_1 as follows.

$$b_1 = \frac{1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1} \tag{A-8}$$

In order to have real roots, ρ_1 must satisfy

$$|\rho_1| \le \frac{1}{2} \tag{A-9}$$

From invertibility condition, b_1 must satisfy

$$|b_1| < 1$$

From (A-7), using the next relation,

$$\begin{aligned} &\left(1 - b_1\right)^2 \ge 0 \\ &\left(1 + b_1\right)^2 \ge 0 \end{aligned}$$

(A-9) always holds. As

$$\alpha = b_1 + 1$$

b_1 is within the range of

$$-1 < b_1 < 0.$$

Finally we get

$$b_1 = \frac{1-\sqrt{1-4\rho_1^2}}{2\rho_1}$$
$$\alpha = \frac{1+2\rho_1-\sqrt{1-4\rho_1^2}}{2\rho_1} \quad \text{(A-10)}$$

which satisfy above condition.

References

[1] Box Jenkins, *Time Series Analysis Third Edition*, Prentice Hall, 1994.

[2] R.G. Brown, *Smoothing, Forecasting and Predict on of Discrete –Time Series*, Prentice Hall, 1963.

[3] Hidekatsu Tokumaru et al., *Analysis and Measurement –Theory and Application of Random data Handling*, Baifukan Publishing, 1982.

[4] Kengo Kobayashi, *Sales Forecasting for Budgeting*, Chuokeizai-Sha Publishing, 1992.

[5] Peter R.Winters, Forecasting Sales by Exponentially Weighted Moving Averages, *Management Science*, **6**(3), (1984), 324-343.

[6] Katsuro Maeda, Smoothing Constant of Exponential Smoothing Method, *Seikei University Report Faculty of Engineering*, **38**, (1984), 2477-2484.

[7] M. West and P.J. Harrison, *Baysian Forecastingand Dynamic Models*, Springer-Verlag, New York, 1989.

[8] Steinar Ekern, Adaptive Exponential Smoothing Revisited, *Journal of the Operational Research Society*, **32**, (1982), 775-782.

[9] F.R. Johnston, Exponentially Weighted Moving Average (EWMA) with Irregular Updating Periods, *Journal of the Operational Research Society*, **44**(7), (1993), 711-716.

[10] Spyros Makridakis and Robeat L. Winkler, Averages of Forecasts; some

Empirical Results, *Management Science*, **9**(9), (1983), 987-996.

[11] Naohiro Ishii et al., Bilateral Exponential Smoothing of Time Series, *Int. J. System Sci*., **12**(8), (1991), 997-988.

[12] Kazuhiro Takeyasu and Keiko Nagata, Estimation of Smoothing Constant of Minimum Variance with Optimal Parameters of Weight, *International Journal of Computational Science*, **4**(5), (2010),411-425.

[13] Kazuhiro Takeyasu, Keiko Nagata and Yuki Higuchi, Estimation of Smoothing Constant of Minimum Variance And Its Application to Shipping Data With Trend Removal Method, *Industrial Engineering & Management Systems* (IEMS), **8**(4), (2009), 257-263.

[14] Kazuhiro Takeyasu, Keiko Nagata and Yui Nishisako, *A Hybrid Method to Improve Forecasting Accuracy Utilizing Genetic Algorithm And Its Application to Industrial Data*, NCSP'10, Honolulu, Hawaii, USA, (2010).

[15] Kazuhiro Takeyasu, Keiko Nagata and Kana Takagi, *Estimation of Smoothing Constant of Minimum Variance with Optimal Parameters of Weight*, NCSP'10, Honolulu, Hawaii, USA, (2010).

[16] Kazuhiro Takeyasu, Keiko Nagata and Tomoka Kuwahara, *Estimation of Smoothing Constant of Minimum Variance Searching Optimal Parameters of Weight*, NCSP'10, Honolulu, Hawaii, USA, (2010).

[17] Kazuhiro Takeyasu, Keiko Nagata, Mai Ito and Yuki Higuchi, *A Hybrid Method to Improve Forecasting Accuracy Utilizing Genetic Algorithm*, The 11th APEIMS, Melaka, Malaysia, (2010).

[18] Kazuhiro Takeyasu, Keiko Nagata, Kaori Matsumura, *Estimation of Smoothing Constant of Minimum Variance and Its Application to Sales Data*, JAIMS, Honolulu, Hawaii, USA, (2011).

[19] Hiromasa Takeyasu, Yuki Higuchi and Kazuhiro Takeyasu, A Hybrid Method to Improve Forecasting Accuracy in the Case of Bread, *International Journal of Information and Communication Technology Research*, **2**(11), (2012), 804-812.