Journal of Statistical and Econometric Methods, vol.8, no.4, 2019, 101-112
ISSN: 2241-0384 (print), 2241-0376 (online)
Scientific Press International Limited

Evaluating Forecast Performance of SETAR Model using Gross Domestic Product of Nigeria.

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Abstract

This paper examines the structural changes emanating from Gross domestic product of Nigeria from 1980 to 2017. Out of sample forecast performances of non-linear time series SETAR model were examined. All necessary theoretical frameworks were stated and stationarity tests conducted before the model setting. Out-of-sample forecast performances between the standard linear ARIMA model and non-linear SETAR model were compared. The Empirical illustration shows that the non-linear SETAR model has superior forecasting power than linear ARIMA model using Gross domestic product of Nigeria. It suffices to recommend the non-linear model for would be policy makers, investors and academia for forecasting. However, this does not foreclose the fact that the linear ARIMA forecast model could still be used by forecasters in the absence of SETAR and other powerful non-linear model.

Keywords: Gross domestic product of Nigeria, SETAR Model, in and out samples forecast, ARIMA model

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Article Info: *Received:* June 26, 2019. *Revised:* September 25, 2019.
Published online: December 22, 2019.

1. Introduction

Linear methods have dominated forecasting in the past decades. Methods such as the MA and ARMA were quite successful in numerous applications. Their main advantage is that they are easy to develop and implement and simple to understand and interpret. However, these models have a shortcoming as they are unable to capture nonlinearity in data (Makridakis *et al.,* 1982). Linear methods are not capable of representing many nonlinear dynamic patterns such as asymmetry, amplitude dependence and volatility clustering. Since linear models have a weakness in terms of capturing nonlinearity in data sets such as stock price, inflation rate, interest rate and others, researchers have resorted to nonlinear methods such as the Smooth Transition Regressive (STR) model, the Threshold Autoregressive (TAR) model and the Markov switching autoregressive model (MS-AR) (Makridakis *et al.*, 1982). These nonlinear modelling techniques have been suggested in the literature to capture the suggested nonlinearities in economics and financial data.

These three modelling techniques differ from conventional linear econometric models by their assumption of the existence of different segments or regimes, within which the time series may exhibit different behaviour. In the three models listed above, the statement of the problem is how these three models do when applied to model and forecast the five closing stock prices, and how the estimated models compare in terms of efficiency and performance. Most of the empirical econometric modeling work in financial and Economic data assumes that relationships are linear. Economic theory plays a passive role on this issue, and thus most applied research finds it convenient to assume linearity. Non linear specification is regarded as a reliable way of representing data in Economics and Financial data. For instance, stock returns appears to be correlated when the volatility is rather low than when it is high. This same behavior is exhibited in exchange rate series (Franses and Dijk, 2000). To accommodate this kind of dynamic behavior using time series data, regime-switching models (RSM) have been introduced (Priestley, 1980 & 1988; Granger and Terasvirta, 1993). Recently threshold autoregressive (TAR) model has become a frequently used model in Agricultural economics literature. TAR model assumes that the regime is determined by a threshold value. The empirical existence of a threshold seems plausible in various economic settings.

2. Literature review

TAR model was initially proposed by Tong (1978) and Tong and Lim (1980) at an Ordinary Meeting of the Royal Statistical Society meeting. The threshold idea was thus conceived in 1977 and Tong put the idea into practice which meant a huge amount of computer experimentation. The first paper presented was the SETAR (Self-exciting threshold autoregressive) model. Then it became more general in the further researches. Tasy (1989) carried on suggesting a simple yet widely applicable model-building procedure for threshold autoregressive models and a test for threshold nonlinearity. Then LeBaron (1992) demonstrated that different levels of volatility can be regarded as the regime-determining process. One year later, Kräger and Kugler (1993) argued that exchange rates might show regime-switching behavior and found that the significant threshold effects, estimated by SETAR models, affected the exchange rates for five currency exchange rates. Till the year of 1998, more econometricians put their attention on the ergodicity/stationarity problem. De Gooijer (1998) considered regime-switching to the MA model and used validation criteria on SETAR model selection. Clements and Smith (2001) evaluated forecasts from SETAR models of exchange rates and compared them with traditional random walk measures. Hansen (2001) used Chow test in testing unknown structural change timing. Boero and Marrocu (2002) showed clear gains from the SETAR model over the linear competitor, on MSFEs evaluation of point forecasts, in sub-samples characterized by stronger non-linear models. Boero (2003) studied the out-of-sample forecast performance of SETAR models in Euro effective exchange rate. The SETAR models have been specified with two and three regimes, and their performance has been assessed against that of a simple linear AR model and a GARCH model. Kapetanios and Yongcheol (2006) distinguished a unit root process from a globally stationary three-regime SETAR process.

An ARIMA model can be considered as a special type of regression model-in which the dependent variable has been stationarized and the independent variables are all lags of the dependent variable and/or lags of the errors. In this study, we quote Box-Jenkins approach to modeling ARIMA processes which was announced by Box and Jenkins in 1970. An ARIMA process is a mathematical model used for forecasting. Box-Jenkins modelling involves identifying an appropriate ARIMA process, fitting it to the data, and then using the fitted model for forecasting. One of the attractive features of the Box-Jenkins approach to forecasting is that ARIMA processes are a very rich class of possible models and it is usually possible to find a process which provides an adequate description to the data. All these years, ARIMA forecasting models for economic variables were broadly developed, estimated, and then used for ex-post and ex-ante forecasts.

3. Theoretical framework

3.1 Stationary SETAR Models

The SETAR model is a convenient way to specify a TAR model because q_t is defined simply as the dependent variable y_t. In this case, the process can be formally written as

$$y_t = \begin{cases} \phi_{0,1} + \phi_{1,1} y_{t-1} + \phi_{2,1} y_{t-2} + +\phi_{p1,1} y_{t-p1} + \varepsilon_t & if \ \text{y}_{t-1} \leq c \\ \phi_{0,2} + \phi_{1,2} y_{t-1} + \phi_{2,2} y_{t-2} + +\phi_{p2,1} y_{t-p2} + \varepsilon_t & if \ \text{y}_{t-1} > c \end{cases}$$

3.2 Smooth transition regression models

Smooth Transition Regression models are a set of nonlinear models that incorporates both the deterministic changes in parameters over time and the regime switching behaviour within the time series data (van Dijk, et.al., 2002). The general STR model for a time series $X_t \quad t = 1, 2, 3, \cdots n$ is:

$$X_t = \left(\alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-i} \right) + \left(\beta_0 + \sum_{i=1}^{p} \beta_i X_{t-i} \right) G\left(y_{t-d}, \gamma, c \right) + \varepsilon_t$$

Where $G\left(y_{t-d}, \gamma, c \right)$ is the transition function with y_{t-d} as the transition variable which determines the switching point, d is the decay parameter γ is the smoothing parameter that determines the smoothness of the transition variable, c is the threshold parameter, $\alpha_0, \alpha_1, \cdots, \alpha_p$ and $\beta_0, \beta_1, \cdots, \beta_p$ are the parameters of the two autoregressive components of the model with optimal lag length p, and ε_t is an error term. The two most popular transition functions are the logistic smooth and exponential functions given, respectively, by

Logistic Function $G\left(y_{t-d}, \gamma, c \right) = \dfrac{1}{1 + \exp\left\{ -\left(y_{t-d} - c \right) \right\}}, \gamma > 0$

Exponential Function $G\left(y_{t-d}, \gamma, c \right) = \dfrac{1}{1 + \exp\left\{ -\left(y_{t-d} - c \right)^2 \right\}}, \gamma > 0$

3.3 ARIMA Models

Consider a standard ARIMA(p,q) model

$$X_t = \phi_0 + \sum_{i=1}^{p} \phi_1 X_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_1 X_{t-j}$$

where p denotes the order of Autoregressive (AR) model and q denotes the order of Moving Average (MA) model. $ARIMA(p,d,q)$ model is a generalization of $ARIMA(p,q)$ to handle non-stationary time series. A common approach to control the non-stationarity is the use of differencing $\Delta' t = \Delta_t - \Delta_{t-1}$ and the parameter d in $ARIMA(p,d,q)$ stands for its degree of integration. $ARIMA$ is a unit-root non-stationary model with strong memory.

3.4 Order determination

Akaike Information Criterion (AIC) is used in the determination of the order (Akaike, 1974). The same applies to Final Prediction Error (F.P.E.) (Parzen (1974). For the k^{th} autoregressive model, the FPE criterion is given by

$$FPE_{(k)} = \sigma_k^2 [1 - k / N]$$

where σ_k^2 is the unbiased estimator of ⑨^2 using the kth order model, that is

$$\sigma_k^k = RSS_k / (N - K)$$

Similarly, for a p^{th} order model,

$$AIC_{(p)} = N \ln \sigma_p^2 + 2p$$

$$\sigma_p^2 = \frac{1}{N} \sum_{i=1}^{N} (x_t - \sum_{i=1}^{p} \phi_i x_{t-i})^2$$

is the maximum likelihood estimate of the residual variance after fitting the AR(p). In practice, we specify a maximum lag L and fit successively AIC(1), AIC(2), …. The most accurate model for the data will have the minimum FPE or AIC. AIC is known to perform better than FPE.

3.5 Test for linearity

A zero-mean stationary stochastic process $\left(X_t\right)$ is said to be generated by an autoregressive model of order k, denoted by AR(k), if it satisfies the difference equation

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + e_t$$

where $\{e_t\}$ is a white noise process with variance σ^2. Here, $\{e_t\}$ will be assumed to be a Gaussian process.

Suppose, in a multiple linear regression, the response variable is given by Y and there is a set of explanatory variables, say $\{X_1, X_2, ..., X_k\}$. The full linear regression model is, given a set of derivations on $\{Y, X_1, X_2, ..., X_k\}$

$$Y_i = a_1 X_{1i} + a_2 X_{2i} + ... + a_k X_{ki} + e_i, i = 1,2,..., N$$

where $\{e_t\}$ are usually assumed to be independently distributed as $N(0, \sigma^2)$. The problem is to search for that subset of explanatory variables which best explains the variation in Y.

3.6 Forecast Equations used

$\hat{y}_{t+\tau}(t)$ is a point forecast of the series at time $t+\tau$ given the series has been observed from 1 to t Statistically speaking,

$$\hat{y}_{t+\tau}(t) = E(y_{t+\tau} \mid y_1, y_2, .., y_t)$$

Since ARMA models build upon the series $\{a_t\}$, the properties of $\{a_t\}$ needs to be revisited. In particular, $a_1, a_2, a_3, ...$ are independent and that future values of $a's$ are independent of the present and the past values of $y's$, i.e., a_{t+1} is independent of $y_t, y_{t-1}, ...$

One-step forecast

First, we have $y_{t+1} = y_t + a_{t+1} - \theta_1 a_t$

$$\hat{y}_{t+1}(t) = E(y_{t+1} \mid y_1, y_2, ..., y_t) = E(y_t + a_{t+1} - \theta_1 a_t \mid y_1, ..., y_t) = y_t + 0 - \hat{\theta}_1 \hat{a}_t = y_t - \hat{\theta}_1 \hat{a}_t$$

Two-step forecast

$$y_{t+2} = y_{t+1} + a_{t+2} - \theta_1 a_{t+1} \Longrightarrow \hat{y}_{t+2} = \hat{y}_{t+1}(t) + E(a_{t+2}) - \hat{\theta}_1 E(a_{t+1}) = \hat{y}_{t+1}(t)$$

4. Methodology/ Data analysis

4.1 Descriptive statistics

Table 1: Summary Statistics of Nigeria gross domestic product

Mean	Median	Std. dev.	Skewness	Kurtosis	Jarque-Bera	Observation
3.2447	9.1500	2.4693	0.4843	1.8123	43.4500 (0.0000)	300

4.2 Unit Root Tests

Augmented Dickey fuller test was used for two-unit root tests. At level the series was not stationary, but at first difference the series appeared to be stationary, thereby paving way for the estimation of parameters of the models involved. The results of unit root tests are displayed in Tables 2 and 3 below.

Table 2: Unit root test at level.

Null Hypothesis: GDP has a unit root				
Exogenous: Constant				
Lag Length: 0 (Automatic - based on SIC, maxlag=17)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			0.201297	0.9725
Test critical values:	1% level		-3.444890	
	5% level		-2.867845	
	10% level		-2.570192	
*MacKinnon (1996) one-sided p-values.				

Table 3: Unit root test at first difference.

Null Hypothesis: D(STOCK) has a unit root				
Exogenous: Constant				
Lag Length: 0 (Automatic - based on SIC, maxlag=17)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-21.66831	0.0000
Test critical values:	1% level		-3.444923	
	5% level		-2.867859	
	10% level		-2.570200	
*MacKinnon (1996) one-sided p-values				

4.3 Asymmetric unit root and linearity tests

Table 4: Asymmetric unit root and linearity tests

	TAR	SETAR
P_1	–0.692 (2,873)	–0.633 (–2.635)
P_2	–0.812 (–4.895)	–0.834 (–5.041)
P	1	1
ϕ	13.455 (0.0000)	13.730 (0.0000)
$P_1 = P_2$	0.221 (0.639)	0.611 (0.437)
$\Phi(\bullet)$	9.233 (0.321)	9.254 (0.321)
AIC	–222.688	–223.074

P_1 and P_2 are coefficient of first lag values of each regime and in parenthesis are t-statistics. P shows the number of lags in a model, ϕ represents F-statistics for the null hypothesis $H_0 : P_1 = P_2 = 0$.
The values related to $P_1 = P_2$ show the F-statistics for the null hypothesis $H_0 : P_1 = P_2 = 0$. $\Phi(\bullet)$ is the Ljung Box Q-statistics and the bracket values are P values of corresponding test values. AIC is calculated as $T^* \log(SSR) + 2^* n$
As observed from the above Table, all of the series reject the null hypothesis of a unit root at 5% significance level for both TAR and SETAR models. This means that all of the variables are stationary.
Below we present the estimated linear AR model for gross domestic product

$$Z_t = \underset{(0.041)}{0.334} - \underset{(0.009)}{0.166} Z_{t-1} - \underset{(0.009)}{0.117} Z_{t-2} + \varepsilon_t$$

JB = 38.908 (0.000)	$SSR = 1273.220$	$\mu_{resid} = 0.000$
Q(12) = 3.984 (0.985)	AIC = 1514.503	Kurtosis = 2.038
ARCH(12) = 1.268 (0.240)	$\sigma^2_{resid} = 6.062$	Skewness= -0.260

From the p values, all coefficients are significant. Even though, second lagged growth of industrial production index is not significant at 5% significance level but significant at 10% significance level, we include it according to AIC.
For the nonlinear SETAR model, threshold variable is selected from two candidate lagged variables. By using Chan (1993) methodology, optimum threshold variable is determined as ΔZ_{t-1} and optimum threshold value is found -0.359.

The estimated non-linear SETAR model for gross domestic product is given below.

$$Z_t = I_t\left(\underset{(0.692)}{0.104} - \underset{(0.034)}{0.226}Z_{t-1}\right) + \left(I - I_t\right)\left(\underset{(0.180)}{0.368} - \underset{(0.052)}{0.353}Z_{t-1}\right)$$

$$I_t = \begin{cases} 1 & \text{if} \quad \Delta Z_{t-1} \geq -0.359 \\ 0 & \text{if} \quad \text{otherwise} \end{cases}$$

JB = 26.603 (0.000) $SSR = 1204.440$ $\mu_{resid} = 0.000$

Q(12) = 3.629 (0.989) AIC = 1504.785 Kurtosis = 1.668

ARCH(12) = 3.629 (0.989) $\sigma^2_{resid} = 5.735$ Skewness = -0.246

Compared to the linear AR model, SSR, AIC and variance of residuals support the nonlinear M-TAR model. This means that the estimated nonlinear model describes the GDP index better that the linear model.

4.4 Out of Samples Forecast Performance of SETAR Models using Nigerian GDP Data

Table 5: SETAR models using Nigerian GDP data

Date	Actual	Lower Forecast limit	Upper Forecast limit
2006	18,564.59	17,231.03	18,423.98
2007	20,657.32	18,503.5	22.190.3
2008	24,296.33	19503.5	24,490.3
2009	24,794.24	19903.9	25,349.7
2010	54,612.26	49,503.5	55,490.5
2011	62,980.40	59,875.1	73,490.7
2012	71,713.94	70,607.5	73,493.2
2013	80,092.56	79,781.2	81,234.3
2014	89,043.62	88,356.0	91,590.7
2015	94,144.96	92.621.3	96,123.1
2016	97,253.02	94,287.9	99,651.7
2017	101,542.01	99,761.7	103,201.9

Table 5 shown above indicates that at both lower and upper levels, SETAR model predict the data very well.

5. Conclusion

Nonlinear models can capture asymmetries more properly by analyzing series regime by regime. Therefore, nonlinear approaches become more preferable and popular in empirical investigations. Furthermore, studies indicating asymmetric structures of many macroeconomic variables in response to downturns and recoveries in a business cycle have also strengthened the popularity of nonlinear approaches. Furthermore, intensive investigations on nonlinear dynamics also trigger the development of asymmetric unit root tests. Various studies prove the fact that traditional unit root tests are inadequate to investigate stationarity for asymmetric dynamics. They only consider the linear stationarity case. Moreover, their powers decrease substantially for asymmetric structures. Therefore, development of a new unit root test which considers the existence of nonlinear stationarity becomes essential for asymmetric dynamics. Hence, developments of asymmetric unit root tests have gained considerable velocity as the nonlinear models become popular in econometric literature.

In the scope of this study, we analyzed Nigeria GDP data to mimic the usual approach. First conventional unit root tests are performed, and then asymmetric unit root test proposed by Enders and Granger is applied in order to test for stationarity. Traditional and asymmetric unit root tests support consistent stationarity results for growth of GDP. We estimate SETAR models since their dynamics show SETAR type of adjustment according to the symmetry tests. We also constructed linear AR models for these variables in order to find the best fitted model. Nonlinear models exhibit better performance than the linear ones for both variables according to Akaike information criteria, sum of squared residuals and variance of residuals of estimated nonlinear models. In other words, nonlinear approaches represent the dynamics of both variables more properly than the linear ones.

Lastly the use of SETAR model is recommended for would be forecasters as its forecast ability at both lower and upper levels is fantastic.

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