Communications in Mathematical Finance, vol.9, no.1, 2020, 1-12
ISSN: 2241-1968 (print), 2241-195X (online)
Scientific Press International Limited, 2020

# Pricing Lookback Option Using Multinomial Lattice

**Edouard Singirankabo**[1], **Philip Ngare**[2] **and Carolyne Ogutu**[3]

**Abstract**

Lookback options is one of the most famous path-dependent exotic options in financial market, whose payoffs depend on the extremum of an underlying asset during the contract life time. Many works have been conducted in pricing Lookback option using continuous time model which is not better compared to numerical methods in pricing path dependent options in discrete situation. This paper tagets to contribute the concept of option pricing with multinomial lattice in pricing Lookback options. Quadrinomial lattice is constructed using moment matching technique. The results obtained in pricing floating lookback option are compared to well known Black-Scholes model.

**Keywords:** Lookback option; Quadrinomial lattice; Black-Scholes; Moment-matching; Relative entropy

[1] Department of Mathematics, Pan African University, Institute of Basic Sciences, Technology, and Innovation, Kenya. E-mail: nkabo12edus@gmail.com
[2] School of Mathematics, University of Nairobi. E-mail: pngare@uonbi.ac.ke
[3] School of Mathematics, University of Nairobi.
E-mail: cogutu@uonbi.ac.ke

Article Info: *Received* : September 10, 2019. *Revised* : October 2, 2019.
*Published online* : January 5, 2020.

# 1 Introduction

Lookback options' holder can make maximum profit which comes in the form of buying at a cheapest rate and selling at the highest rate. Lookback options help investors to minimize regrets and provide them with essential information on stock's behaviour over time. Lookback options have two types: Floating Lookback option and Fixed lookback option. One has strike price which is floating and determined at maturity while another has a strike price which is fixed at the purchase. Floating strike lookback option provides the right not obligation to the holder to buy or sell an asset at its lowest (highest) price during option lifetime. The observed price is used as the strike price. Fixed lookback option is an option in which its strike price is known in advance. Lookback option has been introduced by Goldman [3] since that time no works have been done using multinomial lattice in pricing Lookback option. Lookback options are one of the Exotic options which have been introduced because standard models used in continuous time can not do conveniently the same work for path dependent options. For this reason, numerical methods were created including lattice.

In 1979 Cox, Ross and Rubinstein introduced two-state lattice approach which was easy and powerful tool to determine initial option price [2]. In 1986, Boyle introduced a trinomial tree model as modification of CRR model in case of single variable. Boyle [1] extended the work to two underlying state variables. His model based on moment matching technique. In 2014, Hu Xiaoping [7] studied a trinomial markov tree model where the stock price were modeled by the first order markov progress. Their results show that trinomial tree is very fast and very easy to be implemented compared to binomial markov tree. Kenneth Kiprotich Langat et al [4] studied trinomial lattice method. They concidered the concept of random walk as the path followed by the underlying stock price, obtained results are compared to Binomial and Black-Scholes model. Carolyne Ogutu at al [5] priced Asian options using Moment-matching on multinomial lattice. In that study, underlying asset follows Merton-Bates Jump-diffusion model. They constructed lattice using moment-matching technique where $\alpha$ was considered as distance in between going up and down of underlying asset. This paper provide a concept of pricing floating lookback option in case where there is more than one martingale measure and it proceeds as follow: section two is preliminary notes, section three is main results, section four is conclusion

# 2 Preliminary Notes

## 2.1 Moment matching on Quadrinomial lattice

Let $T \in (0, +\infty)$, $t \in [0, T]$ and $(\Omega, F)$ and$(\Omega_1, F_1)$ be two measurable space. A collection of Random variable $S(.) = S(t)_{t \in [s,T]}$ such that $S(t) = \Omega \longrightarrow \Omega_1$ is called a stochastic process.

Consider an underlying stock $S_t$ to be a stochastic random variable with $S_t = S_{t-1}Z$ where $Z$ is a discrete random variable defined as follows

$$Z = \begin{cases} \omega_1 & \text{with probability} \quad p_1 \\ \omega_2 & \text{with probability} \quad p_2 \\ \omega_3 & \text{with probability} \quad p_3 \\ \omega_4 & \text{with probability} \quad p_4 \end{cases} \tag{1}$$

Such that $\omega_1 \neq \omega_2 \neq \omega_3 \neq \omega_4$ and $\omega_1 > \omega_2 > \omega_3 > \omega_4$. Matching the moments of a random variable $X$ with a discrete random variable $D$ where $E[X] = m_1$ gives

$$D_t = m_1 + S_t \qquad \text{where} \quad t \in (0, T] \tag{2}$$

Consider $t = 1$ then $S_1 = S_0 Z$ therefore, equation (2) becomes $D_1 = m_1 + S_1$. Applying moment-matching technique yields

$$\begin{cases} E(S_1^0) = p_1 + p_2 + p_3 + p_4 = \mu_0 \\ E(S_1) = E[S_0 Z] = S_0\omega_1 p_1 + S_0\omega_2 p_2 + S_0\omega_3 p_3 + S_0\omega_4 p_4 = \mu_1 \\ E(S_1^2) = E[(S_0 Z)^2] = S_0^2\omega_1^2 p_1 + S_0^2\omega_2^2 p_2 + +S_0^2\omega_3^2 p_3 + S_0^2\omega_3^4 p_4 = \mu_2 \\ E(S_1^3) = E[(S_0 Z)^2] = S_0^3\omega_1^3 p_1 + S_0^3\omega_2^3 p_2 + S_0^3\omega_3^3 p_3 + S_0^3\omega_4^3 p_4 = \mu_3 \end{cases}$$

To solve the above linear system of equations some methods have been proposed but the easiest one is the construction of a Vandermonde matrix [8].

Therefore,

$$\begin{pmatrix}\mu_0\\ \mu_1\\ \mu_2\\ \mu_3\end{pmatrix} = \begin{pmatrix}1 & 1 & 1 & 1\\ S_0\omega_1 & S_0\omega_2 & S_0\omega_3 & S_0\omega_4\\ (S_0\omega_1)^2 & (S_0\omega_2)^2 & (S_0\omega_3)^2 & (S_0\omega_4)^2\\ (S_0\omega_1)^3 & (S_0\omega_2)^3 & (S_0\omega_3)^3 & (S_0\omega_4)^3\end{pmatrix}\begin{pmatrix}p_1\\ p_2\\ p_3\\ p_4\end{pmatrix} \tag{3}$$

Since the elements of the above Vandermonde matrix are distinct, the inverse exist.

**Theorem 2.1.** *For a Vandermonde matrix $V_4$ with elements defined in (3) then, elements of inverse are given by*

$$(V_4^{-1})_{ij} = \frac{(-1)^{j-i}\sigma_{4-j,i}}{\prod_{k=1,k\neq i}^{4} S_0(\omega_k - \omega_i)}$$

*where*

$$\sigma_{j,i} = \sum_{1\leq m_1<m_2<\ldots<m_j\leq N}\prod_{n=1}^{j} S_0\omega_i(1-\delta_{m_n,i}), \delta_{a,b} = \begin{cases}1, & a=b,\\ 0, & a\neq b.\end{cases}$$

Matching the lattice to the first three moments yields

$$\overrightarrow{p} = V_4^{-1}\overrightarrow{\mu}$$

with $\overrightarrow{p}$ and $\overrightarrow{\mu}$ are vectors contain jump probabilities and moments respectively.

$$p_i = \sum_{j=1}^{4}(V^{-1})_{ij}\mu_{j-1} = \sum_{j=1}^{4}\frac{(-1)^{j-i}\sigma_{4-j,i}}{\prod_{k=1,k\neq i}^{4} S_0(\omega_k-\omega_i)}\mu_{j-1} \tag{4}$$

Replacing $i$ and $j$ in (4) we can get $p_1$, $p_2$, $p_3$ and $p_4$ such that the adjacent matrix in Theorem 2.1 is determined by equating the inverse of Vandermonde matrix obtained by writting the above probabilities in matrix form with the inverse of Vandermonde matrix mentioned in (3)

$$\sigma_{1,1} = \frac{[\omega_4(\omega_3+\omega_4) - \omega_2(\omega_2+\omega_3)]S_0}{\omega_2-\omega_4} \qquad \sigma_{1,2} = \frac{[(\omega_3+\omega_4)\omega_4 - (\omega_1+\omega_3)\omega_1]}{(\omega_4-\omega_1)}$$

$$\sigma_{1,3} = \frac{[-(\omega_2+\omega_4)\omega_4 + (\omega_1+\omega_2)\omega_1]}{(\omega_4-\omega_1)} \qquad \sigma_{1,4} = \frac{[-(\omega_2+\omega_3)\omega_3 + (\omega_1+\omega_2)\omega_1]}{(\omega_1-\omega_3)}$$

$$\sigma_{2,1} = \frac{[(\omega_2+\omega_3)\omega_4^2 - (\omega_3+\omega_4)\omega_2^2]S_0^2}{\omega_2-\omega_4} \qquad \sigma_{2,2} = \frac{[(\omega_1+\omega_3)\omega_4^2 - (\omega_3+\omega_4)\omega_1^2]S_0^2}{\omega_4-\omega_1}$$

$$\sigma_{2,3}=\frac{[-(\omega_1+\omega_2)\omega_4^2+(\omega_2+\omega_4)\omega_1^2]S_0^2}{(\omega_4-\omega_1)}\quad \sigma_{2,4}=\frac{[-(\omega_1+\omega_2)\omega_3^2+(\omega_2+\omega_3)\omega_1^2]S_0}{(\omega_1-\omega_3)}$$

$$\sigma_{3,1}=-\omega_2\omega_3\omega_4S_0^3 \qquad \sigma_{3,2}=\omega_1\omega_3\omega_4S_0^3$$

$$\sigma_{3,3}=\frac{\omega_1\omega_2\omega_4S_0^3(\omega_1-\omega_3)(\omega_2-\omega_3)}{(\omega_2-\omega_4)(\omega_4-\omega_1)}\quad \sigma_{3,4}=\omega_1\omega_2\omega_3S_0^3$$

$$\sigma_{0,1}=1 \qquad \sigma_{0,2}=1$$

$$\sigma_{0,3}=-1 \qquad \sigma_{0,4}=1$$

By replacing sigma by its values, the probabilities become

$$p_1=\frac{-\omega_2\omega_3\omega_4\mu_0}{(\omega_2-\omega_1)(\omega_3-\omega_1)(\omega_4-\omega_1)}-\frac{[(\omega_2+\omega_3)\omega_4^2-(\omega_3+\omega_4)\omega_2^2]\mu_1}{S_0(\omega_2-\omega_4)(\omega_2-\omega_1)(\omega_3-\omega_1)(\omega_4-\omega_1)}$$
$$+\frac{[-\omega_2(\omega_2+\omega_3)+\omega_4(\omega_3+\omega_4)]\mu_2}{S_0^2(\omega_2-\omega_4)(\omega_2-\omega_1)(\omega_3-\omega_1)(\omega_4-\omega_1)}+\frac{\mu_3}{S_0^3(\omega_2-\omega_1)(\omega_3-\omega_1)(\omega_4-\omega_1)}$$

$$p_2=\frac{-\omega_1\omega_3\omega_4\mu_0}{(\omega_1-\omega_2)(\omega_3-\omega_2)(\omega_4-\omega_2)}+\frac{[(\omega_1+\omega_3)\omega_4^2-(\omega_3+\omega_4)\omega_1^2]\mu_1}{S_0(\omega_4-\omega_1)(\omega_1-\omega_2)(\omega_3-\omega_2)(\omega_4-\omega_2)}$$
$$-\frac{[\omega_4(\omega_3+\omega_4)-\omega_1(\omega_1+\omega_3)]\mu_2}{S_0^2(\omega_4-\omega_1)(\omega_1-\omega_2)(\omega_3-\omega_2)(\omega_4-\omega_2)}+\frac{\mu_3}{S_0^3(\omega_1-\omega_2)(\omega_3-\omega_2)(\omega_4-\omega_2)}$$

$$p_3=\frac{\omega_1\omega_2\omega_4\mu_0}{(\omega_2-\omega_4)(\omega_4-\omega_1)(\omega_4-\omega_3)}-\frac{[-(\omega_1+\omega_2)\omega_4^2+(\omega_2+\omega_4)\omega_1^2]\mu_1}{S_0(\omega_4-\omega_1)(\omega_1-\omega_3)(\omega_2-\omega_3)(\omega_4-\omega_3)}$$
$$+\frac{[-\omega_4(\omega_2+\omega_4)+\omega_1(\omega_1+\omega_2)]\mu_2}{S_0^2(\omega_4-\omega_1)(\omega_1-\omega_3)(\omega_2-\omega_3)(\omega_4-\omega_3)}+\frac{\mu_3}{S_0^3(\omega_1-\omega_3)(\omega_2-\omega_3)(\omega_4-\omega_3)}$$

$$p_4=\frac{-\omega_1\omega_2\omega_3\mu_0}{(\omega_1-\omega_4)(\omega_2-\omega_4)(\omega_3-\omega_4)}+\frac{[-(\omega_1+\omega_2)\omega_3^2+(\omega_2+\omega_3)\omega_1^2]\mu_1}{S_0(\omega_1-\omega_3)(\omega_1-\omega_4)(\omega_2-\omega_4)(\omega_3-\omega_4)}$$
$$-\frac{[-\omega_3(\omega_2+\omega_3)+\omega_1(\omega_1+\omega_2)]\mu_2}{S_0^2(\omega_1-\omega_3)(\omega_1-\omega_4)(\omega_2-\omega_4)(\omega_3-\omega_4)}+\frac{\mu_3}{S_0^3(\omega_1-\omega_4)(\omega_2-\omega_4)(\omega_3-\omega_4)}$$

One can show that $p_1+p_2+p_3+p_4=1$ by putting together similar terms.

## 2.2 Assuption to have positive probablitities

1. $\omega_4<1+r<\omega_1$
2. $S_0\omega_4<\mu_1<S_0\omega_1$

3. $\frac{\mu_1 S_0(\omega_1+\omega_2+2\omega_3)-S_0^2\mu_0\omega_2(\omega_1+\omega_3)}{2} < \mu_2 < S_0(\omega_1+\omega_3)\mu_1 - \omega_1\omega_3 S_0^2\mu_0$

4. $\frac{-\omega_1\omega_2\omega_4 S_0^3(\omega_1-\omega_3)^2(\omega_2-\omega_3)\mu_0-[-(\omega_1+\omega_2)\omega_4^2+(\omega_2+\omega_4)\omega_1^2](\omega_2-\omega_4)(\omega_1-\omega_3)S_0^2\mu_1}{2(\omega_1-\omega_3)(\omega_2-\omega_4)(\omega_4-\omega_1)}$
$- \frac{[-\omega_4(\omega_2-\omega_4)+\omega_1(\omega_1+\omega_2)](\omega_2-\omega_4)(\omega_1-\omega_3)S_0\mu_2-\omega_1\omega_2\omega_3 S_0^3(\omega_1-\omega_3)(\omega_2-\omega_4)(\omega_4-\omega_1)\mu_0}{2(\omega_1-\omega_3)(\omega_2-\omega_4)(\omega_4-\omega_1)}$
$- \frac{[-(\omega_1+\omega_2)\omega_3^2+(\omega_2+\omega_3)\omega_1^2](\omega_2-\omega_4)(\omega_4-\omega_1)S_0\mu_1-[-\omega_3(\omega_2+\omega_3)+\omega_1(\omega_1+\omega_2)](\omega_2-\omega_4)(\omega_4-\omega_1)S_0\mu_2}{2(\omega_1-\omega_3)(\omega_2-\omega_4)(\omega_4-\omega_1)}$
$< \mu_3 <$
$\frac{\omega_2\omega_3\omega_4 S_0^3(\omega_2-\omega_4)(\omega_4-\omega_1)\mu_0+[(\omega_2+\omega_3)\omega_4^2-(\omega_3-\omega_4)\omega_2^2](\omega_4-\omega_1)S_0^2\mu_1}{2(\omega_2-\omega_4)(\omega_4-\omega_1)}$
$- \frac{[-\omega_2(\omega_2+\omega_3)+\omega_4(\omega_3+\omega_4)](\omega_4-\omega_1)S_0\mu_2+\omega_2\omega_3\omega_4 S_0^3(\omega_4-\omega_1)(\omega_2-\omega_4)\mu_0}{2(\omega_2-\omega_4)(\omega_4-\omega_1)}$
$+ \frac{[(\omega_1+\omega_3)\omega_4^2-(\omega_3+\omega_4)\omega_1^2](\omega_2-\omega_4)S_0^2\mu_1+[\omega_4(\omega_3-\omega_4)-\omega_1(\omega_1+\omega_3)](\omega_2-\omega_4)S_0\mu_2}{2(\omega_2-\omega_4)(\omega_4-\omega_1)}$

## 2.3 Relative entropy martingale measure

Since we are not allowed to price with these real world probabilities, we have to find an equivalent martingale measure of underlying asset $S_t$ which is a probability measure $Q$ defined on $(\Omega, F)$. To do so, one needs to define a relative entropy of $Q$ with respect to $P$ as follows

$$R(Q||P) = \sum_{i=1}^{N} q_i ln(\frac{q_i}{p_i}) \tag{5}$$

[6]. We impose a probability distribution $q$ on a set of stock prices $S_0\omega_1, ..., S_0\omega_4$ such that the following hold

$$\sum_{i=1}^{4} q_i = 1 \qquad \text{and} \qquad \sum_{i=1}^{4} q_i\omega_i = S_0$$

By defining a set of equivalent martingale measure as

$$M_e = \left\{ q \in V : \sum_{i=1}^{4} q_i = 1, \sum_{i=1}^{4} q_i\omega_i = S_0, q > 0 \right\}$$

With $V : \Re^n \longrightarrow \Re$. One can demonstrate if the relative entropy is convex, then the problem is manipulated using Lagrange multipliers methods as follows

$$\begin{cases} L(q,\gamma_1,\gamma_2) = \sum_{i=1}^{N} q_i ln(\frac{q_i}{p_i}) + \gamma_1 B_1 + \gamma_2 B_2 \\ \text{S.t} \quad B_1 = \sum_{i=1}^{N} q_i - 1, B_2 = \sum_{i=1}^{N} q_i\omega_i - S_0 \end{cases}$$

This Lagrange equation is minimized with respect to $q$ such that the partial derivative $\frac{\partial L}{\partial q_i}$ equal to zero for all $i \in N$. Then yields,

$$ln(\frac{q_i}{p_i}) + 1 + \gamma_1 + \gamma_2 S_0 \omega_i = 0$$

By arranging

$$q_i = \frac{p_i exp(\eta S_0 \omega_i)}{\sum_{i=1}^{N} p_i exp(\eta S_0 \omega_i)} = \frac{p_i exp(\eta S_0 \omega_i)}{\mathbb{E}[exp(\eta S_0 \omega_i), p]} \tag{6}$$

One can define a function of $\eta$ as follows

$$f(\eta) = \frac{\omega_1 p_1 exp(\eta S_0 \omega_1) + \omega_2 p_2 exp(\eta S_0 \omega_2) + \omega_3 p_3 exp(\eta S_0 \omega_3) + \omega_4 p_4 exp(\eta S_0 \omega_4)}{p_1 exp(\eta S_0 \omega_1) + p_2 exp(\eta S_0 \omega_2) + p_3 exp(\eta S_0 \omega_3) + p_4 exp(\eta S_0 \omega_4)} \tag{7}$$

By studying limit of this function one can get

$\lim_{\eta \longrightarrow -\infty} f(\eta) = \omega_4$, $\lim_{\eta \longrightarrow +\infty} f(\eta) = \omega_1$

# 3 Main Results

## 3.1 Pricing Floating Lookback option

Let $y_0 = 1$, $\omega_1 = 2.2$, $\omega_2 = 1.5$, $\omega_3 = 0.8$, $\omega_4 = 0.1$, $\mu_0 = 1$ and $r = 0$ from assumption to have positive probabilities in quadrinomial, one can get $1 < \mu_1 < 2.2$, $3.315 < \mu_2 < 4.54$ and $9.645 < \mu_3 < 13.52$. Take $\mu_1 = 2.1$, $\mu_2 = 4.5$ and $\mu_3 = 9.73$ then probabilities become $p_1 = 0.8810$, $p_2 = 0.1020$, $p_3 = 0.0102$ and $p_4 = 0.0068$. Referring to Figure 1, we find $f(\eta^*) = y_0$ by trial and error. Let $\eta^* = -2.6$ then neutral probabilities in (6) should be $q_1 = 0.25$, $q_2 = 0.18$, $q_3 = 0.11$ and $q_4 = 0.46$ respectively.

Since neutral probabilities are available one can price floating Lookback option with the following payoffs

$$PLC = max\left(S_T - min(S_t), 0\right); PLP = max\left(max(S_t) - S_T, 0\right) \tag{8}$$

for Call and Put, respectively.

Referring to previous studies like [7],[4] one can go backward to determine initial option price with considering the following payoffs for Call and Put respectively

$$f_1 = max(S_0\omega_1 - min(S_0, S_0\omega_1), 0) \qquad f_1 = max(max(S_0, S_0\omega_1) - S_0\omega_1, 0)$$

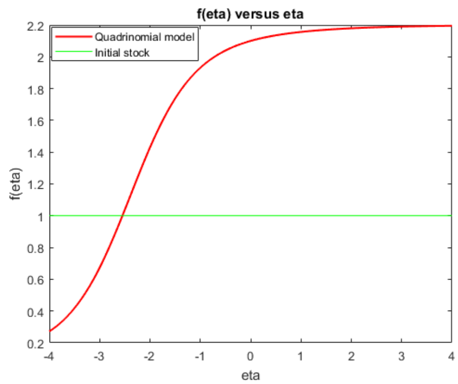


Figure 1: $f(\eta)$ versus $\eta$ in Quadrinomial

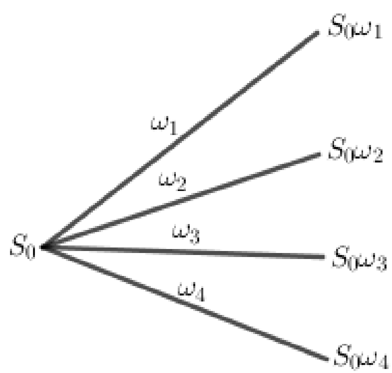


Figure 2: Quadrinomial lattice

$$f_2 = max(S_0\omega_2 - min(S_0, S_0\omega_2), 0) \qquad f_2 = max(max(S_0, S_0\omega_2) - S_0\omega_2, 0)$$

$$f_3 = max(S_0\omega_3 - min(S_0, S_0\omega_3), 0) \qquad f_3 = max(max(S_0, S_0\omega_3) - S_0\omega_3, 0)$$

$$f_4 = max(S_0\omega_4 - min(S_0, S_0\omega_4), 0) \qquad f_4 = max(max(S_0, S_0\omega_4) - S_0\omega_4, 0)$$

Then, the value of holding the option is

$$f = \frac{1}{1+r}(q_1 f_1 + q_2 f_2 + q_3 f_3 + q_4 f_4)$$

Therefore, one can determine multiperiod quadrinomial lattice as follows

**Example 3.1.** *Let* $S_0 = 1$, $r = 0.175$, $K = 2$, $T = 1yr$, $\omega_1 = 2.2$, $\omega_2 = 1.5$, $\omega_3 = 0.8$ *and* $\omega_4 = 0.1$. *We divided* $1year$ *into four period of three months say* $T = \frac{3}{12}$, $T = \frac{6}{12}$, $T = \frac{9}{12}$ *and* $T = 1yr$. *By computing initial option price at each period in Quadrinomial lattice for Call and Put respectively yield*

Table 1: Multiperiod Quadrinomial lattice for Call and Put options

| | $T=\frac{3}{12}$ | $T=\frac{6}{12}$ | $T=\frac{9}{12}$ | $T=1yr$ |
|---|---|---|---|---|
| Call | 0.39 | 0.565 | 0.642 | 0.667 |
| Put | 0.436 | 0.825 | 1.178 | 1.495 |

We compare this results to numerical results obtained from Black-Scholes model.

## 3.2 Black-Scholes model

$$u=\omega_1=e^{\sigma\sqrt{T}}$$

The Black Scholes formula for Call and Put options are given by

$$C=SN(d_1)-Ke^{-r(T-t)}N(d_2)$$

$$P=Ke^{-r(T-t)}N(-d_2)-SN(-d_1)$$

Where

$$d_1=\frac{ln(\frac{S}{K})+(r+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{(T-t)}} \quad , \quad d_2=\frac{ln(\frac{S}{K})+(r-\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{(T-t)}}=d_1-\sigma\sqrt{(T-t)}$$

**Example 3.2.** *Consider $\omega_1=2$, $y_0=1.2$, $r=0.175$, $K=1$ and $T=1yr$. One can determine volatility as time changes. Let discretisize maturity time into four times $T=\frac{3}{12}$, $T=\frac{6}{12}$, $T=\frac{9}{12}$ and $T=1$.*

Therefore, by determining the initial option price for Call and Put options using Black-Scholes model yields

Table 2: Option prices determined using Black Scholes model

| | $T=\frac{3}{12}$ | $T=\frac{6}{12}$ | $T=\frac{9}{12}$ | $T=1yr$ |
|---|---|---|---|---|
| Call | 0.49 | 0.51 | 0.55 | 0.68 |
| Put | 0.13 | 0.184 | 0.27 | 0.44 |

## 3.3 Results and interpretation

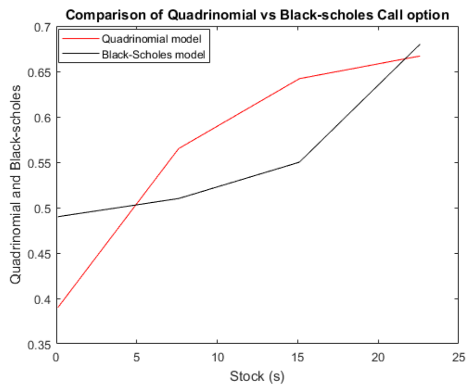


Figure 3: Quadrinomial Versus Black-Scholes model in Call option

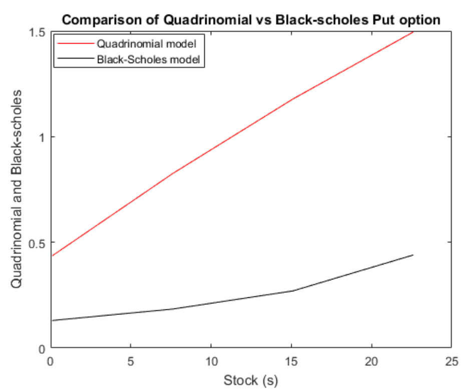


Figure 4: Quadrinomial Versus Black-Scholes model in Put option

Figure 3-4, show that Quadrinomial lattice is doing well compared to Black-Scholes model. Investors always wish to make a benefit at the end of the day, their dreams become true when Put option prices are greater than Call option prices. This paper demonstrated that in pricing Lookback option as one of Exotic options, it is better to use discrete time model like lattice than continous model. From the results we noticed that with quadrinomial model, investors are able to make a significant benefit while using Black-Scholes model investors are not allowed to exercise option within $T = 1yr$.

# 4 Conclusion

This paper develops a quadrinomial lattice approach in pricing floating Lookback Option. Moment-matching technique was introduced on lattice and obtained a system of linear equations in which easiest way to solve the system is to use Vandermonde matrix [8]. For further improvement of this study, one can construct moment matching technique and define discrete random variable $D$ with time period $t$ greater than one.

**ACKNOWLEDGEMENTS** The first author wishes to address thanks to the African Union together with the Pan African University, Instutute for Basic Sciences Technology and Innovation for supporting this research

# References

[1] P. P. Boyle, A lattice framework for option pricing with two state variables, *Journal of Financial and Quantitative Analysis*, **23**(1), (1988), 1-12.

[2] J. C. Cox, S. A. Ross, and M. Rubinstein. Option pricing: A simplified approach, *Journal of financial Economics*, **7**(3), (1979), 229-263.

[3] M.B. Goldman, H.B. Sosin, and M.A. Gatto, Path dependent options: buy at the low, sell at the high, *Journal of Finance*, **34**(5), (1979), 1111-1127.

[4] K.K. Langat, J.I. Mwaniki, and G.K. Kiprop, Pricing options using trinomial lattice method, *Journal of Finance and Economics*, **7**(3), (2019), 81-87.

[5] C. Ogutu, K. Lundengard, S. Silvestrov and P. Weke, Pricing asian options using moment matching on a multinomial lattice,*In AIP Conference Proceedings*, **1637**, (2014), 759-765.

[6] C.S. Ssebugenyi, I.J. Mwaniki and V.S. Konlack, On the minimal entropy martingale measure and multinomial lattices with cumulants,*Applied Mathematical Finance*, 20(4), (2013),359-379.

[7] H. Xiaoping, G. Jiafeng, D. Tao, C. Lihua and C. Jie, Pricing options based on trinomial markov tree, *Discrete Dynamics in Nature and Society*, 2014, 2014.

[8] B. Ycart, A case of mathematical eponymy: the vandermonde determinant, *arXiv preprint arXiv*, (2012), 1204.4716.