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The impact of covariate distribution characteristics on the power in logistic regression models: a simulation study

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Abstract

Continuous covariates in a logistic regression model have been often divided into categories to avoid a potential non-linearity, especially when covariates do not follow normal distributions. However, categorization may lead a considerable loss of power depending on the covariate distribution shape. Therefore, we investigate the impact of the covariate distribution characteristics on the power in logistic regression models when continuous covariates are converted to categorical covariates.

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We consider the uniform, bell-shaped, right-skewed, and left-skewed distributions and assume that the relationship between the original continuous covariate and the corresponding logit outcome is linear. Continuous covariates are categorized into quantiles (median, quartile, or quintiles). The statistical power and regression coefficients are estimated using simulations for continuous covariates and categorical covariates.

When continuous covariates were converted to categorical covariates, the power decreased for any covariate distribution shape. In particular, the increase in the number of categorized groups led to a decrease in power. However, the ranking order of powers among the four distributions were not changed owing to categorization.

Although the power decreases because of categorization, the impact of covariate distribution characteristics on the power in logistic regression models may not be changed by categorization.

Mathematics Subject Classification: 62E99

Keywords: Categorization; Logistic regression model; Power; Distribution shape; Wald test

1 Introduction

1.1 Background

Binary logistic regression models are commonly used to assess the association between outcomes and covariates in the field of clinical and epidemiological studies and represent a powerful class of tools to adjust for the effect of multiple confounding factors [1, 2]. Categorization has been frequently applied to a continuous covariate to avoid a potential non-linearity, particularly when the continuous covariates do not follow a normal (Gaussian) distribution among the individuals with events and also without events. It is more customary to group continuous covariates into quantiles—most often tertiles, quartiles, or quintiles [3 - 4]. However, categorization may lead to a considerable loss of power depending on the covariate distribution shape. Therefore, we aimed to clarify the impact of the covariate distribution characteristics on the power in logistic regression models when continuous covariates are changed to categorical covariates using quintiles. In this study, we considered four typical distributions: the uniform distribution, bell-shaped distribution, right-skewed distribution, and left-skewed distribution. We used some representative percentile-based categorizations by median, quartiles, and quintiles. The powers and regression coefficients were estimated using Monte Carlo simulations and were compared among the four distributions.

1.2 Methods

A Monte Carlo simulation was performed to assess the influence on the estimated coefficients, size and power when a continuous covariate was converted to a categorical covariate. We focused on logistic regression models with a single covariate and a dichotomous outcome.

1.2.1 Original logistic regression model for generating simulation data

The logistic regression model considered in this study is expressed as follows [5]:

$$\pi(x) = P(Y \mid x) = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}} \tag{1}$$

where Y is an outcome variable, x is an observational value of the continuous covariate of X, and β_h (h = 0,1) is an unknown parameter. The continuous covariate X is categorized into a categorical covariate with k categories using (k – 1) quantiles, and this is referred to as a design variable. Here, a reference group forms the lowest category. In this study, we set k = 1, 2, 4, and 6, where k = 1 means the original continuous covariate. The design variable is denoted as D_j, and the corresponding coefficient is denoted as β_j, j = 1, 2 ,…, (k – 1). The logistic regression model with the design variables can then be represented by the following formula [5]:

$$\pi(D_j) = P(Y \mid D_j) = \frac{e^{\beta_0 + \sum_{j=1}^{k-1} \beta_j D_j}}{1 + e^{\beta_0 + \sum_{j=1}^{k-1} \beta_j D_j}}, \quad D_j = 0,1 \tag{2}$$

The log odds, or logit, are defined in terms of $\pi(D_j)$ as follows:

$$\text{Logit}(\pi(D_j)) = \ln\left(\frac{\pi(D_j)}{1-\pi(D_j)}\right) = \beta_0 + \sum_{j=1}^{k-1} \beta_j D_j$$

1.2.2 Data generation

Table 1 summarizes input parameters, notations, and possible values of the simulation.

The continuous covariate X motivates from the number of cigarettes smoked per day, the length of hospital stay, and the amount of daily alcohol consumption. Values of X were created by the following procedure. We first assumed that X follows one of the four distributions shown in Figure 1 and categorized the interval (0, 60) into the three groups consisting of (0, 20), (20, 40), and (40, 60). Then, the numerical numbers with the frequencies n_1, n_2, and n_3 designated in Figure 1 were generated according to a uniform distribution in each group. Figure 1 illustrates the four distribution types.

For the uniform distribution, the same sample sizes were assigned to all groups, as shown in Figure 1-(a). For the centralized-shaped distribution (a bell-shaped distribution), the middle interval was assigned the largest sample size. The ratio of the sample sizes in each sub-group was set to 3:9:3, as shown in Figure 1-(b). For the declining distribution (a left-skewed distribution), the lowest interval was assigned the largest sample size, and the highest interval was assigned the smallest sample size. The ratio of sample sizes in each group was set to 10:4:1 from the lowest to the highest interval, as shown in Figure 1-(c). For the uprising distribution (a right-skewed distribution), the ratio of sample sizes in each group was inversely set to 1:4:10 from the lowest to the highest interval, as shown in Figure 1-(d).

Then, with y_i and x_i (i = 1,…, n) denoting independent observations, the binary outcome Y of individuals with X = x_i was generated using $\pi(x_i)$ and a random number from the **uniform** distribution on the interval (0, 1). If $\pi(x_i)$ was less than the corresponding random number, then $y_i = 1$, (the event occurred); otherwise, $y_i = 0$ (the event did not occur).

1.3 Simulation

The continuous covariate X that follows the uniform, centralized-shaped, declining, or uprising distribution was artificially divided into k categories (k = 2, 4, 6) using (k – 1) quantiles. Logistic regression analysis was performed for a model that includes a continuous covariate or a set of design variables. The Wald test was performed for the null hypothesis of $\beta = 0$. This process was repeated 10,000 times. The proportion of tests in which the p values were less than 0.05 is defined as the power when $\beta_1 = 0.025$; otherwise, it is defined as the size when $\beta_1 = 0.000$.

There is an incidence of no occurrences of convergence. If the data are completely or partially separated, convergence does not occur because one or more parameters in the model become theoretically infinite, and it may not be possible to obtain reliable maximum likelihood estimates [6]. The problem of non-convergence was solved by simply ignoring a sample that produced an occurrence of no convergence. All simulations were carried out using SAS software Ver.9.1.3.

2 Results

2.1 Size

The average values of the regression coefficient, standard error, and estimated size for various conditions are summarized in Table 2. We summarize the detailed results of the simulation for only N=300 in Table 2.

When k = 1, the sizes were nearly equal to 0.05 for all conditions, and their coefficients were also shown to have true values of 0.00. On the other hand, excessive categorization (k = 6) led to a slight decrease in size for any P, N and distribution shape of X. Particularly, the size is less than 0.02 for almost all distribution shapes when P = 0.9 or 0.1 and N = 300. There were no differences that depended on the distributions shape. Overall, no inflations of type I errors were observed.

2.2 Power

Simulations were performed to estimate the powers under a fixed regression coefficient β_1 = 0.025 with P = 0.1, 0.2, 0.5, 0.8, or 0.9, and N = 300, 600, or 900. The detailed results of these simulations are summarized in Tables 3-1, 3-2, and 3-3.

The average coefficient values were correctly estimated for all k, P, and N. The powers differed for each of the four distributions of X. When k = 1, Tables 3-1, 3-2, and 3-3 exhibited power characteristics relating the covariate distribution shapes and P. When P = 0.1 and 0.2, the order of the distributions from highest to lowest power was the uniform, declining, centralized-shaped, and uprising distributions. When P = 0.5, the order was the uniform, centralized-shaped, declining, and uprising distributions. When P = 0.8 and 0.9, the uniform distribution exhibited the highest power, followed by the uprising, centralized-shaped, and declining distributions, in that order. All four distributions exhibited an increased power from P = 0.1 to P = 0.5, but this then decreased from P = 0.5 to P = 0.9. More precisely, the powers of the uniform and centralized distributions exhibited the same changes in power, and they exhibited almost the same power for P and (1 – P). For example, the powers of the uniform distribution for P = 0.2 and P = 0.8 were 0.8456 and 0.8447, respectively, and the powers of the centralized-shaped distribution for P = 0.1 and P = 0.9 were 0.4351 and 0.4393, respectively, as listed in Table 3-1. The declining and uprising distributions exhibited almost an identical power for P = 0.5, and the power of the declining distribution for P and that of the uprising distribution for (1 – P) were almost the same. As a result, the ranks of the declining and uprising distributions were exchanged when P < 0.5 and P > 0.5. For example, the power of the declining distribution for P = 0.9 and the power of the uprising distribution for P = 0.1 were 0.3148 and 0.3193, respectively. The power of the declining distribution for P = 0.8 and the power of the uprising distribution for P = 0.2 were 0.5742 and 0.5665, respectively. These trends were also observed for k = 2, 4, and 6. The ranking order of powers among the four distributions were not changed owing to categorization.

3 Discussion

Our study has identified the power characteristics relating the covariate distribution shapes and event proportions. When the logit outcome and covariates are linearly associated, symmetric distributions have the same power for both P and (1 – P). Furthermore, if the left- and right-skewed asymmetric distributions are perfect mirror images of one another, the power of the left-skewed distribution for P is the same as that of the right-skewed distribution for (1 – P). Moreover, the right-skewed distribution has a smaller power than the left-skewed distribution for P < 0.5 when the logit outcome and covariates are positively associated. Conversely, for P > 0.5, the left-skewed distribution produces a smaller power than the right-skewed distribution. These characteristics are applicable for both continuous covariates and categorical covariates. These suggest that sample size from the formulas based on normal distributions may be over- or under-estimated for skewed distributions, nonetheless the general sample-size formulas or criteria for logistic regression models assume that continuous covariates follow a normal distribution [7−9].

When k = 1, the coefficients of all distributions are approximately 0.025. The coefficients in the logistic regression model with a maximum likelihood estimate are properly estimated for all covariate distribution shapes. Therefore, these power characteristics are related to the variance of the Wald test. The mechanism of power loss is clarified in the appendix. These characteristics were also observed for k = 2, 4, and 6. This may be due to our percentile-based categorization—an area in which many observations were concentrated was narrowly divided and produced many estimates. This follows the characteristics of the continuous distribution of X. Therefore, we supposed that the tendency of the changes in power would be similar to the continuous case.

When continuous covariates were converted to categorical covariates, the power decreased for any covariate distribution shapes. In particular, increasing the number of categorized groups led to a decrease in power for all distributions. This is due to the reduction in sample size for a categorized group.

3.1 Limitations of this study

A limitation of the present study is that our investigation only considered Wald statistics. Three general statistical analyses are available: the likelihood-ratio test, score test, and Wald test. Although these approaches would produce different values of the power, the characteristic of power loss may well be the same as with the Wald statistics. We applied the Wald test to estimate the power because it is routinely used as a significance test for the logistic regression coefficients. We examined only a single covariate to clarify the impact of the covariate distribution characteristics on the power in the logistic regression model. Although we need to examine more than one covariate, these characteristics between shapes of covariates distributions and P may be applicable to the existence of many independent covariates.

4 Conclusions

We clarified the characteristics of the power in terms of the relationship between their statistical power and the covariate distribution shapes in a logistic regression analysis. Both continuous and categorical variables may have the same characteristics. We specifically caution against for the sample size obtained from formulae based on normal distributions when a continuous covariate has skewed distributions. It is recommended to adjust sample-size, accounting for characteristics between shapes of covariates distributions and event proportions.

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Appendix

Suppose we have a sample of n independent observations of the pair (x_i, y_i), i = 1, 2, … , n, where x_i and y_i denote the values of the independent variable for the ith subject of the continuous covariate and the dichotomous outcome variable, 0 or 1, respectively. The contribution to the likelihood function is $\pi(x_i)$ for those pairs (x_i, y_i), where y_i = 1, and the contribution to the likelihood function is $1 - \pi(x_i)$ for those pairs where y_i = 0, where the quantity $\pi(x)$ is defined in Eq. (1). A convenient way to express the contribution to the likelihood function for the pair (x_i, y_i) is

Then, the logarithm of the likelihood function is

$$l(\beta_0,\beta_1)=\sum_{i=1}^{n} y_i(\beta_0+\beta_1 x_i)-\sum_{i=1}^{n}\ln(1+e^{\beta_0+\beta_1 x_i})$$

In particular, we test the hypothesis $H_0 : \beta_1 = 0$ concerning the significance of a single continuous coefficient by calculating the ratio of the estimate to its standard error.

$$\mathrm{W}=\frac{(\hat{\beta}_1)^2}{Var(\hat{\beta}_1)}$$

The variance of the maximum likelihood estimate is given by the **inverse** of the **Fisher information**

$$Var(\hat{\beta}_1)=\frac{\displaystyle\sum_{i=1}^{n}\frac{e^{\hat{\beta}_0+\hat{\beta}_1 x_i}}{(1+e^{\hat{\beta}_0+\hat{\beta}_1 x_i})^2}}{\left(\displaystyle\sum_{i=1}^{n}\frac{e^{\hat{\beta}_0+\hat{\beta}_1 x_i}}{(1+e^{\hat{\beta}_0+\hat{\beta}_1 x_i})^2}\right)*\left(\displaystyle\sum_{i=1}^{n}\frac{x_i^2 e^{\hat{\beta}_0+\hat{\beta}_1 x_i}}{(1+e^{\hat{\beta}_0+\hat{\beta}_1 x_i})^2}\right)-\left(\displaystyle\sum_{i=1}^{n}\frac{x_i e^{\hat{\beta}_0+\hat{\beta}_1 x_i}}{(1+e^{\hat{\beta}_0+\hat{\beta}_1 x_i})^2}\right)^2}$$

$$=\frac{1}{\displaystyle\sum_{i=1}^{n}x_i^2\hat{\pi}(x_i)(1-\hat{\pi}(x_i))-\left[\frac{\left(\displaystyle\sum_{i=1}^{n}x_i(\hat{\pi}(x_i)(1-\hat{\pi}(x_i))\right)^2}{\displaystyle\sum_{i=1}^{n}\hat{\pi}(x_i)(1-\hat{\pi}(x_i))}\right]}$$

Figures legends

Figure 1 – Four types of distributions for the simulation.

Tables

Table 1 Simulation conditions

Table 2 Estimated size of the Wald test for $\beta_1 = 0.000$ and N = 300

Table 3-1 Estimated power of the Wald test for $\beta_1 = 0.025$ and N = 300

Table 3-2 Estimated power of the Wald test for $\beta_1 = 0.025$ and N = 600

Table 3-3 Estimated power of the Wald test for $\beta_1 = 0.025$ and N = 900

Table 1 Simulation conditions

Input parameters	Notations	Possible values	
Sample size	N	300, 600, 900	
Ratio of sample-size of sub-group		Uniform Distribution	5:5:5
(0, 20):(20,40): (40,60)		Centralized-shaped Distribution	3:9:3
		Declining Distribution	10:4:1
		Uprising Distribution	1:4:10
Event proportion	P	0.1, 0.2, 0.5, 0.8, 0.9	
Number of categories	k	1 (continuous), 2, 4, 6	
Regression coefficients (covariate)	β_1	0.000 0.025	

Table 2: Estimated size of the Wald test for $\beta_1 = 0.000$ and N = 300

Event proportion		Uniform			Centralized-shaped			Declining			Uprising		
	k	coefficient	SE	Size	coefficient	SE	Size	coefficient	SE	Size	coefficient	SE	Size
0.1	1	0.0001	0.0114	0.0491	0.0001	0.0141	0.0505	-0.0009	0.0147	0.0485	0.0010	0.0147	0.0490
	2	0.0023	0.3966	0.0448	-0.0006	0.3961	0.0467	0.0084	0.3958	0.0451	-0.0048	0.3960	0.0441
	4	0.0048	0.5796	0.0251	-0.0013	0.5783	0.0272	0.0099	0.5797	0.0273	-0.0035	0.5786	0.0291
		0.0015	0.5797		-0.0012	0.5786		0.0169	0.5789		-0.0065	0.5789	
		0.0076	0.5793		-0.0017	0.5787		0.0102	0.5799		-0.0073	0.5789	
	6	0.0054	0.7315	0.0169	0.0047	0.7290	0.0168	0.0086	0.7315	0.0170	0.0005	0.7287	0.0162
		0.0114	0.7305		-0.0022	0.7306		0.0138	0.7301		-0.0025	0.7291	
		0.0065	0.7310		0.0009	0.7300		0.0148	0.7301		-0.0074	0.7299	
		0.0103	0.7307		-0.0090	0.7319		0.0191	0.7297		0.0007	0.7286	
		0.0053	0.7318		0.0025	0.7301		0.0121	0.7309		-0.0114	0.7308	
0.2	1	0.0001	0.0085	0.0477	0.0001	0.0105	0.0500	-0.0004	0.0109	0.0459	0.0006	0.0109	0.0462
	2	0.0060	0.2920	0.0508	0.0043	0.2921	0.0500	0.0035	0.2921	0.0464	0.0034	0.2923	0.0488
	4	-0.0028	0.4181	0.0433	0.0049	0.4182	0.0390	-0.0022	0.4182	0.0388	0.0024	0.4183	0.0397
		0.0090	0.4173		0.0079	0.4180		0.0037	0.4178		0.0041	0.4182	
		0.0007	0.4178		0.0065	0.4181		0.0014	0.4180		0.0051	0.4181	
	6	-0.0022	0.5180	0.0318	0.0049	0.5185	0.0301	0.0065	0.5186	0.0335	0.0063	0.5196	0.0327
		-0.0064	0.5186		0.0017	0.5188		0.0006	0.5193		0.0042	0.5196	

		0.0103	0.5172		0.0062	0.5186		0.0064	0.5185		0.0113	0.5188	
		0.0021	0.5176		0.0085	0.5182		0.0111	0.5183		0.0099	0.5190	
		-0.0033	0.5183		0.0054	0.5186		0.0009	0.5192		0.0025	0.5196	
0.5	1	-0.0001	0.0067	0.0489	0.0000	0.0084	0.0504	-0.0001	0.0105	0.0495	0.0001	0.0086	0.0487
	2	-0.0045	0.2317	0.0481	0.0000	0.2317	0.0524	-0.0029	0.2920	0.0513	0.0018	0.2317	0.0511
	4	-0.0027	0.3289	0.0463	-0.0018	0.3289	0.0477	0.0025	0.4177	0.0412	-0.0033	0.3288	0.0435
		-0.0052	0.3289		-0.0028	0.3288		-0.0001	0.4176		0.0019	0.3289	
		-0.0067	0.3289		0.0008	0.3288		-0.0032	0.4174		-0.0015	0.3289	
	6	0.0009	0.4042	0.0407	-0.0005	0.4042	0.0447	-0.0020	0.5179	0.0346	0.0006	0.4042	0.0435
		0.0015	0.4042		-0.0025	0.4042		0.0032	0.5185		-0.0004	0.4043	
		0.0004	0.4042		-0.0023	0.4042		0.0020	0.5184		0.0055	0.4043	
		-0.0028	0.4043		-0.0038	0.4042		-0.0025	0.5183		-0.0013	0.4043	
		-0.0091	0.4042		0.0028	0.4042		-0.0058	0.5178		0.0015	0.4043	
0.8	1	-0.0001	0.0085	0.0489	-0.0001	0.0105	0.0495	0.0007	0.0109	0.0483	-0.0005	0.0108	0.0484
	2	-0.0028	0.2922	0.0502	-0.0029	0.2920	0.0513	0.0066	0.2923	0.0457	0.0014	0.2919	0.5050
	4	-0.0042	0.4182	0.0385	0.0025	0.4177	0.0412	0.0006	0.4176	0.0418	-0.0017	0.4173	0.4000
		-0.0016	0.4185		-0.0001	0.4176		0.0061	0.4180		-0.0023	0.4173	
		-0.0084	0.4180		-0.0032	0.4174		0.0080	0.4182		0.0041	0.4177	
	6	-0.0077	0.5190	0.0309	-0.0020	0.5179	0.0346	0.0020	0.5181	0.0298	-0.0041	0.5182	0.0336
		-0.0061	0.5190		0.0032	0.5185		0.0034	0.5177		-0.0063	0.5177	
		-0.0062	0.5192		0.0020	0.5184		0.0087	0.5185		-0.0049	0.5180	

		-0.0032	0.5196		-0.0025	0.5183		0.0082	0.5185		0.0007	0.5185	
		-0.0120	0.5187		-0.0058	0.5178		0.0097	0.5187		-0.0020	0.5183	
0.9	1	0.0002	0.0114	0.0472	0.0001	0.0141	0.0490	0.0010	0.0147	0.0460	-0.0010	0.0147	0.0466
	2	0.0047	0.3956	0.0417	0.0049	0.3960	0.0449	-0.0025	0.3955	0.0437	0.0006	0.3958	0.0452
	4	0.0070	0.5777	0.0265	0.0033	0.5782	0.0269	0.0032	0.5781	0.0257	-0.0020	0.5784	0.0277
		0.0055	0.5776		0.0101	0.5790		0.0010	0.5779		-0.0013	0.5787	
		0.0091	0.5780		0.0026	0.5782		-0.0027	0.5774		0.0006	0.5790	
	6	0.0072	0.7283	0.0171	0.0044	0.7290	0.0174	-0.0042	0.7292	0.0156	0.0062	0.7286	0.0156
		0.0090	0.7286		0.0003	0.7283		0.0023	0.7310		0.0089	0.7293	
		0.0050	0.7281		0.0131	0.7305		-0.0010	0.7298		0.0002	0.7274	
		0.0155	0.7299		0.0025	0.7283		-0.0031	0.7297		0.0089	0.7295	
		0.0119	0.7293		0.0053	0.7299		-0.0076	0.7289		0.0054	0.7289	

Table 3-1: Estimated power of the Wald test for β_1 = 0.025 and N = 300

Event prppotion	k	Uniform coefficient	SE	Power	Centralized-shaped coefficient	SE	Power	Declining coefficient	SE	Power	Uprising coefficient	SE	Power
0.1	1	0.0255	0.0116	0.6038	0.0255	0.0142	0.4351	0.0246	0.0131	0.4908	0.0269	0.0168	0.3193
	2	0.7672	0.4050	0.4692	0.5639	0.3995	0.2724	0.5722	0.4007	0.2846	0.5153	0.3986	0.2413
	4	0.3941	0.6983	0.3228	0.3608	0.6622	0.1819	0.2070	0.6603	0.2258	0.4694	0.6624	0.1383
		0.7964	0.653		0.5732	0.6382		0.4314	0.6316		0.6869	0.6389	
		1.1854	0.6235		0.9465	0.6054		0.9202	0.5879		0.8850	0.6215	
	6	0.2244	0.9088	0.1945	0.3201	0.8683	0.1164	0.1269	0.8488	0.1441	0.3616	0.8743	0.0732
		0.4694	0.8665		0.4709	0.8436		0.2526	0.8272		0.5828	0.8407	
		0.7354	0.8288		0.6071	0.824		0.3909	0.8043		0.7185	0.8221	
		1.0073	0.7975		0.7690	0.8032		0.6227	0.7735		0.8633	0.8044	
		1.2663	0.7755		1.1251	0.7670		1.0424	0.7304		0.9847	0.7921	
0.2	1	0.0254	0.0087	0.8456	0.0253	0.0107	0.6590	0.0249	0.0102	0.6914	0.0259	0.0122	0.5665
	2	0.7546	0.2980	0.7319	0.5530	0.2949	0.4703	0.5463	0.2949	0.4593	0.5126	0.2940	0.4167
	4	0.3922	0.4818	0.6436	0.3431	0.4625	0.4078	0.1970	0.4586	0.4277	0.4547	0.4617	0.3352
		0.7781	0.4588		0.5596	0.4493		0.4068	0.4447		0.6702	0.4494	
		1.1599	0.4440		0.9126	0.4334		0.8775	0.4228		0.8581	0.4409	
	6	0.2655	0.6334	0.5333	0.3467	0.6024	0.3323	0.1294	0.5865	0.3485	0.3993	0.6081	0.2421
		0.5297	0.6070		0.4929	0.5894		0.2676	0.5731		0.6241	0.5889	
		0.7922	0.5873		0.6446	0.5780		0.3939	0.5621		0.7653	0.5791	

		1.0556	0.5725		0.7875	0.5686		0.6321	0.5452		0.8877	0.5716	
		1.3078	0.5623		1.1343	0.5516		1.0247	0.5259		1.0209	0.5647	
0.5	1	0.0252	0.0070	0.9574	0.0252	0.0087	0.8358	0.0254	0.0090	0.8193	0.0253	0.0090	0.8148
	2	0.7466	0.2358	0.8896	0.5433	0.2339	0.6404	0.5199	0.2337	0.6045	0.5190	0.2337	0.6006
	4	0.3712	0.3365	0.8499	0.3431	0.3334	0.6109	0.1901	0.3322	0.5842	0.4548	0.3339	0.5798
		0.7554	0.3365		0.5529	0.3334		0.3958	0.3316		0.6614	0.3345	
		1.1329	0.3425		0.8923	0.3373		0.8492	0.3366		0.8467	0.3366	
	6	0.2563	0.4188	0.7879	0.3273	0.4135	0.5478	0.1346	0.4103	0.5116	0.3836	0.4148	0.5097
		0.5010	0.4155		0.4792	0.4123		0.2562	0.4089		0.6108	0.4137	
		0.7649	0.4156		0.6218	0.4124		0.3845	0.4084		0.7373	0.4142	
		1.0173	0.4188		0.7597	0.4134		0.6067	0.4095		0.8620	0.4155	
		1.2713	0.4257		1.0963	0.4201		0.9978	0.4178		0.9902	0.4177	
0.8	1	0.0252	0.0087	0.8447	0.0254	0.0107	0.6640	0.0260	0.0122	0.5742	0.0250	0.0102	0.6937
	2	0.7497	0.2979	0.7239	0.5536	0.2945	0.4713	0.5137	0.2941	0.4144	0.5474	0.2946	0.4611
	4	0.3805	0.3770	0.6410	0.3560	0.3860	0.4013	0.1876	0.3864	0.3397	0.4731	0.3889	0.4298
		0.7631	0.4040		0.5686	0.4008		0.4023	0.4003		0.6828	0.4045	
		1.1536	0.4436		0.9172	0.4326		0.8620	0.4415		0.8795	0.4223	
	6	0.2593	0.4503	0.5261	0.3430	0.4642	0.3285	0.1232	0.4689	0.2405	0.3976	0.4618	0.3531
		0.5090	0.4676		0.4918	0.4756		0.2571	0.4784		0.6343	0.4814	
		0.7736	0.4917		0.6353	0.4886		0.3914	0.4896		0.7617	0.4935	
		1.0361	0.5226		0.7973	0.5054		0.6225	0.5123		0.8982	0.5092	

		1.3065	0.5622		1.1323	0.5499		1.0209	0.5655		1.0314	0.5261	
0.9	1	0.0257	0.0117	0.6066	0.0256	0.0142	0.4393	0.0270	0.0168	0.3148	0.0250	0.0131	0.5046
	2	0.7701	0.4061	0.4727	0.5692	0.3997	0.2855	0.5211	0.3986	0.2415	0.5683	0.4002	0.2794
	4	0.3972	0.4945	0.3230	0.3730	0.5133	0.1928	0.1913	0.5137	0.1340	0.5080	0.5200	0.2240
		0.7964	0.5506		0.5926	0.5438		0.4188	0.5427		0.7171	0.5508	
		1.1926	0.6256		0.9545	0.6062		0.8815	0.6210		0.9276	0.5875	
	6	0.2649	0.5850	0.0200	0.3527	0.6157	0.1158	0.1311	0.6283	0.0734	0.4287	0.6118	0.1376
		0.5391	0.6251		0.5177	0.6410		0.2553	0.6464		0.6713	0.6519	
		0.7980	0.6716		0.6570	0.6666		0.4035	0.6706		0.7954	0.6755	
		1.0617	0.7273		0.8073	0.6954		0.6272	0.7120		0.9226	0.7013	
		1.2676	0.7759		1.1254	0.7657		0.9761	0.7881		1.0561	0.7312	

Table 3-2: Estimated power of the Wald test for $\beta_1 = 0.025$ and N = 600

Event prppotion	k	Uniform coefficient	SE	Power	Centralized-shaped coefficient	SE	Power	Declining coefficient	SE	Power	Uprising coefficient	SE	Power
0.1	1	0.0253	0.0081	0.8947	0.0252	0.0099	0.7286	0.0249	0.0091	0.7726	0.0258	0.0116	0.6063
	2	0.7600	0.2824	0.7818	0.5549	0.2782	0.5209	0.5590	0.2786	0.5182	0.5006	0.2774	0.4317
	4	0.3845	0.4731	0.7086	0.3490	0.4493	0.4577	0.1972	0.4460	0.5052	0.4656	0.4481	0.3666
		0.7806	0.4443		0.5624	0.4334		0.4111	0.4287		0.6681	0.4342	
		1.1589	0.4251		0.9197	0.4128		0.8898	0.4003		0.8552	0.4232	
	6	0.2644	0.6308	0.6007	0.3542	0.5935	0.3784	0.1313	0.5751	0.4271	0.4053	0.5997	0.2616
		0.5278	0.6003		0.5063	0.5771		0.2653	0.5596		0.6394	0.5764	
		0.7992	0.5753		0.6472	0.5643		0.4024	0.5453		0.7632	0.5660	
		1.0544	0.5573		0.7980	0.5519		0.6283	0.5255		0.8913	0.5562	
		1.3147	0.5429		1.1474	0.5297		1.0392	0.4987		1.0185	0.5476	
0.2	1	0.0252	0.0061	0.9895	0.0251	0.0075	0.9219	0.0251	0.0072	0.9364	0.0254	0.0085	0.8728
	2	0.7484	0.2096	0.9558	0.5483	0.2073	0.7577	0.5436	0.2073	0.7524	0.5061	0.2067	0.6938
	4	0.3818	0.3341	0.9393	0.3397	0.3214	0.7412	0.1871	0.3191	0.7747	0.4451	0.3206	0.6700
		0.7612	0.3187		0.5493	0.3127		0.3926	0.3098		0.6541	0.3126	
		1.1383	0.3087		0.8998	0.3019		0.8674	0.2947		0.8397	0.3068	
	6	0.2649	0.4326	0.9065	0.3294	0.4127	0.6891	0.1296	0.4025	0.7229	0.3841	0.4162	0.5806
		0.5205	0.4163		0.4802	0.4038		0.2523	0.3947		0.6079	0.4038	
		0.7755	0.4036		0.6180	0.3968		0.3772	0.3876		0.7379	0.3978	

		1.0262	0.3942		0.7694	0.3902		0.6101	0.3766		0.8642	0.3927	
		1.2828	0.3871		1.1032	0.3792		1.0097	0.3633		0.9880	0.3883	
0.5	1	0.0251	0.0049	0.9996	0.0252	0.0061	0.9889	0.0251	0.0063	0.9851	0.0251	0.0063	0.9818
	2	0.7429	0.1664	0.9939	0.5480	0.1651	0.9153	0.5197	0.1650	0.8819	0.5199	0.1650	0.8841
	4	0.3731	0.2370	0.9943	0.3411	0.2348	0.9241	0.1839	0.2340	0.9033	0.4486	0.2353	0.9011
		0.7502	0.2370		0.5544	0.2348		0.3913	0.2336		0.6571	0.2357	
		1.1268	0.2411		0.8946	0.2376		0.8433	0.2371		0.8427	0.2371	
	6	0.2516	0.2941	0.9884	0.3318	0.2906	0.9014	0.1232	0.2884	0.8623	0.3811	0.2915	0.8651
		0.5021	0.2919		0.4740	0.2899		0.2474	0.2874		0.6031	0.2908	
		0.7574	0.2919		0.6197	0.2899		0.3771	0.2871		0.7328	0.2911	
		1.0051	0.2941		0.7656	0.2906		0.5963	0.2878		0.8543	0.2920	
		1.2589	0.2987		1.0939	0.2952		0.9825	0.2935		0.9828	0.2935	
0.8	1	0.0251	0.0061	0.9887	0.0251	0.0075	0.9217	0.0255	0.0085	0.8741	0.0249	0.0072	0.9335
	2	0.7469	0.2095	0.9532	0.5509	0.2073	0.7604	0.5065	0.2068	0.6921	0.5393	0.2073	0.7426
	4	0.3724	0.2645	0.9400	0.3501	0.2710	0.7522	0.1893	0.2710	0.6673	0.4658	0.2729	0.7627
		0.7517	0.2826		0.5642	0.2812		0.3960	0.2801		0.6706	0.2833	
		1.1355	0.3086		0.8995	0.3017		0.8435	0.3071		0.8606	0.2949	
	6	0.2515	0.3150	0.9056	0.3358	0.3245	0.6902	0.1289	0.3277	0.5790	0.4035	0.3235	0.7091
		0.5033	0.3267		0.4814	0.3321		0.2546	0.3337		0.6220	0.3355	
		0.7579	0.3420		0.6291	0.3410		0.3829	0.3407		0.7498	0.3438	
		1.0139	0.3615		0.7751	0.3511		0.6100	0.3556		0.8777	0.3530	

		1.2767	0.3866		1.1017	0.3789		0.9899	0.3881		1.0014	0.3630	
0.9	1	0.0254	0.0081	0.8923	0.0253	0.0099	0.7246	0.0260	0.0117	0.6139	0.0247	0.0091	0.7713
	2	0.7590	0.2824	0.7828	0.5585	0.2783	0.5191	0.5078	0.2774	0.4460	0.5572	0.2782	0.5195
	4	0.3819	0.3417	0.7096	0.3639	0.3546	0.4620	0.1969	0.3551	0.3727	0.4793	0.3575	0.5012
		0.7655	0.3765		0.5776	0.3738		0.4061	0.3724		0.6929	0.3779	
		1.1700	0.4264		0.9238	0.4126		0.8610	0.4222		0.8855	0.3994	
	6	0.2593	0.4018	0.5975	0.3410	0.4209	0.3791	0.1313	0.4279	0.2603	0.4043	0.4181	0.4155
		0.5127	0.4253		0.5028	0.4375		0.2663	0.4406		0.6357	0.4426	
		0.7753	0.4559		0.6450	0.4541		0.3962	0.4543		0.7691	0.4595	
		1.0509	0.4963		0.7958	0.4735		0.6263	0.4827		0.9065	0.4781	
		1.3199	0.5435		1.1454	0.5295		1.0207	0.5450		1.0348	0.4980	

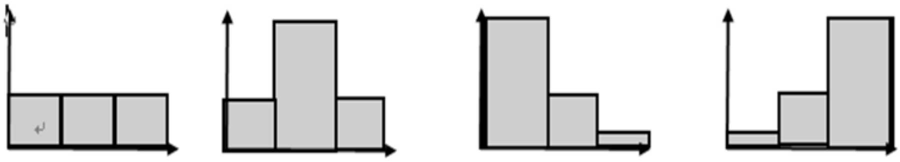
Table 3-3: Estimated power of the Wald test for $\beta_1 = 0.025$ and N = 900

Event prppotion		Uniform			Centralized-shaped			Declining			Uprising		
	k	coefficient	SE	Power	coefficient	SE	Power	coefficient	SE	Power	coefficient	SE	Power
0.1	1	0.0251	0.0066	0.9742	0.0250	0.0081	0.8780	0.0249	0.0074	0.9086	0.0258	0.0095	0.8075
	2	0.7524	0.2293	0.9191	0.5528	0.2263	0.6927	0.5550	0.2263	0.6982	0.5078	0.2257	0.6212
	4	0.3839	0.3799	0.8889	0.3393	0.3618	0.6641	0.1930	0.3591	0.7177	0.4536	0.3616	0.5720
		0.7679	0.3578		0.5556	0.3490		0.3991	0.3458		0.6635	0.3500	
		1.1446	0.3425		0.9032	0.3329		0.8806	0.3229		0.8539	0.3412	
	6	0.2665	0.4997	0.8331	0.3326	0.4727	0.6040	0.1274	0.4585	0.6462	0.3880	0.4783	0.4673
		0.5187	0.4771		0.4864	0.4597		0.2602	0.4465		0.6190	0.4600	
		0.7807	0.4584		0.6349	0.4490		0.3885	0.4361		0.7512	0.4511	
		1.0379	0.4440		0.7724	0.4403		0.6166	0.4203		0.8776	0.4437	
		1.2883	0.4331		1.1145	0.4229		1.0217	0.3994		1.0071	0.4369	
0.2	1	0.0252	0.0050	0.9993	0.0252	0.0061	0.9876	0.0251	0.0058	0.9892	0.0253	0.0069	0.9718
	2	0.7485	0.1707	0.9943	0.5510	0.1689	0.9085	0.5457	0.1690	0.8995	0.5058	0.1685	0.8583
	4	0.3819	0.2712	0.9931	0.3398	0.2612	0.9122	0.1882	0.2595	0.9242	0.4448	0.2606	0.8605
		0.7597	0.2589		0.5524	0.2540		0.3986	0.2519		0.6519	0.2542	
		1.1358	0.2508		0.8996	0.2454		0.8647	0.2399		0.8384	0.2495	
	6	0.2538	0.3492	0.9866	0.3352	0.3341	0.8895	0.1286	0.3266	0.8999	0.3849	0.3366	0.8005
		0.5100	0.3361		0.4794	0.3274		0.2556	0.3200		0.6027	0.3269	
		0.7640	0.3260		0.6247	0.3215		0.3880	0.3141		0.7359	0.3220	

		1.0163	0.3183		0.7689	0.3165		0.6075	0.3058		0.8572	0.3181	
		1.2681	0.3127		1.1021	0.3076		1.0048	0.2951		0.9817	0.3145	
0.5	1	0.0250	0.0040	1.0000	0.0252	0.0050	0.9989	0.0251	0.0052	0.9990	0.0251	0.0052	0.9986
	2	0.7402	0.1358	0.9990	0.5455	0.1347	0.9826	0.5189	0.1346	0.9689	0.5180	0.1346	0.9699
	4	0.3740	0.1932	0.9999	0.3436	0.1915	0.9982	0.1888	0.1909	0.9852	0.4538	0.1918	0.9838
		0.7477	0.1932		0.5533	0.1915		0.3927	0.1905		0.6571	0.1922	
		1.1230	0.1965		0.8916	0.1937		0.8439	0.1933		0.8426	0.1933	
	6	0.2500	0.2396	0.9998	0.3309	0.2368	0.9832	0.1265	0.2350	0.9733	0.3838	0.2376	0.9746
		0.4990	0.2378		0.4772	0.2362		0.2535	0.2342		0.6060	0.2369	
		0.7508	0.2378		0.6184	0.2362		0.3776	0.2340		0.7300	0.2372	
		1.0007	0.2396		0.7619	0.2368		0.5990	0.2346		0.8562	0.2379	
		1.2515	0.2433		1.0894	0.2405		0.9824	0.2391		0.9798	0.2391	
0.8	1	0.0251	0.0050	0.9994	0.0252	0.0061	0.9858	0.0253	0.0069	0.9707	0.0250	0.0058	0.9873
	2	0.7475	0.1708	0.9939	0.5504	0.1689	0.9112	0.5052	0.1685	0.8607	0.5424	0.1690	0.8999
	4	0.3767	0.2154	0.9921	0.3516	0.2206	0.9164	0.1884	0.2207	0.8577	0.4668	0.2221	0.9208
		0.7527	0.2300		0.5621	0.2286		0.3939	0.2280		0.6733	0.2306	
		1.1357	0.2509		0.8995	0.2453		0.8386	0.2495		0.8623	0.2399	
	6	0.2514	0.2564	0.9849	0.3397	0.2640	0.8983	0.1245	0.2665	0.8058	0.3950	0.2629	0.8947
		0.5002	0.2657		0.4805	0.2699		0.2509	0.2714		0.6178	0.2727	
		0.7562	0.2780		0.6249	0.2768		0.3794	0.2770		0.7446	0.2792	
		1.0100	0.2935		0.7715	0.2849		0.6003	0.2885		0.8746	0.2868	

		1.2680	0.3130		1.1016	0.3073		0.9808	0.3144		0.9989	0.2949	
0.9	1	0.0252	0.0066	0.9732	0.0252	0.0081	0.8817	0.0256	0.0095	0.7965	0.0250	0.0074	0.9072
	2	0.7524	0.2291	0.9168	0.5584	0.2265	0.7030	0.5028	0.2257	0.6078	0.5581	0.2264	0.6998
	4	0.3797	0.2773	0.8928	0.3512	0.2874	0.6686	0.1910	0.2884	0.5620	0.4773	0.2900	0.7128
		0.7599	0.3047		0.5695	0.3030		0.3966	0.3020		0.6917	0.3063	
		1.1477	0.3422		0.9131	0.3334		0.8468	0.3409		0.8798	0.3230	
	6	0.2535	0.3251	0.8355	0.3388	0.3410	0.6094	0.1344	0.3468	0.4548	0.4037	0.3380	0.6549
		0.5094	0.3440		0.4838	0.3525		0.2565	0.3559		0.6380	0.3577	
		0.7660	0.3674		0.6367	0.3665		0.3906	0.3670		0.7708	0.3707	
		1.0268	0.3966		0.7823	0.3813		0.6089	0.3881		0.8962	0.3842	
		1.2928	0.4331		1.1245	0.4238		1.0025	0.4364		1.0248	0.3996	

Figure 1. Four types of distributions for the simulation



(a) Uniform distribution 5 : 5 : 5

(b) Centralized-shaped distribution 3 : 9 : 3

(c) Declining distribution 10 : 4 : 1

(d) Uprising distribution 1 : 4 : 10