Mega Publishing Limited
Journal of Risk & Control, 2014, 1(1), 31-49 | Dec 30, 2014

Hedging Effectiveness of Applying Constant and Time-Varying Hedge Ratios: Evidence from Taiwan Stock Index Spot and Futures

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Abstract

This paper investigates the market-risk-hedging effectiveness of the Taiwan Futures Exchange (TAIFEX) stock index futures using daily settlement prices for the period from July 21, 1998 to December 31, 2010. The minimum variance hedge ratios (MVHRs) are estimated from the ordinary least squares regression model (OLS), the vector error correction model (VECM), the generalized autoregressive conditional heteroskedasticity model (GARCH), the threshold GARCH model (TGARCH), and the bivariate GARCH model (BGARCH), respectively. We employ a rolling sample method to generate the time-varying MVHRs for the out-of-sample period, associated with different hedge horizons, and compare across their hedging effectiveness and risk-return trade-off. In a one-day hedge horizon, the TGARCH model generates the greatest variance reduction, while the OLS model provides the highest rate of risk-adjusted return; in a longer hedge horizon, the OLS generates the largest variance reduction, while the BGARCH model provides the best risk-return trade-off. We find that the selection of appropriate models to measure the MVHRs depends on the degree of risk aversion and hedge horizon.

JEL classification numbers: F37, G13, G15
Keywords: Index Futures; Hedge Ratio; VECM model; GARCH model; Multivariate-GARCH model

1 Introduction

Following the subprime crisis, financial risk management has played an important role in investment decisions and asset allocations. Indeed, one of the key components of risk management is how to hedge, with hedging through trading index futures being one of the main functions of derivative markets. Hedgers who hold cash assets trade in the futures markets in order to reduce their risk of adverse price movements, and the reliable

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Article Info: *Received*: November 20, 2014. *Revised*: December 20, 2014
Published online : December 30, 2014

computation of the hedge ratio substantially affects the effectiveness of a hedge. Thus, the core of a successful hedging activity depends on the computation of hedge ratio. There are many different approaches to calculate the hedge ratio, such as the simplest one-to-one, the well-known ordinary least squares regression (OLS), and a series of other more complicated models, introduced by various researchers, to solve this problem. Yet the question remains: which one is better (or the best) for the task to measure hedging performance?

Using the traditional OLS model for estimating the hedge ratio may suffer from the problems of serial correlation in the residuals (Herbst et al., 1993) and heteroskedasticity in spot-futures price series (Park and Switzer, 1995). Taking into account the spot-futures cointegrating relationship is actually indispensible for an effective hedge. According to Ghosh (1993a; 1993b), ignoring the cointegration could result in underestimating the minimum variance hedge ratio (MVHR). This study adopts the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model in attempts to circumvent these problems.

To explore further for better alternatives, we thus also employ the relatively more advanced bivariate GARCH (BGARCH) and threshold GARCH (TGARCH) models, respectively, to compute the hedge ratio in conjunction with the "rolling sample method" (also called the moving window method). Based upon data series of Taiwan stock index futures traded during 1998-2010, we compute the hedge ratios via different models, and thus conduct the out-of-sample analysis to compare across their "hedging effectiveness" and "risk-return trade-off" measures.

According to the Futures Industry Association (FIA), the trading volume of the Taiwan Futures Exchange (TAIFEX) during 2009 was 135,125,695 contracts and ranked 18th in the world. During 2010, the total trading volume rose to 139,792,891 contracts and ranked 17th. According to TAIFEX, the trading volume of stock index futures increased from 24,625,062 contracts to 25,332,827 contracts during 2010. Aside from stock index futures contracts, there are various other derivatives in the futures market for hedging, which further increase the trading volume. Up to now, the trading volume in Taiwan's futures market still continues to grow. Both hedgers and investors can use hedge strategies to make their investing portfolios not only more flexible and less risky, but also to generate greater risk-adjusted return. Taiwan's stock index futures market has become so growingly popular to the investing public that it deserves a detailed analysis on its hedging performance.

Our study has some major contributions. First, different from prior works on those index futures markets in various developed countries, our research focus is switched onto emerging markets such as Taiwan, and update the data coverage to a more recent 1998-2010 horizon. Second, we use models such as bivariate GARCH and TGARH, not employed in previous studies, to specify the relationship between stock index spot prices and stock index futures prices and to estimate the hedge ratios combining with the rolling sample method, which is not adopted by any other published studies. Third, following the key methodologies of Yang and Allen (2005), we incorporate the risk-return trade-off and different hedge horizons to compare the hedge performance, but our empirical results, somehow differ from Yang and Allen (2005), suggest that there exists a risk-return trade-off reflecting the importance of the degree of risk aversion and hedge horizon, both of which do play an influential role in determining the MVHRs.

2 Literature Review

We review the existing literature for the variety of hedging performance measurements and modelling designs. The simplest hedge strategy is the traditional one-to-one, i.e., the so-called naïve hedge. Hedgers who own a spot market position just need to take up a futures position that is equal in size, but opposite in sign, to the spot market position, i.e. the hedge ratio is equal to -1. The price risk will be eliminated if the magnitude of price changes in the spot market is exactly the same with those in the futures market. However, the correlation between spot and futures returns is not perfectly linear in practice, and hence the optimal hedge ratio is almost bigger than -1.

The beta hedge ratio is related to the portfolio's beta. In order to fully hedge the price risk, the number of futures contracts needs to be adjusted by the portfolio's beta. Under a beta hedge strategy, the optimal hedge ratio is bigger than or equal to -1. The "naïve" and "beta" hedges are considered the most traditional in financial market risk management.

Johnson (1960) first introduced the MVHR to calculate the optimal hedge ratio, varying from the traditional hedge methods by applying modern portfolio theory to the hedging problem. He offered the definition to return and risk in terms of mean and variance of return. The hedge ratio calculated under the minimum portfolio variance assumption is the optimal hedge ratio, which is also called the MVHR. The MVHR (h^*) is computed as follows:

$$h^* = - X_F / X_S = - Cov(\Delta S, \Delta F) / Var(\Delta F), \tag{1}$$

where X_F and X_S represent the relative dollar amount invested in futures and spot stock index inderespectively, $Cov(\Delta S, \Delta F)$ is the covariance of spot and futures price changes, and $Var(\Delta F)$ is the variance of futures price changes.

The MVHR can also be calculated by regressing the spot price changes on futures price changes, and the coefficient of the futures price changes is the MVHR. The negative sign of Equation (1) reveals that if hedgers want to hedge their long positions in the spot market, then they have to short futures contracts. Johnson also proposed a measure of the hedging effectiveness of the hedged position in terms of the variance reduction, expressed as follows:

$$[Var(U) - Var(V)] / Var(U), \tag{2}$$

where $Var(U)$ and $Var(H)$ is the variance of a un-hedged and a hedged portfolio, respectively.

Figlewski (1984) calculated the risk minimizing hedge ratio by OLS on historical U.S. S&P 500 spot and futures returns to analyze the hedge effectiveness of stock index futures. He found that hedge ratios computed by ex-post MVHRs outperformed the beta hedge ratios, and that both time to maturity and hedge duration were important factors.

Junkus and Lee (1985) also used the OLS conventional regression model to calculate the optimal hedge ratios, and to investigate the hedging effectiveness of U.S. stock index futures by alternative hedging strategies. They argued that the use of MVHR assessment is the best strategy to reduce the risk of adverse price movement.

Ghosh (1993a,1993b) argued that the conventional OLS approach does not take account of the lead and lag relationships between U.S. stock index prices and corresponding stock index futures prices and is not well specified in estimating the hedge ratio. He used the Error Correction Model (ECM) to overcome this problem and showed that the impact of contract expiration and hedging effectiveness is little. Ghosh found that *if* there existed cointegration between spot and futures prices, and the regression model did not contain the error correction term to take account of the cointegration effect, then the estimated MVHR would be biased downwards due to misspecification. Holmes (1996) applied the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) to estimate optimal hedge ratios of U.K. FTSE-100 stock index. In his investigation he found that based on MVHRs, the optimal hedge ratio calculated by conventional OLS outperforms those estimated by an ECM or a GARCH (1,1) approach. He further pointed out that hedging effectiveness increased with an increase in hedge duration.

Butterworth and Holmes (2001) used the Least Trimmed Squares Approach to estimate optimal hedge ratios of U.K. FTSE-Mid 250 stock index futures contracts. They compared the ratios with those obtained from the FTSE-100 stock index, and figured out that the FTSE-Mid 250 index futures contract outperforms the FTSE-100 index futures contract when hedging cash portfolios.

Chou et al. (1996) examined hedge ratios with different time horizons of Japan’s Nikkei Stock Average (NSA) index spot and futures contract by the conventional OLS model and ECM. After comparing the in-sample and out-of-sample performances, the conventional OLS is superior to the ECM approach under the in-sample performance, but the ECM outperformed the conventional OLS approach under the out-of-sample performance.

Lypny and Powalla (1998) investigated the hedging effectiveness of the German stock index DAX futures. They showed that the hedge ratios taking account of the time-varying conditional variance and computed by GARCH (1,1) approach are the optimal hedge ratios.

Based on the summary of aforementioned research works on the stock index futures markets in developed countries, we can easily consider that the hedge ratios estimated by the complicated econometric model such as GARCH may not always reduce the most variation of return. When we only take account of risk-return trade-off, the easier model such as OLS may usually bring a higher risk-adjusted return. To our knowledge, only a few papers have compared the MVHR based on the variance reduction and risk-return trade-off at the once; yet none of those studies focuses on the emerging markets such as Taiwan’s stock index futures. As such, we employ several models, from the easiest Ordinary Least Square (OLS) to the Bivariate GARCH Model, in order to calculate MVHRs and further evaluate them by following the methodology of Yang and Allen (2005).

3 Model and Estimation Methodology

In attempt to find the most appropriate model for estimating optimal hedge ratios in Taiwan stock index futures, five different models are employed to compute the optimal hedge ratios respectively, and then be compared across their hedging performance. The hedging performance is measured by a) the percentage variance reduction from the hedged portfolio to the un-hedged portfolio, and b) the risk-return trade-off.

Model 1: Conventional OLS Regression Model

This model is just a linear regression of change in spot prices on changes in futures prices. Let S_t and F_t be logged spot and futures prices, respectively, and the one period MVHR (h*) can be estimated from the expression:

$$\Delta S_t = c + \beta \Delta F_t + \varepsilon_t, \tag{3}$$

where c is the intercept, ε_t is the error term from OLS estimation, ΔS_t and ΔF_t represent corresponding spot and futures price changes, and the slope coefficient β is the MVHR.

Model 2: Vector Error Correction Model (VECM)

According to Herbst et al. (1989), if the residuals obtained from Model 1 are autocorrelated, then the result may be Model 1's invalidity. In order to take account of serial correlation, the spot and futures prices are modeled under a bivariate-VAR framework as follows:

$$\begin{aligned} \Delta S_t &= c_s + \textstyle\sum_{i=1}^{k} \beta_{si} \Delta S_{t-i} + \textstyle\sum_{i=1}^{k} \beta_{si} \Delta F_{t-i} + \varepsilon_{st}, \\ \Delta F_t &= c_f + \textstyle\sum_{i=1}^{k} \beta_{fi} \Delta S_{t-i} + \textstyle\sum_{i=1}^{k} \beta_{fi} \Delta F_{t-i} + \varepsilon_{ft} \end{aligned} \tag{4}$$

Where c is the intercept, β_s and β_f are positive parameters, ε_{st} and ε_{ft} are "independently identically distributed"(IID) random vectors. k is the optimal lag length and begins from one, and is added up by one until the serial correlation of residuals is got rid of the mean equations. The MVHR is:

$$h^* = \mathrm{Cov}(\varepsilon_{st}, \varepsilon_{ft}) / \mathrm{Var}(\varepsilon_{ft}). \tag{5}$$

When the sets of series carry a cointegration relationship, as shown by Engle and Granger (1987),the data contain a valid "Error Correction" representation. It is obvious that Equation (3) ignores the relationship that the two series are cointegrated, which is further addressed in Ghosh (1993b), Lien and Luo (1994), Lien (1996), and Lien et al. (2014). They jointly showed that if the two price series are found to be cointegrated, then a VAR model should be estimated along with the error-correction term, which takes account of the long-run equilibrium between spot and futures price movements. Thus, Equation (4) is modified into:

$$\begin{aligned} \Delta S_t &= c_s + \textstyle\sum_{i=1}^{k} \beta_{si} \Delta S_{t-i} + \textstyle\sum_{i=1}^{k} \beta_{si} \Delta F_{t-i} - \lambda_s Z_{t-1} + \varepsilon_{st}, \\ \Delta F_t &= c_f + \textstyle\sum_{i=1}^{k} \beta_{fi} \Delta S_{t-i} + \textstyle\sum_{i=1}^{k} \beta_{fi} \Delta F_{t-i} + \lambda_f Z_{t-1} + \varepsilon_{ft}, \end{aligned} \tag{6}$$

where c_s and c_f are the intercept, β_{si},β_{fi}, λ_s and λ_f are positive parameters, ε_{st} and ε_{ft} are white noise disturbance terms. Z_{t-1} refers to the error-correction term, which measures how the dependent variable adjusts to the previous period's deviation from long-run equilibrium as $Z_{t-1} = S_{t-1} - \alpha F_{t-1}$, where α is the cointegrating vector.

Equation (6) is a bivariate VAR (k) model in first differences augmented by the error-correction terms $\lambda_s Z_{t-1}$ and $\lambda_f Z_{t-1}$.The speed of adjustment depends on λ_s and λ_f, causing the response of S_t and F_t, respectively, to the previous period's deviation from long-run equilibrium. The constant hedge ratio can be similarly calculated using Equation (5).

Model 3: GARCH Model

Bollerslev (1986) introduced the GARCH (1,1) model to parameterize volatility as a function of unexpected information shocks to the market. A standard GARCH (1,1) model is expressed as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{7}$$

where σ_t^2 is the conditional variance, α_0 is the mean, ε_{t-1}^2 (the ARCH term) and σ_{t-1}^2 (the GARCH term) refer to, respectively, the lag of the squared residual from the mean equation and the last period's forecast variance capturing the news about volatility from the previous period. The more general forms of GARCH (p, q) compute σ_t^2 from the most recent p observations on ε_t^2 and the most recent q estimates of the variance rate. Values of ($\alpha_1 + \beta_1$) close to or even larger than unity mean that the persistence in volatility is high. If there is a large positive shock ε_{t-1}, such that ε_{t-1}^2 is large, then the conditional variance σ_t^2 increases. Such a shock fades away if ($\alpha_1 + \beta_1$) is less than unity, but persists into the long run if it is greater than or equal to unity.

Model 4: Threshold GARCH (TGARCH) Model

Glosten et al. (1993) developed TGARCH, which is also called GJR-GARCH. They added the asymmetric term to expand the GARCH model to capture the asymmetric leverage effect rather than quadratic. A standard TGARCH (1, 1, 1) is presented as:

$$\begin{aligned} &\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 D_{t-1} + \beta_1 \sigma_{t-1}^2, \\ &D_{t-1} = 1 \text{ if } \varepsilon_{t-1} < 0, \\ &D_{t-1} = 0 \text{ if } \varepsilon_{t-1} \geq 0, \end{aligned} \tag{8}$$

where α_0, α_1, γ, and $\beta_1$are constant parameters, D_{t-1} is a dummy variable, ε_{t-1} represents the good or bad news impact, and the threshold is zero.

The more general GARCH (p, q, r) computes σ_t^2 from the most recent p observations onε^2, the most recent q estimates of the variance rate, and the most recent r unexpected impacts. Since the asymmetric term $\gamma \varepsilon_{t-1}^2 D_{t-1}$ is included, the model will be asymmetric if $\gamma \neq 0$. The presence of leverage effects can be tested by the hypothesis $\gamma < 0$. After running the appropriate regression, if γ is positive and statistically different from zero, it implies that negative shocks generate more volatility than positive shocks (good news).

Model 5: Bivariate GARCH (BGARCH) Model

Park and Bera (1987) and Pagan (1996) both pointed out that heteroskedasticity (or ARCH effects) in the second movements partly invalidates hedge ratio estimates. Thus, we employ Bollerslev et al. (1988) VECM-GARCH model to take account of the ARCH effects in the residuals.

Engle (1982) and Bollerslev (1986) developed the ARCH model to examine the second movement of financial and economic time series. Bollerslev et al. (1988) generalized the univariate GARCH model to the BGARCH model by simultaneously modeling the conditional variance and covariance of two interacted series. Since the estimated conditional variance and covariance of spot and futures prices vary over time, hedge ratios are also different from time to time. Bollerslev (1986) assumed that covariance matrices are diagonal and the correlation between the conditional variances is constant, so as to reduce some of the large number of parameters, which need to be estimated in the

model. However, Bera and Roh (1991) tested the constant correlation assumption and found the assumption unrealistic for many financial time series.
Bollerslev et al. (1988) develop the Diagonal Vector (DVEC) model, which likes the constant correlation model, but allows for a time-varying conditional variance. In the DVEC model, the off-diagonals in covariance matrices are also set to zero, and so the condition variance depends only on its own lagged variances and lagged squared residuals. Accordingly, the diagonal expression of the conditional variance element scan be presented as:

$$
\begin{aligned}
h_{ss,t} &= c_{ss} + \alpha_{ss} (\varepsilon_{s,t-1})^2 + \beta_{ss} h_{ss,t-1},\\
h_{sf,t} &= c_{sf} + \alpha_{sf} (\varepsilon_{s,t-1})(\varepsilon_{s,t-1}) + \beta_{sf} h_{sf,t-1},\\
h_{ff,t} &= c_{ff} + \alpha_{ff} (\varepsilon_{f,t-1})^2 + \beta_{ff} h_{ff,t-1}.
\end{aligned}
\tag{9}
$$

Equation (9) incorporates a time-varying conditional correlation coefficient between index spot and futures prices, thus making the resulting BGARCH time-varying hedge ratios more realistic.

4 Data and Preliminary Analysis

4.1 Data

We use data collected by *Info Winner Plus*, which is a local data vendor, containing the closing prices (CP) of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) and the settlement prices (SP) of the corresponding TAIEX Futures on a daily basis for the period of July 21, 1998 to December 31, 2010.In all estimations the futures contract nearest to expiration is used. Following previous studies, no adjustment is made for dividends and we use the changes in logarithms of both spot and futures prices for analysis. There are a total of 3,146 observations, but only the first 2,644 observations (07/21/1998 – 12/31/2008) are used for measuring the MVHRs, leaving the remaining 502 observations (01/01/2009 – 12/31/2010) for the out-of-sample forecast.
Figure I plots the logarithm of CP and SP, and we find that the two series are highly correlated. Just in case that a cointegration relationship might exist between the two sets, we conduct the ADF test, KPSS test, and Johansen test.

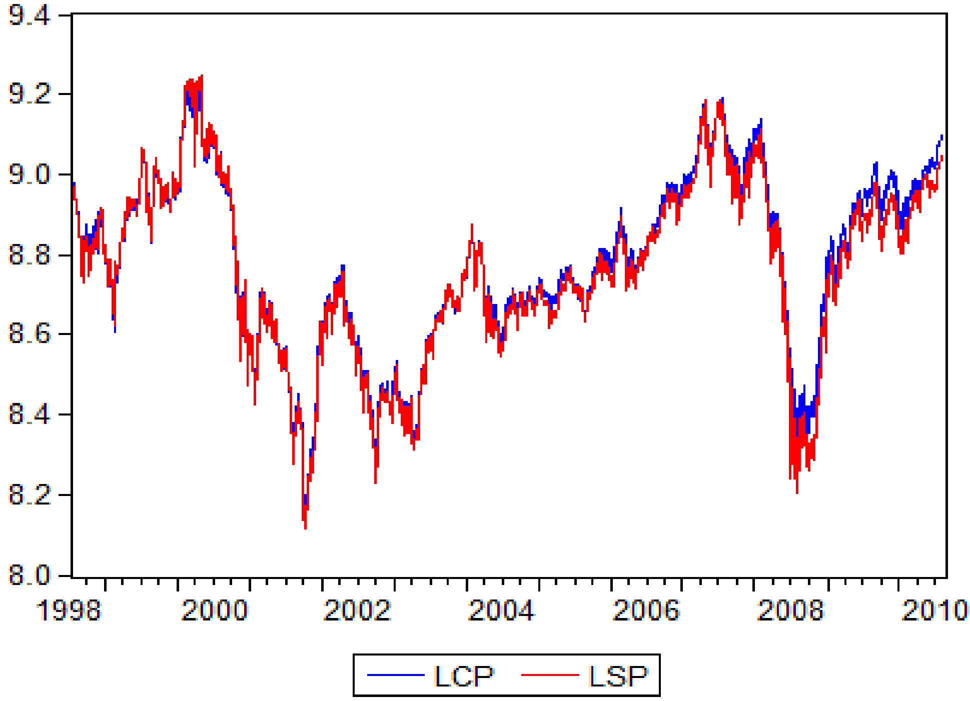

Figure I: The Logarithm of Spot Closing Prices(LCP) and Futures Settlement Price(LSP) Series on Taiwan Stock Market Index

4.2 Tests of Unit Roots and Cointegration

Tests for the existence of a unit root are performed by conducting the Augmented Dickey-Fuller (1979) ADF tests. The KPSS tests proposed by Kwiatkowski et al. (1992) are employed to complement the ADF tests, since the power of such tests are questioned by Schwert (1987) and DeJong and Whiteman (1991). The null hypothesis for the ADF test is that a series contains a unit root or it is non-stationary at a certain level. However, the null hypothesis for the KPSS test is that a series is stationary around a deterministic trend, and the alternative hypothesis is that the series is difference stationary.

The series is represented as the sum of deterministic trend, random walk, and stationary error:

$$y_t = \xi t + r_t + \varepsilon_t,$$

where $r_t = r_{t-1} + u_t$, and u_t is IID $(0, \sigma_u^2)$. The test is a Lagrange Multiplier (LM) test of the hypothesis that r_t has zero variance, which means that $\sigma_u^2 = 0$. In this case, r_t becomes a constant and then the series $\{y_t\}$ is trend stationary. The test is based on the statistic:

$$LM = (1/T^2) \sum_{t=1}^{T} S_t^2 / \sigma_s^2,$$

where $S_t^2 = \sum_{t=1}^{T} e_t$, e_t is the residual term from the regression of series y_t on a intercept, σ_s^2 is the estimation value of the variance of e_t, and T is the sample size. If the value of LM is large enough, the null of stationary for the KPSS test is rejected.

Table 1 reports the results of unit roots tests of logarithmic levels and first differences of stock prices and stock index futures prices. This table indicates that both series are non-stationary under their level, since the ADF t-statistic is insignificant and the LM-statistic is significant. After being differentiated once, the ADF t-statistic changes to being significant and the LM-statistic becomes insignificant, so that the two differentiated series turn to being stationary and the logged spot and logged futures prices are I (1) processes. According to Enders (1995), when two series are both I (1) processes, there may exist cointegration between them.

Table 1: Tests for Unit Roots

	ADF Tests t-statistic	**KPSS Tests LM-statistic**	
Neither Trend nor Intercept			
LCP LSP DLCP DLSP	-0.545780 -0.599671 -12.20973*** -12.80585***		
Critical Values			
Level ADF	1% -2.565842	5% -1.940944	10% -1.616618
Trend and Intercept			
LCP LSP DLCP DLSP	-2.041124 -2.000197 -12.21948*** -12.81896***	0.809612*** 0.773838*** 0.110228 0.101630	
Critical Values			
Level ADF KPSS	1% -3.961534 0.216	5% -3.411517 0.146	10% -3.12762 0.119
Intercept			
LCP LSP DLCP DLSP	-2.061830 -2.017024 -12.21791*** -12.81586***	0.885481*** 0.795064*** 0.105976 0.098758	
Critical Values			
Level ADF KPSS	1% -3.432645 0.739	5% -2.86244 0.463	10% -2.567294 0.347

Notes: For the ADF tests, *** represents that the series is stationary at the 99% confidence level; for the KPSS tests, *** means that the series is non-stationary at the 99% confidence level. LCP and LSP are the logarithm of spot closing and futures settlement prices, respectively. DLCP and DLSP are the differenced logarithm of spot and futures prices, respectively.

Table 2 shows the results of the Johansen and Juselius (1990) cointegration test and the supplement model selection-criteria method. The former tests the hypothesis of r cointegrating vectors versus (r+1) cointegrating vectors (the maximum eigenvalue test), and the latter tests for the existence of r cointegrating vectors (the trace test), both of them are undertaken on logarithmic spot and futures prices. Under the null hypothesis of no cointegrating vector, both tests strongly reject the null hypothesis; however, under the hypothesis that there exists a single cointegrating vector, both tests fail to reject it. After testing, we figure out that there exists a cointegration relationship between the series with rank of one. The result resembles that of the model selection-criteria method, in which the statistic of each criterion (AIC for Akaike Information Criterion, SBC for Schwarz Bayesian Criterion) reaches the largest value when the cointegrating rank equals one.

Table 2: Tests for Cointegration

H0	**H1**	**Eigenvalue Test**	**Trace Test**	**H0**	**H1**
		LR-statistic	95% Critical Value	LR-statistic	95% Critical Value
r = 0	r < 1	123.1953**	19.38704	r = 0	r < 1
r = 1	r < 2	3.571712	12.51798	r = 1	r < 2

Choice of the Number of Cointegrating Relations Using Model Selection Criteria

Rank	AIC	SBC
r = 0	-12.42866	-12.38857
r = 1	-12.45316#	-12.40193#
r = 2	-12.45082	-12.38845

Notes: Cointegration LR Test Based on Maximum Eigen value of the Stochastic Matrix and Trace of the Stochastic Matrix. r represents the number of linearly independent cointegrating vectors. Trace statistic $=-T\sum_{i=r+1}^{n}\ln(1-\lambda_i)$; Eigenvalue statistic $= -T\ln(1-\lambda_{r+1})$, where T is the number of observations in Johansen and Juselius (1990). AIC = Akaike Information Criterion, SBC = Schwarz Bayesian Criterion.# marks the largest statistic value for a certain criterion. ** denotes the significance level of 5%.

5 Empirical Results

5.1 Results from Models 1, 2, 3, 4, and 5

The estimation of Equation (3), with the OLS being applied, is presented as follows:

$$\Delta S_t = -0.00003087 + 0.7837\,\Delta F_t + e_t,$$

where $\Delta S_t = \mathrm{Ln}(CP_t/CP_{t-1})$, $\Delta F_t = \mathrm{Ln}(SP_t/SP_{t-1})$, and e_t is the residual of the regression. The estimated MVHR is 0.7837, which is significant at the 99% level, and R^2 is 0.8341. However, the model results exhibit problems of both serial correlation and heteroskedasticity. To minimize such problems in our time-series data and to improve the consistency of the OLS estimations, we further employ Newey-West (1987) estimators, with the results being corrected as:

$$\Delta S_t = -0.00007320 + 0.6129\,\Delta F_t + e_t.$$

Table 3: Estimates of Vector Error Correction Model

	DLCP		DLSP	
	Coefficient	Std. Error	Coefficient	Std. Error
Cointegrating Equation (Z_{t-1})	0.0311* (λ_s)	0.0175	0.0861*** (λ_f)	0.0202
DLCP (-1)	0.0013	0.0511	0.2537***	0.0589
DLCP (-2)	0.0594	0.0494	0.2185***	0.0571
DLSP (-1)	0.0503	0.0443	-0.2244***	0.0511
DLSP (-2)	-0.0237	0.0432	-0.1391**	0.0499

Cointegrating Relationship

	LCP_{t-1}	LSP_{t-1}
Coefficient	1.0000	-1.001494

Notes: This table report the results estimated from the VECM model in Equation (6). The coefficients of cointegration equation are λ_i and λ_j in Equation (6). The DLCP (.) and DLSP (.) represent the coefficients of each lag from 1 to 10 for the differenced logarithm of spot and futures prices, respectively. The statistically significant coefficients are marked with *, **, and *** to show each coefficient's significance at 90%, 99%, and 99.9% level, respectively. The cointegration relationship is LCP_{t-1} = -**1.001494**LSP_{t-1}.

According to Schwarz's Bayesian Information Criterion (BIC), the appropriate lag length of the bivariate VECM model is ten.[4] Tables 3 and 4 show the associated VECM test results, indicating that for both equations, the coefficients of the error correction term are statistically significant. Since $\lambda_s<\lambda_f$ (0.0311 vs. 0.0861), the spot price series S_t have a slower speed of adjustment to the previous period's deviation from the long-run equilibrium than do the index futures price series. Such findings suggest that the futures price has to adjust itself to the spot price on the delivery date.

Table 4: ARCH LM Test and White Heteroskedasticity Test on the Residuals from VECM

ARCH LM Test	χ^2	Prob.
e_{st}	257.2876	0.0000
e_{ft}	239.9920	0.0000
White Heteroskedasticity Test		
$e_{st} . e_{st}$	265.1495	0.0000
$e_{st} . e_{ft}$	461.5788	0.0000
$e_{ft} . e_{ft}$	298.0486	0.0000

Notes: The ARCH LM Test is Engle (1982)'s Lagrange Multiplier (LM) Statistic for Autoregressive Conditional Heteroskedasticity under the null hypothesis of no ARCH effect. The White Heteroskedasticity Test tests for heteroskedasticity in the residuals, and the asymptotically distribution of test statistic is χ^2 under the null hypothesis of no heteroskedasticity. e_{st} and e_{ft} represent the respective residuals of ΔS_t and ΔF_t from VECM in Equation (6).

[4] The results for the VECM order selection can be provided upon request.

Figure II plots the two streams of residuals from Equation (6), exhibiting volatility clustering, but the mean seems constant around zero.

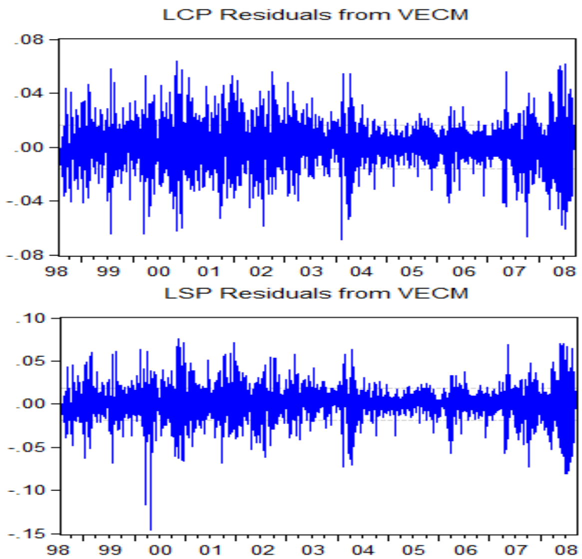

Figure II: The Plot of Residuals from VECM

According to Mandelbrot (1963)and Engle (1982), there exists an autoregressive conditional heteroskedastic (ARCH) effect. We thus apply the other three models- GARCH (2,2), TGARCH (2,2,1), and BGARCH (1,1) - to correct for the presence of heteroskedasticity.[5]We also incorporate the error correction term into Model 1 as the mean equation for the above three models. The estimation results are reported in Tables 5 and 6.

Specifically in Table 5, the parameters α_1, α_2 and $\beta_1$are significant at the 1% level for the GARCH (2,2) model. Testing for the ARCH (1) and ARCH (2) effects, we do not reject the null hypothesis of no ARCH effects at the 5% level, and thus heteroskedasticity is corrected for. The sum of ARCH and GARCH coefficients (α_1 + β_1+ α_2) is 0.9522, very close to unity and showing that old shocks have an impact on current variance and this effect is permanently remembered. In addition, a TGARCH (2,2,1) model was estimated and all the parameters are significant at the 1% level except β_2. As with the GARCH (2,2) model, the test for ARCH (1) and ARCH (2) effects are both insignificant to show the correction of the heteroskedasticity. Since the leverage effect term γ is positive and statistically significant at the 1% level, there exists a leverage effect in which negative shocks (bad news) generate more volatility than positive shocks (good news).

[5]The results for the GARCH, TGARCH, and BGARCH order selection based on AIC can be provided upon request.

Table 5: Results from the GARCH model and TGARCH model

GARCH (2,2)				
Variable	Coefficient	Std. Error	*z*-Statistic	Prob.
ΔF_t	0.838993	0.004048	207.2426	0.0000
$(\varepsilon_{t-1})^2$	0.252835	0.024464	10.33513	0.0000
$(\varepsilon_{t-2})^2$	-0.202426	0.024097	-8.400428	0.0000
$(\sigma_{t-1})^2$	0.904331	0.057481	15.73278	0.0000
$(\sigma_{t-2})^2$	0.047304	0.053973	0.876446	0.3808
Adjusted R^2	S.E. of regression	ARCH test (*p*-value)		
0.834513	0.006602	0.6448		
TGARCH (2,2,1)				
Variable	Coefficient	Std. Error	*z*-Statistic	Prob.
ΔF_t	0.836930	0.004214	198.6161	0.0000
$(\varepsilon_{t-1})^2$	0.239832	0.024890	9.635541	0.0000
$(\varepsilon_{t-1})^2 D_{t-1}$	0.036295	0.007434	4.882214	0.0000
$(\varepsilon_{t-2})^2$	-0.214821	0.024691	-8.700459	0.0000
$(\sigma_{t-1})^2$	0.886279	0.051624	17.16795	0.0000
$(\sigma_{t-2})^2$	0.071109	0.049223	1.444638	0.1486
Adjusted R^2	S.E. of regression	ARCH test (*p*-value)		
0.834936	0.006593	0.5793		

Notes: This table reports the estimates from the GARCH model in Equation (7) and TGARCH model in Equation (8). $\Delta F_t = Ln(SP_t / SP_{t-1})$, and the coefficient of ΔF_t is the minimum variance hedge ratio.

We finally examine a BGARCH to correct for heteroskedasticity, and the results are presented in Table 6. We use the diagonal-vech model and matrix-diagonal model to estimate all coefficients c_{ij}, α_{ij}, and β_{ij} simultaneously and all estimates are positive definite and significant at the 1% level. Moreover, the sum of each equation is close to unity (for example, $c_{ss} + \alpha_{ss} + \beta_{ss} = 0.98733$), showing the persistence of shockimpacts. The minimum-variance hedge ratios, measured by the coefficients of ΔF_t, amount to 0.838993, 0.836930 and 0.794622 for GARCH(2,2), TGARCH(2,2,1) and BGARCH(1,1) estimations, respectively; and all such estimates are significant at the 0.01 level.

Table 6: Results from the BGARCH (1,1) Model

Variable	Coefficient	Std. Error	*z*-Statistic	Prob.
ΔF_t	0.794622	0.005350	148.5275	0.0000
c_{ss}	0.000004	0.000001	7.788508	0.0000
c_{sf}	0.000004	0.000001	8.526319	0.0000
c_{ff}	0.000005	0.000001	8.997206	0.0000
α_{ss}	0.069656	0.004956	14.05472	0.0000
α_{sf}	0.071881	0.004877	14.73837	0.0000
α_{ff}	0.077761	0.005119	15.19184	0.0000
β_{ss}	0.917671	0.005025	182.6101	0.0000
β_{sf}	0.913125	0.005087	179.5131	0.0000
β_{ff}	0.908600	0.006332	143.4949	0.0000

Notes: This table reports the results estimated from the BGARCH model in Equation (9). c_{ss}, c_{sf} and $c_{ff}$are constants; α_{ss}, α_{sf} and α_{ff} are coefficients of the squared error terms; β_{ss}, β_{sf} and β_{ff} are coefficients of the conditional variances and covariances. $\Delta F_t=Ln(SP_t/ SP_{t-1})$, and the coefficient of ΔF_t is the minimum variance hedge ratio.

5.2 Hedging Effectiveness Comparison

So far five models have been employed in our study to estimate the MVHR. To evaluate the hedging effectiveness and forecasting accuracy of each model, we introduce a rolling sample method to estimate the five respectivetime-varying MVHRs for the out-of-sample period. As aforementioned in Section 4, our complete sample time series consist of a total of 3,146 daily observations, in which the first 2,644 observations (07/21/1998 – 12/31/2008) are used for measuring the MVHRs, and the remaining 502 observations (01/01/2009 – 12/31/2010) for the out-of-sample forecast. According to Baillie and Myers (1991) and Park and Bera (1987), the returns on the portfolio can be expressed as:

$$r_u = \Delta S_{t+1} - \Delta S_t,$$
$$r_h = \Delta S_{t+1} - \Delta S_t - h^* (\Delta F_{t+1} - \Delta F_t), \qquad (10)$$

where r_u is the return of un-hedged portfolios, r_h is the return of hedged portfolios, and h^* is the MVHR. The mean and variance of un-hedged and hedged portfolios can be obtained as follows:

$$E(U) = E(r_u),$$
$$E(H) = E(r_h),$$
$$Var(U) = Var(r_u),$$
$$Var(H) = Var(r_h), \qquad (11)$$

Following Johnson (1960), we compute hedging effectiveness by Equation (2) to compare hedging performances. In addition to hedging effectiveness, the risk-return trade-off is also compared with in various hedge horizons of one-day, one-week, and one-month, presuming the performance may vary (Lien and Tse, 1999). Under the rolling sample method, all estimated MVHRs vary with time, while transaction costs remain constant and thus their effects are ignored.

The mean of the hedge ratio is the average of the time-varying hedge ratio estimated by each model during the out-of-sample period. The mean and variance of the return of the portfolio, and percentage in variance reduction, are calculated by Equations (11) and (2), respectively.

Table 7 summarizes the comparisons of the MVHR estimated by alternative methods. There are several issues noteworthy. Firstly, under a one-day hedge horizon, a trade-off between risk and return occurs. Although the OLS model generates the highest daily return, its resulting variance is also the largest. On the other hand, the TGARCH model yields the smallest variance and the largest variance reduction, but this is accompanied by a smaller daily return.

Table 7: Hedging Performances Comparison

Hedge Horizons	Mean of the Hedge Ratio	Mean of the Return of the Portfolio	Variance of the Return of the Portfolio	Percentage in Variance Reduction
One-Day				
UN-HEDGE	0	0.133470%	0.017286%	0%
NAÏVE	1	-0.004281%	0.002249%	86.99%
OLS	0.7852763	**0.025222%**	0.001566%	90.94%
VECM	0.7986924	0.023367%	0.001546%	91.06%
GARCH (2,2)	0.8373481	0.018014%	0.001534%	91.13%
TGARCH (2,2,1)	0.8359688	0.018078%	**0.001530%**	**91.15%**
BGARCH (1,1)	0.8059272	0.022686%	0.001663%	90.38%
One-Week				
UN-HEDGE	0	0.680767%	0.080844%	0%
NAÏVE	1	-0.023044%	0.006157%	92.38%
OLS	0.8634001	0.071162%	**0.003719%**	**95.40%**
VECM	0.8871256	0.053476%	0.003858%	95.23%
GARCH (2,2)	0.8760147	0.063974%	0.003760%	95.35%
TGARCH (2,2,1)	0.8800202	0.059750%	0.003793%	95.31%
BGARCH (1,1)	0.8491698	**0.083348%**	0.004340%	94.63%
One-Month				
UN-HEDGE	0	2.791744%	0.487517%	0%
NAÏVE	1	-0.255422%	0.029286%	93.99%
OLS	0.9067244	0.001005%	**0.018851%**	**96.13%**
VECM	0.9518758	-0.012258%	0.022328%	95.42%
GARCH (2,2)	0.9663665	-0.166920%	0.024636%	94.95%
TGARCH (2,2,1)	0.9710556	-0.166824%	0.024698%	94.93%
BGARCH (1,1)	0.8894014	**0.119150%**	0.020180%	95.86%

Under the one-week and one-month hedge horizons, the BGARCH model provides the highest return, while the OLS model generates the smallest variance and largest variance reduction. Hence, which model is more appropriate for hedging purpose seemingly depends on the investor's degree of risk aversion. Secondly, within one-week and one-month hedge horizons, the OLS model has the largest variance reduction (similar to

Holmes, 1996), but it is not the case for the one-day hedge horizon (different from Lypny and Powalla, 1998). Such inconsistency in findings may be attributed to that prior researchers did not use the rolling sample method that we have employed. Hence, our evidence suggests that the same model could lead to different hedging performances under various hedge horizons (in line with Lien and Tse, 1999), but which specific model should be considered superior for a specific hedge horizon does not have a clear-cut answer. Thirdly, a longer hedge horizon is associated with a greater average of MVHR and variance reduction, no matter which model is adopted (supporting Holmes, 1996). Finally, the average MVHR of the OLS model, which does not account for cointegration, is smaller than the other models (consistent with Ghosh 1993a, 1993b).

6 Summary and Conclusions

This study uses a variety of models to estimate MVHRs and thus examine the hedging effectiveness of the TAIFEX stock index futures. Besides investigating across alternative hedge models over the 07/21/1998 – 12/31/2008 sample estimation period, we implement the rolling sample method to evaluate time-varying MVHRs of various models for the 01/01/2009 – 12/31/2010 out-of-sample forecast period. To examine each model's appropriateness for measuring MVHRs, we conduct cross-model comparison of hedging performance in terms of hedging effectiveness and risk-return trade-off.

In the one-day hedge horizon, the TGARCH model generates the largest variance reduction, whereas the OLS model provides the highest rate of risk-adjusted return. In the longer hedge horizon, however, the OLS generates the largest variance reduction, while the BGARCH model provides the highest rate of risk-adjusted return. As the risk-return trade-off occurs, the investor's degree of risk aversion plays an important role in choosing the appropriate model to measure the MVHRs. Such findings differ from Yang and Allen (2005) and other prior literatures, possibly due to that most of those previous studies did not adopt the rolling sample method to get time-varying hedge ratios for constant hedge ratio models such as the OLS. The earlier studies, instead, focus on the "constant" and "time-varying" hedge ratio such as BGARCH.

Our study estimates all MVHRs by each model varying with time, and we also take account of different hedge horizons, which in turn affect the hedging performance (supporting Figlewski, 1984; Lien and Tse, 1999). A longer hedge horizon accompanies a larger average of MVHR and variance reduction, which is consistent with earlier findings (Chou et al., 1996; Holmes, 1996;Kenourgios et al., 2008). This evidence indicates that as the hedge horizon increases, the variance of the return of spot and futures prices also increases; but the increased variance of the return of futures prices is smaller than that of spot prices. Thus, the hedge ratio will grow larger so as to hedge the more volatile spot prices. Finally, the average MVHR of the OLS model, which does not account for cointegration, is smaller than the other models, and this is in line with both previous empirical findings(e.g., Yang and Allen, 2005; Kenourgios et al., 2008) and the underlying theorems (Ghosh, 1993a, 1993b). In summary, our findings, collected from an emerging market of stock index futures, can meaningfully extend the scope of similar research works which have been mainly focusing on those presumably more developed and efficient markets. In the implementation process of hedging stock market risk, the application appropriateness of various models could be mixed, with being country/region/market depth-specific in some aspects yet consistent in some others. Further studies are needed to investigate other futures markets and contract types, and/or to incorporate additional measurements of hedging performance (e.g., Pennings and Meulenberg,1997).

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