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What are the benefits of globally invested mutual funds? Evidence from statistical arbitrage models

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Abstract

Even though correlations between different economies' stock markets have empirically increased over time, it would have been advantageously to invest in developing countries' stock markets such as the Indian stock market, instead of investing in the US-stock market when considering the overall market returns of the last decade. Anticipating beneficial asset allocations is challenging since higher returns are basically associated with higher risks. The estimation procedure which is employed in this study to construct globally invested portfolios is based on cointegration analysis. The forecast period covers 11 years. All constructed portfolios show a strong cointegration relationship with the benchmark in the back-testing period while generating statistically significant abnormal returns between 2.44%-11.96% p.a. The Sharpe ratios are estimated to be 0.1447-0.4261 and clearly higher in comparison to the benchmark's Sharpe ratio of 0.0373 within the out-of-sample period running from 08-01-1999 – 07-31-2010.

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1 Introduction

An important issue of modern portfolio management is to invest globally. A relevant reason to invest in foreign stocks has been that markets have historically tended to move independently; when one is up, the other might be down, and vice versa. In statistical terms, such markets are said to lack correlation. Investing in different assets being uncorrelated reduces the portfolio risk by diversification. Even though [4] figure out, that correlations between different economies' stock markets have empirically increased over time, it would have been advantageously to invest in India's stock market instead of the US-stock market when comparing the overall return concerning the last 14 years, for instance. Figure 1 shows the developments of two selected investment funds in price levels. While the investment fund DWS Nordamerika invests mainly in the US-stock-market, the DWS India is invested in India, only. Even though the correlation between those two assets was on average 0.44 from 08-01-1996 to 05-31-2010, the realized annual return of the investment fund DWS India was 20.30% in comparison to 5.99% which was the corresponding figure of the DWS Nordamerika.

Since thirty years ago, the advantage of investing internationally was according to [21] not a new concept. However, [7] report that 94% of American investors who buy stocks invest in domestic stocks, only. The corresponding figure among Japanese and British investors is 98%, respectively, 82%. This phenomena, often referred to as "home bias", can in future periods be a determining factor concerning differences of the investors' future consumption possibilities.

As a consequence of growing interdependence among the international markets due to growing international trade, investment flows and increasing international financial transactions, the benefits of international portfolio diversification may be lowered, but in accordance to [15] still verifiable. Assuming a high correlation among international stock markets, the benefits of a globally invested stock portfolio may not rest upon risk minimization aspects, but on the possibility to gain higher returns. However, recent realized returns can be highly misleading estimates of expected future returns. But is it possible to forecast abnormal returns or, respectively, to apply investment strategies that exploit potential advantageous future trends of different international stock markets? An investment strategy could be, for instance, to allocate the same weight to every asset employed. The risk of such a portfolio should be lower but the returns remain uncertain. As stock markets exhibit random walk behavior, there exists ex ante no mechanism to build reasonable expectations concerning the forecast period.

In the following, ten portfolios are constructed which are invested in global stock markets. As proxies for stock markets eight different mutual funds are employed which are mainly invested in domestic, respectively, foreign stock markets. The portfolios are estimated by cointegration analysis and show strong cointegration relationships with the benchmark even within forecast period running from 08-01-1999 to 07-31-2010. Even though these portfolio allocation strategies are associated with an increase in portfolio-risk, the estimated Sharpe-ratios (i.e. 0.1447-0.4253) concerning the out-of-sample period are clearly higher than the benchmark's Sharpe-ratio which is estimated to be equal to 0.0373 within the corresponding period.

2 Background

The statistical tool to construct portfolios that may outperform the underlying stock index, respectively, anticipate advantageously future trends involved in stochastic processes, such as asset prices, can be based on enhanced index tracking resting basically upon studies by [20]. Then, self-financing statistical arbitrage portfolios can according to [1] and [6] be set up as the difference between the portfolio tracking the enhanced benchmark and the ordinary benchmark.

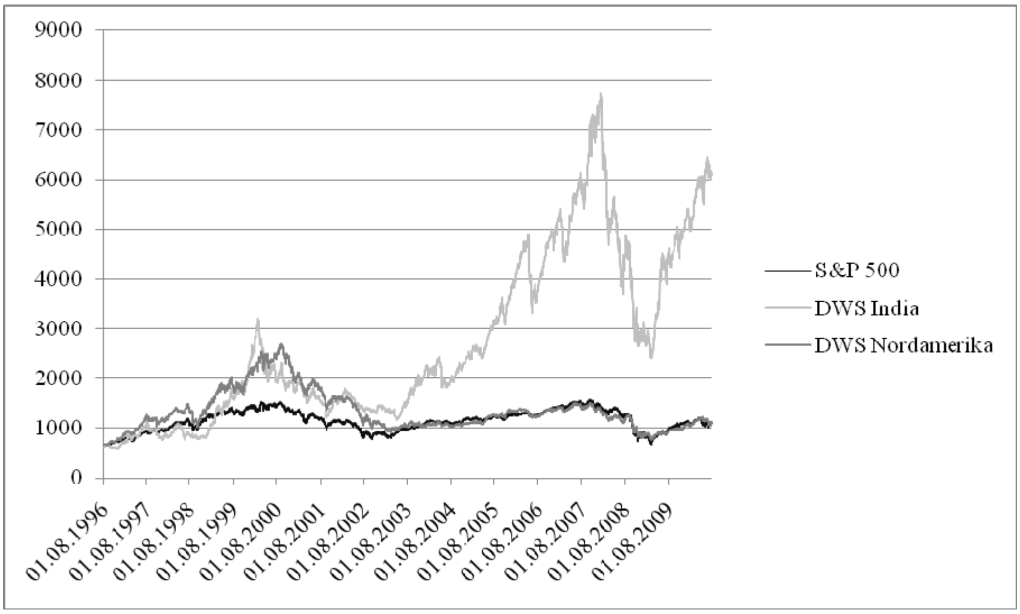

Figure 1: Standardized time series of the DWS Nordamerika and the DWS India from 1996-2010[2]

The statistical arbitrage portfolio is expected to generate returns according to the

[2] Figure 1 plots the data from 08-01-1996 to 05-31-.2010. The daily data is available for free on the funds provider's homepage https://www.deami.de/dps/default.aspx.

spread between benchmark and enhanced benchmark with low volatility as mentioned by [1]. Going long on the portfolio tracking the enhanced index while going short on the underlying benchmark will in line with [3] result in a zero-cost statistical arbitrage portfolio exhibiting a conditional expected payoff which is nonnegative. Thereby, this zero-cost trading strategy involves an adjustment of the portfolio-beta such that the factor risk cancels out across the long and short position. In contrast to pure arbitrage opportunities, statistical arbitrage strategies exhibit negative payoffs only stochastically.

[1] argue that correlation based statistical arbitrage strategies such as introduced by [3] may be very sample specific and unstable out-of-sample, especially under volatile market conditions. Their empirical analysis shows that the tracking error of correlation based models usually exhibits out-of-sample random walk behavior. As correlation is according to [1] a short-run measure, hedging strategies resting upon correlation analysis commonly require frequent rebalancing involving high trading costs. [1] mention furthermore that correlation lacks stability as there is no mechanism ensuring the reversion of the hedge to the underlying.

[1] apply hedging strategies based on cointegration analysis. In contrast to correlation, cointegration methods work directly on portfolio values and make no assumption concerning stationarity of the asset values. Cointegration refers to the fact that financial time series share common stochastic trends causing them to move toward the long-term equilibrium after every shock. Cointegration is according to [1] and [8] a property of some integrated stochastic processes: If two or more integrated time series are cointegrated, a linear combination relationship being stationary is said to exist. In the context of asset allocation theory, whether the value series of a fixed weight portfolio of assets with integrated prices is stationary, the assets will in line with [12] exhibit a cointegrated set. The set of these asset weights which generate such a portfolio is called the cointegrating vector.

In contrast to correlation, cointegration is a long-run measure ensuring the portfolio to revert to the underlying over the longer term. Since prices are long-memory stochastic processes, cointegration is in accordance to [8] able to explain assets' long-run behavior. [1] show that cointegration based statistical arbitrage strategies exhibit even out-of-sample stationary tracking errors that are uncorrelated with the stock market. They consider six plus/minus benchmarks by adding and subtracting annual returns of 5%, 10%, and 15% to, respectively, from the reconstructed Dow Jones Industrial Average Index (DJIA) returns, uniformly distributed over time.[3] Then, they construct portfolios tracking these artificial benchmarks. The statistical arbitrage strategy is set up by being long on the plus and short on the minus tracking portfolio. Their empirical analysis including 10 years out-of-sample (i.e. 1993-2003) shows that narrow spreads such as 5% hedged exhibit the best performance resulting in statistically significant abnormal returns.

Furthermore, [2] analyze the performance of different long/short strategies developed in the S&P 100 stock index. Thereby, their models imply an extensive search for the best long-short combination over a large number of portfolios constructed on cointegration relationships and optimized on model parameters such as training period, targeted tracking error, as well as the number of assets included in the portfolio. The outcomes, even if based on a black box selection algorithm, show evidence that cointegration-based optimization procedures can ensure stable out-of-sample alphas with low volatility while being uncorrelated with the underlying stock market.

In another application of cointegration analysis to investment management that is relevant to this line of research, [5] constructs cointegration based index-tracking portfolios in order to replicate the Swedish leading stock index

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[3] The DJIA is a price weighted index showing how 30 large publicly owned companies based in the United States have traded during a standard trading session in the stock market.

OMX 30.[4] Thereby, the cointegration based tracking portfolios exhibit better Sharpe- and Treynor-Ratios in comparison to correlation based models. Even though [5] focus does not lay on arbitrage, the cointegration based index-tracking portfolios exhibit out-of-sample statistically significant abnormal returns between 7.62%-7.97% p.a. above the Swedish stock market. Furthermore, [6] analyzes trading strategies which are aimed at exploiting stock markets' stochastic oscillations occurring due to different states of the business cycle. Thereby, he constructs artificial indices based on transformed prices processes being extended by switching linear trend-stationary stochastic processes. The constructed cointegration-optimal portfolio shows within the out-of-sample period statistically significant abnormal net-returns of 6.83% p.a. As long as cointegration holds and the linear trend term concerning the cointegration relationship is stable over time (i.e. within the forecast period), the statistical arbitrage is according to [6] equal to a trend-stationary stochastic process incorporated in the data.

In the following contribution it is analyzed how high the performance of cointegration based portfolio allocation strategies is while investing in different economies. Thereby, the artificial indices tracked are enhanced with ten different factors between 5%-50% p.a., uniformly distributed over time which is also in line with [1]. Instead of employing different countries' stock indices, investment funds are employed investing mainly in different economies. Thus, these time series which are employed in order to replicate the artificial indices need not to be adjusted for the exchange rate, as the mutual funds taken into account are traded in EUR, only. Table 1 shows the investment funds of the company DWS which are employed to estimate the models. DWS Investments, a member of Deutsche Bank Group, is one of the world's leading asset management companies (see www.dws.com).

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[4] The Swedish leading stock index OMX 30 measures the performances of 30 companies in the Swedish stock market which exhibit the highest market capitalization.

3 Econometric Methodology

In line with [1] and [6] the cointegration approach will be applied to track enhanced benchmarks constructed by adding an annual excess return of δ% to the underlying index returns, uniformly distributed over daily returns. Self-financing statistical arbitrage portfolios can be constructed as difference between two portfolios tracking a plus benchmark and the ordinary benchmark.

Table 1: DWS Investment funds and different economies[5]

Asset *i*	Investment fund	ISIN	Mainly invested economy (%)
1	DWS Akkumula	DE0008474024	USA (28.30%)
2	DWS Emerging Asia	LU0045554143	China (26.50%)
3	DWS Eurorenta	LU0003549028	Germany (28.80%)
4	DWS India	LU0068770873	India (100%)
5	DWS Lateinamerika	LU0055698269	Brasilia (59.40%)
6	DWS Nordamerika	DE0008490897	USA (85.70%)
7	DWS Osteuropa	LU0062756647	Russia (65.10%)
8	DWS Top 50 Europa	DE0009769729	Europa (100%)

[5] Note: The percentage concerning the main invested economy is based on the 12. June 2010.

Then, the zero-cost trading strategy is expected to generate abnormal returns according to the plus spread with low volatility. In contrast to [1] who employ log prices, in the following, transformed time series are employed as proposed by [6]. Given the returns R_{it}, where $R_{it} = (P_{it} - P_{it-1}) \cdot 100 / P_{it-1}$ and P_{it} denotes the price of mutual funds i at time t, where the index $i = 1,...,I$ denotes the mutual funds employed as proxies for different economies (see table 1), and given the index returns $R_t^{S\&P500}$ with $R_t^{S\&P500} = (P_t^{S\&P500} - P_{t-1}^{S\&P500}) \cdot 100 / P_{t-1}^{S\&P500}$ the time series are re-trended such that

$$y_t = \alpha_0 + \sum\nolimits_{j=1}^{t} R_j^{S\&P500} \tag{1}$$

$$x_{it} = \alpha_0 + \sum\nolimits_{j=1}^{t} R_{ij} , \tag{2}$$

where α_0 is a constant term. Testing for integration shows that these transformations do not change the order of integration, as shown by [6]. Both time series y_t and x_{it} are integrated of order one, that is $y_t \sim I(1)$ and $x_{it} \sim I(1)$, as it is the case for the ordinary time series in price levels or the logarithm of the latter. However, taking the first difference Δy_t and Δx_{it}, respectively, it is ended up with the ordinary return series $R_t^{S\&P500}$ and R_{it} again. Employing such transformations enables according to [6] to price the out-of-sample time series of the estimated portfolios with Vector-Error-Correction-Models (VECM), where the linear trend parameter equals the abnormal returns being generated (see equation 7).

The optimization method which is employed in order to figure out cointegration-optimal allocation weights concerning the in-sample-periods is Quasi-Maximum Likelihood-Estimation (QMLE), given by

$$\log L(\theta, t) = -\frac{T}{2}\log(2 \cdot \pi) - \frac{T}{2}\log\sigma^2 - \frac{1}{2}\sum_{t \in T}\left(\frac{\varepsilon_t^2}{\sigma^2}\right), \tag{3}$$

where $\varepsilon_t = y_t^* - \sum_{i=1}^{I} a_i \, x_{it}$.

Thereby, the vector y_t^* denotes the transformed index prices, enhanced with the factors $\delta_j = \delta_1, ..., \delta_J$ as following:

$$y_t^* = \alpha_0 + \sum_{j=1}^{t} R_j^{S\&P500} + t \cdot \delta_j / T_A , \tag{4}$$

where T_A denotes the number trading days within one year and $t = 1, ..., T$ is the in-sample period used to estimate the portfolio weights. As a consequence, the Maximum-Likelihood functions will estimate different weight allocations for each enhanced index employed. The estimated cointegration-optimal weights are held constant one year ahead and will be iteratively re-estimated while employing a rolling time-window of three years which is also in line with [1] who argue that at least three years of daily data are necessary to estimate stable cointegration-optimal weights. According to [5], the estimated parameters are restricted to sum up to one and to be positive as given by equations (5) and (6),

$$\sum_{i=1}^{I} \hat{a}_i = 1 \tag{5}$$

$$\hat{a}_i > 0 \quad \text{for} \quad i = 1, ..., I . \tag{6}$$

In order to price the realized portfolios (i.e. out-of-sample performance) accurately, it is essential to employ an appropriate asset pricing model. [6] suggests employing the VECM in order to price the portfolios' out-of-sample processes. He argues that the long run beta estimators within the VECM framework are superconsistent as long as the constructed portfolios have a cointegration relationship with the benchmark. Furthermore, as a result of the latter, long-run forecasts regarding the portfolios' out-of-sample behaviour lead to more sufficient estimates, as the cointegration relationship causes the portfolio and the benchmark to be tied together through the beta.

Thereafter all portfolios are tested whether they have also within the forecast period a cointegration relationship the benchmark. Thereby, the cointegration test as proposed by [17], [18] and [19] will be employed. If cointegration holds, the out-of-sample portfolio processes can be estimated by employing the VECM which is in line with [9], [10] and [11] given by

$$\Delta Y_t = \alpha \left(\beta\,' Y_{t-1} - \mu_0 - \mu_1 t\right) + \sum_{j=1}^{l} \Gamma_j \Delta Y_{t-1} + u_t, \tag{7}$$

where $\Delta Y_t = \left(R_{jt}^{*}, R_t^{S\&P500}\right)$, $Y_t = \left(P_{jt}^{*}, y_t\right)$ with $P_{jt}^{*} = \hat{a}_1 x_{1t} + ... + \hat{a}_I x_{It}$, Γ_j are parameter matrices and $u_t \sim N\left(0, \Sigma\right)$. The index $j = 1, ..., J$ denotes the constructed portfolios concerning the corresponding enhancement factors $\delta_j = \delta_1, ..., \delta_J$ (see equation 4). The cointegration-optimal weights $\hat{a}_1, ..., \hat{a}_I$ which are individual for every portfolio (see appendix) are iteratively re-estimated every year. After estimating the VECM of each out-of-sample portfolio process, the estimated linear trend term $\hat{\mu}_1$ will be tested for significance where the pair of hypothesis is given by

$$H_0 : \hat{\mu}_{j1} = 0 \qquad \text{vs.} \qquad H_1 : \left|\hat{\mu}_{j1}\right| > 0 .$$

According to the seminal work of [9] and [11], the corresponding test statistic λ_j will be chi-square distributed with one degree of freedom. Under the null hypothesis, the estimated processes do not exhibit abnormal returns, whereas the latter will be statistically significant if the null hypothesis is rejected. As transformed price series are employed, the generated annual abnormal returns are with respect to the VECM framework given by $\left|\hat{\mu}_{j1}\right| \cdot T_A$ as long as daily data is taken into account.

4 Results

The investment funds data is available for free on the index provider’s homepage https://www.deami.de/dps/default.aspx. Data of the S&P 500 index is downloaded from the database yahoo. Index notations of US-indices may deviate from price notations in European countries due to red-letter days, for instance.

After adjustments $T_A = 244$ trading days on average p.a. can be taken into account exhibiting price notations for both, the US-stock index and the mutual funds employed. Operating with long-run data involves that researchers may face a survivorship bias. That means that there are only certain investment funds available for which daily data is available until 08-01-1996. The in-sample period used to estimate the portfolio weights concerning the initial allocation runs from 08-01-1996 to 07-31-1999 including three years of highly frequented daily observations. The overall out-of-sample period runs from 08-01-1999 to 07-31-2010 including 2684 daily observations. In line with [1], [5] and [6] the cointegration-optimal weights are iteratively re-estimated every year and held constant one year ahead. Hence, the portfolios are rebalanced ten times. In the appendix the weight allocations of all constructed portfolios can be considered.

All transformed prices are integrated stochastic processes of order one. The enhancement factors δ_j differ concerning each constructed portfolio and run from 5%-50% so that ten different portfolios are estimated tracking individual artificial enhanced indices. Thereby, the constant term α_0 is set equal to 100 for all transformed asset price processes as well the artificial S&P 500 indices. Apart from that another portfolio is estimated having equally allocated weights (see table 2 portfolio 11). Thereby, it is assumed that the annual turnover of portfolio 11 is equal to 30%. Table 2 shows that portfolio 10 that tracks the most aggressive artificial enhanced index accounting for an index enhancement factor of 50% p.a., exhibits compared to all other estimated portfolios or, respectively, the benchmark, the highest annual returns (i.e. 13.60% p.a.) as well as the highest Sharpe-ratio (i.e. 0.4253) in the back-testing period.

Portfolios 1 and 2 tracking artificial indices enhanced by a factor of 5%, respectively, 10% in annual terms exhibit 4.43% and 8.27% higher returns out-of-sample compared to the underlying benchmark. Increasing the enhancement factor from 10% to 50%, however, results in additional returns of only 4.51% p.a. while involving 3.18 percentage points in higher portfolio risk. All portfolios, apart from portfolio 11, exhibit a beta that is higher or equal to 2 and can be considered as “offensive investment strategies”. Portfolio 11 which is an equally weighted portfolio is closest to the market risk and exhibits an estimated beta of $\left|\hat{\beta}\right| = 1.25$ within the out-of-sample period.

Table 2: Statistical properties of the constructed portfolios

Portfolio	Enhancement	Returns p.a.	Volatility p.a.	Average Turnover p.a.	Net returns p.a.*	Sharpe-Ratio	Estimated $\left\|\hat{\beta}\right\|$** (VECM)
1	5%	5.25%	22.46%	39.94%	3.25%	0.1447	2.08 (9.20)
2	10%	9.09%	24.33%	48.58%	6.67%	0.2741	2.07 (10.12)
3	15%	9.56%	25.32%	57.11%	6.70%	0.2648	2.07 (14.86)
4	20%	13.07%	26.85%	48.48%	10.65%	0.3966	2.02 (13.53)
5	25%	13.06%	26.74%	46.14%	10.75%	0.4020	2.04 (14.20)
6	30%	12.40%	27.00%	49.35%	9.93%	0.3678	2.03 (14.85)
7	35%	13.25%	27.96%	40.92%	11.20%	0.4006	2.00 (13.99)
8	40%	13.54%	27.49%	38.52%	11.61%	0.4223	1.98 (13.63)
9	45%	13.35%	27.40%	38.91%	11.40%	0.4161	2.01 (13.56)
10	50%	13.60%	27.51%	37.98%	11.70%	0.4253	2.00 (13.11)
11	-	6.92%	15.64%	30.00%	5.42%	0.3465	1.25 (14.22)
S&P 500	-	0.82%	22.00%	-	-	0.0373	-

* Accounting for 5% as being the maximum transaction fee of the overall trading volume.
** Including 2 lags in accordance to the HQ-Criteria (p-values in parenthesis).

Table 2 shows that additional abnormal returns increase on a decreasing rate while the portfolio volatilities decrease for portfolios being enhanced with a factor above 35%. Even though portfolio 11 does not rest upon any optimization procedure, it has a cointegration relationship with the benchmark while generating abnormal net-returns of 5.42% p.a. above the benchmark (i.e. corresponding to 6.92% net returns p.a., see Table 2).

Table 3 shows the results of testing for cointegration. All constructed portfolios exhibit a cointegration relationship with the underlying benchmark as long as the test statistic involves a linear trend term (all p-values < 5%). The VECMs for all constructed portfolios are estimated for asset pricing in the forecast period (i.e. 08-01-1999 to 07-31-2010) including a lag-order of 2 as suggested by the HQ criterion. Testing the linear trend parameters for significance shows that all constructed portfolios generate abnormal returns within the out-of-sample period. However, increasing the enhancement factor gives only marginal increasing abnormal returns (see column 5 in Table 3 and Figure 2) while the deviations from the enhancement factors increase for both, the generated annual returns and the estimated trend parameters.

5 Discussion

The following main outcomes can be recorded: First, the constructed global portfolios exhibit the same stochastic properties like the underlying stock index (i.e. S&P 500) and, hence, may act as a hedge of the latter as suggested by [6]. Second, pricing the cointegration-optimal portfolio processes in the back-testing period shows that the trend components are highly statistically significant and account for expected abnormal returns between 2.44-11.96% p.a. (see Table 3).

Consequently, by employing a rolling time window of only three years of daily data, it is possible to figure out trend-stationary stochastic processes which are implicitly involved in the data. The stochastic trends are stable during the overall 11-years out-of-sample period. The higher the enhancement factor of the artificial index tracked the more the deviations between the returns p.a. and the corresponding enhancement factor (see Figure 2).

Different economies show various patterns of economic development, respectively, growth which may be reflected by their stock market's developments, for instance, as mentioned by [13] and [14].

Table 3: Testing for cointegration and statistical significance of the abnormal returns

Portfolio	p-value for testing cointegration (including a constant)***	p-value for testing cointegration (including a trend term)***	Testing the trend parameter $\lvert\hat{\mu}_1\rvert$ for significance (p-value in parenthesis)****	Estimated trend parameter $\lvert\hat{\mu}_1\rvert$ (in annual terms)
1	0.1797	0.0069	5.07 (0.0244)	2.44% (2.25)
2	0.3203	0.0321	48.78 (0.0000)	7.08% (6.98)
3	0.3620	0.0025	106.47 (0.0000)	7.08% (10.32)
4	0.1861	0.0033	233.86 (0.0000)	11.22% (15.29)
5	0.2053	0.0015	256.51 (0.0000)	11.22% (16.02)
6	0.2486	0.0010	243.69 (0.0000)	10.49% (15.61)
7	0.2161	0.0023	264.53 (0.0000)	11.47% (16.26)
8	0.1962	0.0031	271.61 (0.0000)	11.71% (16.48)
9	0.1926	0.0033	251.02 (0.0000)	11.47% (15.84)
10	0.1797	0.0069	77.48 (0.0000)	11.96% (15.88)
11	0.2993	0.0163	118.87 (0.0000)	4.64% (10.90)
S&P 500	-	-	-	-

*** Lütkepohl and Saikkonen test for cointegration.
**** The corresponding test-statistic is chi-square distributed with one degree of freedom.

Stock markets of strongly growing economies exhibit usually higher returns as well as higher risks in comparison to well developed economies' stock markets. This information is according to [6] and [8] implicitly cached in the assets' price processes. The economies which are accounted for in this study are represented by various mutual funds being primarily invested in different countries. Depending on how high the enhancement factor concerning the artificial index tracked is chosen the higher the uncertainty between realized abnormal returns and expected abnormal returns within the forecast period. Even though deviations between enhancement factor and realized abnormal returns are increasing, all constructed portfolios show strong cointegration relationships with the benchmark. Thereby, they involve a trend stationary stochastic process offering statistical arbitrage opportunities.

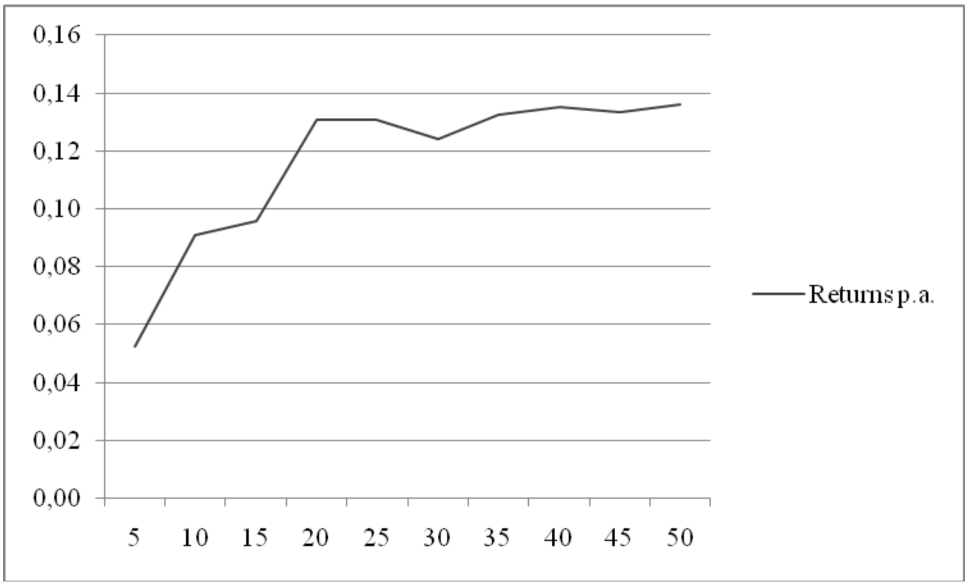

Figure 2: Enhancement factors and realized returns p.a.

[1] select stocks according to their price ranking, starting with the highest-priced stocks while [2] employ black-box algorithms in order to search for the best long-short combination over a large number of stocks. In this study though, eight different assets, employed as proxies for various economies, are taken into account only due to the "survivorship bias". Furthermore, a rolling time window of three years of daily data is employed to run iteratively the optimization procedures which is also in line with [1], [5] and [6] who employ between three and four years of high frequented daily data to estimate cointegration-optimal portfolios. In contrast to [1] who consider higher frequented rebalancing strategies between two weeks and one year, in the analysis presented here the estimated

cointagration-optimal weights are held constant over a one-year-period. This is also in line with [5] who analyzes semi-annual, annual and buy-and-hold strategies based on cointegration analysis, spanning ten years out-of-sample. The outcome of [1] studies that the tracking-error of cointegration-optimal allocation strategies is stationary even within the out-of-sample period used for back-testing the models can be supported, as cointegration holds for all constructed portfolios.

The more extremely the tracked artificial index is enhanced the less stable is according to [1] the cointegration relationship within the forecast period. Even though [1] investigate unstable cointegration relationships for trading spreads of larger than +/-5% p.a., the models suggested here exhibit excess returns up to 12.78% p.a. (i.e. 13.60% gross-returns p.a., see table 2). This outcome is also in line with [5] who estimates a cointegration based stock portfolio in the context of a buy-and-hold strategy exhibiting abnormal returns of 7.62% above the Swedish stock index while the empirical volatility is 1.19 percentage points lower in comparison to the benchmark.

While [1] suggest a rebalancing frequency of 10 days while trading the spread of the long/short portfolios, the constructed portfolios in this study are rebalanced only once a year during the overall out-of-sample period used for back-testing the models. Rebalancing a fund of funds portfolio may be expansive due to issue fees up to 5% p.a. Even though the spread of 10% p.a. is according to [1] stable and statistically significant out-of-sample, their trading strategy is not beneficial when accounting for trading costs. If average trading costs of 0.75% p.a. for each portfolio are taken into account, for instance (i.e. for rebalancing the long- and short-position), rebalancing 25 times a year[6] (i.e. every 10 days) will result in trading costs of 18.75% p.a. and, as a consequence, the profit will become negative (i.e. -8.75% on average). The trading strategies suggested here, however,

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[6] Here it is assumed that the investor faces on average 250 trading days a year.

exhibit net-returns between 3.25%-11.70% while accounting for trading costs depending on the portfolio's turnover.

[1] as well as [3] price the out-of-sample portfolios by using their corresponding ordinary return series as introduced first by [16]. This asset-pricing approach may be reasonable when operating stationary distributions. However, operating with integrated time series having a cointegration relationship with the benchmark may require more sophisticated asset-pricing. Therefore, in this study price processes, respectively, transformed price processes are in line with [6] priced by applying the VECM methodology. In the VECM framework the abnormal returns being generated correspond to a trend-stationary stochastic process. Testing for significance suggests that the abnormal returns estimated by the VECMs are statistically significant. Even though the trend parameters are statistically significant, there exists uncertainty about the magnitude concerning the realized abnormal returns within the forecast period.

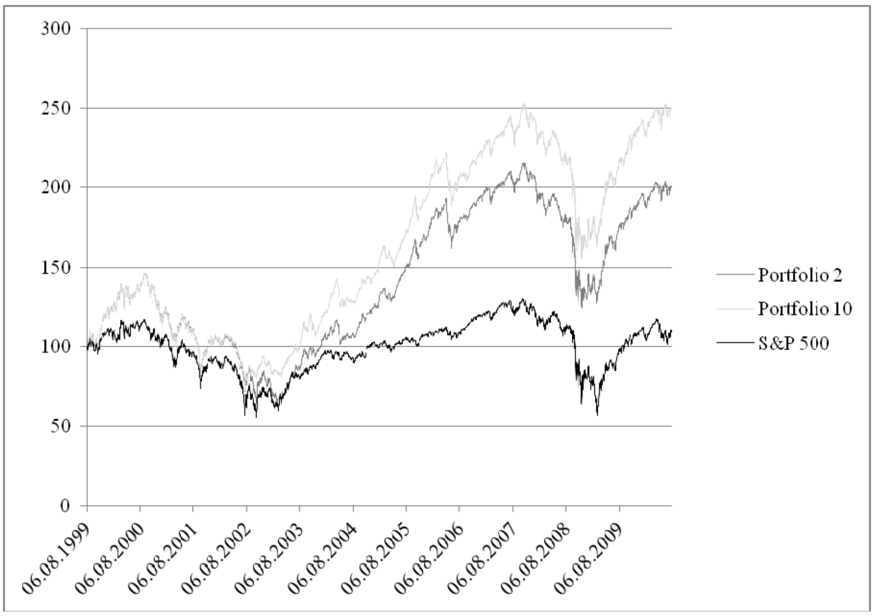

Figure 3: The S&P 500 and portfolios 2 and 10 within the forecast period 08-01-1999 to 07-31-2010

Tracking artificial indices of 5% and 10% results in gross returns of 5.25% and 9.09% in annual terms (i.e. the corresponding abnormal net returns are 3.25% and 6.67%; see Table 2), whereas larger enhanced indices result in only marginally increasing abnormal returns.

The distribution of the statistical arbitrage according to [6] a trend-stationary stochastic process with expectation $t \cdot \hat{\mu}_1$. Furthermore, exhibit 3 shows that the portfolio volatility is decreasing for portfolios tracking enhancement factors above 35% while the generated returns are still marginally increasing. This phenomenon is the reason why the highest Sharpe-ratio is associated with the portfolio tracking the highest enhanced artificial index. This outcome can be seen as anomaly.

6 Concluding Remarks

The benefits of investing globally are clear, even though investments in foreign countries are risky. Especially developing countries exhibit good possibilities to participate in their growth by investing in their stock markets, for instance. In the long run, however, there is a tendency of assimilation between economies. Roughly spoken, in the long run growth rates of developed countries are assumed to decrease while growth rates of developing countries are assumed to increase over time. However, information such as common stochastic trends that move away from each other (i.e. a trend-stationary stochastic process) is cached in the price processes. Such developments can be exploited by investors and enable statistical arbitrage opportunities. Statistical arbitrage strategies based on cointegration-optimal weight allocations enable the investors to anticipate advantageous developments of foreign economies, respectively, foreign stock markets in future periods. Thereby, the statistical arbitrage corresponds to the stationary linear trend which behaves relatively stable over time, depending on the enhancement factors. A merit of cointegration-optimal weight allocations is that these methods entail the portfolio’s error term to revert to the mean which means that in future periods the error term will move towards the trend, whereas the trend depends on the time.

The more economies are involved in the optimization procedure the more accurately should be the weight allocation, basically. In July 2008 the mutual fund company DWS launched the investment funds DWS Invest Africa LC which is mainly invested in South Africa and Egypt, whereas other investment companies offer mutual funds which may invest even in other geographic areas. Instead of employing mutual funds acting as proxies for different economies being accounted for in this study, one could also employ other financial instruments, such as certificates, in order to replicate artificial indices. The knowledge of possibilities to anticipate stochastic trends implicitly involved in the assets’ price information establishes a wide area of research with respect to asset management.

The outcome that returns of portfolios tracking artificial benchmark enhanced by more than 35% p.a. increase while the portfolio volatilities decrease can be seen as anomaly that needs to be analyzed further. Apart from that it is not clear what the optimal rebalancing moment is. In this study all portfolios are rebalanced annually which is also in line with other studies. As long as the trend-stationary stochastic process is stable, there is actually no need to rebalance the portfolio. Consequently, there is also need of research concerning the optimal rebalancing moment in the presence of cointegration-optimal portfolio allocations.

References

[1] C. Alexander and A. Dimitriu, Indexing and Statistical Arbitrage, *The Journal of Portfolio Management,* **31**, (2005), 50-63.

[2] C. Alexander, I. Giblin and W. Weddigton, Cointegration and Asset Allocation: A New Active Hedge Fund Strategy, *Research in International Business and Finance,* **16**, (2001), 65-90.

[3] O. Bondarenko, Statistical Arbitrage and Securities Prices, *The Review of Financial Studies,* **16**, (2003), 875-919.

[4] T. Gklezakou and J. Mylonakis, Links and interdependence of developed stock markets under global economic crisis conditions, *Journal of Financial Services Marketing,* **14**, (2010), 314-327.

[5] K. Grobys, Correlation versus Cointegration: Do Cointegration based Index-Tracking Portfolios perform better? Evidence from the Swedish Stock-Market, *German Journal for Young Researchers,* **2**(1), (2010), 72-78.

[6] K. Grobys, Do Business Cycles exhibit Beneficial Information for Portfolio Management? An Empirical Application of Statistical Arbitrage, *Review of Finance and Banking,* **2**(1), (2010), 43-59.

[7] K. French and J. Poterba, Investor Diversification and International Equity Markets, *American Economic Review Papers and Proceedings,* **81**, (1991), 222-226.

[8] C.W.J Granger and T. Terasvita, *Modelling Nonlinear Economic Relationships*, Chapter 5, Oxford, Oxford University Press, 1993.

[9] S. Johansen, Statistical analysis of cointegration vectors, *Journal of Economic Dynamics and Control,* **12**, (1988), 231-254.

[10] S. Johansen and K. Juselius, Maximum likelihood estimation and inference on cointegration with applications to the demand for money, *Oxford Bulletin of Economics and Statistics,* **52**(2), (1990), 169-210.

[11] S. Johansen, Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregression residuals, *Econometrica,* **59**(6), (1991), 1551-1581.

[12] D.G. Kaiser and R. Füss, The tactical and strategic value of hedge fund strategies: a cointagration approach, *Financial Markets and Portfolio Management*, **21**, (2007), 425-444.

[13] R. Levine and S. Zervos, Stock Market Development and Long-Run Growth, *World Bank Economic Review,* **10**(2), (1996), 323-339.

[14] R. Levine and S. Zervos, Stock Markets, Banks, and Economic Growth, *The American Economic Review,* **88**(3), (1998), 537-558.

[15] B.F. Yavas, Benefits of International Portfolio Diversification, *Graziadio Business Report,* **10**(2), (2007).

[16] W.F. Sharpe, A Simplified Model for Portfolio Analysis, *Management Science*, **9**(2), (1963), 277-293.

[17] P. Saikkonen and H. Lütkepohl, Testing for the cointegrating rank of a VAR process with an intercept, *Econometric Theory,* **16**, (2000), 373-406.

[18] P. Saikkonen and H. Lütkepohl, Testing for the cointegrating rank of a VAR process with structural shifts, *Journal of Business & Economic Statistics,* **18**, (2000), 451-464.

[19] P. Saikkonen and H. Lütkepohl, Trend adjustment prior to testing for the cointegrating vector autoregressive process, *Journal of Time Series Analysis*, **21**, (2000), 435-456.

[20] P.A. Samuelson, Proof that Properly Anticipated Prices Fluctuate Randomly, *Industrial Management Review,* **6**(2), (1965), 41-49.

[21] B. Solnik, Why Not Diversify Internationally Rather Than Domestically, *Financial Analysts Journal,* **51**(1), (1995), 89-94.

Appendix

Enhancement 5%		**Forecast period**					
	Asset	08-01-1999-07-31-2000	08-01-2000-07-31-2001	08-01-2001-07-31-2002	08-01-2002-07-31-2003	08-01-2003-07-31-2004	08-01-2004-07-31-2005
	1	0,0000	0,0000	0,0261	0,3811	0,5641	0,3624
	2	0,0000	0,0000	0,1558	0,0232	0,0000	0,0000
	3	0,0000	0,0000	0,0000	0,0000	0,0009	0,1162
	4	0,0000	0,0000	0,0000	0,0000	0,0007	0,1129
	5	0,0000	0,0000	0,0000	0,0000	0,3478	0,2195
	6	0,9987	1,0000	0,8181	0,5955	0,0006	0,0000
	7	0,0000	0,0000	0,0000	0,0000	0,0830	0,1889
	8	0,0013	0,0000	0,0000	0,0001	0,0029	0,0001
	Asset	08-01-2005-07-31-2006	08-01-2006-07-31-2007	08-01-2007-07-31-2008	08-01-2008-07-31-2009	08-01-2009-07-31-2010	
	1	0,4035	0,3210	0,0018	0,0200	0,3253	
	2	0,0000	0,0000	0,3786	0,0632	0,0120	
	3	0,0139	0,0741	0,0002	0,0011	0,0006	
	4	0,0153	0,2066	0,0002	0,0011	0,0006	
	5	0,3335	0,0007	0,0000	0,3580	0,2027	
	6	0,0000	0,0010	0,0000	0,0000	0,0078	
	7	0,2301	0,3362	0,0000	0,0003	0,1599	
	8	0,0037	0,0604	0,6193	0,5563	0,2910	

Enhancement 10%		**Forecast period**					
	Asset	08-01-1999-07-31-2000	08-01-2000-07-31-2001	08-01-2001-07-31-2002	08-01-2002-07-31-2003	08-01-2003-07-31-2004	08-01-2004-07-31-2005
	1	0,0000	0,0431	0,7057	0,8867	0,3052	0,0731
	2	0,0000	0,0000	0,0420	0,0000	0,0000	0,0000
	3	0,0000	0,0002	0,0000	0,0000	0,0000	0,0207
	4	0,0000	0,0016	0,0000	0,0000	0,0000	0,3080
	5	0,0000	0,0000	0,0000	0,0000	0,3730	0,2527
	6	1,0000	0,9550	0,2522	0,0000	0,0000	0,0000
	7	0,0000	0,0000	0,0000	0,1133	0,3219	0,3455
	8	0,0000	0,0001	0,0000	0,0000	0,0000	0,0000
	Asset	08-01-2005-07-31-2006	08-01-2006-07-31-2007	08-01-2007-07-31-2008	08-01-2008-07-31-2009	08-01-2009-07-31-2010	
	1	0,1800	0,1071	0,0004	0,0080	0,3623	
	2	0,0000	0,0000	0,3790	0,0848	0,0002	
	3	0,0152	0,1354	0,0005	0,0242	0,0084	
	4	0,0153	0,1288	0,0005	0,0250	0,0056	
	5	0,4074	0,0008	0,0002	0,6525	0,5393	
	6	0,0000	0,0000	0,0000	0,0000	0,0048	
	7	0,3765	0,5828	0,0002	0,0205	0,0687	
	8	0,0056	0,0450	0,6191	0,1849	0,0107	

Enhancement 15%		Forecast period					
	Asset	08-01-1999-07-31-2000	08-01-2000-07-31-2001	08-01-2001-07-31-2002	08-01-2002-07-31-2003	08-01-2003-07-31-2004	08-01-2004-07-31-2005
	1	0,0000	0,0010	0,9422	0,2675	0,0865	0,0060
	2	0,0000	0,0000	0,0042	0,0000	0,0000	0,0000
	3	0,0000	0,0002	0,0000	0,0000	0,0000	0,2549
	4	0,0000	0,0003	0,0000	0,0000	0,0000	0,2223
	5	0,0000	0,0000	0,0001	0,0088	0,2911	0,0157
	6	1,0000	0,9984	0,0535	0,0000	0,0000	0,0000
	7	0,0000	0,0000	0,0000	0,5259	0,6223	0,5003
	8	0,0000	0,0000	0,0001	0,1977	0,0001	0,0009
	Asset	08-01-2005-07-31-2006	08-01-2006-07-31-2007	08-01-2007-07-31-2008	08-01-2008-07-31-2009	08-01-2009-07-31-2010	
	1	0,0661	0,0000	0,0015	0,0000	0,0000	
	2	0,0000	0,0000	0,9209	0,1049	0,0000	
	3	0,0079	0,1027	0,0052	0,0297	0,0139	
	4	0,0087	0,1121	0,0052	0,0272	0,0019	
	5	0,4076	0,0000	0,0586	0,4832	0,9841	
	6	0,0000	0,0000	0,0000	0,0000	0,0000	
	7	0,5094	0,7851	0,0024	0,3548	0,0000	
	8	0,0003	0,0000	0,0062	0,0002	0,0000	

Enhancement 20%		Forecast period					
	Asset	08-01-1999-07-31-2000	08-01-2000-07-31-2001	08-01-2001-07-31-2002	08-01-2002-07-31-2003	08-01-2003-07-31-2004	08-01-2004-07-31-2005
	1	0,0000	0,0000	0,9408	0,0451	0,0105	0,0000
	2	0,0000	0,0000	0,0575	0,0000	0,0000	0,0000
	3	0,0000	0,0000	0,0000	0,0000	0,0097	0,0380
	4	0,0000	0,0000	0,0000	0,0000	0,0106	0,3963
	5	0,0000	0,0000	0,0016	0,0000	0,0670	0,0285
	6	1,0000	1,0000	0,0001	0,0000	0,0000	0,0000
	7	0,0000	0,0000	0,0000	0,9549	0,9020	0,5372
	8	0,0000	0,0000	0,0000	0,0000	0,0002	0,0000
	Asset	08-01-2005-07-31-2006	08-01-2006-07-31-2007	08-01-2007-07-31-2008	08-01-2008-07-31-2009	08-01-2009-07-31-2010	
	1	0,0012	0,0002	0,0006	0,0003	0,0000	
	2	0,0000	0,0000	0,6475	0,0973	0,0000	
	3	0,0307	0,0155	0,0277	0,0004	0,0007	
	4	0,0294	0,0126	0,0225	0,0006	0,0010	
	5	0,3737	0,0753	0,2099	0,8564	0,9983	
	6	0,0000	0,0003	0,0001	0,0001	0,0000	
	7	0,5646	0,8961	0,0293	0,0438	0,0000	
	8	0,0003	0,0000	0,0625	0,0012	0,0000	

Enhancement 25%	Asset	Forecast period 08-01-1999-07-31-2000	08-01-2000-07-31-2001	08-01-2001-07-31-2002	08-01-2002-07-31-2003	08-01-2003-07-31-2004	08-01-2004-07-31-2005
	1	0,0000	0,0001	0,8656	0,0000	0,0000	0,0000
	2	0,0000	0,0000	0,0578	0,0000	0,0000	0,0000
	3	0,0000	0,0046	0,0000	0,0000	0,0000	0,0195
	4	0,0000	0,0037	0,0001	0,0000	0,0000	0,2187
	5	0,0000	0,0000	0,0738	0,0000	0,0000	0,0000
	6	1,0000	0,9917	0,0000	0,0000	0,0000	0,0000
	7	0,0000	0,0000	0,0003	1,0000	1,0000	0,7618
	8	0,0000	0,0000	0,0024	0,0000	0,0000	0,0000
	Asset	08-01-2005-07-31-2006	08-01-2006-07-31-2007	08-01-2007-07-31-2008	08-01-2008-07-31-2009	08-01-2009-07-31-2010	
	1	0,0000	0,0000	0,0011	0,0002	0,0003	
	2	0,0000	0,0000	0,5577	0,0486	0,0001	
	3	0,0002	0,0388	0,0024	0,0059	0,0031	
	4	0,0002	0,0355	0,0022	0,0024	0,0048	
	5	0,3993	0,1559	0,2794	0,8107	0,9916	
	6	0,0000	0,0000	0,0000	0,0000	0,0000	
	7	0,6003	0,7697	0,0016	0,1322	0,0000	
	8	0,0000	0,0001	0,1556	0,0001	0,0000	

Enhancement 30%	Asset	Forecast period 08-01-1999-07-31-2000	08-01-2000-07-31-2001	08-01-2001-07-31-2002	08-01-2002-07-31-2003	08-01-2003-07-31-2004	08-01-2004-07-31-2005
	1	0,0000	0,0045	0,7798	0,0000	0,0000	0,0000
	2	0,0000	0,0000	0,0044	0,0000	0,0000	0,0000
	3	0,0000	0,0031	0,0002	0,0000	0,0000	0,0939
	4	0,0000	0,0022	0,0002	0,0000	0,0000	0,0625
	5	0,0000	0,0000	0,2058	0,0000	0,0000	0,0000
	6	1,0000	0,9901	0,0000	0,0000	0,0000	0,0000
	7	0,0000	0,0000	0,0089	1,0000	1,0000	0,8436
	8	0,0000	0,0001	0,0008	0,0000	0,0000	0,0000
	Asset	08-01-2005-07-31-2006	08-01-2006-07-31-2007	08-01-2007-07-31-2008	08-01-2008-07-31-2009	08-01-2009-07-31-2010	
	1	0,0008	0,0000	0,0000	0,0000	0,0000	
	2	0,0000	0,0000	0,7913	0,0060	0,0000	
	3	0,0049	0,1011	0,0004	0,0279	0,0000	
	4	0,0059	0,0941	0,0002	0,0037	0,0000	
	5	0,0287	0,6199	0,1384	0,8477	1,0000	
	6	0,0000	0,0000	0,0000	0,0001	0,0000	
	7	0,9564	0,1847	0,0064	0,1144	0,0000	
	8	0,0032	0,0002	0,0631	0,0001	0,0000	

Enhancement 35%		Forecast period					
	Asset	08-01-1999-07-31-2000	08-01-2000-07-31-2001	08-01-2001-07-31-2002	08-01-2002-07-31-2003	08-01-2003-07-31-2004	08-01-2004-07-31-2005
	1	0,0001	0,0007	0,7171	0,0000	0,0000	0,0000
	2	0,0000	0,0000	0,0094	0,0000	0,0000	0,0000
	3	0,0000	0,0003	0,0003	0,0000	0,0000	0,0472
	4	0,0000	0,0003	0,0003	0,0000	0,0000	0,0458
	5	0,0000	0,0000	0,2156	0,0000	0,0000	0,0000
	6	0,9997	0,9987	0,0001	0,0000	0,0000	0,0000
	7	0,0000	0,0000	0,0570	1,0000	1,0000	0,9070
	8	0,0002	0,0000	0,0003	0,0000	0,0000	0,0000
	Asset	08-01-2005-07-31-2006	08-01-2006-07-31-2007	08-01-2007-07-31-2008	08-01-2008-07-31-2009	08-01-2009-07-31-2010	
	1	0,0000	0,0007	0,0000	0,0000	0,0000	
	2	0,0000	0,0012	0,7652	0,0028	0,0000	
	3	0,0122	0,0009	0,0000	0,0019	0,0000	
	4	0,0078	0,0001	0,0000	0,0024	0,0000	
	5	0,0739	0,3906	0,2073	0,9925	1,0000	
	6	0,0000	0,0010	0,0000	0,0000	0,0000	
	7	0,9060	0,6055	0,0163	0,0003	0,0000	
	8	0,0000	0,0000	0,0111	0,0000	0,0000	

Enhancement 40%		Forecast period					
	Asset	08-01-1999-07-31-2000	08-01-2000-07-31-2001	08-01-2001-07-31-2002	08-01-2002-07-31-2003	08-01-2003-07-31-2004	08-01-2004-07-31-2005
	1	0,0001	0,0332	0,7214	0,0000	0,0000	0,0000
	2	0,0000	0,0000	0,0538	0,0000	0,0000	0,0000
	3	0,0001	0,0001	0,0038	0,0000	0,0000	0,0211
	4	0,0001	0,0001	0,0038	0,0000	0,0000	0,0207
	5	0,0000	0,0000	0,1666	0,0000	0,0000	0,0000
	6	0,9996	0,9664	0,0011	0,0000	0,0000	0,0000
	7	0,0000	0,0000	0,0282	1,0000	1,0000	0,9582
	8	0,0000	0,0002	0,0212	0,0000	0,0000	0,0000
	Asset	08-01-2005-07-31-2006	08-01-2006-07-31-2007	08-01-2007-07-31-2008	08-01-2008-07-31-2009	08-01-2009-07-31-2010	
	1	0,0000	0,0000	0,0001	0,0000	0,0000	
	2	0,0000	0,0000	0,6252	0,0235	0,0000	
	3	0,0023	0,0004	0,0002	0,0138	0,0000	
	4	0,0000	0,0004	0,0005	0,0160	0,0000	
	5	0,0000	0,1049	0,3376	0,9445	1,0000	
	6	0,0000	0,0000	0,0000	0,0000	0,0000	
	7	0,9976	0,8943	0,0059	0,0019	0,0000	
	8	0,0000	0,0000	0,0306	0,0003	0,0000	

Enhancement 45%		Forecast period					
	Asset	08-01-1999-07-31-2000	08-01-2000-07-31-2001	08-01-2001-07-31-2002	08-01-2002-07-31-2003	08-01-2003-07-31-2004	08-01-2004-07-31-2005
	1	0,0000	0,0197	0,9716	0,0000	0,0000	0,0000
	2	0,0000	0,0000	0,0024	0,0000	0,0000	0,0000
	3	0,0000	0,0175	0,0000	0,0000	0,0000	0,0034
	4	0,0000	0,0175	0,0000	0,0000	0,0000	0,0034
	5	0,0000	0,0000	0,0254	0,0000	0,0000	0,0000
	6	1,0000	0,9451	0,0000	0,0000	0,0000	0,0000
	7	0,0000	0,0000	0,0006	1,0000	1,0000	0,9932
	8	0,0000	0,0002	0,0000	0,0000	0,0000	0,0000
	Asset	08-01-2005-07-31-2006	08-01-2006-07-31-2007	08-01-2007-07-31-2008	08-01-2008-07-31-2009	08-01-2009-07-31-2010	
	1	0,0001	0,0000	0,0000	0,0001	0,0000	
	2	0,0000	0,0000	0,6172	0,0387	0,0000	
	3	0,0008	0,0000	0,0006	0,0147	0,0001	
	4	0,0006	0,0000	0,0006	0,0086	0,0001	
	5	0,0018	0,0289	0,2756	0,9358	0,9998	
	6	0,0000	0,0000	0,0000	0,0000	0,0000	
	7	0,9968	0,9710	0,0111	0,0021	0,0000	
	8	0,0000	0,0000	0,0949	0,0000	0,0000	

Enhancement 50%		Forecast period					
	Asset	08-01-1999-07-31-2000	08-01-2000-07-31-2001	08-01-2001-07-31-2002	08-01-2002-07-31-2003	08-01-2003-07-31-2004	08-01-2004-07-31-2005
	1	0,0000	0,0000	0,9656	0,0000	0,0000	0,0000
	2	0,0000	0,0000	0,0035	0,0000	0,0000	0,0000
	3	0,0000	0,0000	0,0000	0,0000	0,0000	0,0014
	4	0,0000	0,0000	0,0000	0,0000	0,0000	0,0014
	5	0,0000	0,0000	0,0275	0,0000	0,0000	0,0000
	6	1,0000	1,0000	0,0000	0,0000	0,0000	0,0000
	7	0,0000	0,0000	0,0029	1,0000	1,0000	0,9973
	8	0,0000	0,0000	0,0004	0,0000	0,0000	0,0000
	Asset	08-01-2005-07-31-2006	08-01-2006-07-31-2007	08-01-2007-07-31-2008	08-01-2008-07-31-2009	08-01-2009-07-31-2010	
	1	0,0008	0,0000	0,0347	0,0000	0,0000	
	2	0,0001	0,0000	0,5025	0,0184	0,0000	
	3	0,0002	0,0007	0,0034	0,0153	0,0000	
	4	0,0002	0,0001	0,0036	0,0167	0,0000	
	5	0,0413	0,2231	0,3502	0,9452	1,0000	
	6	0,0000	0,0000	0,0001	0,0000	0,0000	
	7	0,9573	0,7760	0,0006	0,0040	0,0000	
	8	0,0001	0,0001	0,1049	0,0004	0,0000	