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Multivariate *t*- distribution and GARCH modelling of Volatility and Conditional Correlations on BRICS Stock Markets[1]

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Abstract

We examine the nature of BRICS stock market returns using a *t*-DCC model and investigate whether multivariate volatility models can characterize and quantify market risk. We initially consider a multivariate *normal*-DCC model and show that it cannot adequately capture the fat tails prevalent in financial time series data. We then consider a multivariate *t*- version of the Gaussian dynamic conditional correlation (DCC) proposed by [16] and successfully implemented by [24, 26]. We find that the *t*-DCC model (dynamic conditional correlation based on the *t*-distribution) out performs the *normal*-DCC model. The former passes most diagnostic tests although it barely passes the Kolmogorov-Smirnov goodness-of-fit test.

JEL Classification: C51, C52, G11
Keywords: Correlations and Volatilities; MGARCH (**M**ultivariate **G**eneral **A**uto**r**egressive **C**onditional **H**eteroscedasticity); ML – Maximum Likelihood, Daily returns (standard and devolatized), Multivariate t (*t*-DCC), Kolmogorov-Smirnov test (KS_N), **V**alue **a**t **R**isk (VaR) diagnostics

1 Introduction

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[1]The paper was presented at the 12th *African Finance Journal Conference* in Cape Town, South Africa on May 20-21, 2015. In late 2001, Jim O'Neil, an economist at Goldman Sachs came up an acronym BRIC as a shorthand for the then fast-growing countries Brazil, Russia, India and China. Almost a decade later in December 2010, South Africa joined the group resulting in the acronym BRICS. A few people have suggested that South Africa was added just to represent the African continent. We do not enter that debate.

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Although any grouping of countries (such BRICS) involves some degree of arbitrary selection; the country and population size coupled with economic growth potential often acts as a common framework. There are at least two identifiable strengths to BRICS economies that are worth examining. First, BRICS countries produce 25% of global Gross Domestic Product (GDP), an increase of 15% from 1990. It is estimated that by 2020, they will account for about 37%-38% of global GDP with the current population of 3 billion with income per capita ranging from $7,710 to $13,689. Second, these economies have reasons for creating a club or grouping of their own to act as a counterweight in multilateral diplomacy, particularly in dealing with the U.S. and the EU.

Since BRICS stock markets are now globally integrated, they are likely to be affected by developments in each other's market. For investors, less international correlation between stock market returns mean that investors may reduce portfolio risk more by diversifying internationally instead of wholly investing in the domestic market. Since the level of gains from international diversification to reduce risk depends on the international *correlation structure*, the proposed paper provides empirical estimates. The correlation structure between stock returns is widely used in finance and financial management, to establish efficient frontiers of portfolio holdings. The paper provides time-varying (*dynamic*) conditional correlation estimates of BRICS stock market returns. The fact that stock markets are related, there is likely to volatility across such markets.[3] To account for such effects, the multivariate model estimates a measure of conditional volatility. Thus, we employ a multivariate *t*-DCC model for conditional correlations in returns and conditional volatility.

Table 1 shows the correlation matrix of equity returns. Brazilian equity returns are negatively correlated with Russia (-0.153); Russia and India (-0.171), China and Russia (-0.169) and Russia and South Africa (-0.243). However, Brazilian equity returns are positively correlated with those of India, China and South Africa. It remains to be seen whether these relationships can be captured by conditional correlations from the t-DCC model.

Table 1: Estimated Correlation Matrix of Variables

1498 observations used for estimation from 02-Jan-08 to 27-Sep-13

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	RBR	RRU	RIN	RCH	RSA
RBR (Brazil)	1.0000				
RRU (Russia)	-.15282	1.0000			
RIN (India)	.37028	-.17146	1.0000		
RCH (China)	.37940	-.16858	.56478	1.0000	
RSA (South Africa)	.50155	-.24296	.48157	.49054	1.0000

A few empirical results are noteworthy. First, our results indicate that the *t*-DCC model is preferred over the normal-DCC model in estimating conditional volatilities and

[3]Since volatility is a non-observable variable, it is usually proxied for in two ways: (a) using the square of daily equity returns (r^2_{it}) or (b) the standard error of intra-daily returns (realized volatilities) ($\sigma_{it}^{realized}$) as in (7) below.

correlations. Second, both $\hat{\pi}_N$ and $\hat{z}_\pi$ (tests of the validation of the *t*-DCC model) provide support for the *t*-DCC model despite the 2008 financial crisis. However, the model barely passes the non-parametric Kolmogorov-Smirnov (KS_N) test which tests whether probability transform estimates, $\hat{U}_t$ are uniformly distributed over the range (0, 1).[4] Third, from Figures 1(a) -1(c), it is clear that some stock market equity returns correlations are negatively related: India and Brazil, Russia and Brazil, and China and Russia respectively. Fourth, the model shows that the 2008 and 2009 financial crisis led to sharp spikes in volatility. In 2008, China had the highest spike; in 2009, India had the highest spike and in 2010, South Africa experienced the highest spike. During these periods, Russia had the lowest spike in volatility. Fifth, conditional correlations of Russia (in equity returns) fell during the financial crisis but picked up from July 2010. It shows that despite the 2008 financial crisis, BRICS equity returns (in terms of correlations) are anchored by the process of globalization in which stock markets are now more interdependent than ever. Finally, South African conditional correlations are negatively related to Brazil, India and China but they move together with Russian equity returns. It suggests that South African investors would have been better diversifying into these economies and that BRIC investors would not be investing in South Africa.[5] Of course, the statement excludes any consideration of the effects of exchange rates.[6]

For almost a decade, BRICS economies consistently posted high growth rates as long as foreign capital was cheap, with growing exports, and a strong foreign appetite for emerging market stocks and bonds by developed economies and investors. By late 2013, the reports that the Fed would soon reduce its bond-buying program quickly caused panic runs on the Brazilian real, the South African rand, and the Indian rupee as investors fled the BRICS

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[4]The probability integral transform (PIT) idea is that from a cumulative distribution function (CDF) in terms of one variable, it can be transformed into another CDF in terms of different variable such as $F_x(x) \to F_y(y)$. It is used mostly to generate random variables from continuous distributions. For instance, if *X* has a *U* (0, 1) distribution, then $F_X(x) = x$. Thus the requirement $F_X(x) = F_Y(y)$ in the probability integral transform (PIT) reduces to $x = F_Y(y)$ or $y = F_Y^{-1}(\mathrm{x})$. Since y is an observation from the probability distribution *Y*, this means that one can generate observations from the distribution *Y* by generating *U* (0,1) random variables and applying the $y = F_Y^{-1}(\mathrm{x})$ transformation.

[5]Russian equities (in terms of conditional equity returns) tend to move close together with equities in Brazil, India, and China but not with South African equities.

[6][11] studied exchange rate movements and stock market returns in BRICS countries using a Markov-switching VAR model. They find that stock markets in BRICS countries have more influence on exchange rates during calm and turbulent period.

economies.[7] The BRICS economies have now become the Fragile Five, signifying economies that were too dependent on skittish foreign investment to finance their economic growth. The Fragile Five now confront three major problems. First, the world economic downturn has cooled the global export boom that fueled earlier economic growth. Any growth in the future requires a boost in domestic consumption – a time consuming transition. Second, the high equity returns in BRICS stock markets relied on huge inflows of foreign capital and very little domestic financing. Third, the U.S. Federal Reserve Bank (Fed) and other central banks, pursuing their own interests, held down interest rates, thus masking some of the weaknesses in BRICS finances. As interest rates in developed economies are expected to trend upwards in the near future, BRICS and other emerging markets may experience severe capital flight.

The process of modeling conditional correlations across equity returns and conditional volatilities is a major function of portfolio managers and those tasked with reducing risks under the Value at Risk (VaR) strategies. If there is more than one equity in a portfolio, the use of multivariate models is often suggested. The return to equities are of five BRICS countries (Brazil, Russia, India, China and South Africa). This paper employs a $t-DCC$ model to estimate conditional volatilities and conditional equity returns.

The estimation of conditional volatilities and equity returns is achieved by the DCC (with time-varying correlation estimates) model by assuming a normal or Gaussian distribution of errors in the variance-covariance matrix Σ_{t-1}. In other words, the DCC model solves *the curse of dimensionality* by decomposing the variance-covariance matrix and transforming returns to normality or Gaussianity by dividing equity returns by a volatility measure ($\sigma_{i,t-1}$) to obtain standardized returns (see (6) below). A major shortcoming of this approach is that the Gaussian assumption often fails in financial empirical analysis because of the fat-tailed nature of the distribution of returns. The simple dynamic conditional correlation model ($normal-DCC$) from [16] and [16b] is based on a covariance-based method. This bears the risk of modeling bias but the assumed conditional Gaussian marginal distributions are not capable in mimicking the heavy-tails found in financial time series data observed in markets. Despite this shortcoming, [18] found that the conditional Gaussian distribution fits with V**a**R models with reasonable estimates.

The transformation of equity returns to Gaussianity is critical since correlation as a measure of dependence can be misleading in the presence of non-Gaussian equity returns as in (6) below. [24, 26] and [13] point out that for correlation to be useful as a measure of dependence, the transformation of equity returns should be made approximately Gaussian. The *t*-DCC model uses devolatized returns that very closely approximate Gaussianity. It is based on de-volatized returns as outlined in [24, 26].

The literature on multivariate modelling is quite sizable as reviewed in [5] and [21], the *Riskmetrics* from J.P. Morgan and others and the multivariate generalized autoregressive conditional heteroscedastic specification (MGARCH) from [15]. However, if the portfolio has m equities, the number of unknown parameters in the unrestricted MGARCH tends to

[7]In three phases since late 2008, the Federal Reserve has bought trillions of dollars in bonds using newly created money (quantitative easing) to stimulate the economy.

increase exponentially with m so that estimation is not feasible even for a few equities.[8] The diagonal VEC version of the MGARCH, although better, it still has too many parameters to be estimated. This curse of dimensionality is addressed somehow in [16]'s dynamic conditional correlations model which allows for time-varying correlations in the correlation matrix (D_{t-1}) in (1) below.

The major innovation is the decomposition of the conditional covariance matrix to conditional volatilities and conditional cross-equity returns correlations ($\Sigma_{t-1} = D_{t-1}R_{t-1}D_{t-1}$, see (1) below) where D_{t-1} is a $m\,x\,m$ diagonal matrix of conditional volatilities while R_{t-1} is a symmetric $m\,x\,m$ correlation matrix. The returns to equities is represented by a vector $r_t\ (= m\,x1)$ at time t that have a conditional multivariate $t-$ distribution with mean of μ_{t-1} , a non-singular variance-covariance matrix (Σ_{t-1}), and $v_{t-1} > 2$ degrees of freedom. The cross-equity returns are modelled in terms of a fewer number of unknown parameters which resolves *the curse of dimensionality*. The returns are standardized to achieve Gaussianity. [16] shows that with Gaussianity in innovations, the log-likelihood function of the *normal*-DCC model can be maximized in a two-step procedure. In step 1, m univariate GARCH models are estimated separately and step 2 uses the standardized residuals from step 1 to estimate conditional correlations (R_{t-1}).

Of the first four moments (mean, variance, skewness, and kurtosis), the latter three are unlikely to be satisfied by the assumption of a normal distribution. To capture these properties of financial data (equity returns in this case), the DCC model is combined with a multivariate $t-$ distribution for equity returns where tail properties of return distributions are a primary concern. Under this approach, [16]'s two-step procedure is no longer applicable to a t–DCC specification. Following [24, 26], the obvious approach is to estimate simultaneously all the parameters of the model, including v , the degrees of freedom parameter. This approach solves the *curse of dimensionality* [16] and the absence of Gaussianity (by assuming a t distribution instead).

According to [29], the data on financial series (equities in this case) share some commonalities such as heteroscedasticity; the variation and clustering of volatility over time, and autocorrelation. Since volatility is not observable, the usual way to model this is to adopt [7]'s GARCH framework. To the extent that financial volatilities tend to move together over time and across equity markets (clustering), the relevant model is the multivariate modelling framework with estimates that improve decision-making in areas such as portfolio selection, option pricing, hedging, risk management, and equity pricing.

The generalization of the univariate standard GARCH model include [8]'s VEC and BEKK models, factor models (F-GARCH) from [14], the full factor models (FF-GARCH) from [32], linear combinations of univariate GARCH models including the orthogonal (O-GARCH) model from [1] who use a static principle component decomposition of standard residuals, the generalized orthogonal (GO-GARCH) from [31], nonlinear combinations of univariate models which include the constant conditional correlation(CCC-GARCH), the dynamic conditional correlation (DCC-GARCH) from [30], and [16] respectively. [19]

[8]The unrestricted model allows for the estimation of λ_{1i} and λ_{2i} (conditional volatility parameters) for $i = 1, \ldots\ldots, 5$ and ϕ_1 and ϕ_2 (mean-reverting conditional correlations parameters).

suggests a slightly different model from those above in the form of a generalized dynamic covariance (GDC-GARCH) model.

According to [24, 26], there are 53 different specifications of Σ_{t-1} that can be categorized into 8 different model types such as the equal-weighted moving average (EQMA), the exponential-weighted moving average (EWMA), mixed moving average (MMA), generalized exponential –weighted moving average (GEWMA), constant correlation (CCC), the asymmetrical dynamic conditional correlation (ADCC) from [10], and the dynamic conditional correlation (t-DCC) from [24, 26] (Table 2). [26, 27] modified [16]'s DCC model by basing it on the stochastic process of the conditional correlation matrix on *devolatized* residuals rather than on *standardized* residuals. Standardized residuals are obtained by dividing residuals by the conditional standard deviations from the a first-stage GARCH (p, q) model, while devolatized residuals are found by dividing residuals by the square root of the *k*-day moving average of squared residuals.

Table 2: Different Specifications of Σ_{t-1}

	Model Types	**Major Innovation**
1.	Equally-Weighted Moving Average(EQMA)	Without intra-daily observations, a simple estimate of $\Sigma_{1t} = \frac{1}{n_0}\sum_{s=1}^{n_0} r_{t-s}r'_{t-s}$ with n_0 = last observations
2.	Exponential-Weighted Moving Average (EWMA)	One parameter is the popular *Riskmetrics* estimate of Σ_{t-1} made popular by J.P. Morgan
3.	Mixed Moving Average (MMA)	The conditional variances are calculated are calculated as in the equal-weighted MA model
4.	Generalized Exponential-Weighted Moving Average (GEWMA)	Generalization of the two=parameter EWMA.
5.	Constant Conditional Correlation (CCC) –[9]	Assumes that the one-step ahead conditional correlations are constant.
6.	Dynamic Conditional Correlation (DCC)- [16]	Conditional correlations are allowed to be time-varying.
7.	Asymmetric Dynamic Conditional Correlation (ADCC) – [10]	Allows the possibility of asymmetric effects on conditional variances and correlations.
8.	*t*-Dynamic Conditional Correlation (*t*-DCC)[24, 25, 26, 27]	*t*-DCC is based on the stochastic process of the conditional correlation matrix on devolatized residuals.

All these models can be organized into two groups for the convenience. Models 5 -8 can be grouped together as Group 1 whereas models 1- 4 can be viewed as Group 2. The different multivariate volatility models are all special cases of MGARCH and the associated conditional covariance matrix by Σ_{it} .

The paper is organized as follows. Section 2 presents the $t-DCC$ used to provide estimates of conditional volatilities and equity returns using devolatized equity returns. Section 3 offers a brief discussion on recursive relations for real time analysis. Section 4 details the maximum likelihood (ML) estimation of the *normal*-DCC and *t*-DCC model. Section 5 presents VaR diagnostics such as tests of serial correlation and uniform distributions. Section 6 is the empirical application to devolatilized returns. Section 7 presents ML estimates of the *t*-DCC models in subsections: (a) equity-specific estimates; (b) post-estimation evaluation of the *t*-DCC model, and (c) recursive estimates and the V**a**R diagnostics. Section 8 presents the evolution of equity return volatilities and correlations. Section 9 concludes.

2 *t-DCC* Model or Modelling Dynamic Conditional Volatilities and Correlations of Equity Returns

We use equity returns which are standardized by realized volatilities (7) rather than GARCH (1, 1) volatilities (6). Returns in (7) are more likely to be approximately Gaussian than standardized returns [2, 3]. Since we employ daily data and have no access to intra-daily data, we follow [24, 26] in getting an estimate of σ_{it} that uses contemporaneous daily returns and their lagged values as in (9). The $t-DCC$ estimation is applied to five equity indexes over the period 01 January 2008 to 27 September 2013. The sample is split into an estimation sample (2008 to 2011) and an evaluation sample (2012 to 2013). The results show a strong rejection of the $normal-DCC$ model in favor of the $t-DCC$ model (partly based on the log-likelihood for the normal distribution is -8606.4 (Tables 4) while that of the t-DCC model is -8508.0 (Tables 5)). When subjected to a series of diagnostic tests, it passes a number of VaR tests over the evaluation sample. The data comes from the Financial Times Stock Equities (FTSE) for each BRICS country. Some data for Russia and South Africa came from Yahoo Finance!

We now offer a *t*-DCC model as formulated by [24, 26, and 27] from the work by [9] and [16].

$$\Sigma_{t-1} = D_{t-1}R_{t-1}D_{t-1} \tag{1}$$

where

$$D_{t-1} = \begin{pmatrix} \sigma_{1,t-1}.......... \\ ....\sigma_{2,t-1}....0 \\ ....0.....\sigma_{3,t-1}.... \\ \qquad\qquad\qquad\qquad \sigma_{m,t-1} \end{pmatrix}$$

$$R_{t-1} = \begin{pmatrix} 1 & \rho_{12,t-1} & \rho_{13,t-1} & \cdots & \rho_{1m,t-1} \\ \rho_{21,t-1} & 1 & \rho_{23,t-1} & \cdots & \rho_{2m,t-1} \\ . & & . & & . \\ . & & & . & . \\ . & & & \ddots & . \\ \vdots & \cdots & \cdots & & \rho_{m-1,m,t-1} \\ \rho_{m,1,t-1} & \cdots & \rho_{m,m-1,t-1} & & 1 \end{pmatrix}$$

$R_{t-1} = (\rho_{ij,t-1}) = (\rho_{ji,t-1})$ is the symmetric $m\,x\,m$ correlation matrix and D_{t-1} is $m\,x\,m$ diagonal matrix with $\sigma_{i,t-1}, \mathrm{i} = 1, 2, \ldots, \mathrm{m}$ representing the conditional volatility of the i-*th* equity return. That is, $\sigma^2_{i,t-1} = V(r_{it} \mid \Omega_{t-1})$ and conditional pair-wise equity return correlations are represented by $\rho_{ij,t-1} = \dfrac{Cov(r_{it}, r_{jt} \mid \Omega_{t-1})}{\sigma_{i,t-1}\sigma_{j,t-1}}$ where Ω_{t-1} is the information set available at $t-1$. Note that when for $i = j$, $\rho_{ij,t-1} = 1$.

[9] considered a correlation matrix where $R_{t-1} = R$ which defines a constant correlation matrix (CCC) while [16] allows R_{t-1} to be time-varying, suggesting a class of multivariate models known as the dynamic conditional correlation models (DCC). [9]'s multivariate GARCH model assumes that the one-step ahead conditional correlations are constant. [16] relaxed the assumption of constant conditional correlation of the CCC model of [9]. The conditional variances of individual equity returns are estimated as univariate GARCH (p, q) specifications, and the diagonal matrix is formed with their square roots. [10] generalized the DCC model by allowing for the possibility of asymmetric effects on conditional variances and correlations.

[28] proposed an alternative model which uses a conditionally heteroscedastic model where unobserved common factors are assumed to be heteroskedastic and assumes that the number of common factors are less than the number of equities. The decomposition of the variance-covariance matric Σ_{t-1} is critical to the estimation of conditional volatilities and correlation. That is, Σ_{t-1} allows for the separate specification of conditional volatilities and conditional cross-equity returns correlations. One uses the GARCH (1, 1) to model $\sigma_{i,t-1}{}^2$ as

$$V(r_{it} \mid \Omega_{t-1}) = \sigma_{i,t-1}{}^2 = \bar{\sigma}_i{}^2(1 - \lambda_{1i} - \lambda_{2i}) + \lambda_{1i}\sigma^2{}_{i,t-2} + \lambda_{2i} r^2{}_{i,t-1} \qquad (2)$$

where $\bar{\sigma}_i^2$ is the **un**conditional variance of the of the i-*th* equity return. In the event that $\lambda_{1i} + \lambda_{2i} = 1$, the **un**conditional variance ceases to exist in which case we have an integrated GARCH (IGARCH) model that is heavily used by finance practitioners and the model is similar to the "exponential smoother" as applied to $r_{it}^{2'} s^2$. That is,

$$\sigma^2{}_{i,t-1}(\lambda_i) = (1-\lambda_i)\sum_{s=1}^{\infty}\lambda_i^{s-1} r^2{}_{i,t-s}, \qquad 0<\lambda_i<1, \text{ Or} \tag{3}$$

In recursive form,

$$\sigma^2{}_{i,t-1}(\lambda_i) = \lambda_i \sigma^2{}_{i,t-2} + (1-\lambda_i) r^2{}_{i,t-1} \tag{4}$$

[16] suggested that cross-equity correlations estimates can use the following exponential smoother applied to "standardized returns" to obtain Gaussianity.

$$\hat{\rho}_{ij,t-1}(\phi) = \frac{\sum_{s=1}^{\infty}\phi^{s-1} z_{i,t-s} z_{j,t-s}}{\sqrt{\sum_{s=1}^{\infty}\phi^{s-1} z^2{}_{i,t-s}}\sqrt{\sum_{s=1}^{\infty}\phi^{s-1} z^2{}_{\mathrm{j},t-s}}} \tag{5}$$

The standardized returns are represented by

$$z_{it} = \frac{r_{it}}{\sigma_{i,t-1}(\lambda_i)} \tag{6}$$

The unknown parameters that must be estimated are given by $\lambda_1, \lambda_2, \ldots\ldots, \lambda_m$, and ϕ which have been subject to [16]'s two-step procedure. The first stage involves fitting a GARCH (1, 1) model separately to m equities. The second step estimates the coefficient of conditional correlations, ϕ by Maximum Likelihood (ML) methods assuming that equity returns are conditionally Gaussian. However, [24, 26] point to two major disadvantages of the two-step procedure. First, the normality assumption never holds in daily or weekly returns and it has a tendency to under-estimate portfolio risk.[9] Second, without Gaussianity, the two-step procedure is inefficient.

Pair-wise correlations based on realized volatilities

[24, 25, and 26] base the specification of cross correlation of volatilities on devolatized returns defined by (7) below. Suppose the realized volatility ($\sigma_{it}^{realized}$) of the i *-th* equity return in day t is defined as standard returns (r_{it}) divided by realized volatilities ($\sigma_{it}^{realized}$) to yield

$$\tilde{r}_{it} = \frac{r_{it}}{\sigma_{it}^{realized}} \tag{7}$$

In (7), devolatilized returns are $\tilde{r}_{it}$ while in (6), standardized returns are represented by z_{it}. Hence, the conditional pair-wise return correlations based on devolatized returns is given by

[9]The use of daily data has its cost. For example, there is no accounting for the non-synchronization of daily returns across equity markets in different time zones. The use of weekly or monthly data deals with this issue.

$$\tilde{\rho}_{ij,t-1}(\phi)=\frac{\sum_{s=1}^{\infty}\phi^{s-1}\tilde{r}_{i,t-s}\tilde{r}_{j,t-s}}{\sqrt{\sum_{s=1}^{\infty}\phi^{s-1}\tilde{r}^{2}_{i,t-s}}\sqrt{\sum_{s=1}^{\infty}\phi^{s-1}\tilde{r}^{2}_{j,t-s}}}$$, such that $-1<\tilde{\rho}_{ij,t-1}(\phi)<1$ for all values of $|\phi|<1$. (8)

[24, 26] offer an alternative formulation of $\rho_{ij,t-1}$ that makes use of realized volatilities as in (8). There is empirical support for this approach that daily returns on foreign exchange assets and stock market returns standardized by realized volatility are approximately Gaussian [2, 3].

Since we do not have intraday data for the equities examined here, we provide a simple estimate of σ_{it} based on daily returns that take into account all contemporaneous values of $\tilde{r}_{it}$.

$$\tilde{\sigma}^{2}_{it}(p)=\frac{\sum_{s=0}^{p-1}r^{2}_{i,t-s}}{p} \tag{9}$$

where p is the lag order which should be chosen very carefully. [24, 26] emphasize that the choice of p is critical since the chosen value must be such that it transforms r_{it} into a Gaussian process. The non-Gaussian behavior found in daily returns is mainly due to jumps in the return process for many markets as reported in [24, 25, and 4]. A choice of p well above 20 does not allow for possible jumps in data to be adequately reflected in $\tilde{\sigma}^{2}_{it}(p)$, while values of p well below makes r_{it} to behave as an indicator type looking function [29]. [24, 25, 26 and 27] note that $\tilde{\sigma}^{2}_{it}(p)$ is not equivalent to the standard rolling historical estimate of σ_{it} given by

That is, $\tilde{\sigma}^{2}_{it}(p)$ - $\hat{\sigma}^{2}_{it}(p)$ = $\dfrac{r_{it}^{\,2}-r^{2}_{i,t-p}}{p}$ (9b)

when implementing real time analysis, as in recursive formulae augments used in the estimation and evaluation process. It seems that the inclusion of current squared returns $r_{it}^{\,2}$ (9) in the estimation of $\tilde{\sigma}^{2}_{it}$ is important in transforming non-Gaussian returns r_{it} into Gaussian $\tilde{r}_{it}$ returns.

3 Recursive Relations for Real Time Analysis

The computation of $\rho_{ij,t-1}$ in (5) and (8) as noted by [16] is given by

$$\tilde{\rho}_{ij,t-1}(\phi)=\frac{q_{ij,t-1}}{\sqrt{q_{ii,t-1}q_{jj,t-1}}} \tag{10}$$

where

$$q_{ij,t-1} = \phi q_{ij,t-2} + (1-\phi)\tilde{r}_{i,t-1}\tilde{r}_{j,t-1} \quad (11)$$

It is important to that $\tilde{\rho}_{ij,t-1}$ is positive definite as the covariance as the typical element of the matrix $q_{ij,t-1}$ is a positive definite.

The recursive formula for $\tilde{\rho}_{ij,t-1}(\phi)$ is the same as in (5) except that (10) uses devolatized returns while (5) uses standardized returns (z_{it}). We note that in the above models for pair-wise correlations, $\rho_{ij,t-1}$, these are non-mean reverting. The general specification for pair-wise correlations is given by

$$q_{ij,t-1} = \bar{\rho}_{ij}(1-\phi_1-\phi_2) + \phi_1 q_{ij,t-2} + \phi_2\tilde{r}_{i,t-1}\tilde{r}_{j,t-1} \quad (12)$$

where $\bar{\rho}_{ij}$ is the **un**conditional correlation of r_{it} and r_{jt} with the restriction that $\phi_1+\phi_2<1$ (mean reversion). There is an expectation that $\phi_1+\phi_2$ will be very close to one. The non-reverting mean case is a special case of $\phi_1+\phi_2=1$. However, it is not possible to be certain that $\phi_1+\phi_2<1$ or not. On the other hand, it is possible to estimate **un**conditional correlations, $\bar{\rho}_{ij}$ by using an expanding window. In the empirical part of the paper, we consider both; the mean reverting and non-mean reverting cases and compare two specifications of conditional correlations using standardized and devolatized returns.

With m daily equity returns in the $m\,x1$ vector, r_t over period $t=1,2,\ldots\ldots,T$, $T+1,\ldots\ldots,T+N$, we use the first T_0 observations to calculate (9) to start the initialization recursive in (12) and obtain estimates of $\bar{\sigma}_i^2$ and $\bar{\rho}_{ij}$ in (2) and (12) respectively. Suppose s is the starting point of the recent sample of observations for estimation within the estimation sample (2008 to 2011). Then it follows that $T>s>T_0>\rho$ where ρ is the size of estimation window so that the estimation window is, $T_e=T-s+1$. Thus, the remaining observations, N (2012 to 2013) can be used for evaluating the *t*-DCC model. Thus, the whole sample equals S_e+S_{ev}. With a rolling window of size w, then $s=T+1-w$ so that the whole estimation can be moved into the future with an update frequency of h.

Mean-Reverting Conditional Correlations

For the mean-reverting case, we need estimates of the unconditional volatilities and correlation coefficients from (13) and (14) below.

$$\bar{\sigma}_{i,t}^2 = \frac{\sum_{\tau=1}^{r} r^2{}_{i\tau}}{t} \quad (13)$$

$$\bar{\rho}_{ij,t} = \frac{\sum_{\tau=1}^{t} r_{i\tau} r_{j\tau}}{\sqrt{\sum_{\tau=1}^{t} r^2{}_{i\tau}} \sqrt{\sum_{\tau=1}^{t} r^2{}_{j\tau}}}, \tag{14}$$

The index t represents the end of available estimation sample which may be recursively rolling or expanding [24, 25, and 26].

4 Maximum Likelihood Estimation of the *normal*-DCC and the *t*-DCC Model

In the non-mean reverting specifications, (2) and (12), the *t*-DCC model has $2m + 3$ unknown parameters made up of $2m$ coefficients $\lambda_1 = (\lambda_{11}, \lambda_{12}, \ldots\ldots, \lambda_{1m})'$ and $\lambda_2 = (\lambda_{21}, \lambda_{22}, \ldots\ldots, \lambda_{2m})'$ that enter the individual equity returns volatilities, and the two coefficients ϕ_1 and ϕ_2 that enter conditional correlations plus the degrees of freedom (v) of the multivariate t distribution.

Following [29], for testing that one of the equity returns has non-mean reverting volatility, let λ_{i1} and λ_{i2} be parameters for the conditional volatility equation of the $i-th$ equity, the relevant test is

$H_0 : \lambda_{i1} + \lambda_{i2} = 1$ against $H_A : \lambda_{i1} + \lambda_{i2} < 1$

Under H_0, the process is non-mean reverting and the unconditional variance for the equity does not exist.

In (2) and (12), parameters $\bar{\sigma}_i^2$ and $\bar{\rho}_{ij}^2$ are unconditional volatilities and return correlations and could be estimated using the initialization sample (13) and (14). In the non-mean reverting case, the intercepts in (2) and (12) cease to exist.

Suppose we denote the unknown coefficients as follows.

$\theta = (\lambda_1, \lambda_2, \phi_1, \phi_2, v)`$

Given a sample of observations on returns, $r_1, r_2, \ldots\ldots, r_t$ available at time t, the t log-likelihood function based on decomposing (1) is given by

$$l_t(\theta) = \sum_{\tau=s}^{t} f_\tau(\theta), \tag{15}$$

where $s < t$ is beginning date for the estimation window.[10] With the $t-DCC$ model, $f_\tau(\theta)$ is the density of the multivariate distribution with v degrees of freedom that can be written in terms of $\Sigma_{t-1} = D_{t-1} R_{t-1} D_{t-1}$ as

[10]There is no need to write out the log- likelihood function for a normal distribution since it is only estimated here to show that the results from *t*-DCC are preferred to those from the *normal*-DCC model.

$$f_{\tau}(\theta)=\frac{m}{2}\ln(\pi)-\frac{1}{2}\ln|\mathrm{R}_{\tau-1}(\theta)|-\ln|\mathrm{D}_{\tau-1}(\lambda_1,\lambda_2)|+\ln[\Gamma(\frac{m+v}{2})/\Gamma(\frac{v}{2})]$$
$$-\frac{m}{2}\ln(v-2)-(\frac{m+v}{2})\ln[1+\frac{e_{\tau}'\mathrm{D}^{-1}_{\tau-1}(\lambda_1,\lambda_2)R^{-1}_{\tau-1}(\theta)\,\mathrm{D}^{-1}_{\tau-1}(\lambda_1,\lambda_2)e_{\tau}}{v-2}] \quad (16)$$

where $e_{\tau}=r_{\tau}-\mu_{\tau-1}$ and

$$\ln|\mathrm{D}_{\tau-1}(\lambda_1,\lambda_2)|=\sum_{i=1}^{m}\ln[\sigma_{i,\tau-1}(\lambda_{1i,}\lambda_{2i})] \quad (17)$$

As pointed out by [24, 26], surveys by [5] and [21], the multivariate *t* density is usually written in terms of a scale matrix. However, if we assume that $v>2$, then it means that Σ_{t-1} exists to permit the scale matrix to be written in terms of Σ_{t-1}.
In [16], R_{t-1} depends on λ_1 and λ_2 in addition to ϕ_1 and ϕ_2 (based on standardized returns) but the specification here is based on devolatilized returns has R_{t-1} depending only on ϕ_1 and ϕ_2 plus the p -the lag order that is used in the devolatization process. The ML estimate of ϕ based on sample observations $r_1, r_2,\ldots\ldots, r_t$ are computable by maximizing $l_t(\theta)$ with respect to ϕ represented by $\hat{\theta}_t$ or simply as

$$\hat{\theta}_t = Arg\max_{\theta}\{l_t(\theta)\},\ \text{for}\ t=T, T+h, T+2h,\ldots, T+N, \quad (18)$$

where h , the estimation is update frequency and N is the length of the evaluation sample. Note that the standard errors of ML estimates are calculated from the following asymptotic expression.

$$Co\hat{v}(\hat{\theta}_t)=\{\sum_{\tau=s}^{t}[\frac{-\partial^2 f_{\tau}(\theta)}{\partial\theta\partial\theta'}]_{\theta=\theta_t}\}^{-1}$$

The model is reasonable to estimate in that the number of unknown coefficients of the MGARCH model increases as a *quadratic* function of m while in the standard DCC model, it rises *linearly* with m . This fact notwithstanding, the simultaneous estimation of all parameters of the DCC model can and do often gives rise to convergence problems or to a local maxima of the likelihood function $l_t(\theta)$. However, if the standard returns are conditionally Gaussian, it is possible to resort to [16]'s two-stage estimation, albeit with some loss in estimation efficiency. In the multivariate *t* distribution adopted here, the degrees of freedom (v) is the same across all equity returns whereas under the two-stage estimation procedure, separate $t-GARCH(1,1)$ can easily lead to different estimates of v [11].

[11][24, 25, 27] note that the marginal distributions found in a multivariate $t-$distribution with v are also $t-$distributed with the same v .

5 Diagnostic Tests of the t-DCC Model

Suppose one has a portfolio with m equities with r_t as a *vector* of returns with $m\,x\,1$ *vector* of predetermined weights w_{t-1}. The returns to such a portfolio would be

$$\rho_t = w'_{t-1} r_t \tag{19}$$

If the interest is calculating the capital **V**alue **a**t **R**isk (VaR) of a portfolio at t-1 with probability $(1-\alpha)$, represented by $VaR(w_{t-1},\, \alpha)$ this requires that

$$\Pr[w'_{t-1} r_t < -VaR(w_{t-1},\, \alpha) \,|\, \Omega_{t-1} |] \le \alpha$$

Under assumptions, the conditional on Ω_{t-1}, then (equity returns) $w'_{t-1} r_t$ or ρ have a Student $t-$ distribution with mean of $w'_{t-1}\mu_{t-1}$ and variance $w'_{t-1}\Sigma_{t-1} w_{t-1}$ with v degrees of freedom. Thus,

$$z_t = \sqrt{\frac{v}{v-2}}\left(\frac{w'_{t-1} r_t - w'_{t-1}\mu_{t-1}}{\sqrt{w'_{t-1}\Sigma_{t-1} w_{t-1}}}\right)$$

which is conditional on Ω_{t-1} and also has a $t-$ distribution with v degrees of freedom with mean $E(z_t \,|\, \Omega_{t-1}) = 0$ and $Var(z_t \,|\, \Omega_{t-1}) = v/v-2.$ With the cumulative distribution function (CDF) of a Student t with v degrees of freedom represented by $F_v(z)$, the $VaR(w_{t-1},\, \alpha)$ is the solution to

$$F_v = \left(\frac{-VaR(w_{t-1},\alpha) - w'_{t-1}\mu_{t-1}}{\sqrt{\frac{v-2}{2}(w'_{t-1}\Sigma_{t-1} w_{t-1})}}\right) \le \alpha$$

However, since $F_v(z)$ is a continuous and monotonic function of z, then

$$\left(\frac{-VaR(w_{t-1},\alpha) - w'_{t-1}\mu_{t-1}}{\sqrt{\frac{v-2}{2}(w'_{t-1}\Sigma_{t-1} w_{t-1})}}\right) = F_v^{-1}(\alpha) = -c_\alpha$$

where c_α is a $\alpha\%$ critical value from the Student t-distribution with v degrees of freedom. The out-of-sample V**a**R forecast puts $\alpha = 0.99.$ Thus,

$$VaR(w_{t-1},\alpha) = \tilde{c}_\alpha \sqrt{w'_{t-1}\Sigma_{t-1} w_{t-1}} - w'_{t-1}\mu_{t-1} \tag{20}$$

where $\tilde{c}_\alpha = c_\alpha \sqrt{\frac{v-2}{v}}$

Following [24, 25, 25, 27], [12] and [17], the test of the validity of the $t-DCC$ is calculated recursively by using the V**a**R indicators denoted by (d_t)

$$d_t = I(w'_{t-1} r_t + VaR(w_{t-1},\, \alpha)) \tag{21}$$

where $I(B)$ an indicator function that is equal to 1 if $B > 0$ and zero otherwise. The indicator statistics can be computed in-sample or preferably based on recursive out-of-sample one-step ahead forecasts of Σ_{t-1} and μ_{t-1} for pre-determined preferred set of portfolio weights w_{t-1} . In an out-of-sample exercise, the parameters of the mean returns variables (θ) and volatility variables (λ) can be fixed at the start of the evaluation exercise or changed with an update frequency of h periods. Suppose we an evaluation sample, $S_{eval} = \{r_t, t = T+1, T+2, \ldots\ldots, T+N\}$ then the mean hit rate [MHR] is

$$\hat{\pi}_N = \frac{1}{N}\sum_{t=T+1}^{T+N} d_t \tag{22}$$

With a $t-DCC$ model, the estimated mean hit rate, $\hat{\pi}_N$, has a mean of $(1-\alpha$) and variance ($\frac{\alpha(1-\alpha)}{N}$) and the resulting *standardized statistic* is

$$z_\pi = \frac{\sqrt{N[\hat{\pi}-(1-\alpha)]}}{\sqrt{\alpha(1-\alpha)}} \tag{23}$$

This expression has a standard normal distribution if the evaluation sample size N (in our case, 455 observations) is very large. According to [20, 4] and [25, 26], the z_π statistic provides evidence of the performance of Σ_{t-1} and μ_{t-1} in an average **un**conditional setting. On the other hand, [6] has suggested an alternative conditional evaluation procedure based on probability integral transforms.

$$\hat{U}_t = F_v\left(\frac{w'_{t-1}r_t - w'_{t-1}\hat{\mu}_{t-1}}{\sqrt{\frac{v-2}{v}w'_{t-1}\hat{\Sigma}_{t-1}w_{t-1}}}\right), \quad t = T+1, T+2, \ldots\ldots, T+N \tag{24}$$

If the $t-DCC$ model is correctly specified, under the null hypothesis the probability integral transforms estimates (PIT), $\hat{U}_t$ should be not be serially correlated and should have a uniform distribution over the range $(0,1)$ and is testable. The serial correlation property of $\hat{U}_t$ can be tested by Lagrange multiplier tests by running OLS of $\hat{U}_t$ on an intercept and lagged values of $\hat{U}_{t-1}, \hat{U}_{t-2}, \ldots\ldots, \hat{U}_{t-s}$ [Table 7]. In this case, the maximum lag length, s can be determined by the AIC information criteria. The uniform distribution of $\hat{U}_t$ over t can be tested using the Kolmogorov-Smirnov (KS_N) statistic defined as $KS_N = \sup_x |F_{\hat{U}}(x) - U(x)|$ where $F_{\hat{U}}(x)$ is the empirical cumulative distribution function (CDF) of $\hat{U}_t$ for $t = T+1,\ T+2, \ldots\ldots,\ T+N$ and $U(x) = x$ is the CDF of the *iid* U (0, 1). If the value of the KS_N statistic is large, it would show that the CDF is not similar to the uniform distribution assumed in the $t-DCC$.[12] However, if the estimated value of KS_N is below the critical value (say 5%), then it does support the validity of the $t-DCC$.

[12]For more details on the Kolmogorov-Smirnov test and critical values, see [22] and [23].

6 Empirical Application to devolatized Equity Returns

Table 3: Standardized and Devolatized Equity Returns

	Standardized Equity Returns				Devolatized Equity Returns			
	Mean	Std. Dev	Skewness	Excess Kurtosis[13]	Mean	Std. Dev	Skewness	Excess Kurtosis
Brazil	-0.087	2.701	-0.636	8.935	- 0.031	1.0123	-0.021	-0.160
Russia	0.094	3.473	-0.532	10.135	0.066	1.0269	-0.312	-0.195
India	0.078	2.728	-0.549	8.121	0.036	1.0172	-0.224	-0.177
China	0.113	3.428	-0.741	7.870	0.065	1.005	-0.350	-0.132
South Africa	0.074	2.622	-0.458	4.578	0.034	1.008	-0.219	-0.253

The rate of return is calculated as follows. If the price of equity is S_t then the returns are defined as $\Delta S_t = \ln(\frac{S_t}{S_{t-1}})\,\text{x}100$. Table 3 provides summary statistics of equity returns in percentage terms arrived at by setting $\rho = 20$ which transformed returns (r_t) into approximate Gaussian to obtain devolatilized returns. Standardized returns or non-devolatized returns shows kurtosis ranging from 4.5 to 10.13 – all well above 3, the value expected in a Gaussian distribution. The excess kurtosis is highest for Russia and lowest for South Africa. On the other hand, all five devolatized returns show excess kurtosis that ranges from -0.160 to -0.253. The excess kurtosis for Russia has fallen from 10.13 (standardized) to -0.195 (devolatized). Similarly, the standard deviations for standardized returns range from 2.62 to 3.743 while for devolatized returns, they are all close 1.00 which allows one to compare equity returns across all BRICS countries.[14]

7 ML estimates of the t-DCC Models

[24, 26] point out that weekly or daily returns approximately have mean zero serially uncorrelated processes which make it possible to assume that $\mu_{t-1} = 0$. The *t*-DCC model is estimated for 5 five BRICS equity returns over the period 01-Jan-2008 to 27-Sept-2013. The estimation period is 30-Jan-2008 to 30-Dec-2011 (1023 observations) and we use 455 observations (02-Jan-2012 to 27-Sept-2013) for the evaluation of estimated volatilities and

[13]It is a statistical term that describes a probability or return distribution that has a kurtosis coefficient that is larger than the coefficient associated with a normal distribution, which is around 3. A higher value usually indicates that the probability of obtaining an extreme value in the future is higher than a lower level of kurtosis.

[14]The evidence exists as well in the form of QQ plots (not included here) which fits returns with the normal density. The plots compare the distribution of returns to the normal distribution (represented by a straight line in QQ plots). Comparing QQ plots for standardized returns and devolatized returns, the QQ plots for devolatized returns tend to lie closer to the diagonal of a normal distribution than standardized plots. Russia's distribution was the only QQ plot that was slightly off the diagonal line. Standardized returns tended to lie way-off the diagonal line.

correlations model. The V**a**R and distribution diagnostics are used to assess the results from the model.

We estimated the unrestricted version of the DCC (1, 1) assuming a normal distribution (*normal* DCC) with asset-specific volatility parameters $\lambda_1 = (\lambda_{11}, \lambda_{12}, \ldots\ldots, \lambda_{1m})'$ and $\lambda_2 = (\lambda_{21}, \lambda_{22}, \ldots\ldots, \lambda_{2m})'$ with common conditional correlations, ϕ_1 and ϕ_2. In the paper, $m = 5$ and there are no restrictions on decay factors (different volatility for each variable and same for the correlation decay factor). Table 4 presents the maximum likelihood estimates of λ_{i1}, λ_{i2} for five equity returns and ϕ_1 and ϕ_2. We note that all the equity-specific returns are highly significant with their sum all close to unity. The log-likelihood value is -8848.4. This value is important since we will compare it to the log-likelihood value from the *t*-DCC model.

Table 4: Results from the *Normal-DCC-GARCH* multivariate model

Equity Index	λ_1	λ_2	$\lambda_1 + \lambda_2$	$1 - \lambda_1 - \lambda_2$
Brazil (FTSE 100 returns)= rib	0.88687 (51.4491)	0.10551 (6.8752)	0.99238	0.00762
Russia (FTSE 100 returns) =rru	0.90689 (69.6704)	0.082182 (7.5107)	0.989072	0.010928
India (FTSE100 returns) = rin	0.90709 (65.5531)	0.086223 (6.9970)	0.993313	0.006687
China (FTSE 100 returns) = rch	0.91192 (67.1305)	0.088387 (6.7286)	0.995793	0.004207
South Africa (FTSE 100 returns) =rsa	0.90827	0.083487	0.991757	0.008243
	ϕ_1	ϕ_2	$\phi_1 + \phi_2$	
	0.98483 (417.349)	0.011280 (7.1186)	0.99611	
Value of the log-likelihood = -8848.4				

Note: λ_{1i} and λ_{2i} are the equity-specific volatility parameters.

Table 5: Results from the *t-DCC-GARCH* multivariate model

Equity Index	λ_1	λ_2	$\lambda_1 + \lambda_2$	$1 - \lambda_1 - \lambda_2$	$\rho - value$
Brazil (FTSE 100 returns)= rbr	0.90451 (46.3046)	0.087542 (5.1155)	0.992052	0.007948	0.000
Russia (FTSE 100 returns) =rru	0.90994 (59.3676)	0.078049 (6.1920)	0.987989	0.012011	0.000
India (FTSE100 returns) = rin	0.91832 (63.7770)	0.074587 (5.9389)	0.992907	0.007093	0.000
China (FTSE 100 returns) = rch	0.91765 (62.8273)	0.076140 (5.8393)	0.99379	0.00621	0.000
South Africa (FTSE 100 returns) =rsa	0.91362 (53.6865)	0.074819 (5.3731)	0.988439	0.011561	0.00
$\phi_1 + \phi_2$	ϕ_1	ϕ_2	v	Log-likelihood Value	
0.994955	0.98301 (325.5349)	0.011945 (6.1483)	8.9779 (9.1914)	-8508.0	

The unrestricted version of the DCC (1, 1) is estimated assuming a *t*- distribution (*t*-DCC) with asset-specific volatility parameters $\lambda_1 = (\lambda_{11}, \lambda_{12}, \ldots\ldots, \lambda_{1m})'$ and $\lambda_2 = (\lambda_{21}, \lambda_{22}, \ldots\ldots, \lambda_{2m})'$ with common conditional correlations, ϕ_1 and ϕ_2 and v degrees of freedom. There are no restrictions on decay factors (different volatility for each variable but the same for the correlation decay factor). Table 5 presents all the maximum likelihood estimates of λ_{i1} , λ_{i2} for five equity returns and ϕ_1 and ϕ_2 . We note that all the equity-specific returns are highly significant with their sum all close to unity. The log-likelihood value from the *t*-DCC model is -8508.0 and it is larger than the value from Table 4. The degrees of freedom is 9.1914, well below the value of 30 that is expected for a multivariate normal distribution. As a check on the results in Table 5, similar results were obtained when we estimated a *t*-DCC model on residuals obtained when a regression of equity returns is on returns on their past values. As the in Table 4, specific-equity returns estimates of the volatility and correlation decay parameters are highly significant and close to estimates obtained in Table 4.

Table 6: Results for non-mean reverting volatility from the *t-DCC-GARCH* multivariate model

Equity Index	$1-\lambda_1-\lambda_2$ Estimate	Standard-Error	t-statistic	Variance Matrix
Brazil (FTSE 100 returns)= rbr	0.0079994	0.0030902	2.5886	0.9550E-5
Russia (FTSE 100 returns) =rru	0.012099	0.0044621	2.7116	0.1991E-4
India (FTSE100 returns) = rin	0.0070998	0.0027202	2.6101	0.7399E-5
China (FTSE 100 returns) = rch	0.0062000	0.0022165	2.7972	0.4913E-5
South Africa (FTSE 100 returns) =rsa	0.011599	0.0043472	2.6682	0.1890E-4

Table 6 presents tests for non-mean reversion. The sum of estimates of λ_{1i} and λ_{2i} are almost unit. The hypothesis that $H_0 : \lambda_{i1} + \lambda_{i2} = 1$ (Integrated GARCH) against mean reversion ($H_0 : \lambda_{i1} + \lambda_{i2} < 1$) is rejected for all five equities. This means that BRICS stock market returns show significant mean-reverting volatility for all equities in these economies. Similarly, in Table 6, $\hat{\phi}_1 + \hat{\phi}_2 = 0.994955$ which suggests very slow but statistically mean reverting conditional correlations.

The evaluation sample from 02-Jan-2012 to 27-Sept-2013 tests are based on the probability integrals transform (PIT), $\hat{U}_t$ as defined by (24). Under the null hypothesis, if the t-DCC model is correctly specified, then $\hat{U}_t$ has no serial correlation and it is uniformly distributed over (0, 1). $\hat{U}_t$ is obtained by considering an equal-weighted portfolio of all five BRICS equity returns as defined by (19) with a risk tolerance of $\alpha = 0.1$. To test the null hypothesis that $\hat{U}_t$,s are not serial correlated, we use the Lagrange Multiplier test. The value of the CHSQ (12) = 14.6899[.259] and the F Statistic = F (12,442) = 1.2289[.260] as reported in Table 7. Given these values, it is clear the t-DCC model specification passes the test.

Table 7: Test of Serial Correlation of Residuals (OLS case)

Dependent variable is U-Hat
List of variables in OLS regression:
Intercept
455 observations used for estimation from 02-Jan-12 to 27-Sep-13

Regressor	Coefficient	Standard Error	T-Ratio [Prob]
OLS RES(-1)	-0.0039942	.047466	-.084149[.933]
OLS RES(-2)	-0.038424	.047307	-.81223[.417]
OLS RES(-3)	0 .082767	.047311	1.7494[.081]
OLS RES(-4)	-0.060729	.047477	-1.2791[.202]
OLS RES(-5)	-0.013246	.047436	-.27924[.780]
OLS RES(-6)	-0.020254	.047498	-.42642[.670]
OLS RES(-7)	0.049075	.047659	1.0297[.304]
OLS RES(-8)	-0.076828	.047763	-1.6085[.108]
OLS RES(-9)	0 .0095552	.047809	.19986[.842]
OLS RES(-10)	0.039533	.047648	.82970[.407]
OLS RES(-11)	0.089915	.047680	1.8858[.060]
OLS RES(-12)	-0.068241	.047876	-1.4254[.155]

Lagrange Multiplier Statistic CHSQ (12) = 14.6899[.259]
F Statistic F (12,442) = 1.2289[.260]

U-Hat denotes the probability integral transform.
Under the null hypothesis, U-Hat should not display any serial correlation.

In Figure 2, the Kolmogorov-Smirnov test (KS_N) is applied to $\hat{U}_t$ to determine whether the probability integrals transform (PIT) are from a uniform distribution. The value of the KS_N statistic is 0.061220 which is barely within the 5% critical value of 0.063758. This means that the null hypothesis that the sample's cumulative density function (CDF) is similar to the uniform distribution cannot be rejected, although barely. Figure 4 shows the histograms of the probability integral transform variable $\hat{U}_t$ with minor violations of a uniform distribution.

In Figure 5, we test whether there is any violation of the Value at Risk (VaR) constraint as this test focuses on the tail properties of equity returns. With a tolerance probability of $\alpha = 0.01$, Figure 5 shows the risks in these emerging markets shows spikes in June and September 2013 when the U.S. Federal Reserve Bank indicated that it might begin reducing liquidity (quantitative easing). This announcement sent shock waves in the emerging markets (BRICS) as the U.S. market looked better for investors.

Table 8: Mean V**a**R Exceptions and the Associated Diagnostic Test Statistics
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Mean Hit Rate ($\hat{\pi}_N$ statistic) =0.99780 with expected value of 0.99000

Standard Normal Test Statistic ($\hat{z}_\pi$) = 1.6726[.094]

In Table 8 there is an additional test of V**a**R violations under a tolerance probability of $\alpha = 0.01$. The $\hat{\pi}$ statistic has a value equal to 0.998 which is very close to its expected value of 0.990. Similarly, the F statistic is 1.229 with a p-value of 0.260. These results provide support for the validity of *t*-DCC model. The $\hat{z}_\pi$ statistic is not significant at p = 0.094.

Table 9: Use of Cross Conditional Correlations in Investment Decisions

Figure	Country-to-Country Correlation (Negative)	Country to invest in	Do not Invest in
1A	India and Russia	Indians can invest in South Africa, Brazil and China	Indians should NOT invest in Russia
1B	Brazil and Russia	Brazilians can invest in India, South Africa and China	Brazilians should NOT invest in Russia
1C	China and Russia	Chinese can invest in India, South Africa and Brazil	Chinese should NOT invest in Russia
8 and 9	South Africa and Russia	South Africans can invest in India, China and Brazil	South Africans should NOT invest in Russia
6 and 7	Russia with all other Stock Markets in India, China and South Africa	By 2013, India, Brazil, and China look good for investment in stocks	Not in Brazil (2011-12) Not in India (mid-2012) South Africa (avoid 2010) India (avoid 2010)

Table 9 provides a summary of how investors in BRICS countries might use results from conditional correlations to determine where to invest in terms of stock market performance. While the results are tentative, they do suggest that investors in South Africa, Brazil, China and India avoid investing in the Russian stock market.[15] Figures 6 and 7 indicate that South

[15]Incidentally, the negative view of Russian stocks has nothing to do with the current crisis in Ukraine. The analysis in this paper ends on September 27, 2013 – long before the start of the Russian-Ukraine crisis. In fact, the current situation would tends to strengthen the idea that Russian stocks should be unattractive to investors in China, India, Brazil and South Africa.

Africa was not favorable in 2010, Brazil in 2011-12, India in 2010 and in mid-2012. However, by 2013 India, Brazil, and China had become favorable destinations for stock market investments.

8 Evolution of Equity Return Volatilities and Correlations

The conditional equity returns on equities for BRICS countries are shown in Figures1 (a) - 1(c), Figure 3, and Figures 6-9. Figure 1(a) shows conditional correlations of the Indian stock market with other members in the BRICS countries. Indian and Russian equity returns show negative correlation but positive correlation with Brazil, China, and South Africa. Figure 1(b) shows negative conditional correlation between India equity returns with Russia but positive correlation with Brazil, China, and South Africa. Figure 1(c) shows negative conditional correlation between Chinese equity returns with Russia but positive correlation with Brazil, China, and South Africa. The Chinese-Russian conditional correlations improve until January 10, 2011 but then deteriorate thereafter.

Figure 3 shows that the 2008 and 2009 financial crisis led to sharp spikes in volatility. In 2008, China had the highest spike; in 2009, India had the highest spike, and in 2010, South Africa experienced the highest spike. In these periods, Russia had the lowest spike in volatility. Figure 5 shows that volatility risk increased between May 16 and 24 July 2013 and around September 27, 2013. From December 2012, it appears as if these emerging markets exhibited little variations in equity volatility as foreign investors sought higher returns in these markets. In Figure 6, conditional correlations of Russia (in equity returns) fell during the financial crisis but picked up from July 2010 (Figure 7). It shows that despite the 2008 financial crisis, BRICS equity returns (in terms of correlations) are anchored by the process of globalization in which stock markets are now more interdependent than ever. The estimation period shows how the Russian equity returns move together. This fact is true even during the depth of the financial crisis.

Finally, in Figures 8 and 9, South African conditional correlations were negatively related to the Russian stock market but positively related to Brazil, China, and India stock markets. These results hold true for both the estimation period and the evaluation sample. The negative relationship between South African and Russian equity returns improved until May 2011 but deteriorated again from mid-September 2013.

All said, equity markets with negative conditional correlation within BRICS can diversify their portfolio by investing in stock markets that have positive conditional correlations. The maximum eigenvalues show that there has been a decline in volatility. Figure 10 represents the extent of volatilities across all 5 BRICS equity returns given by the maximum 5 x 5 matrix of equity volatilities. It captures a high spike at the beginning of 2009 that reflects the financial crisis.

9 Concluding Remarks

The paper tested the idea that devolatized returns are a better approach to understanding the volatility of asset markets than standardized returns so widely used in portfolio decision making and risk management [24, 25, and 26]. Given that the modelling of conditional volatilities and correlations across stock market returns is a critical function of investing and portfolio management in a global economy, [24, 25, 26] suggest that devolatilized returns within a multivariate *t*-DCC model capture the fat tail properties of financial time series since transforming returns by realized volatility makes the innovations Gaussian. This is a key concept in the application of the *t*-DCC model. The paper applied this approach to the estimation of conditional correlations and volatilities for BRICS equity returns. Our results indicate that the *t*-DCC model is preferred over the normal-DCC model in estimating conditional volatilities and correlations. Second, both $\hat{\pi}_N$ and $\hat{z}_\pi$ [tests for serial correlation and a uniform distribution] provide support for the *t*-DCC model despite the 2008 financial crisis. However, the model barely passes the non-parametric Kolmogorov-Smirnov (KS_N) test that tests whether probability transform estimates, $\hat{U}_t$ are uniformly distributed over the range (0, 1). From Figures 1a -1c, it is clear that some stock market equity return correlations are negatively related: India and Brazil, Russia and Brazil, and China and Russia respectively. The model shows that in 2008 and 2009 financial crisis led to sharp spikes in volatility (Figure 3). Overall, the results track well correlations and volatilities in BRICS equity returns.

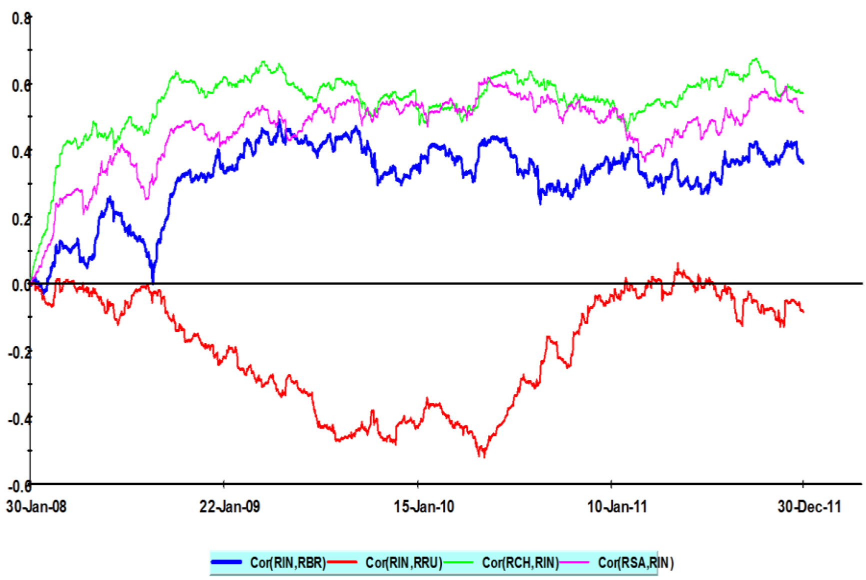

Figure 1A: Plot of conditional correlations (negative correlation equity returns: India and Russia)

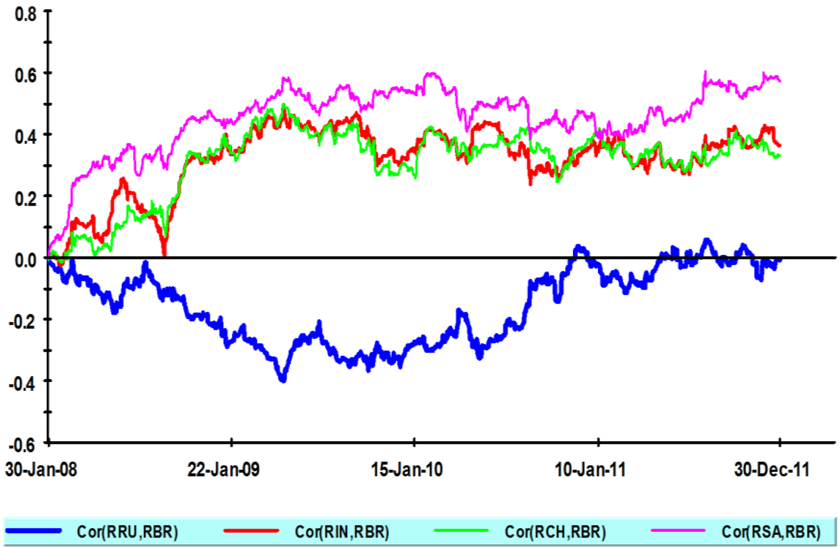

Figure 1B: Plot of correlations (negative equity returns correlation: India and Brazil)

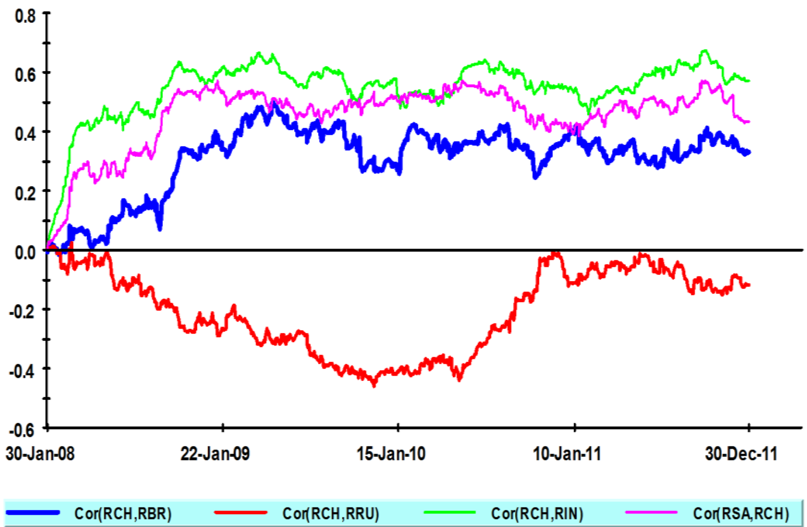

Figure 1C: Plot of correlations (negative equity returns correlation: Russia and China)

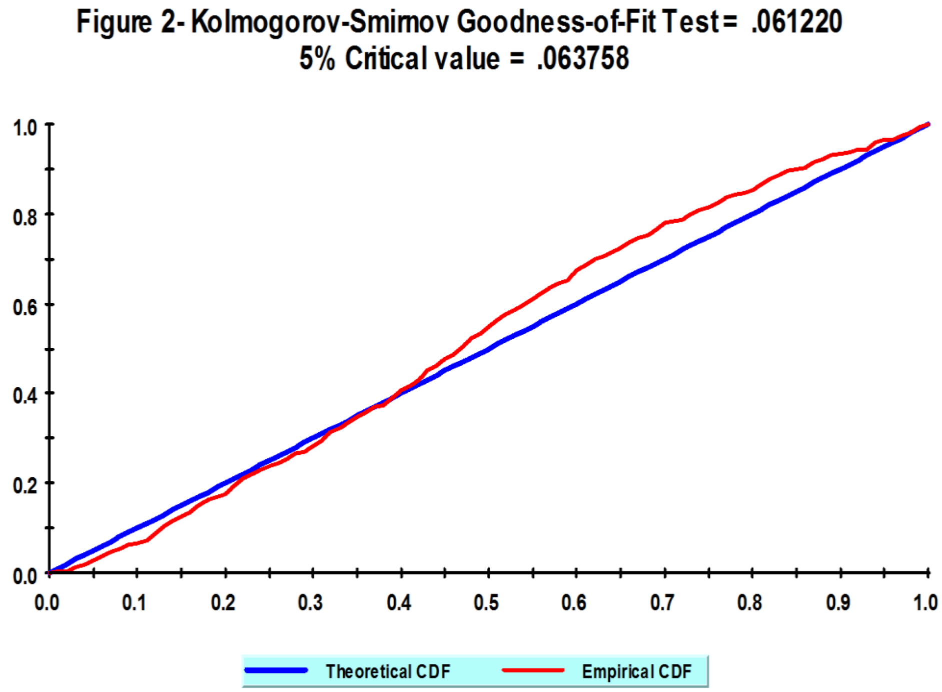

Figure 2: The Kolmogorov-Smirnov goodness-of-fit test for the full t-DCC model over the evaluation sample.

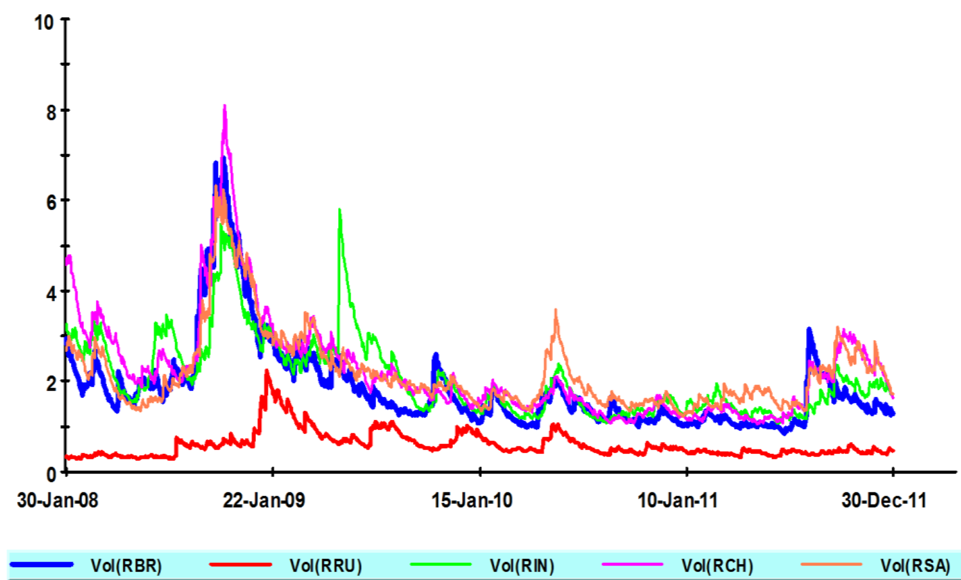

Figure 3:Plot of BRICS conditional volatilities, 30-Jan-2008 to 30-Dec-2011

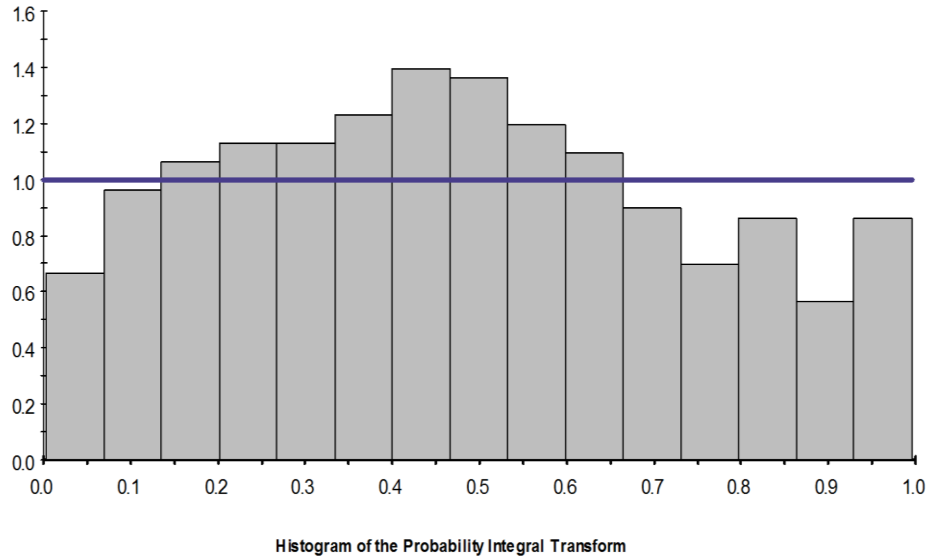

Figure 4:Histogram of the probability integral transform

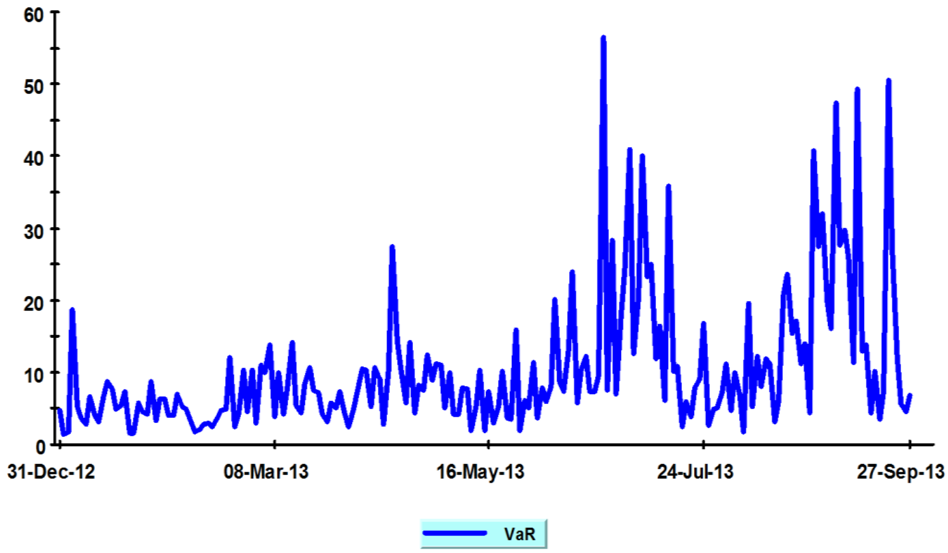

Figure 5: Plot of VaR for the evaluation period, Dec. 2012 to Sept. 2013

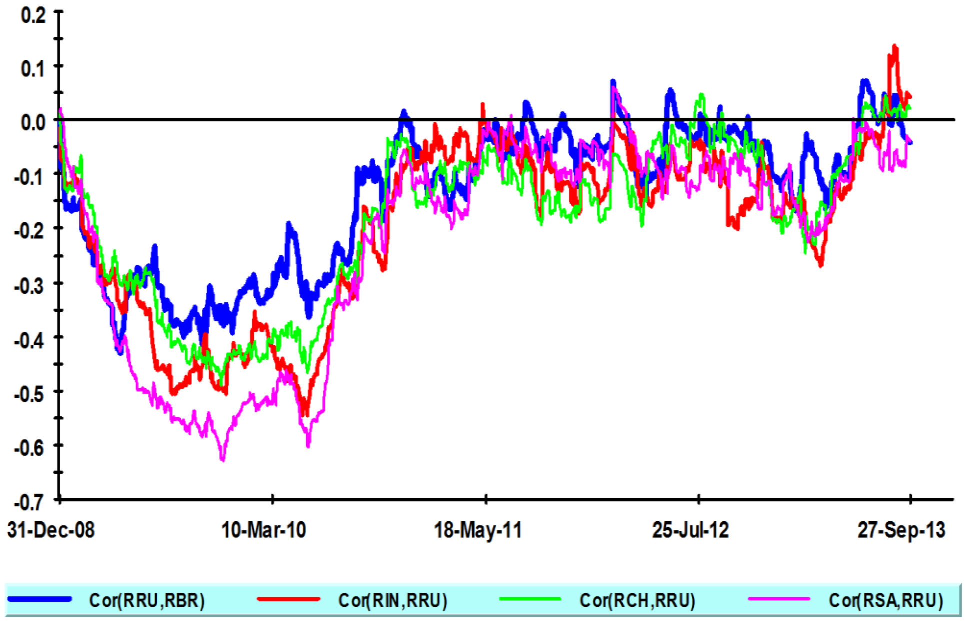

Figure 6: Plot of conditional correlations of Russian equities (evaluation period)

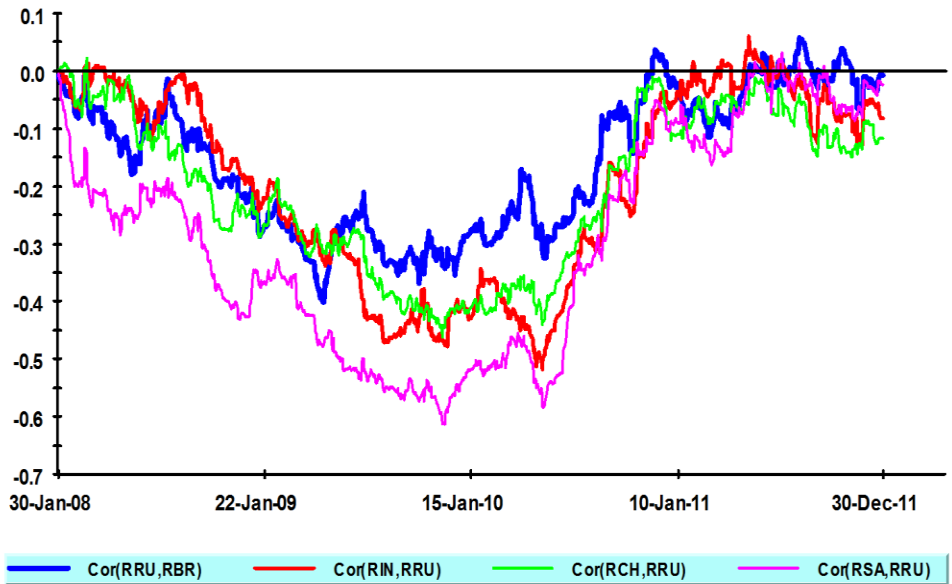

Figure 7: Plot of conditional correlations of Russian equities (estimation period)

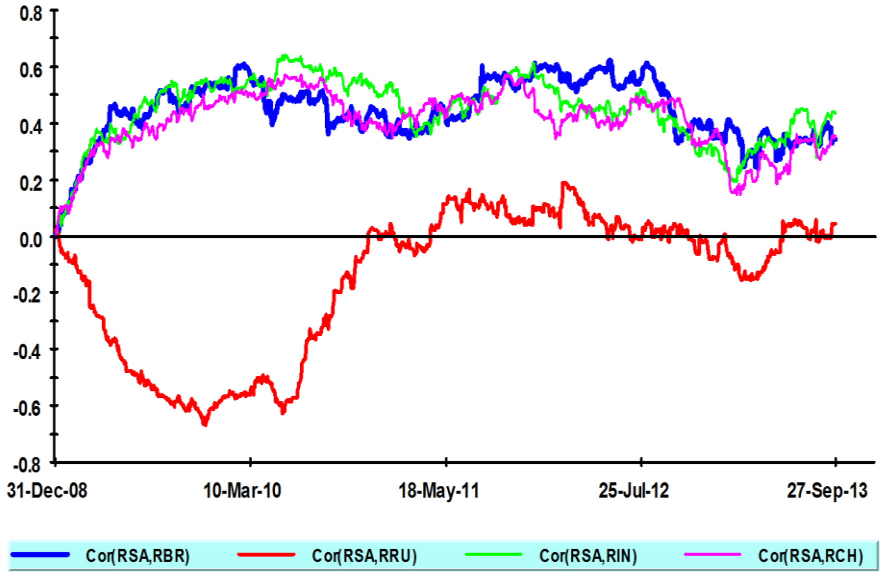

Figure 8: Plot of conditional correlations of South African equity returns (evaluation period)

In order to reduce the impact of the impact of the initial initialization on the plots of correlations, the initial estimates for 11 months in 2008 are excluded.

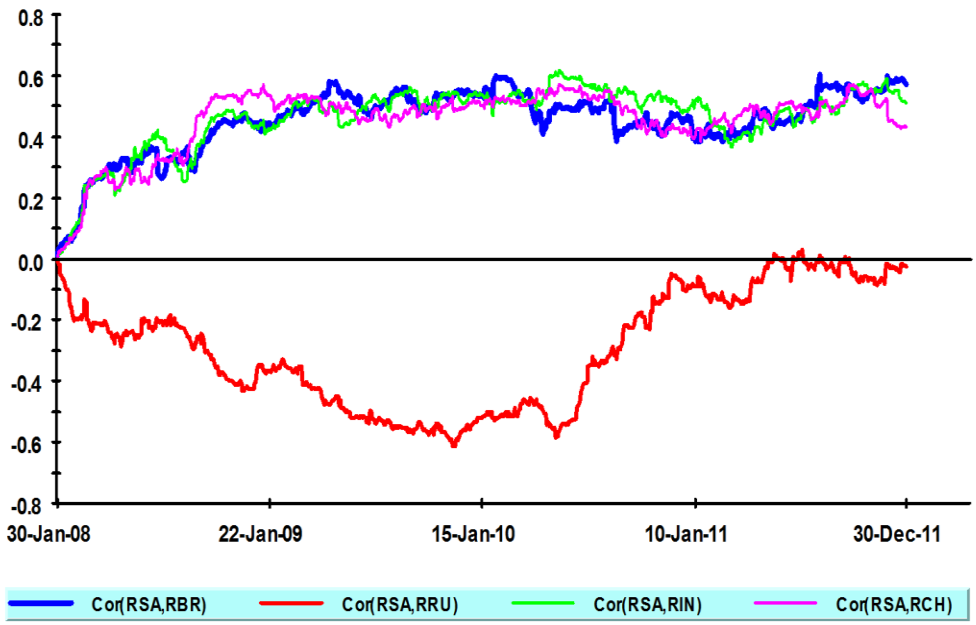

Figure 9:Plot of conditional correlations of South African equities (estimation period)

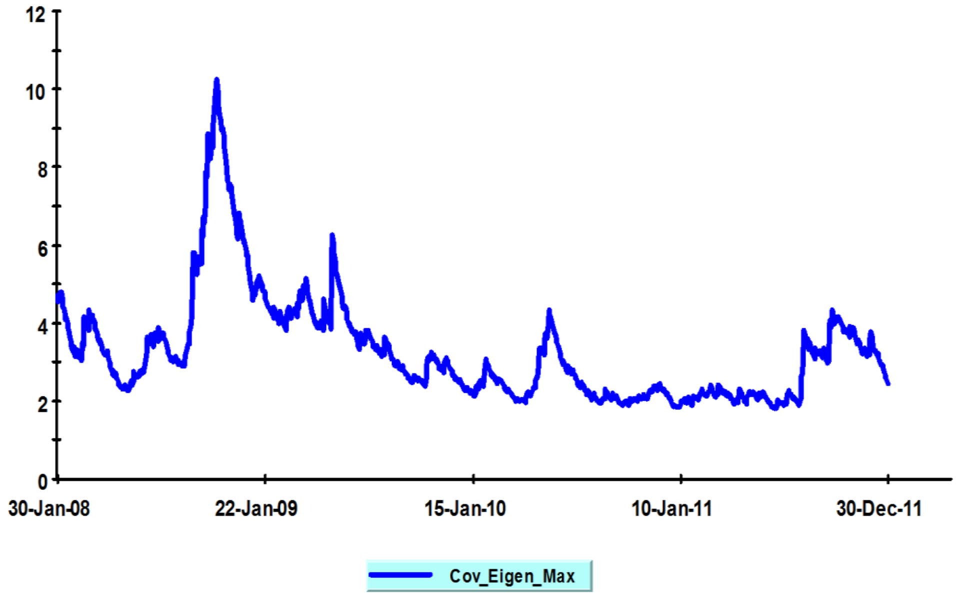

Figure 10: Maximum eigenvalues of the 5x5 matrix of volatilities

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