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Optimal Work Effort, Income and Wage

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Abstract

We study work effort with its various determinants such as the educational level of the worker, the minimum or start-up salary as well as the initial endowment of the worker. By means of optimization we find that optimal work effort depends directly on the initial income available to the worker, with a higher income reducing the effort of the worker. We also find that a higher initial wage and a reward parameter per work effort discourage workers to exert more effort on the job. Firms set optimal wages disregarding reward for work effort with more productive workers receiving higher wages and exerting more effort at the optimum.

JEL classification numbers: J2, J24, J31

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1 Introduction

Some of the literature on labor economics is dedicated to job search with Stigler [15, 16] and Alchian and Allen [1] being the first to study search effort in the conditions of costly information and high uncertainty. McCall [12] put this essential problem in a mathematical framework with relevance to reservation wage or the lowest wage the worker is willing to accept. Reservation wage may differ for two jobs of different characteristics or two individuals, thus leading to compensating wage differentials between different types of jobs. Mortensen [13], Burdett [3], Pissarides [14], and Van den Berg [17] analyze the behavior of unemployment [14], wage determination [3], job duration, job turnover [18], quit rates [3], and unemployment insurance and employment protection [14]. Kahn [9] investigates the relationship between search time and resulting wage as well as the duration span of unemployment.

In his model of optimal human capital investment Becker [2] studies how ability and family wealth affect the distribution of lifetime earnings. Family income determines human capital investment since families pay for the post-secondary schooling required to gain additional skills. Wealthy agents can obtain better education that promises higher future earnings. Becker defines this difference in financing education "unequal opportunity." Since poor individuals face higher opportunity cost of financing their education, they are likely to remain less educated.

Faber and Gibbons [5] and Jovanovic [8] develop learning models in the theory of earnings distributions by which jobs give workers information about their relative talents for different types of jobs. Workers gain from work experience by receiving information about their skills that can boost future earnings. Low-income workers accept low-paid jobs and tend to gain less valuable experience at the workplace. This decreases the quality of their sorting process and ultimately lowers their lifetime earnings. High-income workers employed in highly-skilled positions gain more from work experience which improves their opportunities of finding even better jobs.

Some studies dealing with minimum and equilibrium wage go beyond the traditional supply and demand analysis of the labor market and account for price effects. Fields [6] proposes a more complicated model of minimum wage in a two-sector economy, in which one sector is free of minimum wage. Gillespie [7] argues that if the demand for the good produced by the firm is very inelastic, management can offset the negative effects of the higher wage floor by raising prices without the need to fire employees. Katz [10], Katz and Krueger [11], and Card and Krueger [4] see the labor market as monopsonistic in that employers have greater market power in setting wages than employees. This monopsony could be the result of employer collusion or some natural factors such as segmented markets, search costs, information costs, imperfect mobility, etc. and represents a type of market failure by which employees are paid less than their marginal value.

We study work effort with its various determinants such as the educational level of the worker, the minimum salary, as well as the initial endowment of the worker. We perceive work effort as the skillfulness of labor, not as its marginal product. Thus, work effort can be defined as the amount of work, effort and hardships the worker endures on the job. At the same time, the marginal product of each worker can be equated to the outcome of his activities. While the latter is equivalent to product and is product-oriented, the former is process-oriented. By means of optimization we find that optimal work effort depends directly on the initial income available to the worker, with a higher income reducing the effort of the worker. We also find that a higher initial wage and a reward parameter per work effort discourage workers to exert more effort and to try harder on the job.

The paper is organized as follows: Part 1 introduces the reader into the literature. Part 2 reveals the effect of educational level, minimum wage and initial work effort on total work effort, overall wage, and worker’s income. Part 3 discusses the relationship between work effort and worker's income assumed to be exogenous. Part 4 reveals optimal work effort and wage under the conditions of profit maximization. Part 5 extends these results to the unconstrained case of profit maximization where two individuals are employed, one more productive than the other. The paper ends with conclusions.

2 A Simple Job Market Equilibrium Model

We use a job market equilibrium model in which demand for labor D_o is exogenously determined while supply S is positively related to wage w and effort e on the job. Thus, the more efforts workers exert, the greater the overall supply of labor. On the other hand, supply of labor is assumed to depend negatively on the worker's income m. Thus individuals are subject to the income effect and richer individuals tend to supply much less of their labor, therefore, $S_m < 0$.

$$S(w,e,m) = D_o \qquad S_w > 0 \qquad S_e > 0 \qquad S_m < 0$$

Wage starts from a base level w_o unrelated to effort. At the same time, it depends positively on the educational level s_o, as well as the effort e of the individual worker. Thus harder working and more educated individuals are rewarded with an increase in the overall wage.

$$w = w_o + g(e, s_o) \qquad g_e > 0 \qquad g_{s_o} > 0$$

Finally, work effort depends on its initial, autonomous level e_o as well as on the income m of the worker. Starting from a higher initial level of effort hard working individuals tend to exert a higher overall work effort. A higher income m may reduce overall work effort for two reasons. First, being rich, high-income workers have low opportunity cost of losing their job so they are likely to exert less effort. Second, income may be an indicator of talent and intellect which, furthermore, reduce the need for strenuous efforts.[2] Therefore,

$$e = e_o + h(m) \qquad\qquad h_m < 0$$

Solving the equations in the form of implicit functions,

$$S(w, e, m) - D_o = 0$$

$$w - w_o - g(e, s_o) = 0$$

$$e - e_o - h(m) = 0$$

we write off the following matrix equation differentiating the endogenous variables with respect to the educational level, first, and applying the implicit-function theorem.

$$\begin{bmatrix} S_w & S_e & S_m \\ 1 & -g_e & 0 \\ 0 & 1 & -h_m \end{bmatrix} \begin{bmatrix} \dfrac{\partial w}{\partial s_o} \\ \dfrac{\partial e}{\partial s_o} \\ \dfrac{\partial m}{\partial s_o} \end{bmatrix} = \begin{bmatrix} 0 \\ g_{s_o} \\ 0 \end{bmatrix} \qquad\qquad |J| = S_w g_e h_m + S_e h_m + S_m < 0$$

The Jacobian is positive providing for a unique set of solutions. We solve conveniently by matrix inversion.

$$C = \begin{bmatrix} g_e h_m & h_m & 1 \\ h_m S_e + S_m & -h_m S_w & -S_w \\ g_e S_m & S_m & -g_e S_w - S_e \end{bmatrix} \qquad C' = \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix}$$

$$J^{-1} = \frac{1}{|J|} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix}$$

$$= \frac{1}{(S_w g_e h_m + S_e h_m + S_m)} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix}$$

[2] We can paraphrase this with the well-known saying that smart people are lazy.

$$\begin{bmatrix} \dfrac{\partial w}{\partial s_o} \\ \dfrac{\partial e}{\partial s_o} \\ \dfrac{\partial m}{\partial s_o} \end{bmatrix} = \frac{1}{(S_w g_e h_m + S_e h_m + S_m)} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix} \begin{bmatrix} 0 \\ g_{s_o} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{g_{s_o}(h_m S_e + S_m)}{S_w g_e h_m + S_e h_m + S_m} \\ -\dfrac{g_{s_o} h_m S_w}{S_w g_e h_m + S_e h_m + S_m} \\ -\dfrac{g_{s_o} S_w}{S_w g_e h_m + S_e h_m + S_m} \end{bmatrix}$$

Differentiating implicitly with respect to educational level s_o we obtain that it affects wage positively. As expected, educated people receive a higher wage, where the exact effect is

$$\frac{\partial w}{\partial s_o} = \frac{g_{s_o}(h_m S_e + S_m)}{S_w g_e h_m + S_e h_m + S_m} > 0$$

With respect to effort we find that more educated individuals need not work as hard as less educated ones. This may be because of higher productivity of labor for more educated, qualified individuals. Uneducated people have to exert strenuous efforts to achieve the same results as skillful, educated workers. Thus education has an adverse effect on work effort.

$$\frac{\partial e}{\partial s_o} = -\frac{g_{s_o} h_m S_w}{S_w g_e h_m + S_e h_m + S_m} < 0 \qquad \frac{\partial m}{\partial s_o} = -\frac{g_{s_o} S_w}{S_w g_e h_m + S_e h_m + S_m} > 0$$

Furthermore, higher education has a positive effect on worker’s income. This could be because a low level of qualification does not promise a high wage to the worker and, consequently, leads to a lower level of overall income. But this may also be because individuals who cannot afford higher education due to low initial endowment are likely to remain poor. This is consistent with Becker’s “unequal opportunity” treatment. Since poor individuals face higher opportunity cost of financing their education, they are likely to remain less educated even when they have the same personal abilities as rich individuals. However, remaining less educated, they remain poor as well. With respect to initial wage w_o,

$$\begin{bmatrix} \dfrac{\partial w}{\partial w_o} \\ \dfrac{\partial e}{\partial w_o} \\ \dfrac{\partial m}{\partial w_o} \end{bmatrix} = \frac{1}{(S_w g_e h_m + S_e h_m + S_m)} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{h_m S_e + S_m}{S_w g_e h_m + S_e h_m + S_m} \\ -\dfrac{h_m S_w}{S_w g_e h_m + S_e h_m + S_m} \\ -\dfrac{S_w}{S_w g_e h_m + S_e h_m + S_m} \end{bmatrix}$$

Logically, a higher initial wage affects overall wage favorably. Individuals with better reputation, education and work experience who start at a higher wage are likely to receive a higher total wage. But this positive effect may stem from the fact that individuals starting at a higher wage are perceived as high earners, promising workers and talented people. Those starting at a low level may be perceived as less promising, less ambitious, and less talented. Therefore, the very perception of the worker's qualities, skills and talent may be a determinant of his wage. When a person accepts a low initial pay and a low-prestige job, he may be perceived as a low-potential worker and is likely to remain low-paid.

$$\frac{\partial w}{\partial w_o} = \frac{h_m S_e + S_m}{S_w g_e h_m + S_e h_m + S_m} > 0 \qquad \frac{\partial e}{\partial w_o} = -\frac{h_m S_w}{S_w g_e h_m + S_e h_m + S_m} < 0$$

A person starting with a higher initial salary exerts less effort on the job. A higher initial wage unrelated to work effort discourages workers to work. But it may also be that more talented workers starting at a higher salary have to invest less effort in the production process, as they can perform the task more efficiently. Thus, initial wage is indicative of the qualities of the worker.

$$\frac{\partial m}{\partial w_o} = -\frac{S_w}{S_w g_e h_m + S_e h_m + S_m} > 0$$

A higher initial wage increases the worker's income. This may again be related to the talent of the worker, with greater talent or skills rewarded by a higher initial wage and increasing the total income of the worker. Since initial wage is unrelated to work effort, the income it generates is merely the result of good work reputation, credentials and habits. Thus a higher start-up wage promises greater overall income to the worker. With respect to initial effort e_o we solve

$$\begin{bmatrix} \frac{\partial w}{\partial e_o} \\ \frac{\partial e}{\partial e_o} \\ \frac{\partial m}{\partial e_o} \end{bmatrix} = \frac{1}{(S_w g_e h_m + S_e h_m + S_m)} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{g_e S_m}{S_w g_e h_m + S_e h_m + S_m} \\ \frac{S_m}{S_w g_e h_m + S_e h_m + S_m} \\ -\frac{g_e S_w}{S_w g_e h_m + S_e h_m + S_m} \end{bmatrix}$$

Autonomous work effort e_o affects gross salary positively. Since this initial work effort is a signal of quality for the employer, it increases gross wage. Thus hard working individuals receive a higher wage than laggards. Logically, higher autonomous effort increases overall work effort. Hard working individuals starting at a higher autonomous effort with good work credentials exert a greater amount of overall effort. It turns out that hard working individuals hired as such end up exerting more work effort than others irrespective of the pay level, reward or income available to them.

$$\frac{\partial w}{\partial e_o} = \frac{g_e S_m}{S_w g_e h_m + S_e h_m + S_m} > 0 \qquad \frac{\partial e}{\partial e_o} = \frac{S_m}{S_w g_e h_m + S_e h_m + S_m} > 0$$

Greater initial level of effort e_o guarantees a higher income for the worker as well. This may be because a person who starts as a good worker is likely to remain such and try harder on the job. As a result, harder working people have more income.

$$\frac{\partial m}{\partial e_o} = -\frac{g_e S_w}{S_w g_e h_m + S_e h_m + S_m} > 0$$

Solving with respect to exogenous demand D_o,

$$\begin{bmatrix} \frac{\partial w}{\partial D_o} \\ \frac{\partial e}{\partial D_o} \\ \frac{\partial m}{\partial D_o} \end{bmatrix} = \frac{1}{(S_w g_e h_m + S_e h_m + S_m)} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{g_e h_m}{S_w g_e h_m + S_e h_m + S_m} \\ \frac{h_m}{S_w g_e h_m + S_e h_m + S_m} \\ \frac{1}{S_w g_e h_m + S_e h_m + S_m} \end{bmatrix}$$

we find that market demand has a positive effect on equilibrium wage expressed as

$$\frac{\partial w}{\partial D_o} = \frac{g_e h_m}{S_w g_e h_m + S_e h_m + S_m} > 0 \,.$$

3 Income and Work Effort

We have so far revealed the endowment of the worker as the dependent variable. More specifically, we demonstrated how income m depends on worker education, minimum wage or autonomous work effort. To see the effect of initial income as the exogenous variable, we solve our job market equilibrium model in the single-equation case where

$$D(w,e,s_o) = S(w,s_o,m_o) \qquad D_w < 0 \qquad D_e > 0 \qquad D_{s_o} > 0$$

$$S_w > 0 \qquad S_{s_o} < 0 \qquad S_{m_o} < 0$$

Both demand and supply depend on equilibrium wage w and education s_o. However, demand is also positively related to work effort e. Thus a higher effort on the part of workers stimulates firms to demand more labor. On its own, supply is discouraged by a higher income or initial endowment of the worker illustrating the income effect. Rearranging in the form of an implicit function and applying the implicit-function rule, we obtain a number of comparative-static derivatives:

$$D(w,e,s_o) - S(w,s_o,m_o) = 0$$

By implicit differentiation,

$$\frac{\partial w}{\partial s_o} = -\frac{D_{s_o} - S_{s_o}}{D_w - S_w} > 0 \qquad \frac{\partial e}{\partial s_o} = -\frac{D_{s_o} - S_{s_o}}{D_e} < 0$$

we obtain that the educational level has a positive effect on equilibrium wage and negative on the amount of work effort which is consistent with our previous results. Skillful and educated workers need not work as hard as uneducated ones.

Exogenous income affects equilibrium wage positively. This could be because a higher initial endowment may be associated with greater talent, work effort or education leading to a higher wage. At the same time, more income reduces work effort. This can be because of lack of motivation and low opportunity cost of retaining the job for rich people. But it may also be the result of human capital and talent. Being more productive, rich individuals need not exert as much effort as poor ones. At the same time, poor workers have to try a lot harder on the job.

$$\frac{\partial w}{\partial m_o} = -\frac{(-S_{m_o})}{D_w - S_w} = \frac{S_{m_o}}{D_w - S_w} > 0 \qquad \frac{\partial e}{\partial m_o} = -\frac{(-S_{m_o})}{D_e} = \frac{S_{m_o}}{D_e} < 0$$

4 Optimal Work Effort and Wage

We express profit as a function of work effort e such that

$$\pi(e) = pq(e) - e\left[w_o + g(e)\right]$$

where the price p the firm charges is assumed to be constant and gross wage can be presented as $w = w_o + g(e)$ with greater work effort rewarded by a higher wage at a constant rate, i.e., $g_e > 0$, and $g_{ee} = 0$. Thus the total production cost that represents spending on labor is subtracted from total revenue. The production function is subject to diminishing returns to work effort, that is, $q'(e) = \frac{dq}{de} > 0$ and $q''(e) < 0$. By first-order condition of profit maximization,

$$\pi'(e^*) = pq'(e^*) - w_o - g(e^*) - e^* g_e = 0$$

The second-order condition

$$\pi''(e^*) = pq''(e^*) - 2g_e - e^* g_{ee} < 0$$

proves maximum profit. By implicit differentiation we find the effect of minimum wage on work effort

$$\frac{de^*}{dw_o} = -\frac{(-1)}{pq''(e^*) - 2g_e - e^* g_{ee}} = \frac{1}{pq''(e^*) - 2g_e} < 0$$

A higher minimum or initial wage definitely reduces work effort by discouraging people to work. Using a specific wage function such as $w = w_o + \beta e$ where β is a positive reward parameter for work effort, we have

$$\pi(e) = pq(e) - e(w_o + \beta e) = pq(e) - w_o e - \beta e^2$$

$$\pi'(e^*) = pq'(e^*) - w_o - 2\beta e^* = 0,$$

giving optimal work effort to the firm as

$$e^* = \frac{pq'(e^*) - w_o}{2\beta}$$

The second-order condition proves maximum profit, or

$$\pi''(e^*) = pq''(e^*) - 2\beta < 0$$

By the implicit-function rule, we can see the effect of the reward parameter and wage on optimal work effort. Both reward for work effort β and start-up wage w_o tend to reduce the amount of effort invested at the workplace. Therefore, workers rewarded the most or starting from the highest pay level are most likely to shirk.

$$\frac{\partial e^*}{\partial \beta} = -\frac{(-2e^*)}{pq''(e^*) - 2\beta} = \frac{2e^*}{pq''(e^*) - 2\beta} < 0$$

$$\frac{\partial e^*}{\partial w_o} = -\frac{(-1)}{pq''(e^*) - 2\beta} = \frac{1}{pq''(e^*) - 2\beta} < 0$$

5 Optimal Work Effort and Wage for Two Individuals

Let us assume that the firm hires two workers, one exerting a high work effort e_1 and the other exerting a lower work effort e_2. Different work efforts do not imply different education and professional skills but just the amount of effort or work done at the work place necessary to achieve certain results. Let the wage per unit of work effort be $w_i(e_i) = w_{io} + \beta_i e_i$ for each of the workers, that is, gross wage and work effort are positively related. There is a reward parameter β_i for good performance and effort. It shows the degree to which total wage is affected by work effort. Total wage is also positively related to minimum or initial wage w_{io} that does not depend on work effort but indicates the worker's credentials, qualities or education when starting on the job. Thus initial wage w_{io} works as a separating differential between two individuals of unequal qualities. The wage of each individual can be expressed as

$w_1(e_1) = w_{1o} + \beta_1 e_1$ wage of the first, high-effort worker,

$w_2(e_2) = w_{2o} + \beta_2 e_2$ wage of the second, low-effort worker.

We expect $w_{1o} > w_{2o}$ or higher start-up wage for the first, more productive worker who starts as the better one. Expressing total profit to the employer,

$$\pi(e_1, e_2) = pq(e_1, e_2) - (w_{1o} + \beta_1 e_1)e_1 - (w_{2o} + \beta_2 e_2)e_2$$
$$= pq(e_1, e_2) - w_{1o}e_1 - \beta_1 e_1^2 - w_{2o}e_2 - \beta_2 e_2^2$$

By first-order condition,

$$\pi_1 = \frac{\partial \pi}{\partial e_1} = pq_1 - w_{1o} - 2\beta_1 e_1^* = 0 \qquad \pi_2 = \frac{\partial \pi}{\partial e_2} = pq_2 - w_{2o} - 2\beta_2 e_2^* = 0$$

The first-order condition gives the optimal work effort for both workers,

$$e_1^* = \frac{pq_1 - w_{1o}}{2\beta_1} \qquad e_2^* = \frac{pq_2 - w_{2o}}{2\beta_2},$$

where we need $pq_1 > w_{1o}$ and $pq_2 > w_{2o}$ for positive effort. The optimal work effort depends on initial wage, the marginal revenue product of the worker and the reward parameter. Thus, a higher initial wage and reward parameter reduce the incentives for good performance. A more productive worker is likely to exert more effort on the job than a less productive one. This is because by exerting more effort the more productive worker receives a higher pay. Thus the more productive worker is less likely to shirk and is subject to the substitution effect by which he tends to work harder in order to receive a higher salary. Expressing optimal profit,

$$\pi_{\max} = \pi(e_1^*, e_2^*) = pq(e_1^*, e_2^*) - (w_{1o} + \beta_1 e_1^*)e_1^* - (w_{2o} + \beta_2 e_2^*)e_2^*$$
$$= pq - \left[w_{1o} + \frac{\beta_1 (pq_1 - w_{1o})}{2\beta_1} \right] \frac{(pq_1 - w_{1o})}{2\beta_1} - \left[w_{2o} + \frac{\beta_2 (pq_2 - w_{2o})}{2\beta_2} \right] \frac{(pq_2 - w_{2o})}{2\beta_2}$$
$$= pq - \frac{(pq_1 + w_{1o})}{2} \frac{(pq_1 - w_{1o})}{2\beta_1} - \frac{(pq_2 + w_{2o})}{2} \frac{(pq_2 - w_{2o})}{2\beta_2}$$
$$= pq - \frac{p^2 q_1^2 - w_{1o}^2}{4\beta_1} - \frac{p^2 q_2^2 - w_{2o}^2}{4\beta_2} = pq - \frac{p^2 q_1^2}{4\beta_1} + \frac{w_{1o}^2}{4\beta_1} - \frac{p^2 q_2^2}{4\beta_2} + \frac{w_{2o}^2}{4\beta_2}$$

Analyzing optimal profit, we see that the more productive worker costs more to the employer, as his marginal product exceeds that of the other. A higher reward parameter β_i increases profit to the firm with the reward parameter for the first worker contributing more to profit since $\beta_1 > \beta_2$. Assuming same rewards and initial wages, we have

$$\pi_{\max} = \pi(e_1^*, e_2^*) = pq - \frac{p^2q_1^2 - w_o^2}{4\beta} - \frac{p^2q_2^2 - w_o^2}{4\beta} = pq - \frac{p^2q_1^2}{4\beta} - \frac{p^2q_2^2}{4\beta} + \frac{w_o^2}{2\beta}$$

Again, a higher reward parameter increases profit to the firm. The more productive worker costs more to the firm thus bringing profit down, everything else same, although he contributes more to total output and revenue. Optimal wage for both workers is

$$w_1(e_1^*) = w_{1o} + \frac{\beta_1(pq_1 - w_{1o})}{2\beta_1} = \frac{w_{1o} + pq_1}{2}$$

$$w_2(e_2^*) = w_{2o} + \frac{\beta_2(pq_2 - w_{2o})}{2\beta_2} = \frac{w_{2o} + pq_2}{2}$$

Note that optimal wage is not related to the reward parameter β_i. It depends on autonomous, start-up wage and the contribution of each worker to the production process. Since the more productive worker is likely to start at a higher salary and contributes more through his higher marginal product, he will definitely receive a higher wage. Furthermore, the more productive worker exerts more effort on the job because this increases his overall salary. Having a higher productivity, the more efficient worker has the potential to obtain a higher wage which is why he is trying harder. Although the reward parameter does not affect optimal wage, it is essential in increasing total profit since with a positive effort the profit is positively related to β_i. Thus, the higher the reward parameter, the higher the profit of the firm is.

Optimal work effort and wage would increase with the amount of capital and the efficiency of management. Since optimal work effort and wage are positively related to marginal product and marginal product is higher with better and more machinery used, this increases the optimal efforts exerted by both workers as well as the optimal wage for each of them. Increases in the efficiency of management and improvements in the coordination of productive activities also increase optimal work effort and wage. As a second-order condition of profit maximization we form a Hessian with the following second derivatives:

$\pi_{11} = pq_{11} - 2\beta_1 < 0 \qquad \pi_{12} = pq_{12} = \pi_{21} \qquad \pi_{22} = pq_{22} - 2\beta_2 < 0$

$$|H| = \begin{vmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{vmatrix} = \begin{vmatrix} pq_{11} - 2\beta_1 & pq_{12} \\ pq_{12} & pq_{22} - 2\beta_2 \end{vmatrix},$$

where

$|H_1| = pq_{11} - 2\beta_1 < 0$, and $|H_2| = |H| = (pq_{11} - 2\beta_1)(pq_{22} - 2\beta_2) - p^2 q_{12}^2 > 0$, or

$(pq_{11} - 2\beta_1)(pq_{22} - 2\beta_2) > p^2 q_{12}^2$

is a condition that insures maximum profit.

6 Conclusion

Using a simple job market equilibrium model we find that gross wage is related positively to the educational level of the worker, his initial salary and his autonomous work effort unrelated to any reward or initial endowment. Overall work effort is affected positively by autonomous work effort but negatively by education and the initial wage. Thus people with better education and starting at a higher pay level have a separating differential from less educated ones, starting with no credentials. The worker’s income increases with start-up wage, education, and autonomous work effort. Thus education is a promise for accumulating wealth at the workplace. Assuming income to be exogenous, we find that it increases gross wage while decreasing work effort. Lastly, firms set optimal wages disregarding reward for work effort with more productive workers clearly receiving higher wages. A higher reward for work effort and a start-up pay level discourage workers to try hard on the job.

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