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Forecasting Chinese Mortality Based on the Long-run Equilibrium

Liu Xiangdong[1] and Fan Yangyang[2]

Abstract

Human mortality, which reflect the deaths' extent, is one of the key research of Population Science and Population Economics. Accurately mortality forecasting can lay the foundation of pricing longevity risk bonds. Based on Lee-Carter model, this paper considers mortality correlations and investigates the long-run equilibrium of mortality rates between China mainland and Taiwan province for mortality forecasts. Differing from the traditional ARIMA model which is based on the limited data of China, the paper proposes a VECM model for the mortality time index forecasts after the co-integration test. Minimum mean square prediction errors (MSPE) is used for criteria, our results show that the forecasting based on the VECM model is better. Mortality rates under the multi-region framework can provide important reference for further study of pricing a multi-region longevity bonds.

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[1] School of economics, Jinan University. E-mail: tliuxd@jnu.edu.cn
[2] School of economics, Jinan University. E-mail: yoyofan96@163.com

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1 Introduction

Human mortality rates, which measure the death toll per unit time for a certain size of the population, reflect the extent of deaths, and are closely related to the population health, population quality because it affect population age structure. Accurately mortality forecasting is one of the key researches of Population Science and Population Economics, and it can also lay the foundation of identifying and quantifying longevity risk and pricing longevity risk bonds. With the development of the economy, medical advance, the application of public health in new technology and the enhancing of the education levels, mortality rates of China continue to decrease. Longer life expectancy aggravates the aged tendency of population, thus putting government, life-insurance companies, enterprises and people themselves enormous pressure under further pressure when they make relevant pension policy or retirement plans. Therefore, forecasting mortality rates precisely plays a particular important role for both government and enterprises.

The most classical model for mortality forecast is the Lee-Carter model[1] which proposed by Lee and Carter in 1992. The Lee-Carter model which considers the dynamics of mortality with clear parameters can be understood easily. Later on, some scholars improve and expand the Lee-Carter model according to the feather of different countries. Brouths et al. (2002)[2] embed Poisson model in Lee-Carter model for prediction of age-sex specific death rates. Renshaw and Habeman (2003)[3] add age-specific enhancement to Lee-Carter model for mortality forecast. Czado et al. (2005)[4] build Logarithmic bilinear Poisson regression model and forecast mortality by Bayesian estimation. Renshaw and Habeman (2006)[5]build the cohort-based extension to the Lee-Carter model for mortality reduction factors. Delwarde et al. (2007)[6] propose different ways to estimate the three parameters in Lee-Carter model. Wang and Cai (2008)[7] point out that the Lee-Carter model is suitable for Chinese mortality forecast and they also use the ARIMA model to simulate stochastic effects of period. Li et al. (2010)[8] get the results that the weighted least square method are the best method for estimation of parameters in Lee-Carter model. Wang and Huang (2011)[9] use bayesian information criterion and likelihood ratio test to show that an expansion of Carins-Blake-Dowd model have a better accurate predictions. Wang and Ren (2012)[10] offer a model under the "two stochastic process aiming at the limited data of China. Wang, Zhu and Fu (2013)[11] note that mortality is a typical functional data, and they propose a functional prediction model for mortality.

However, the extended forms of Lee-Carter model can't use more information of the data. Yang et al. (2011)[12] build a coherent mortality model for a group of populations across countries based on the Lee-Carter model. Yang et al. (2013)[13] verify that there exist the long-run equilibrium relationships between US mortality and UK mortality and then propose the mortality forecast model under a multi-country framework based on the Lee-Carter model. The correlations of multi-country (region) can help find out more information about mortality data, and co-integration can investigate the long-run equilibrium relationships among the countries or regions. This paper not only shows the trends of mortality for China mainland and Taiwan province, but also investigates the correlations between them. Co-integration test is applied to find out the long-run equilibrium relationships of the mortality time index. After the test, the vector error correlation model (VECM) which is based on the correlations between China mainland and Taiwan province is built to predict the mortality. Comparing results of VECM model with the results that predict by traditional ARIMA models, we can get a better model for mortality forecast.

2 Mortality modeling under a multi-region framework

2.1 Lee-Carter model

The Lee-Carter model[1] can be shown as

$$\ln m_{x,t} = a_x + b_x k_t + e_{x,t} \tag{1}$$

Where $m_{x,t}$ is the mortality force at age x in year t, the parameters b_x and k_t are subject to $\sum_{t=t_1}^{t=t_n} k_t = 0$ and $\sum_{x=x_1}^{x_k} b_x = 1$, parameter a_x is the means of $\ln m_{x,t}$ across years which describes the average pattern mortality, k_t is the mortality time index which is the time trend of mortality rates, b_x is the relative change speed of mortality at age x which can influence k_t, $e_{x,t}$ is the residual term at age x and time t which reflects the influences not captured by the models. Based on the above hypothesis, a_x can be calculate by the formula $\hat{a}_x = (1/n)\sum_{t=t_1}^{t_n} \ln m_{x,t}$ in which estimate $\hat{a}_x$ is an effective squares error estimate. For k_t and b_x, this paper uses method of MSPE which is $\min \sum_{x,t} (\varepsilon'_{x,t})^2 = \min[\sum_{x,t} (\ln m_{x,t} - \hat{a}_x - \hat{b}_x \hat{k}_t)^2]$ to get the values. Parameter k_t can be calculated by the formula $\hat{k}_t = \sum_x (\ln m_{x,t} - \hat{a}_x)$, and b_x can be calculated by $b_x = (\sum_{t=1}^{T} \hat{k}_t (\ln m_{x,t} - \hat{a}_x)) / \sum_{t=1}^{T} \hat{k}_t^2$.

The two parameters k_t and b_x are obtained by successively two steps, so the estimation of k_t do not need to re-adjusted.

2.2 Lee-Carter modeling under a multi-region framework

When Chinese scholars predict mortality rates, they usually use the original data of China to make maximum use of the data. However, the predictions of mortality are limited by the few data. As Sharon(2013)[7] has pointed out, mortality across countries has some correlations. It is shown that the mortality correlations among UK men, UK women, US men and US women are positive. This paper considers the mortality correlations between China mainland and Taiwan province to find out whether the correlations can enhance the precision of the prediction.

2.2.1 Mortality modeling based on the multi-region correlations

To make maximum use of the data of China mainland and Taiwan province, we employ the Lee-Carter model (1992) and Sharon's (2013) [7] model. If we consider N regions and research men and women individually, there are $2N$ mortality rates. That is, $m_{x,t}^{j}, j=1,\cdots,2N$ is the mortality rates at age x for j group in year t. We can show the model as below:

$$\ln m_{x,t}^{j} = a_x^{j} + b_x^{j} k_t^{j} + e_{x,t}^{j}, \quad j=1,\cdots,2N \tag{2}$$

To present the multi-region mortality model better, we use matrix form:

$$\begin{bmatrix} \ln m_{x,t}^{1} \\ \ln m_{x,t}^{2} \\ \vdots \\ \ln m_{x,t}^{2N} \end{bmatrix} = \begin{bmatrix} a_x^{1} \\ a_x^{2} \\ \vdots \\ a_x^{2N} \end{bmatrix} + \begin{bmatrix} b_x^{1} & 0 & 0 & 0 \\ 0 & b_x^{2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & b_x^{2N} \end{bmatrix} \times \begin{bmatrix} k_t^{1} \\ k_t^{2} \\ \vdots \\ k_t^{2N} \end{bmatrix} + \begin{bmatrix} e_{x,t}^{1} \\ e_{x,t}^{2} \\ \vdots \\ e_{x,t}^{2N} \end{bmatrix} \tag{3}$$

The matrix form can be written as $\ln M_{x,t} = a_x + b_x K_t + e_{x,t}$, where $\ln M_{x,t} = [\ln m_{x,t}^{1} \cdots \ln m_{x,t}^{2N}]'$ is a $2N$ -by-1 vector which represent the mortality of men and women from the N regions; $a_x = [a_x^{1},\cdots,a_x^{2N}]'$ is a $2N$ -by-1 vector which reflect the average pattern mortality; b_x defines a $2N$ -by- $2N$ diagonal matrix beginning with vectors $(b_x^{1},\cdots,b_x^{2N})$ in the left upper corner; $K_t = [k_t^{1},\cdots,k_t^{2N}]'$ represents a $2N$ -by-1 time trend vector of mortality; $e_{x,t} = [e_{x,t}^{1},\cdots,e_{x,t}^{2N}]$ is a $2N$ -by-1 vector of random error terms.

For the random error terms, we assume $e_x^j = (e_{x,s}^j)_{s=0}^{t+T}$ is a variable which mean is 0 and volatility rate is σ_x^j. That is, the difference of error terms for at age x for j group in year t can be defined as $\Delta e_{x,t}^j = e_{x,t}^j - e_{x,t-1}^j$ which is not rely on $\Delta e_{x,s}^j, s \neq t$ and obeys normal distribution with mean 0 and standard derivation σ_x^j.

So the difference vector of error terms at age x for j group can be denoted by $\Delta e_{x,t} = [\Delta e_{x,t}^1, \cdots, \Delta e_{x,t}^{2N}]'$. We let $\sigma_{x,y}^{j,h} = Cov(\Delta e_{x,t}^j, \Delta e_{y,t}^h)$ to represent the covariance of $\Delta e_{x,t}^j$ and $\Delta e_{y,t}^h$. Similarly, σ_x^j and σ_y^h can be denoted by $\sigma_x^j = \sqrt{\sigma_{x,x}^{j,j}} = \sqrt{Cov(\Delta e_{x,t}^j, \Delta e_{x,t}^j)}$ and $\sigma_y^h = \sqrt{\sigma_{y,y}^{h,h}} = \sqrt{Cov(\Delta e_{y,t}^h, \Delta e_{y,t}^h)}$. We define the log mortality force correlation coefficient between people whose age is x and in j group and people whose age is y and in h group as follows:

$$\rho_{x,y}^{j,h} = \frac{\sigma_{x,y}^{j,h}}{\sigma_x^j \sigma_y^h}, \quad j,h = 1,\ldots,2N. \tag{4}$$

From the equation (4), we can define the covariance matrix of difference of error terms as $\Theta_{xy} = (\sigma_{xy}^{j,h})_{2N\times 2N}$. For this paper, we only consider the correlations between China mainland and Taiwan province, so $N=2$.

2.2.2 K_t forecast under a VECM model

For classical Lee-Carter model, scholars usually use traditional ARIMA model to forecast the time series K_t. However, ARIMA model is not suitable for the situation when we consider the mortality correlations between China mainland and Taiwan province. A VECM which contains the multi-region mortality correlations is built to predict K_t. Before the VECM model is established, a co-integration analysis has to be applied to explore whether a common stochastic trend, that is, long-run equilibrium relationship appears in the multi-region mortality time index K_t .

If the time series in K_t are all nonstationary, and satisfy the $I(p)$ process, that is, they all have p unit roots, we can say K_t is co-integrated. VECM (p) for K_t can be presented as follows:

$$\Delta K_t = \omega_t + \Pi K_{t-1} + \sum_{d=1}^{p-1} \Gamma_d \Delta K_{t-d} + \varepsilon_t \tag{5}$$

Where Δ represents the first difference; ω_t represents the error correction term based on co-integration equation, and it is a $2N \times 1$ matrix which reflects the degree of time series deviating from the long-run equilibrium relationship; $\Gamma_d (d = 1, \cdots p-1)$ is a $2N \times 2N$ short-term impact matrix; $\varepsilon_t = [\varepsilon_t^1, \cdots, \varepsilon_t^{2N}]'$ is a random fluctuations matrix with 0 mean in which each element is independent; $\Sigma = \left[v_{g,h} \right]_{2N \times 2N}$ is a $2N \times 2N$ covariance matrix of the series $\varepsilon_t = [\varepsilon_t^1, \cdots, \varepsilon_t^{2N}]'$.

For the eq. (5), we can change its form as follows:

$$K_t = \omega_t + \sum_{d=1}^{p} \varphi_d K_{t-d} + \varepsilon_t \tag{6}$$

Comparing to eq. (5), we can know that $\varphi_1 = \Gamma_1 + \Pi + I_{2N}$, $\varphi_d = \Gamma_d - \Gamma_{d-1}, d = 2, \cdots p-1, \varphi_p = -\Gamma_{p-1}$, where I_{2N} is a $2N \times 2N$ matrix.

Based on the above assumption, we can get the mortality time index at time $t+n$, that is, K_{t+n+1} as follows:

$$K_{t+n+1} = \sum_{d=0}^{n} y_1(d-1)\omega_{t+n+1-d} + \sum_{d=1}^{p} y_d(n) K_{t+1-d} + \sum_{d=0}^{n} \theta_d(n) \varepsilon_{t+n+1-d} \tag{7}$$

Where $y_d(h) = y_1(h-1)\varphi_d + 1_{(d<h)} y_{d+1}(h-1);\ d = 1, \cdots, p;\ h = 1, \cdots, n$

$$\theta_d(h) = \theta_d(d) 1_{(d<h)} + y_1\left(h-1\right) 1_{(d=h)}, h = 1, \cdots, n$$

$$y_1(-1) = I_{2N},\quad \theta_0(0) = I_{2N},\quad y_i(0) = \varphi_i, i = 1, \cdots, p$$

2.2.2Mortality forecast based on the correlations

According to the eq. (2), we can get the difference equation as follows:

$$\ln M_{x,t+n+1} - \ln M_{x,t} = b_x(K_{t+n+1} - K_t) + e_{x,t+n+1} - e_{x,t} \quad (8)$$

Where $\ln M_{x,t+n+1}$ is the future mortality for people at age x. When substituting eq. (7) into eq. (8), the future mortality can be represented by the following form:

$$\ln M_{x,t+T} = \ln M_{x,t} + \mu_{t,T}^{x} - \frac{1}{2} D_T^x + \sum_{n=1}^{T} (B_{T-n}^{x} \varepsilon_{t+n} + \Delta e_{x,t+n}) \quad (9)$$

Where D_T^x is the diagonal entries in the matrix $V_{t,T}^x$; $V_{t,T}^x$ is $\ln M_{x,t+n+1}$ conditional on $\Im_t$, and $V_{t,T}^x$ is

$$V_T^x \equiv Var_p(\ln M_{x,T+1} | \Im_t) = (\sum_{n=1}^{T} B_{T-n}^{x} \Sigma B_{T-n}^{x}{}') + T\Theta_{x,x} \quad (10)$$

Where $B_h^x = b_x \theta_h(h)$. $u_{t,T}^x$ in eq. (9) denotes cumulative rates of mortality, which satisfy

$$\ln m_{x,t+n}^{j} = \ln m_{x,t}^{j} + u_{t,n}^{x,j};\ \ n = 1,\ldots,T;\ \ \ j = 1,\ldots,2N \quad (11)$$

$$u_{t,n}^{x} = b_x(\sum_{j=0}^{n-1} (y_1(j-1)\omega_{t+n-j}) - K_t + \sum_{j=1}^{p} (y_j(n-1)K_{t+1-j}) + \frac{1}{2} D_n^x \quad (12)$$

3 Mortality forecast for China

3.1 Data sources and processing methods

The data of this paper mainly have two parts. Data of China mainland which we use in this paper is the age-sex specific mortality rates from year 1994 to 2013. Data from year 1994 to 2006 is from China population statistics yearbook, and data from year 2007 to 2013 is from Chinese population and employment statistics yearbook. Data of year 2000 is from the Fifth census data, and data of 2010 is from census data for 2010. The paper divide in 19 age groups which contain the groups 0-4, 5-9, 10-14, … , 85-89, 90+. In order to keep the data consistent, some data need to be preliminary addressed. Data of 1995, 2000, 2005, and 2010 need to be recalculated for age group in 90 and 90+, and data of 1996 for 90+ can be supplemented by mean of adjacent years' mortality. Data for Taiwan province is from the human mortality database. Corresponding to data of China mainland, we choose age-sex with 19 groups for men and women. Data for 0-4 and 90+ groups need to be recalculated by the formula $m_{x,t} = (D_{x,t} / E_{x,t})$, where $D_{x,t}$ is the death toll of people at age group x in calendar year t, $E_{x,t}$ is number of people who are in the risk.

3.2 Mortality forecast without the correlations

Based on Lee-Carter model, least squares fit (LSF) is used to estimate the parameters a_x, b_x and k_t for data of China mainland. A portion of the estimated values for a_x, b_x appear in Table 1.

Table 1: Parameters estimation of a_x, b_x

Age group	a_x for men	a_x for women	b_x for men	b_x for women
0-4	-5.5567	-5.4010	0.1592	0.1232
5-9	-7.4728	-7.9832	0.0574	0.0657
10-14	-7.6710	-8.0073	0.0446	0.0700
15-19	-7.1805	-7.6474	0.0747	0.0786
…	…	…	…	…

After obtaining estimation of a_x, b_x and k_t, classical ARIMA model need be built to forecast k_t. This paper uses Augmented Dickey-Fuller (ADF) Test to test k_t series, the results of the unit root test appear in Table 2. The results is about ADF test, k_t series of Chinese men and women all have unit root, while the two series become stationary series with 5% critical values after difference disposal. That is, the mortality time index for Chinese men and women all follow an I(1) process.

Table 2: ADF test for China mainland mortality time index k_t

Series	t-Statistic	Test critical values for t-Statistic			P-value
		1%	5%	10%	
CM	0.9909	-4.0044	-3.0989	-2.6904	0.9935
d(CM)	-4.9147	-4.0044	-3.0989	-2.6904	0.0020
CF	0.2432	-3.9204	-3.0656	-2.6735	0.9665
d(CF)	-3.3073	-3.9591	-3.0810	-2.6813	0.0333

Note: Series CM, CF respectively represent mortality time index for men and women of China mainland.

Auto.arima function in R software is used to choose the most suitable ARIMA model for the prediction of k_t series. We use Akaike's information criterion (AIC) to be the selection criteria. In Table 3, we can know that the most suitable model for Chinese men is ARIMA(1,1,0), and for women is ARIMA(0,1,0).

Table 3: Selection criteria of ARIMA models for mortality index

k_t for men	AIC value for women	k_t for women	AIC value for women
ARIMA(2,1,2) with drift	64.9298	ARIMA(1,1,0) with drift	47.8452
ARIMA(0,1,0) with drift	60.8425	ARIMA(0,1,0) with drift	44.7709
ARIMA(1,1,0) with drift	58.2882	ARIMA(0,1,1) with drift	47.8429
ARIMA(0,1,0) with drift	60.76697	ARIMA(0,1,0) with drift	54.0990
ARIMA(2,1,0) with drift	58.4439	—	—
ARIMA(1,1,1) with drift	59.5826	—	—
ARIMA(1,1,0)	62.3316	—	—

With the suitable ARIMA model, we can predict k_t series. Table 4 presents the results of the predicted value for year 2011 to 2013. The mortality for Chinese men and women can be calculated through the predicted value of k_t.

Table 4: Mortality time index k_t forecast under ARIMA models

Gender	Men	Women
k_t in year 2011	-7.1127	-8.6164
k_t in year 2012	-7.2025	-9.5030
k_t in year 2013	-8.1633	-10.3895

3.3 Mortality forecast with the corrections

3.3.1 Parameters' estimation in the Lee-Carter model for Taiwan province

We use the data of Taiwan province to get the estimated value of the parameters a_x, b_x and k_t in the Lee-Carter model. Combine the estimated results of China mainland, Figure 1 is presented. In Figure 1, we can know that each parameter estimation for China mainland and Taiwan province in Lee-Carter model all have the same trend.

Parameter a_x reflects the level index of mortality, and its estimation are bathtub curve with high value in 0-4 group and the aged group. It satisfies the reality of the situation where infants and old people have a higher mortality than other age groups. On the other hand, female is less than male from the trend of both estimation for China mainland and Taiwan province.

For b_x, there are some fluctuation because of the limited data. The value of low age groups is relatively high because the infants are more sensitive to the change of speed of $\ln m_{x,t}$; the values of the aged group are stable and close to zero. The value of male is higher than female for the aged group, and it shows that the population structure of male is more significant than female.

The downward trends of mortality time index K_t reflect that mortality rates have a descending trend. Tends basically are linear which show that the speed of decrease for mortality is relatively stable. From the difference between men and women, we can know that the speed of decrease for mortality of women is faster than men.

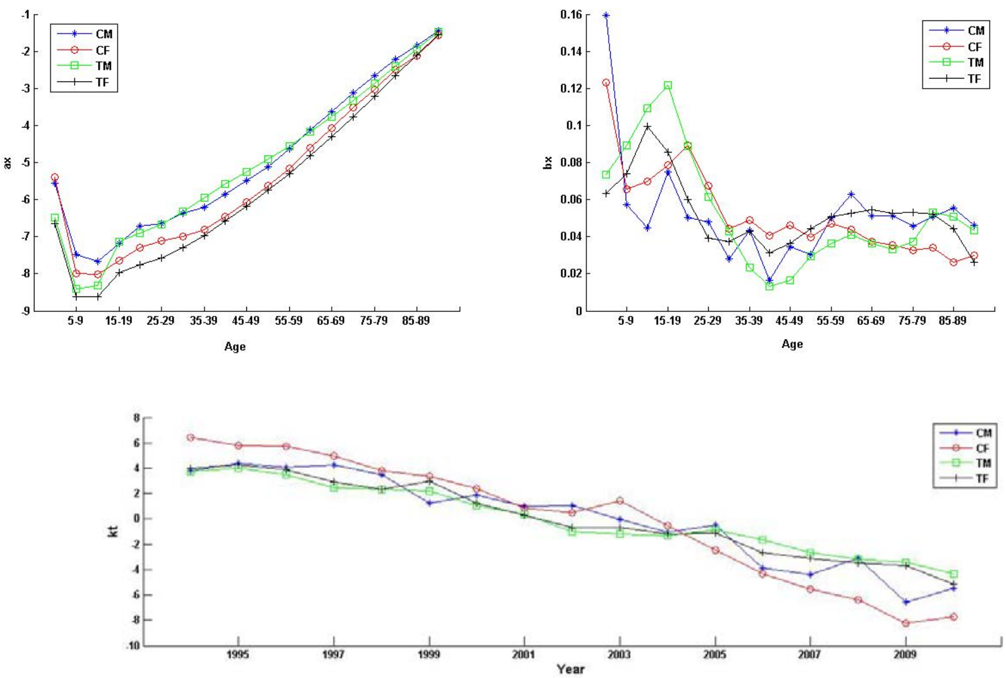

Figure 1: Parameter estimates of a_x, b_x and K_t in Lee-Carter model for CM, CF, TM and TF, 1994-2010.

3.3.2 Co-integration analysis

To forecast the mortality time index across several regions, co-integration analysis is needed to find out the long-run equilibrium relationship. The co-integration contains four steps:

Step 1: Investigate if the k_t^j series for the four groups follows an I(1) process.

Step 2: Determine the lag order.

Step 3: Test for co-integration.

Step 4: Build VECM model

To investigate if the k_t^j series for the four groups follows an I(1) process, we apply the ADF test to find the unit root. Table 5 is the test result of the mortality time index k_t^j for Taiwan province. So all four series follow an I(1) process.

Table 5: ADF test for Taiwan province mortality time index k_t

Series	t-Statistic	Test critical values for t-Statistic			P-value
		1%	5%	10%	
TM	0.0971	-3.9204	-3.0656	-2.6735	0.9548
d(TM)	-3.8739	-3.9591	-3.0810	-2.6813	0.0117
TF	0.2989	-3.9204	-3.0656	-2.6735	0.9702
d(TF)	-4.9488	-3.9591	-3.0810	-2.6813	0.0016

Note: Series TM, TF respectively represent mortality time index for men and women of Taiwan province.

This paper uses three selection criteria to determine the lag order of VECM model for the k_t^j series, and they are the likelihood ratio test, AIC and Schwarz Bayesian Criterion (SBC). These three criteria all aim to minimize. In the Table 6, we can found that three inspection values are all smaller when lag order equals 1 than 2. That is, the VECM(1) is the most appropriate model to forecast k_t^j.

Table 6: Selection criteria for Lag order

Lag	Log-likelihood	AIC	SBC
1	-46.2354	8.2794	9.2451
2	-26.4645	8.3286	10.0279

If we want to build VECM model, the number of co-integration equation must be tested. Table 7 shows the test results which are based on the Johansen co-integration test. When we hypothesize that there is no co-integration equation, the trace statistic is 52.9571 which exceeds the critical value 47.8561 and the p value is 0.0154, so we reject the null hypothesis. That is, there at least one co-integration equation. When the null hypothesis is that there is at most one co-integration equation, we can find that p value is 0.1833 which exceeds 5% significance level. Therefore, there is one co-integration equation.

Table 7: Johansen co-integration test

Hypothesized No.of CE(s)	Eigenvalue	Trace Statistic	Critical Value (5%)	Prob.**
None*	0.8508	52.9571	47.8561	0.0154
At most 1	0.6121	24.4183	29.7971	0.1833
At most 2	0.4055	10.2132	15.4947	0.2647
At most 3	0.1486	2.4127	3.8415	0.1204

Based on the above tests for co-integration, the VECM(1) can be shown as follows:

$$\begin{bmatrix} \Delta k_t^1 \\ \Delta k_t^2 \\ \Delta k_t^3 \\ \Delta k_t^4 \end{bmatrix} = \begin{bmatrix} -1.3360 \\ -1.4414 \\ -0.4049 \\ -0.5829 \end{bmatrix} + \begin{bmatrix} 0.2264 & -0.6464 & -2.0155 & 1.2882 \\ -0.5373 & 0.0336 & -1.1646 & 0.7937 \\ 0.1880 & 0.0197 & 0.1124 & -0.1158 \\ 0.0790 & 0.1359 & 0.2420 & -0.5011 \end{bmatrix} \begin{bmatrix} k_{t-1}^1 \\ k_{t-1}^2 \\ k_{t-1}^3 \\ k_{t-1}^4 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \\ \varepsilon_t^3 \\ \varepsilon_t^4 \end{bmatrix} \quad (13)$$

The lag order of the VECM is 1, that is, $p=1$. We substitute it into eq. (6), eq. (7) and eq. (8) to get the derivative:

$$\varphi_1 = \Pi + I_4,\ \ \theta_1(1) = y_1(0) = \varphi_1,\ \ \theta_2(2) = y_1(0)\varphi_1 = \varphi_1^2 \ \ \cdots\cdots, \theta_n(n) = \varphi_1^n .$$

Through eq. (13), the predictions of k_t can be calculated which are presented in Table 8.

Table 8: Mortality time index k_t forecast under VECM models

Year	CF	CM	TF	TM
2011	-6.8164	-9.8849	-4.4679	-5.0848
2012	-6.9804	-10.3522	-5.2096	-6.1320
2013	-8.3515	-11.4719	-5.6546	-6.4383

Note: CM, CF, TM and TF respectively represent men and women of China mainland and Taiwan province.

With the predictions of k_t series and estimation of parameters a_x, b_x, the mortality of China mainland can be forecasted.

3.4 Analysis of residual error under VECM(1) model

Residual error is the difference of the predicted value and real value, and the analysis of residual error can test the predictions of the model. In Figure 2, the residual errors of mortality for China men and women in year 2011-2013 are presented. The residual error is nearly zero for age groups from 0-4 to 80-84, and age groups 85-90 and 90+ for women is also nearly zero while for men have some fluctuation because of the groups with five years. In general, the difference error have no significant trends when age or year changes.

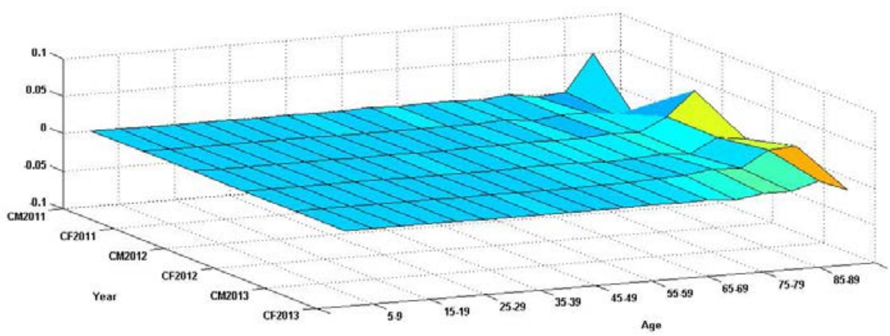

Figure 2: residual error under VECM(1) model

3.5 Result comparing under VECM model and ARIMA model

To compare the results of VECM model and ARIMA model, data of mortality for China men and women in year 2011-2012 are used for testing set, and the minimum mean square prediction errors (MSPE) are used for criteria.

$$MSPE = \frac{1}{H}\sum_{i=1}^{H}\omega_i^2 \tag{14}$$

Table 9 shows the MSPE of men and women in year 2011-2013. When we consider correlations between China mainland and Taiwan province, that is, under VECM model the MSPE are almost smaller than under ARIMA model. In general, the predictions are more accurate when considering correlations.

Table 9: Result comparing under VECM model and ARIMA model

Model	2011 (men)	2011 (women)	2012 (men)	2012 (women)	2012 (men)	2012 (women)
ARIMA	0.0084	0.0003	0.0105	0.0027	0.0071	0.0014
VECM	0.0025	0.0007	0.0034	0.0007	0.0022	0.0003

4 Conclusions

Considering correlations between mortality rates can investigate more useful information for mortality forecast. Based on the limited data of China, various extended models and calculation methods can't investigate more information. This paper combines data of China mainland and Taiwan province. Least square method is used for parameters estimation, and a co-integration test is performed to build a VECM model. Through the model, we can find out the long-run equilibrium relationship and forecast mortality of China mainland. To verified models' validity, data form year 2011 to 2013 are used for test. Comparing the results which are based on the correlations with the traditional ARIMA model, and the minimum mean square prediction errors are used for criteria, we can find that the correlation can contribute to a better model. The model based on the correlations can not only make a more precise prediction for age-sex specific death rates, but also show clearly that the death rates present the tub shape, and both mortality for women and men are decreasing stably while women decrease faster. However, it can have a little negative influence on the old-aged groups with five years for a group. The problem deserve a further study using one year for a group.

Correlations can makes up for the limitations effectively and makes the mortality forecast more accurate. To make full use of the mortality correlations, the results of this paper can lay the foundation for pricing the longevity bond. The trend of declining mortality is worthy of deep research. In the future, survival probability can be predicted through the correlation of two regional mortality, which can be used for making life table to predict the life expectancy of newborns and can be applied to the discussion of longevity risk.

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References

[1] Lee, R.D. and Carter, L. R ., Modeling and Forecasting US Mortality, *Journal of the American Statistical Association*, **87**(417), (1992), 659-671.

[2] Brouhns, Natacha, Michel Denuit, and Jeroen K. Vermunt, A Poisson log-bilinear regression approach to the construction of projected life tables, *Insurance: Mathematics and Economics*, **31**(3), (2002), 373-393.

[3] Renshaw, Arthur E., and Steven Haberman. Lee – Carter mortality forecasting with age-specific enhancement, *Insurance: Mathematics and Economics*, **3**(2), (2003), 255-272.

[4] Czado, Claudia, Antoine Delwarde and Michel Denuit, Bayesian Poisson log-bilinear mortality projections, *Insurance: Mathematics and Economics*, **6**(3), (2005), 260-284.

[5] Renshaw, Arthur E., and Steven Haberman, A cohort-based extension to the Lee–Carter model for mortality reduction factors, *Insurance: Mathematics and Economics*, **38**(3), (2006), 556-570.

[6] Delwarde A., Denuit M. and Partrat C., Negative binomial version of the Lee–Carter model for mortality forecasting, *Applied Stochastic Models in Business and Industry*, **23**(5), (2007), 385-401.

[7] Wang Xiaojun, Cai Zhenggao. New development on mortality projection model, *Statistic Research* (in Chinese), **25**(9), (2008), 80-84.

[8] Li Zhisheng and Liu Hengjia, Estimation and Application of th Lee-Carter model: based on demograghic data of China, *Chinese Journal of Population Science* (in Chinese), **3**, (2010), 46-56.

[9] Wang Xiaojun and Huang Shunlin, Compasion and selection of stochastic mortality models in China, *Population & Economics* (in Chinese), **1**, (2011), 82-86.

[10] Wang Xiaojun and RenWendong, Application of Lee-Carter method in forecasting the mortality of Chinese population with limited data, *Statistic Research* (in Chinese), **29**(6), (2012), 87-94.

[11] Wang Danjie, Zhu Jianping and Fu Rong, Functional prediction model for mortality, *Statistical Research* (in Chinese), **30**(9), (2013), 87-93.

[12] Yang S.S., Yue J.C. and Yeh Y. ., Coherent mortality modeling for a group of populations, *Living to 100 Symposium*, (2011).

[13] Yang S.S. and Wang C.W., Pricing and securitization of multi-country longevity risk with mortality dependence, *Insurance*: *Mathematics and Economics*, **52**(2), (2013), 157-169.