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Monitoring, Loan Rates and Threat of Enterprise Liquidation in a Bank Relationship

Rim Tlili[1]

Abstract

This article explores the impact of the choice of the number of banks on the banking monitoring, the cost of credit and the threat of liquidation of the enterprise. According to the literature, the multiple-banking presents a problem of duplication of the monitoring effort of each bank and the sharing of the monitoring revenue. The choice of the number of banks depends on the advantages and disadvantages of the monitoring. The model developed in this paper is a recovery of the Carletti (2004) to which a new hypothesis was added. This is a joint use of banking monitoring and the threat of liquidation of the company to counter the risk of entrepreneur opportunism. The threat of liquidation of the company, in case of failure of the project, can deter the entrepreneur to save his efforts. The results only confirm those of Carletti. Indeed, it is optimal for the company to be financed from a single bank when the amount of the private benefits that the entrepreneur wants to divert is low. Otherwise, the company has interest to be financed from a single bank if the cost of monitoring is weak and vis-à-vis two banks, if not.

JEL classification numbers: G21; G32
Keywords: single-banking; multiple-banking; hazard moral; monitoring.

1 Introduction

The importance of banks in the financing of enterprises, and particularly the more opaque, has long been recognized. However, the choice of the Multiple-banking remains less well understood. Modern financial intermediation theories assume that the problem of bank hold up (Sharpe, 1990), the risk of liquidity of banking origin (Detragiache et al, 2001) and the risk of unfair support have prompted companies to diversify their sources of Bank funding. It follows that multi-bank enterprises should be of good quality and pay interest rates should be lower than single-banking companies. However, empirical studies show a discrepancy

[1]Ph.D. in Economics, Université Paris Dauphine, 2012, Professor of Finance, the Institut Supérieur de Gestion (Sousse), Department of Finance, since October 2011.

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in the level of results. One explanation for this discrepancy is that the theoretical literature does not explicitly consider banking monitoring intensity by analyzing the cost of credit granted by the Bank. Thus, Padilla and Pagano (1997) emphasized the important role of the banks in terms of production of information to its customers. The reduction of informational problems characterizing the companies requires an enormous effort of research information and Bank monitoring. Von Thadden (1992) introduced the concept of the cost of monitoring but assumed that its level is exogenous and the intensity of the monitoring of the bank is the same whether it is the only to finance the enterprise or it does it with other banks. However, Dewatripoint and Maskin (1995) speculated that the banking monitoring level is endogenous, but they studied it only in the case of the single-banking. The study of Carletti (2004) is the first to examine the relationship between the number of banks of the enterprise and the bank monitoring. Within a framework of analysis similar to Holmstrom and Tirole (1997), Carletti (2004) considered a model in a single period, in which there is an entrepreneur in need of funding. The latter must decide whether he should make an effort to increase the probability of success of a risky investment project. The problem of the moral hazard of the entrepreneur can be improved by the banking monitoring which is supposed to encourage him to exert effort to ensure the success of his investment project.
In this article, I will analyze the impact of the choice of the number of banks on the banking monitoring and the cost of credit charged to the enterprise in the SME financing activity. It seems that the intensity of the banking monitoring affects the optimal choice of the number of banks. To do this, I develop a theoretical modeling of the conditions of the decision to grant credit and incentives of the various factors involved in this relationship based on the model of Carletti (2004). In the model two modes of bank financing are opposed such as the single-banking and the multiple-banking.
In what follows, I will present our model as well as the proposals arising therefrom. The first section focuses on presenting the basic structure of the model. In the second part, I will present the game balance of credit with banking monitoring and the threat of liquidation of the enterprise and, according to the two modes of bank financing. The third section is devoted to the study of the optimal choice of the number of banks.

2 The Basic Structure

I consider a single period economy in which there is a single firm and two banks operating in a perfectly competitive banking sector[2]. All these economic agents are risk-neutral. The entrepreneur has a risky investment project but he has no personal wealth, so he needs external funding. I consider by hypothesis that the financing bank is the only available external funding enterprise and that we face, in our model, two modes of bank financing to the image of Carletti (2004): the single-banking and the multiple-banking. Indeed, the enterprise has the choice between single-banking funding (Bank A or Bank B) and multiple-banking funding limited to two banks[3] (Bank A and Bank B) to finance the investment project.

[2]The banking sector is assumed to be perfectly competitive so that banks have an expectation of profit zero.
[3]To simplify, we limit the multiple-banking to funding accorded by two banks.

The investment project requires an initial endowment of a unit of capital and generates an income $\begin{cases} R \\ 0 \end{cases}$ such as $R \geq 1$**.** Thus, if successful, the project generates a cash flow $R \geq 1$ whereas in case of failure, it generates no cash flow. The probability of success of the project depends on the effort of the entrepreneur during the period of the project. This probability is equal to p_H if the entrepreneur provides great efforts and p_L if he provides weak efforts such as $p_H > p_L$**.** The project is profitable only if the entrepreneur behaves correctly such as $p_H R > 1$. On the other hand, the probability of success of the project is very low when the entrepreneur provides weak efforts such as $p_L R < 1$.

Therefore, the probability of failure of the project is equal to $(1 - p_L)$ that is also the probability that the enterprise is liquidated. In this sense, at the end of the period and in case of success of the project, the bank is paid fully. If, on the contrary, the project realizes a failure, the entrepreneur is in default of payment and the bank has the right to liquidate the enterprise. The net asset value of the enterprise on the market is equal to L such as $0 < L < R$. So when the Bank finances an entrepreneur who fails to honor his commitments, it can however retrieve a part of his placement by proceeding on the liquidation of the enterprise.

The enterprise is thereby solvent only if the entrepreneur behaves properly by providing great efforts such as: $p_H R > 1 > p_L R$, where the idea is that it is optimal for the bank to finance the entrepreneur only if the latter is ready to provide great efforts to ensure the success of the investment project. The problem of moral hazard is introduced by distinguishing between the two types of behavior. The entrepreneur can choose not to conduct themselves properly during the implementation of the project by providing a low effort. Indeed, his behavior depends on the amount of the private benefits that he can extract. It can for example do a strategic default by announcing to his bank that the project has failed by declaring a null result to keep for himself one result noticed *B* equivalent to private beneficiaries. It is, therefore, a problem of information asymmetry as the behavior of the entrepreneur is not observable by the banks without cost.

Moreover, banks compete on their offers of credit agreements and they refinance to the risk-free rate that I assume equal to zero. They agree to finance the firm if they hope to make profit and this only if the entrepreneur behaves properly by providing great efforts. In other words, banks finance the borrowing firm only if:

$$\boldsymbol{p_H(R - r) \geq p_L(R - r) + B} \quad (1)$$

We notice that:

- $\boldsymbol{r}$ is the cost of bank credit [4] paid by the enterprise and charged by banks;
- $\boldsymbol{p_H(R - r)}$ is the entrepreneur's expectation profit if he makes great effort;
- $\boldsymbol{p_L(R - r) + B}$ is the entrepreneur's expectation profit in case he decides to make low efforts in order to make private profits noticed *B.*

The equation (1) translates the idea that banks will be willing to finance the enterprise only when the entrepreneur's expectation profit is higher in case he chooses to provide great efforts during his project. So to have this condition checked, banks must encourage the

[4]The banks offer the company a bank credit at a noted price r, which must cover at least the amount of initial investment equivalent to a unit of capital such as $r = I(1 + i)$. The interest rate i is equal to zero because the banking sector is assumed to be competitive and I is the investment cost.

entrepreneurs to behave properly through the monitoring banking by refusing to be simple fund sponsors of the enterprise. This condition ensures that credit rationing exists since the banks that cannot control the behavior of the entrepreneur, during the realization of the investment project, will not accept to give the capital to the borrower firm. The bank monitoring is therefore indispensable especially if:

$$(\boldsymbol{Rp_H - 1})\left(\frac{p_H - p_L}{p_H}\right) < \boldsymbol{B} \quad (2)$$

Demonstration. See Appendix A.

The hypothesis presented by the equation (2) shows that if the amount of private profit is high enough, the entrepreneur is encouraged to make low effort during the implementation of the project to keep only for himself these private profits. He will be, in this case, indifferent in his choice of funding between the single-banking and the multiple-banking. Under this condition, the banks refuse to finance the enterprise and do that only if the amount of private profits is low and does not go beyond $(Rp_H - 1)\left(\frac{p_H - p_L}{p_H}\right)$, and to encourage the entrepreneur to behave properly. To have this condition checked, banks must monitor the entrepreneur once the credit is granted.

We assume that banks can mitigate the problem of moral hazard of the firm by the threat of liquidation of the enterprise in case of project failure, on the one hand, and by the bank monitoring, on the other hand. However, the acquisition of the information requires a costly investment in monitoring technology. This costly investment course, allows banks to encourage the entrepreneur to increase his effort during the realization of the project. Indeed, at a cost of monitoring, banks observe the project that they propose to finance as well as the behavior of the entrepreneur. They also intervene to provide more efforts in case he decides to change his behavior. The cost of bank credit should now cover the initial investment and the cost of monitoring. Each bank chooses its monitoring intensity M as M ∈ [0, 1]. This is the probability that the bank will encourage the entrepreneur to provide further efforts in the implementation of the project in case of a moral hazard problem of the latter. For example, a value of M zero means the absence of the banking monitoring and a value M equivalent to 1 means that the intensity of monitoring of the Bank is at its maximum level.

The monitoring is expensive; it depends on the intensity of monitoring mobilized M by the bank. Thus, the monitoring implies that the bank must know and control the circuits and the processes that form the structure that it controls. However, the resources and the skills that the bank has are limited; it should therefore manage them well. Increasing the intensity of monitoring requires an increase in the staff that undertakes the monitoring or also trains the staff to adapt it to the new responsibilities. This monitoring requires a cost for the bank, noticed C (M) that is assumed to be quadratic. The total cost of the credit monitoring service has the following form:

$$C(M) = \frac{\boldsymbol{m}}{\boldsymbol{2}}\, \boldsymbol{M^2}$$

with m ∈ [0, 1]: the cost of monitoring and M: the intensity of the bank monitoring. The total cost of the credit monitoring function is an increasing and concave function of the monitoring intensity M and the cost of the monitoring m.

As presented, the model helps to explain the joint use of the banking monitoring and the threat of liquidation of the enterprise in case of project failure. The supervision exercised by the bank and the risk of liquidation of the enterprise were intended to limit the opportunism of the entrepreneur. The threat of liquidation of the enterprise may deter the entrepreneur to save his efforts for the realization of the project. In this context, banks will no longer be simple suppliers of credit to the enterprise and it will be more indifferent in his choice of bank financing between the single-banking and the multiple-banking.

To recap, the sequence of events of the model appears in the following figure:

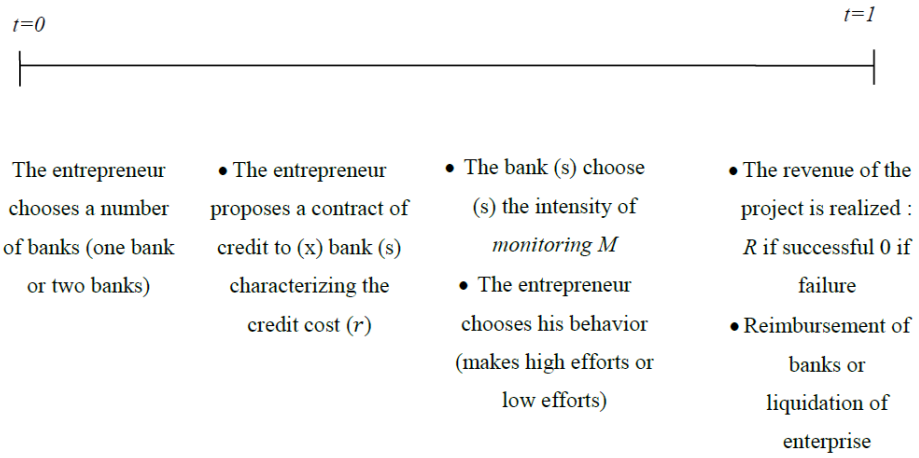

Figure 1: The temporal structure of the model

The analysis of the model framework can be summarized as below:

- In $t = 0$, the firm chooses its number of banks (a single bank or two banks). This choice is observable. The firm subsequently contacts the banks and a two-stage game begins. In the case of single-banking, the firm contacts a bank and proposes a cost of credit r. If the bank refuses the contract credit proposed by the enterprise agreement, the game ends. Otherwise, the project is funded and the firm and the bank simultaneously choose their strategies: the behavior of the enterprise (providing great efforts or low efforts) and the intensity of the M bank monitoring.
- In $t = 1$, the project is carried out. If successful, the firm pays the bank r and keeps the surplus. In contrast, the enterprise will be liquidated by the bank at a price equal to L.

We notice that the game will have the same structure in case the entrepreneur decides to finance its investment project by two banks.
Next, I will examine the balance of these two cases: the single-banking (a single bank) and the multiple-banking (two banks).

3 Game Balance of Credit Offer with Banking Monitoring and the Threat of Liquidation of the Enterprise

I now propose to determine the balance of the game of credit offer. In order to solve this game, I assume that the choice of the number of banks of the enterprise is a datum, and I

analyze the impact of the intensity of the banking monitoring on the optimal choice of the number of banks of the enterprise. The resolution of the model is done by the determination of the balance of the game of credit offer depending on the two choices of bank financing; the single-banking and the multiple-banking of the borrowing firm.

3.1 The Case of Single-banking

It should be noticed that the banking sector is considered as competitive: the bank has an expectancy of profit zero. Let's enquire:
r_1: The cost of bank credit supported by the enterprise
M_1: The intensity of the monitoring of the bank.
Next, I will look for the balance of the game of the single-banking. This balance characterized the logical outcome of this game that is the way rational players should behave: the bank and the enterprise.
The single-banking is a dynamic game with complete information (see appendix B). The bank is player 1 and the enterprise is player 2. The enterprise fixes a cost of credit which allows him to maximize his expected benefit and which also checks the status of benefit zero of the bank. The latter plays the first and chooses between two options. First, it has the ability to stop the game by refusing the debt contract proposed by the enterprise. In this case, the enterprise is not involved. Secondly, the bank can continue the game by deciding to finance the enterprise with the cost of credit proposed by the Commission. In this case, the entrepreneur plays with the bank by choosing a behavior relative to the effort that he provides during the realization of the project. This choice of behavior is not observable, similarly for the choice of the intensity of the bank monitoring, which is done at the same time. The concept of the most appropriate balance is the perfect sub-games of Nash equilibrium (see Appendix C). Let's remember that a Nash equilibrium is defined as a set of strategies like when any player cannot win additional gain by changing unilaterally the strategy. A Nash equilibrium is said to be perfect in sub-games if and only if it is a balance of all sub-games of the considered game. Each sub-game admits at least a balance.
The characteristics of the game of the single-banking are represented by the following figure:

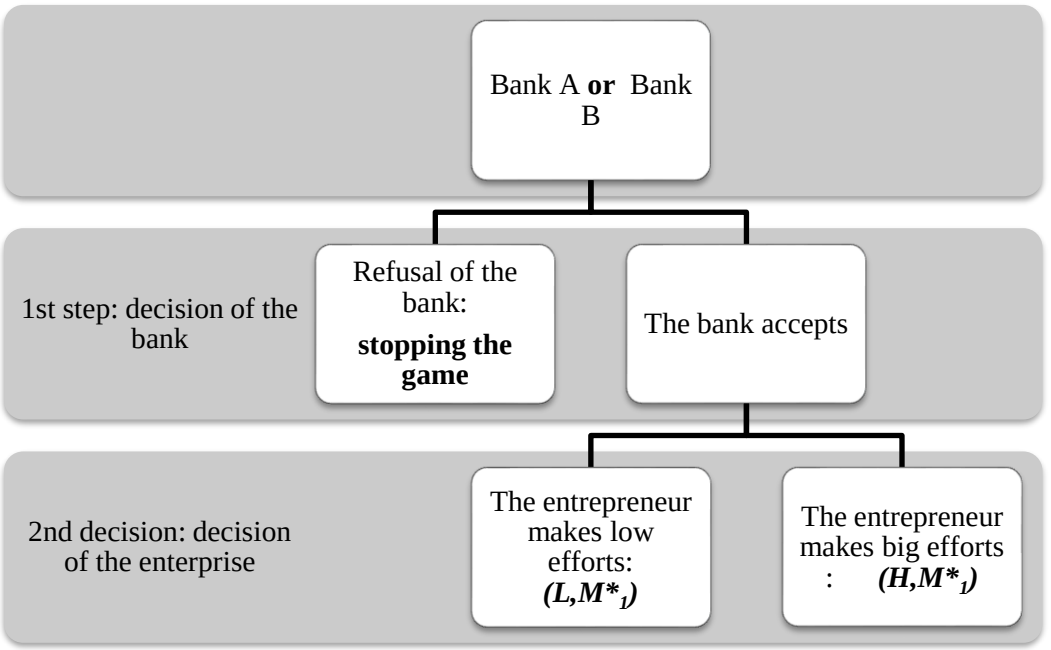

Figure 2: Extensive game form of the single-banking over a period

We notice the existence of two sub-games: the whole game and the sub-game correspond to the node of the enterprise. The balance of the game of the single-banking is defined as follows: the enterprise fixes the cost of credit, which allows maximizing the expected profit. The cost of credit must therefore check the condition of profit zero of the bank allowing the two parts of the contract of debt to anticipate respectively their behavior (H or L) and the intensity of monitoring M. Pure strategies of the two players (the level of effort and intensity of monitoring) constitute a Nash perfect sub-game equilibrium. In the sub-game of the single-banking, the profit expected by the two players can be distinguished according to the strategy adopted by the entrepreneur relative to the effort choice during the realization of the investment project.

The profit expected by the enterprise funded by a single bank according to the effort[5] is defined as follows:

$$\pi_{F1}^{H} = p_H (R - r_1) \quad (3)$$

$$\pi_{F1}^{L} = M_1 p_H (R - r_1) + (1 - M_1) [p_L(R - r_1) + B] \quad (4)$$

The profit expected by the bank according to the effort provided by the entrepreneur is defined as follows:

$$\pi_{B1}^{H} = p_H \, r_1 - 1 - \frac{m}{2} (M_1)^2 \quad (5)$$

$$\pi_{B1}^{L} = M_1 p_H r_1 + (1 - M_1)(p_L \, r_1 + (1 - p_L) L) - 1 - \frac{m}{2} (M1)^2 \quad (6)$$

[5]The indices *H* and *L* denote respectively the big efforts and the weak efforts provided by the entrepreneur during the realization of the investment project.

To the image of Carletti (2004), we obtain the following proposal[6]:

Proposition 1: The game of single-banking accepts a single balance defined in the following way: the project is funded if and only if $\boldsymbol{R \geq r_1^*}$
The characteristics of this equilibrium are:
i) The cost of the credit balance $\boldsymbol{r_1^*}$
ii) if the entrepreneur is of type $\boldsymbol{L}$, the Bank operates to induce him to increase his efforts with the intensity $\boldsymbol{M_1^*} \in [0,1]$

Knowing that:

$$\boldsymbol{M_1^*} = \begin{cases} \mathbf{1} & \boldsymbol{si\ m \leq\ (p_H - p_L) r_1 -\ (1 - p_L) L} \\ \dfrac{(p_H - p_L) r_1 - (1 - p_L) L}{m} & \boldsymbol{si\ m >\ \ (p_H - p_L) r_1 -\ (1 - p_L) L} \end{cases} \quad (7)$$

and

$$\boldsymbol{r_1^*} = \frac{1 + C(M_1^*) - (1 - M_1^*)(1 - p_L) L}{[p_L + M_1^* (p_H - p_L)]} \quad (8)$$

Demonstration: See appendix D.
In accordance with what has been demonstrated by Carletti (2004), in the absence of monitoring, the bank refuses to finance the enterprise. On the other hand, an informed bank may use its ability to follow the evolution of the behavior of the entrepreneur during the realization of the project in order to intervene in case of problem of moral hazard of the latter. As the entrepreneur effort decreases, the informed bank operates to induce him to increase his efforts (the objective is to increase the effort from *H* to *L*), and this in order to increase the probability of success of the project and to avoid the risk of liquidation of the enterprise. In this sense, the cost of credit must cover the costs of the banking monitoring without exceeding the income expected from the project.
Thus, the first proposal exposes the essential role of the bank, as a monitor, in the financing of the enterprise. The bank monitoring is intended to ensure the success of the project. However, the enterprise will be liquidated by the bank in case of project failure.

3.2 The Case of Multiple-banking

To the image of the game of the single-banking, multiple-banking is also a dynamic game with complete information. In this case, two banks finance the enterprise and not one so that each is half of the amount of investment. Subsequently, the firm and the banks choose strategies that allow maximizing their expected profits. The two banks choose their intensity of monitoring at the same time independently. However, the intensity of monitoring of each has an impact on the overall behavior of the entrepreneur. Indeed, if a bank discovers a change in the behavior of the entrepreneur, it will intervene to provide more efforts to ensure the success of the project. This implies that the choice of bank monitoring intensity is a private information but its result is public, observed by all stakeholders of the bank credit market.

[6]However, this proposition takes account of the hypothesis relative to the threat of liquidation of the company in the case of failure of the investment project.

We notice that the cost of credit granted by each bank is equal to $\frac{r_2}{2}$ and consider $\boldsymbol{M_i}$ as the intensity of the monitoring of the Bank *i* with $i = \{A, B\}$.
The characteristics of the multiple-banking game are presented by the following figure.

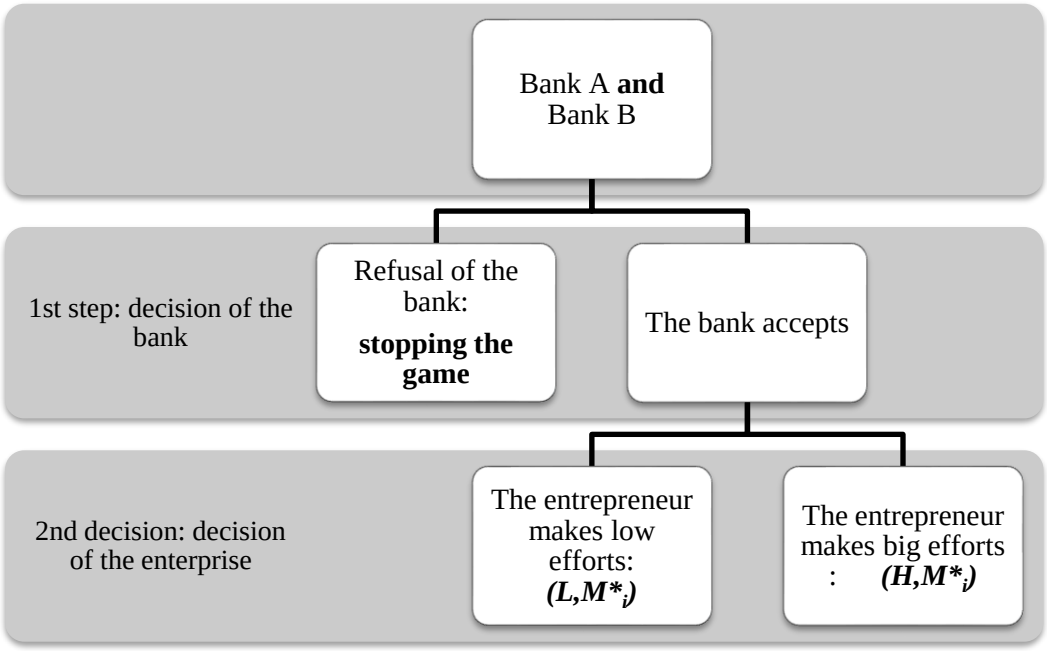

Figure 3: Extensive form of the game of the multiple-banking on a single period

The multiple-banking game takes place in the same way as the game of the single-banking. The only difference lies in the number of banks. In the case of multiple-banking, two banks agree to finance the enterprise under the conditions proposed by the Commission. Each one has the ability to stop the game by refusing the terms of the credit agreement. In this case, the enterprise does not play. On the other hand, the game cannot continue if the two banks agree to finance the enterprise.
The enterprise plays after the two banks by choosing its behavior (*H* or *L*). This choice of behavior is not observable, similarly for the choice of the intensity of monitoring of each bank which is also unobservable. This game admits a Nash perfect sub-game equilibrium. The equilibrium of this game is defined as follows: the enterprise fixes the cost of credit, which allows maximizing the expected profit. The credit cost must therefore check the condition of profit zero of each bank and allow them and the enterprise to anticipate respectively their intensities of monitoring and of behaviors. Pure strategies of the players (the choice of behavior and intensities of monitoring of the two banks) constitute a Nash perfect sub-game equilibrium. The sub-game of the multiple-banking is as follows:
The total intensity of the monitoring of the two banks:

$$\bar{\boldsymbol{M}}_{\boldsymbol{2}} = \boldsymbol{M_A} + \boldsymbol{M_B} - \boldsymbol{M_A}\,\boldsymbol{M_B} \tag{10}$$

$\boldsymbol{M_A}$: The intensity of the monitoring of Bank A
$\boldsymbol{M_B}$: The intensity of the monitoring of Bank B
$\boldsymbol{M_A}\,\boldsymbol{M_B}$**:** duplication of the effort of the monitoring of the two banks A and B

The profit expected by the company funded by 2 banks according to the effort provided by the entrepreneur:

If the effort is *H*, the profit of the company is:

$$\boldsymbol{\pi_{F2}^{H} = p_H\,(R - r_2)} \quad (11)$$

If the effort is *L*, the benefit of the company is:

$$\boldsymbol{\pi_{F2}^{L} = \bar{M}_2\, p_H\,(R - r_2) + (1 - \bar{M}_2)\,[p_L(R - r_2) + B]} \quad (12)$$

The profit expected by each bank according to the effort provided by the entrepreneur:

If the effort is *H*, the profit of the bank is:

$$\boldsymbol{\pi_{Bi}^{H} = p_H\,\frac{r_2}{2} - \frac{1}{2} - \frac{m}{2}\,(M_i)^2} \quad (13)$$

If the effort is *L*, the profit of the bank is:

$$\boldsymbol{\pi_{Bi}^{L} = \bar{M}_2\, p_H\,\frac{r_2}{2} + (1 - \bar{M}_2)(p_L\,\frac{r_2}{2} + (1 - p_L)\frac{L}{2}) - \frac{1}{2} - \frac{m}{2}\,(M_i^*)^2} \quad (14)$$

The equations **(10)**, **(13)** and **(14)** present characteristics of the multiple-banking game. First, the two banks face a duplication of efforts (the second and the third term of the equation **(10)**), since the monitoring of each bank assigns the whole project without being observable. Then, these two banks must share revenues from monitoring ($\frac{r_2}{2}$) in case of success and of net asset value of the business in the event of project failure$\left(\frac{L}{2}\right)$. On the other hand, each bank supports all the cost of monitoring *C(M₂)*. Finally, the two banks benefit from the diseconomies of scale due to the convexity of the function of the monitoring cost.

Proposition 2. The game of the multiple-banking accepts a single symmetric equilibrium according to which the project is financed if and only if R ≥ r_2^*.. The characteristics of this balance are as follows:

i) The cost of the credit balance is r_2^* ;

ii) If the entrepreneur is of type *L*, optimal monitoring intensity of each bank is M_2^* such as:

$$\boldsymbol{M_2^* = \frac{(p_H - p_L)r_2^* - (1 - p_L)L}{(p_H - p_L)r_2^* - (1 - p_L)L + 2m}}. \quad (15)$$

Demonstration. See Appendix E.

I notice that the expression of the cost of the credit balance in the case of the multiple-banking is as follows:

$$\boldsymbol{r_2^* = \frac{1 + 2C(M_2^*) - (1 - \bar{M}_2^*)(1 - p_L)L}{[p_L + \bar{M}_2^*\,(p_H - p_L)]}} \quad (16)$$

To the image of the game of the single-banking, proposition 2 states that the investment of the enterprise project can be financed only in the presence of monitoring. The two banks are monitoring the behavior of the entrepreneur during the realization of the project with a same positive monitoring intensity noted $\boldsymbol{M_2^*}$. in what follows, I consider the case of symmetric equilibrium. The denominator of the expression **(15)** presents the main features of the game of multiple-banking previously discussed. Banks share the results of the monitoring in case of success $\boldsymbol{(p_H - p_L)r_2^*}$ as well as the net asset value of the business in case of failure $\boldsymbol{(1 - p_L)L}$. However, the monitoring effort of each bank duplicate $\boldsymbol{2m}$. All these factors have an impact on the incentive of the monitoring of the two banks. The

total intensity of bank monitoring in the case of the multiple-banking will therefore be less than one bank in the case of single-banking.
The threat of the risk of liquidation of the company in case of project failure of this threat presents two main advantages for the company. Firstly, it can deter the entrepreneur to save his effort once the investment project financing is obtained. Then, the cost of credit as shown in equations **(9)** and **(16)** is a decreasing function of the probability of enterprise liquidation. In other words, without the possibility of liquidation of the company in case of failure of the investment project, the cost of bank credit would be higher.

3.3 Single-banking vs Multiple-banking

In this section, I will compare the two modes of bank financing: the single-banking and the multiple-banking. It is important to notice that in the light of the previous results relative to $\boldsymbol{r_1^*}$and $\boldsymbol{r_2^*}$, credit costs are implicitly determined by the total cost of monitoring, the probabilities of success of the project and the net asset value of the company. As the cost of monitoring m increases and the probability of success decreases, credits $\boldsymbol{r_1^*}$and $\boldsymbol{r_2^*}$ increase in value. In addition, I notice that for a value of **m** near zero, the cost of credit charged by two banks $\boldsymbol{r_2^*}$ would be higher than that charged by a single bank $\boldsymbol{r_1^*}$**.**
To compare these two modes of bank financing, I carry out numerical simulations. In doing so, we pose $\boldsymbol{r_1^* = r_2^* = r^*}$ and I compare the intensity of monitoring ($\boldsymbol{M_1^*}$, $\boldsymbol{M_2^*}$and $\boldsymbol{\bar{M}_2^*}$) as well as the total cost of the monitoring ($\boldsymbol{C(M_1^*)}$ and $\boldsymbol{2C(M_2^*)}$) related to the two financing modes: single-banking and multiple-banking.
The first simulation is to pose that $\boldsymbol{p_L = 0,4}$**,** $\boldsymbol{p_H = 1}$ and $\boldsymbol{L = 0,8}$
The following figure shows the results:

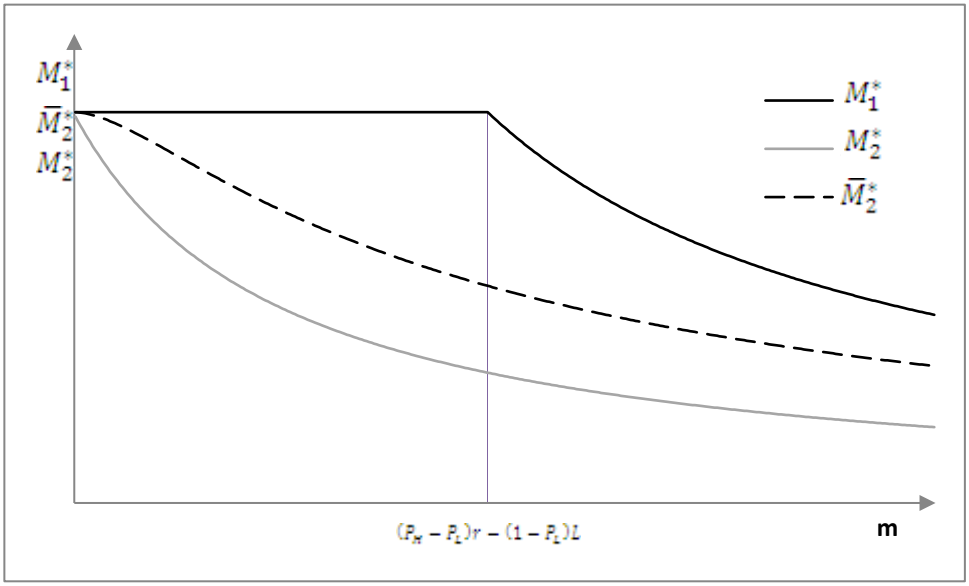

Figure 4: Variation intensities of monitoring M_1^*, M_2^* and $\bar{M}_2^*$ according to the cost of *monitoring* m.

Results show that if the cost of bank credit is the same in both modes of financing, the intensity of monitoring in the case of single-banking funding $\boldsymbol{M_1^*}$ is always greater than the

intensity of the total monitoring $\bar{M}_2^*$ relative to multiple-banking financing. I notice that the duplication of monitoring efforts, on the one hand, and the sharing of profits and the net asset value of the company in case of failure of the project, on the other hand, significantly reduce the incentive of the two banks to monitor the company for the project realization. However, this difference varies according to the cost of monitoring m. Indeed, for low values of $\boldsymbol{m} \leq (p_H - p_L)r_2 - (1 - p_L)L$**,** the intensity of monitoring in the case of single-banking $\boldsymbol{M_1^*}$ is equal to 1 and begins to decrease beyond this value. On the other hand, the intensity of monitoring $\boldsymbol{M_2^*}$ of every bank in case of multiple-banking is a decreasing function of the cost of monitoring $\boldsymbol{m}$. Regarding the total intensity of monitoring of the two banks, it is slightly higher than that of each bank $\boldsymbol{M_2^*}$, while remaining less than $\boldsymbol{M_1^*}$. As a result, the incentive for banks to monitor the company is so low that the cost of monitoring is high especially that multiple-banking financing because of the duplication of monitoring effort that has become very important. This result suggests that a firm is better in terms of high probability of success of the project, if it is financed from a single bank. The advantage of the single-banking is more pronounced for intermediary levels of $\boldsymbol{m}$, but it decreases for low or high cost of monitoring values ***m.***

What about the relationship between the cost of the credit and the total cost of the monitoring of the banks? I also consider that the cost of bank credit is the same in both modes of financing and we pose $\boldsymbol{p_L = 0,4}$**,** $\boldsymbol{p_H = 1}$ and $\boldsymbol{L = 0,8}$. The following figure shows the results of this second simulation which consists in estimating the net asset value of the company and the probability of project success.

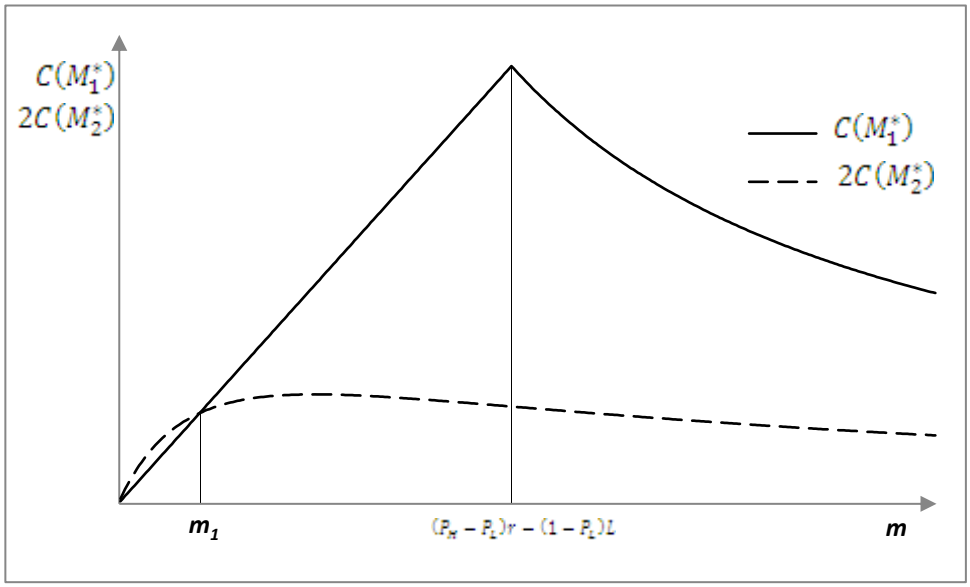

Figure 5: Variation of the total cost of *monitoring* according to m

The results show that for low values of m, the total cost of the monitoring in the case of a single-banking financing $C(M_1^*)$ is less than the total cost of monitoring of two banks $2C(M_2^*)$. If $\boldsymbol{m \leq m_1}$ and in the case of multiple-banking, the intensity of monitoring of each bank is similar to that applied by one bank in a single-banking funding, but overall, the two banks are facing a significant duplication of monitoring efforts. The more the value

of **m** increases, the more the two banks reduce their intensities of monitoring and therefore benefit from very significant scale economies. Indeed, when $\boldsymbol{m}$ varies between $\boldsymbol{m_1}$ *and* $(p_H - p_L)r_2 - (1 - p_L)L$, the intensity of a single banking monitoring $\boldsymbol{M_1^*}$ is at its maximum and $\boldsymbol{C}(\boldsymbol{M_1^*})$ is an increasing function of **m** and reaches its peak when $\boldsymbol{m} = (p_H - p_L)r_2 - (1 - p_L)L$. On the other hand, the intensity of monitoring of the two banks is a decreasing function of **m**, which allows them to benefit from very strong diseconomies of scale and low total cost of monitoring $2C(M_2^*)$. This advantage of the two banks in terms of diseconomies of scale begins to decline in values of $\boldsymbol{m} > (p_H - p_L)r_2 - (1 - p_L)L$. Figure 4 shows that over this interval of **m**, the decline in the intensity of monitoring of a single bank is larger than that of the two banks.

In what follows, I shall proceed to a third simulation to compare between the cost of a credit of a single-banking funding $\boldsymbol{r_1^*}$ and the cost of a multiple-banking funding $\boldsymbol{r_2^*}$. we pose $\boldsymbol{p_L = 0,4,\ p_H = 1}$ et $\boldsymbol{L = 0,8}$, the following figure shows the results of this third simulation.

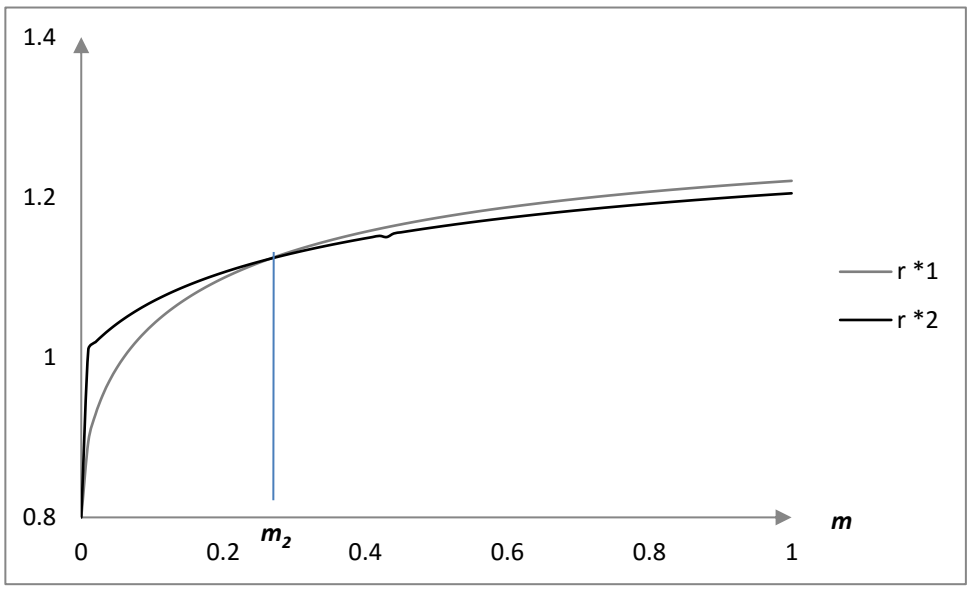

Figure 6: Variation of credit costs according to the cost of *monitoring* m

The equations **(8)** and **(16)** show that the cost of credit of each mode of bank financing depends on the cost of monitoring, the probability of success of the project and the probability of liquidation of the company in case of failure. The simulation results show that the two credit costs vary between 0.8 and 1.22. I find that the cost of minimum credit corresponds to the net asset value of the company. Indeed, without the threat of liquidation of the company in case of project failure, credit costs vary between 1 and 1.5. In this sense, the possibility of liquidation of the company in case of project failure has advantages for the bank as well as for the company. Firstly, the bank will guarantee at least the recovery of a portion of its capital loaned to the company in case of default of payment thereof. On the other hand, the entrepreneur will benefit from lower credit cost and will be prompted to provide great efforts during the realization of the project. In addition, I note that for low cost values of monitoring such as $\boldsymbol{m \leq m_1}$, the cost of the two banks credit is higher than

that of a single bank. This is due to the significant duplication of effort of monitoring in a multiple-banking financing, which therefore increases the total cost of monitoring of the two banks. Beyond $\boldsymbol{m_1}$, the advantage of the multiple-banking for the two banks in terms of diseconomies of scale dominates its limits: the duplication of monitoring efforts, the sharing of profits in case of project success and of sharing the net asset value of the business in case of failure. In this case, the cost of credit of the two banks is lower than that of a single bank.

4 The Optimal Choice of the Number of Banks

To study the optimal choice of the number of banks of the enterprise, I replace the two modes of bank financing monitoring intensity values and the cost of credit by their equilibrium values in equations **(4)** and **(12).** The profits expected by the firm in a single-banking financing and a multiple-banking financing are written respectively as below:

$$\boldsymbol{\pi_{F1}^{L}} = [\boldsymbol{M_1^* p_H R} + (\mathbf{1} - \boldsymbol{M_1^*})(\boldsymbol{p_L R} + (\mathbf{1} - \boldsymbol{p_L})\boldsymbol{L}] + (\mathbf{1} - \boldsymbol{M_1^*})\boldsymbol{B} - \left[\mathbf{1} + \frac{\boldsymbol{m}}{\mathbf{2}}(\boldsymbol{M_1^*})^{\mathbf{2}}\right] \quad (17)$$

$$\boldsymbol{\pi_{F2}^{L}} = [\boldsymbol{\bar{M}_2^* p_H R} + (\mathbf{1} - \boldsymbol{\bar{M}_2^*})(\boldsymbol{p_L R} + (\mathbf{1} - \boldsymbol{p_L})\boldsymbol{L}] + (\mathbf{1} - \boldsymbol{\bar{M}_2^*})\boldsymbol{B} - [\mathbf{1} + \boldsymbol{m}(\boldsymbol{M_2^*})^{\mathbf{2}}] \quad (18)$$

The first term[7] of equations **(17)** and **(18)** reflects the expected financial revenue of the project. The second term represents the private benefit expected by the entrepreneur. The last term represents its expected reimbursement by the bank which is equal to the credit amount and to the total cost of monitoring.

The firm chooses the number of banks that allows to maximize its expected profits **(17)** and **(18).** This choice depends on the difference in the level of bank monitoring and the cost of bank credit between the two modes of bank financing: one or two banks. It is true that the bank monitoring allows the firm to decrease the risk of liquidation of the company and to increase the expected income of the project but at the expense of a small private profit expected by the entrepreneur, on the one hand, and the total cost of monitoring on the other hand.

To determine the optimal choice of the number of banks of the enterprise, I carry out numerical simulations. We pose $\boldsymbol{R} = \mathbf{1.6}$, $\boldsymbol{p_L} = \mathbf{0,4}$, $\boldsymbol{p_H} = \mathbf{1}$ and $\boldsymbol{L} = \mathbf{0,8}$ and I study the variation in profits expected by the firm given by equations **(17)** and **(18)** on the basis of the cost of monitoring **m**. I consider also two different values[8] of the amount of the private benefits $\boldsymbol{B} = \mathbf{0,2}$ et $\boldsymbol{B} = \mathbf{0,35}$. The first simulation results are presented in the figure below.

[7] $[\boldsymbol{M_1^* p_H R} + (\mathbf{1} - \boldsymbol{M_1^*})(\boldsymbol{p_L R} + (\mathbf{1} - \boldsymbol{p_L})\boldsymbol{L}]$ and $[\boldsymbol{\bar{M}_2^* p_H R} + (\mathbf{1} - \boldsymbol{\bar{M}_2^*})(\boldsymbol{p_L R} + (\mathbf{1} - \boldsymbol{p_L})\boldsymbol{L}]$

[8] These two values of $\boldsymbol{B}$ verify the hypothesis of the equation (2).

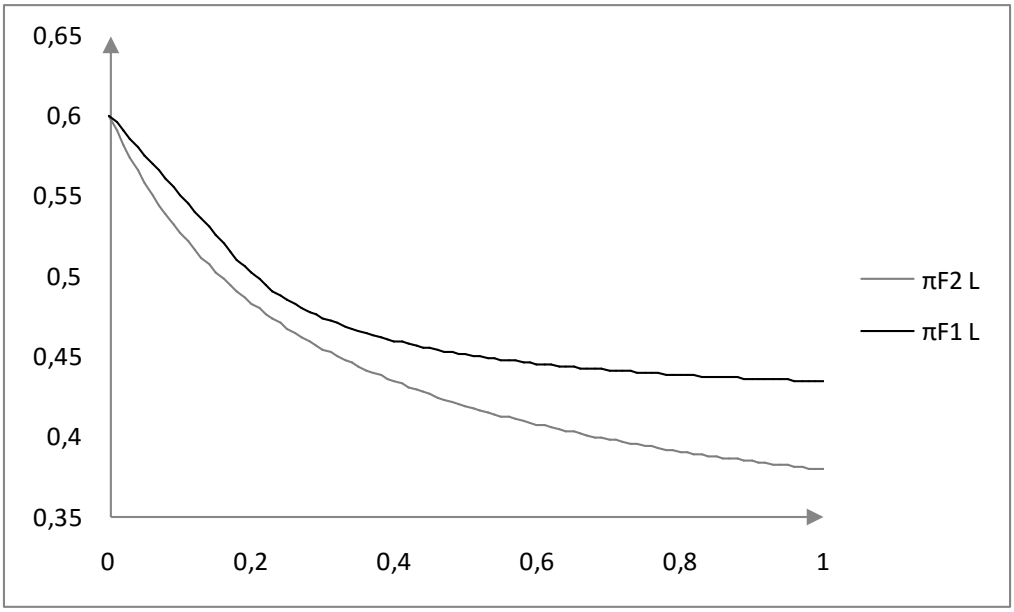

Figure 7: Variation of the expected profits by the firm according to the cost of *monitoring* m when B=0,2

The results show that the hoped profit by a company funded by a single bank is always bigger than the benefit expected by a firm financed by two banks. Indeed, if I consider a small amount of private profits, the company has an interest in opting for a single-banking funding regardless the cost of monitoring ***m***. The low amount of private profits deters the entrepreneur to save effort for the success of the project. However, if moral problems become more important by posing ***B*** = 0,35, the results change as shown in figure 8.

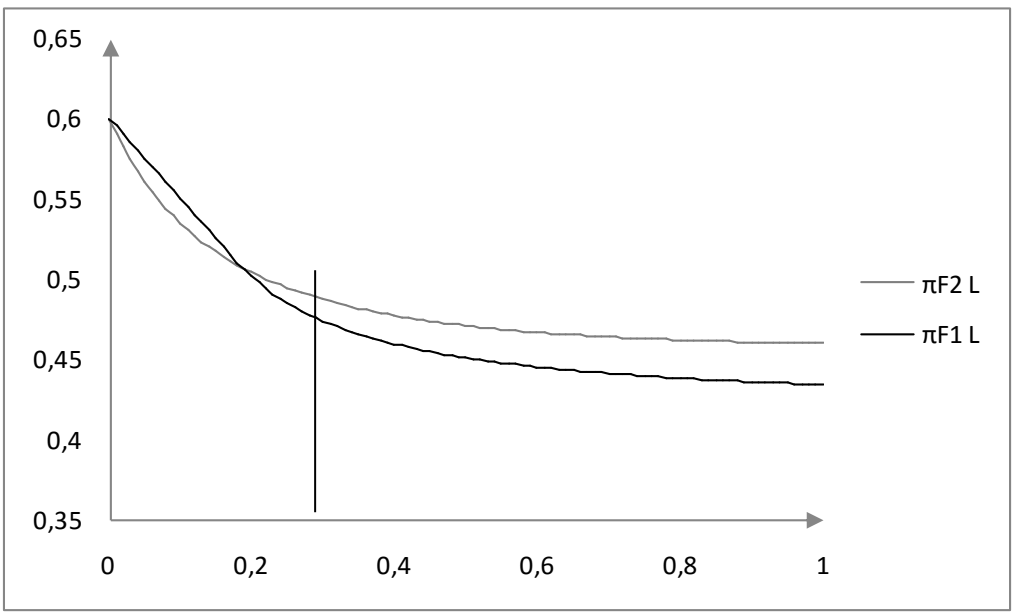

Figure 8: Variation of the expected profits by the firm according to the cost of *monitoring* m when B=0,35

A higher value of private profits increases the risk of opportunistic behavior on the part of the entrepreneur. This also changes the optimal choice of the number of banks of the company. According to the figure above, the expected profit by a single-banking company is bigger than the benefit expected by a multiple-banking company when $m \leq 0,2$. Thus, if the cost of monitoring **m** is low, the multiple-banking seems to be an optimal choice. Indeed, an intensive monitoring of the bank is desirable; it allows to encourage the entrepreneur to provide big efforts with a low total cost of monitoring. On the other hand, funding from two banks costs more to the company given the duplication of effort of monitoring of each bank. However, for values of $m > 0,2$; the total intensity of monitoring of two banks decreases compared to the intensity of monitoring of a single bank. The result is a decrease of expected financial revenue of the project, in favor of an increase of the private hoped benefit by the entrepreneur, and the funding from two banks becomes the best choice. To summarize:

Proposition 4. It is optimal for the company to finance a single bank in the case of small amounts of private profits. Otherwise, the company has interest to finance by a single bank if the cost of monitoring is weak and by two banks, if not.
The firm chooses, thereby, its number of banks based on several variables and especially, the amount of private profits that it can divert **B**, the cost of monitoring **m** as well as the intensity of banking monitoring **M**.

5 Conclusion

In conclusion, I can say that the bank monitoring is a key feature that distinguishes the banks in the financing of SMEs. However, literature did not address the question of the intensity of the banking monitoring and its effect on the cost of financing, the quality of the company and especially the number of banks funding the firm.
Indeed, the absence or the lack of availability and reliability of the information related to these enterprises complicate the financing of investment projects. In this case, the decision to grant credit is mainly conditioned by the monitoring.
The model of Carletti (2004) shows that the multiple-banking presents a problem of duplication of the monitoring effort of each bank and the sharing of the monitoring revenue. It follows that the bank monitoring intensity is higher in the case of single-banking. However, the multiple-banking does not necessarily imply a higher credit cost than that of funding accorded by a single bank. Regarding the optimal choice of the number of banks of the enterprise, the results of the model show that this choice depends on the advantages and disadvantages of the monitoring. Indeed, the banking monitoring encourages the entrepreneur to make efforts thus increasing the probability of the project success and therefore the hoped financial income of the project but at the expense of an increase in the total cost of bank monitoring and the decrease in the amount of private profit expected by the entrepreneur.
The model that I have developed is a recovery of the Carletti (2004) to which I have added a new hypothesis. This is a joint use of banking monitoring and the threat of liquidation of the company to counter the risk of entrepreneur opportunism. The threat of liquidation of the company, in case of failure of the project, can deter the entrepreneur to save his efforts. My results only confirm those of Carletti. Indeed, it is optimal for the company to be financed from a single bank when the amount of the private benefits that the entrepreneur

wants to divert is low. Otherwise, the company has interest to be financed from a single bank if the cost of monitoring is weak and vis-à-vis two banks, if not.

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Appendix

A: Demonstration of the equation (2)

If I consider the bank as a simple provider of credit to the company, its expected profit will be of the following form: $p_H r - 1$. The latter tends to zero, given that the banking sector is assumed to be competitive. As a result, the cost of bank credit to balance that verifies this condition is $r = \frac{1}{p_H}$. If I replace this value of cost of bank credit in equation (1), I obtain:

$$\boldsymbol{p_H(R - \frac{1}{p_H}) \geq p_L(R - \frac{1}{p_H}) + B}$$

$$\boldsymbol{(p_H - p_L)(R - \frac{1}{p_H}) \geq B}$$

$$\boldsymbol{(Rp_H - 1)\left(\frac{p_H - p_L}{p_H}\right) \geq B}$$

In absence of bank monitoring, the bank agrees to finance the investment project of the company only if the amount of private profits that the entrepreneur attempts to extract, checks the inequality above. So as this condition is verified, banks must observe the behavior of the entrepreneur during the project realization in order to counteract the risk of an opportunistic behavior from his part.

B: Complete information dynamic games

A game is qualified as dynamic when it is repeated, played sequentially, which gives the opportunity to at least one of the two players to react to the actions of the other player after having been observed. By complete information, we mean the situation where all the players know all the data of the game: all the players, all the strategies and payment functions.

The description of a dynamic game is often done by a game tree, called the extensive form of the game. To determine the Nash equilibrium for a dynamic game, I must absolutely define the strategies to be adopted by the players. It's the complete action plans indicating the actions of the players in any circumstance. The Nash equilibrium is a state in which no player wishes to change his strategy because of the other strategies adopted by other players. In this case, I talk about a perfect Nash equilibrium.

C: Nash perfect sub-game equilibrium

The Nash equilibrium is defined for simultaneous games to complete information. However, a dynamic game involves an issue with the simultaneity of the choices that is no longer possible. Indeed, in this case, one of the players has the opportunity to respond to the choices of the other players. So it seems optimal to redefine the notion of the strategies to be adopted by the players who must restore the simultaneity of their choice while excluding non-credible equilibriums. As a result, a refinement of the Nash equilibrium is needed. Thus, we obtain a perfect Nash equilibrium. It is perfection in sub-games. Referring to the definition of Selten (1965), a Nash equilibrium is perfect if the players' strategies constitute a Nash equilibrium of all the sub-games of the game.

D: Demonstration of proposition 1

The resolution of the game of single-banking is done on two stages. First, we pose r_1 equal to the cost of bank credit which is a datum and we seek the Nash balance relative to the sub-game: the effort of the entrepreneur and the bank monitoring. Then, I determine the expression of the credit cost r_1 that I have fixed as a datum previously.

The sub-game: the effort of the entrepreneur and the bank monitoring

I seek the Nash equilibrium in pure strategies of this sub-game. Thus, if the entrepreneur elects to provide great efforts (H), the bank maximizes (3.5) and chooses M_1^* which corresponds to 0. This is not an equilibrium value because in the absence of monitoring ($M_1^* = 0$), the bank refuses to finance the enterprise. If the entrepreneur chooses to provide low (L) efforts, the bank chooses to turn the intensity of monitoring M_1^* which allows him to maximize (3.6) such as:

$$\frac{\partial \pi_{B_1}^L}{\partial M_1} = (p_H - p_L)r_1 - (1 - p_L)L - mM_1(r_1) = 0$$

Given that the intensity of monitoring banking M_1 belongs to [0,1], $M_1^*(r_1) = \left\{\min\left(\frac{(p_H - p_L)r_1 - (1-p_L)L}{m}, 1\right)\right\}$

$$M_1^* = \begin{cases} 1 & si\ m \leq (p_H - p_L)r_1 - (1-p_L)L \\ \dfrac{(p_H - p_L)r_1 - (1-p_L)L}{m} & si\ m > (p_H - p_L)r_1 - (1-p_L)L \end{cases}$$

Therefore, if $M_1^* = 1$, the entrepreneur is indifferent between the two choices of behavior *L* and *H*. In contrast, if $M_1^* < 1$the entrepreneur chooses to provide a low effort. It follows that the only sub-game Nash equilibrium is (L, M_1^*).

Determination of r_1^*

The cost of the credit balance r_1^* is the one that cancels the benefit expected by the bank. It is obtained by replacing M by M_1^*. I get a second polynomial degree and we retain the positive solution of r_1^*.

To simplify, we pose :

$$r_1^* = \frac{1 + C(M_1^*) - (1 - M_1^*)(1 - p_L)L}{[p_L + M_1^*(p_H - p_L)]}$$

E: Demonstration of proposition 2

To the image of the single-banking game, the resolution of the game of the multiple-banking is also on two stages. First, I determine the sub-game of Nash equilibrium: the effort made by the entrepreneur / Bank monitoring. Then, I determine the expression of the credit cost r_2 that we have fixed as a datum previously.

The sub-game: the effort made by the entrepreneur / banking monitoring

I seek the Nash equilibrium in pure strategies of this sub-game. It is important to notice that at balance, the company chooses to be of type *L* following the same reasoning presented above for the single-banking game. As a result, the bank chooses a monitoring intensity noted M_A and bank B chooses M_B.

I obtain the following equilibrium condition:

$$\frac{\partial \pi_{B_A}^L}{\partial M_A} = (1 - M_B)\left((p_H - p_L)\frac{r_2}{2} - (1 - p_L)\frac{L}{2}\right) - mM_A = 0$$

It follows that:

$$\boldsymbol{M_A^* = \frac{(p_H - p_L)r_2 - (1 - p_L)L}{2m}(1 - M_B)}$$

In the case of a symmetric equilibrium, the two banks choose the same intensity[9] of *monitoring*:

$$\boldsymbol{M_2^* = M_A^* = M_B^* \rightarrow M_2^* = \frac{(p_H - p_L)r_2^* - (1 - p_L)L}{(p_H - p_L)r_2^* - (1 - p_L)L + 2m}}$$

Determination of $\boldsymbol{r_2^*}$

To find the cost of the credit balance r_2^*, simply cancel the expected profit by the bank and replace M by $\boldsymbol{M_2^* = \frac{(p_H - p_L)r_2^* - (1 - p_L)L}{(p_H - p_L)r_2^* - (1 - p_L)L + 2m}}$. I obtain a third-degree polynomial that takes three possible solutions. We retain the only positive solution.

To simplify, we pose:

$$\boldsymbol{r_2^* = \frac{1 + 2C(M_2^*) - (1 - \bar{M}_2^*)(1 - p_L)L}{[p_L + \bar{M}_2^*(p_H - p_L)]}}$$

[9]In the case of a symmetric equilibrium $(p_H - p_L)r_2^* - p_F L = 2m$.